5

The Stochastic Equations of the US Model

5.1 Introduction

The stochastic equations of the US model are specified, estimated, and tested in this chapter. As noted at the beginning of Chapter 3, extra "theorizing" is involved in going from theory like that in Chapter 2 to empirical specifications. This chapter thus uses the theory in Chapter 2 plus additional theory in the specification of the stochastic equations.

The stochastic equations are listed in Table A.3 in Appendix A, and the variables are defined in Table A.2. The construction of the variables is discussed in Chapter 3. There are 30 stochastic equations in the US model. The empirical results for the equations are presented in Tables 5.1 through 5.30 in this chapter, one table per equation except for equations 19 and 29. (There are no tables for equations 19 and 29.) Each table gives the left hand side variable, the right hand side variables that were chosen for the "final" specification, and the results of the tests described in Chapter 4. The basic tests are 1) adding lagged values, 2) estimating the equation under the assumption of a fourth order autoregressive process for the error term, 3) adding a time trend, 4) adding values led one or more quarters, 5) adding additional variables, and 6) testing for structural stability. Also, the joint significance of the age distribution variables is examined in the household expenditure and money demand equations. Remember that "adding lagged values" means adding lagged values of all the variables in the equation (including the left hand side variable if the lagged dependent variable is not an explanatory variable). As discussed in Section 4.5, this is a test against a quite general dynamic specification. For the autoregressive test, the notation "RHO=4" will be used to denote the fact that a fourth order autoregressive process was used.

It will be seen that only a few of the equations pass all the tests. My experience is that it is hard to find macroeconomic equations that do. If an equation does not pass a test, it is not always clear what should be done. If, for example, the hypothesis of structural stability is rejected, one possibility is to divide the sample period into two parts and estimate two separate equations. If this is done, however, the resulting coefficient estimates are not always sensible in terms of what one would expect from theory. Similarly, when the additional lagged values are significant, the equation with the additional lagged values does not always have what one would consider sensible dynamic properties. In other words, when an equation fails a test, the change in the equation that the test results suggest may not produce what seem to be sensible results. In many cases, the best choice seems to be to stay with the original equation even though it failed the test. My feeling (being optimistic) is that much of this difficulty is due to small sample problems, which will lessen over time as sample sizes increase, but this is an important area for future work. Obviously less confidence should be placed on equations that fail a number of the tests than on those that do not.

The χ^2 value is presented for each test along with its degrees of freedom. Also presented for each test is the probability that the χ^2 value would be whatever it is if the null hypothesis that the additional variables do not belong in the equation is true. These probabilities are labeled "p-value" in the tables. A small p-value is evidence against the null hypothesis and thus evidence against the specification of the equation. In the following discussion of the results, a p-value of less than .01 will be taken as a rejection of the null hypothesis and thus as a rejection of the specification of the equation.[1]

It will be seen that lagged dependent variables are used as explanatory variables in many of the equations. They are generally highly significant, even after accounting for any autoregressive properties of the error terms. It is well known that they can be accounting for either partial adjustment effects or expectational effects and that it is difficult to identify the two effects separately.[2] For the most part no attempt is made in what follows to separate the two effects, although, as discussed in Chapter 4, the tests of the significance

[1] Using a value of .01 instead of, say, .05 gives the benefit of the doubt to the equations, but, as indicated above, the equations need all the help they can get.

[2] See Fair (1984), Section 2.2.2, for a discussion of this.

of the led values are tests of the rational expectations hypothesis.

In testing for the significance of nominal versus real interest rates, some measure of expected future inflation must be used in constructing real interest rate variables. Two measures were used in the following work: the actual rate of inflation in the past four quarters, denoted p_4^e, and the actual rate of inflation (at an annual rate) in the past eight quarters, denoted p_8^e. The price deflator used for this purpose is PD, the price deflator for domestic sales, and so $p_4^e = 100[(PD/PD_{-4}) - 1]$ and $p_8^e = 100[(PD/PD_{-8})^{.5} - 1]$.

The significance of nominal versus real interest rates was tested as follows. Consider RMA, the after tax mortgage rate, which is used in the model as the long term interest rate facing the household sector. Assume that p_8^e is an adequate proxy for inflation expectations of the household sector. If real interest rates affect household behavior, then $RMA - p_8^e$ should enter the household expenditure equations, and if nominal interest rates affect household behavior, then RMA alone should enter. The test of real versus nominal rates was first to estimate the equation with RMA included and then to add p_8^e and reestimate. If real rates instead of nominal rates matter and if p_8^e is a good proxy for actual inflation expectations, then p_8^e should be significant and have a coefficient estimate that equals (aside from sampling error) the negative of the coefficient estimate of RMA. The same reasoning holds for p_4^e.

As will be seen, the p_4^e and p_8^e variables were never significant when added to the household expenditure equations, whereas the nominal interest rate variables were, and so the data support the use of nominal over real interest rates. It could be, of course, that the inflation expectations variables are not good approximations of actual expectations and that if better expectations variables were used they would be significant. This is an open question and an area for future research. It will also be seen below that the real interest rate does affect nonresidential fixed investment, although the estimated effect is small and may be the result of data mining.

The basic estimation period was 1954:1–1993:2, for a total of 158 observations. For the AP stability tests, T_1, the first possible quarter for the break, was taken to be 1970:1 and T_2, the last possible quarter for the break, was taken to be 1979:4. The "break" quarter that is presented in the tables for the AP test is the quarter at which the break in the sample period corresponds to the largest χ^2 value. Although not shown in the tables, it was generally the case that the χ^2 values monotonically rose to the largest value and monotonically fell after that. A * after the AP value in the tables denotes that the value is significant at the one percent level. In other words, a * means that the hypothesis of no break is rejected at the one percent level: the equation fails the stability test.

Tests of the Leads

Three sets of led values were tried per equation. For the first set the values of the relevant variables led once were added. For the second set the values led one through four times were added. For the third set the values led one through eight times were added, with the coefficients for each variable constrained to lie on a second degree polynomial with an end point constraint of zero. To see what was done for the third set, assume that one of the variables for which the led values are used is X_{2i}. Then for the third set the term added is

$$\sum_{j=1}^{8} \beta_j X_{2it+j} \tag{5.1}$$

where $\beta_j = \gamma_0 + \gamma_1 j + \gamma_2 j^2$, $j = 1, \ldots, 8$, $\beta_9 = 0$. The end point constraint of zero implies that $\gamma_0 = -9\gamma_1 - 81\gamma_2$. Given this constraint, the led values enter the equation as

$$\gamma_1 F_{1t} + \gamma_2 F_{2t} \tag{5.2}$$

where

$$F_{1t} = \sum_{j=1}^{8} (j - 9) X_{2it+j} \tag{5.3}$$

$$F_{2t} = \sum_{j=1}^{8} (j^2 - 81) X_{2it+j} \tag{5.4}$$

There are thus two unconstrained coefficients to estimate for the third set. For the second set the equation is estimated under the assumption of a moving average error of order three, and for the third set the equation is estimated under the assumption of a moving average error of order seven.

It may be helpful to review the exact procedure that was followed for the leads test. First, the estimation period was taken to be shorter by one, four, or eight observations. (When values led once are added the sample period has to be shorter by one to allow for the led values; when the values led four times are added the sample period has to be shorter by four; and so on.) The equation with the led values added was then estimated using Hansen's GMM estimator under the appropriate assumption about the moving average process of the error term (zero for leads +1, three for leads +4, and seven for leads +8). The M_i matrix discussed in Section 4.3 that results from this estimation was then used in the estimation of the equation without the led values by Hansen's method for the same (shorter) estimation period. The χ^2 value is then $(SS_i^{**} - SS_i^{*})/T$, as discussed at the beginning of Section 4.5.

The results of adding the led values to the stochastic equations are used in Chapter 11, Section 11.6, to examine the economic significance of the rational expectations assumption in the US model.[3] The question asked in Section 11.6 is: How much difference to the properties of the US model does the addition of the led values make? Two versions of the model are examined. The first consists of the "base" equations in Tables 5.1–5.30, which have no led values in them. This version is called Version 1. The second version consists of the equations with the third set of led values added (i.e., with F_{1t} and F_{2t} added). This version is called Version 2. The particular variables for which led values were used are mentioned in this chapter in the discussion of each equation. For some equations no led values were tried because none seemed appropriate, and so these equations are the same for both Versions 1 and 2.

First Stage Regressors

The first stage regressors (FSRs) that were used for each equation are listed in Table A.7 in Appendix A. The choice of FSRs for large scale models is discussed in Fair (1984), pp. 215–216, and this discussion will not be repeated here.

Autoregressive Errors

Each equation was first estimated under the assumption of a first order autoregressive error term, and the assumption was retained if the estimate of the autoregressive coefficient was significant. In one case (equation 4) a second order process was used in the final specification, and in one case (equation 11) a third order process was used. In the notation in the tables "RHO1" refers to the first order coefficient, "RHO2" to the second order coefficient, and "RHO3" to the third order coefficient.

Previous Version of the US Model

The previous version of the US model is presented in Fair (1984), Chapter 4. The present discussion of the model is self contained, and so this previous material does not have to be read. For the most part the current version of the model is quite similar to the previous version. Three of the main changes are 1) the use of disposable income in the household expenditure equations instead of the wage, price, nonlabor income, and labor constraint variables

[3]This work is an updated version of the material in Section 5 in Fair (1993b).

separately, 2) the use of the age distribution variables, and 3) the different treatment of the interest payment variables of the firm and federal government sectors (equations 19 and 29). In addition, a few more coefficient constraints have been imposed in the current version, and different functional forms have been used in a few cases.

5.2 Household Expenditure and Labor Supply Equations

The two main decision variables of a household in the theoretical model are consumption and labor supply. The determinants of these variables include the initial value of wealth and the current and expected future values of the wage rate, the price level, the interest rate, the tax rate, and the level of transfer payments. The labor constraint also affects the decisions if it is binding.

In the econometric model the expenditures of the household sector are disaggregated into four types: consumption of services (CS), consumption of nondurable goods (CN), consumption of durable goods (CD), and investment in housing (IHH). Four labor supply variables are used: the labor force of men 25–54 ($L1$), the labor force of women 25–54 ($L2$), the labor force of all others 16+ ($L3$), and the number of people holding more than one job, called "moonlighters" (LM). These eight variables are determined by eight estimated equations.

Consider first the four expenditure equations. The household wealth variable in the model is AA, and the lagged value of this variable was tried in each of the equations. The variable is expected to have a positive sign, and if it did not, which occurred in two of the four equations, it was dropped.

The household after tax interest rate variables in the model are RSA, a short term rate, and RMA, a long term rate. RSA was used in the CS equation, and RMA was used in the others. The CS and CN equations are in log form per capita, and the interest rates were entered additively in these equations. The means that, say, a one percentage point change in the interest rate has the same percent change over time in each of the two equations. The CD and IHH equations, on the other hand, are in per capita but not log form, and if the interest rates were entered additively in these equations, the effect of, say, a one percentage point change in the interest rate would have a smaller and smaller percent effect over time on per capita durable consumption and on per capita housing investment as both increase in size over time. Since this does not seem sensible, the interest rate in the CD equation was multiplied by CDA, which is a variable constructed from peak to peak interpolations of

CD/POP. Similarly, the interest rate in the IHH equation was multiplied by $IHHA$, which is a variable constructed from peak to peak interpolations of IHH/POP. Both CDA and $IHHA$ are merely scale variables, and they are taken to be exogenous.

These interest rate variables are nominal rates. As discussed above, the inflation expectations variables p_4^e and p_8^e were added in the testing of the equations to test for real interest rate effects, and the results of these tests are reported below.

The age distribution variables were tried in the four expenditure equations, and they were jointly significant at the five percent level in three of the four, the insignificant results occurring in the IHH equation. They were retained in the three equations in which they were significant. The lagged dependent variable and the constant term were included in each of the four expenditure equations.

Regarding the wage, price, and income variables, there are at least two basic approaches that can be taken in specifying the expenditure equations. The first is to add the wage, price, nonlabor income, and labor constraint variables separately to the equations. These variables in the model are as follows. The after tax nominal wage rate variable is WA, the price deflator for total household expenditures is PH, the after tax nonlabor income variable is YNL, and the labor constraint variable, discussed in Chapter 3, is Z. The price deflators for the four expenditure categories are PCS, PCN, PCD, and PIH.

Consider the CS equation. Under the first approach one might add WA/PH, PCS/PH, YNL/PH, and Z to the equation. The justification for including Z is the following. By construction, Z is zero or nearly zero in tight labor markets (i.e., when JJ is equal to or nearly equal to JJP, where JJ is the actual ratio of worker hours paid for to the total population and JJP is the potential ratio). In this case the labor constraint is not binding and Z has no effect or only a small effect in the equation. This is the "classical" case. As labor markets get looser (i.e., as JJ falls relative to JJP), on the other hand, Z falls and begins to have an effect in the equation. Loose labor markets, where Z is large in absolute value, correspond to the "Keynesian" case. Since Z is highly correlated with hours paid for in loose labor markets, having both WA and Z in the equation is similar to having a labor income variable in the equation in loose labor markets.

The second, more traditional, approach is to replace the above four variables with real disposable personal income, YD/PH. This approach in effect assumes that labor markets are always loose and that the responses to changes in labor and nonlabor income are the same. One can test whether the data support YD/PH over the other variables by including all the variables in the equation and examining their significance. The results of doing this in the four expenditure equations generally supported the use of YD/PH over the other variables, and so the equations reported below use YD/PH. This is a change from the version of the model in Fair (1984), where the first approach was used.

The dominance of YD/PH does not necessarily mean that the classical case never holds in practice. What it does suggest is that trying to capture the classical case through the use of Z does not work. An interesting question for future work is whether the classical case can be captured in some other way. It will be seen below that the Z variable does work in the labor supply equations, where it is picking up "discouraged worker" effects when labor markets are loose.

Some searching was done in arriving at the "final" equations presented below. Explanatory variables lagged once as well as unlagged were generally tried, and variables were dropped from the equation if they had coefficient estimates of the wrong expected sign in both the unlagged and lagged cases. Also, as noted above, each equation was estimated under the assumption of a first order autoregressive error term, and the assumption was retained if the estimate of the autoregressive coefficient was significant. All this searching was done using the 2SLS technique.

Equation 1. CS**, consumer expenditures: services**

The results of estimating equation 1 are presented in Table 5.1. The equation is in real, per capita terms and is in log form. The series for CS is quite smooth, and most of the explanatory power in equation 1 comes from the lagged dependent variable. The disposable income variable has a small short run coefficient (.0570) and a long run coefficient of roughly one [$.991 = .0570/(1 - .9425)$].[4] The short term interest rate is significant. The age variables are jointly significant at the five percent level (but not at the one percent level) according to the χ^2 value. Remember that the coefficient of $AG1$ is the coefficient for people 26–55 minus the coefficient for those 16–25.

[4]Since the equation is in log form, these coefficients are elasticities.

Table 5.1
Equation 1
LHS Variable is $\log(CS/POP)$

Equation			χ^2 Tests			
RHS Variable	Est.	t-stat.	Test	χ^2	df	p-value
cnst	.0870	2.17	Lags	8.53	3	.0362
$AG1$	-.2347	-2.86	$RHO = 4$	1.30	4	.8611
$AG2$	.2293	0.99	T	17.25	1	.0000
$AG3$	.2242	1.14	Leads +1	6.53	1	.0106
$\log(CS/POP)_{-1}$	.9425	29.58	Leads +4	25.60	4	.0000
$\log[YD/(POP \cdot PH)]$	.0570	1.88	Leads +8	28.92	2	.0000
RSA	-.0009	-3.93	p_4^e	3.30	1	.0692
			p_8^e	2.29	1	.1299
			Other[a]	22.63	5	.0004
			Spread	0.82	4	.9362
SE	.00412					
R^2	.999					
DW	2.01					

$\chi^2(AGE) = 10.47$ (df = 3, p-value = .0150)

Stability Test:

AP	T_1	T_2	λ	Break
14.49*	1970:1	1979:4	2.75	1971:4

Estimation period is 1954:1–1993:2.
[a] $\log(AA/POP)_{-1}$, $\log(WA/PH)$, $\log(PCS/PH)$, Z, $\log[YNL/(POP \cdot PH)]$.

Similarly, the coefficient of $AG2$ is the coefficient for people 56–65 minus the coefficient for those 16–25, and the coefficient of $AG3$ is the coefficient for people 66+ minus the coefficient for those 16–25. The age coefficient estimates for the CS equation suggest that, other things being equal, people 26–55 spend less than others (the coefficient estimate for $AG1$ is negative and the other two age coefficient estimates are positive), which is consistent with the life cycle idea that people in their prime working years spend less relative to their incomes than do others.

Consider now the test results in Table 5.1. (Remember that an equation will be said to have passed a test if the p-value is greater than .01.) Equation 1 passes the lags test[5] and the RHO=4 test. These results thus suggest that the dynamic specification of the equation is fairly accurate.

On the other hand, the equation dramatically fails the T test: the time trend is highly significant when it is added to equation 1. This suggests that

[5] Remember that for the lags test all the variables in the equation lagged once are added to the equation (except for the age variables). This means that for equation 1 three variables are added: $\log(CS/POP)_{-2}$, $\log[YD/(POP \cdot PH)]_{-1}$, and RSA_{-1}.

the trending nature of the CS series has not been adequately accounted for in the equation. None of the other specifications that were tried eliminated this problem, and it is an interesting area for future research.

Disposable income was the variable for which led values were tried—in the form $\log[YD/(POP \cdot PH)]$—and the test results show that leads +4 and leads +8 are highly significant. This is thus evidence in favor of the rational expectations assumption. The largest χ^2 value was for 8 leads. This is the equation that is used for Version 2 in Section 11.6 to examine the sensitivity of the model's properties to the use of the led values.

The inflation expectations variables are not significant, which is evidence against the use of real versus nominal interest rates. The additional variables ("Other"), which, as discussed above, are the variables that one might use in place of disposable income, are significant. However, although not shown in the table, the coefficient estimates for the variables are all of the wrong expected sign, and so the version of the equation with these variables added is not sensible. There appears to be too much collinearity among these variables to be able to get sensible estimates.

For the "spread" test in Table 5.1 and in the other relevant tables that follow, the current and first three lagged values of the spread between the commercial paper rate and the Treasury bill rate were added to the equation. For this test the estimation period began in 1960:2 rather than 1954:1 because data on the spread were only available from 1959:1 on.[6] As can be seen, the spread values are not close to being significant, with a p-value of .9362.

Finally, the equation fails the stability test. The AP value is 14.49, which compares with the one percent critical value in Chapter 4 (for 7 coefficients) of 9.50. The largest χ^2 value occurred for 1971:4, which is near the beginning of the test period of 1970:1–1979:4.

Equation 2. CN, consumer expenditures: nondurables

Equation 2 is also in real, per capita, and log terms. The results are presented in Table 5.2. The asset, disposable income, and interest rate variables are significant in this equation, along with the age variables and the lagged dependent variable. Both the level and change of the lagged dependent variable are significant in the equation, and so the dynamic specification is more complicated than that of equation 1. Again, the age coefficients show that people 26–55 spend less than others, other things being equal.

[6] Whenever an estimation period had to be changed for a test, the basic equation was always reestimated for this period in calculating the χ^2 value for the test.

Table 5.2
Equation 2
LHS Variable is $\log(CN/POP)$

Equation			χ^2 Tests			
RHS Variable	Est.	t-stat.	Test	χ^2	df	p-value
cnst	-.1229	-1.41	Lags	9.43	4	.0511
$AG1$	-.4791	-4.14	$RHO = 4$	17.40	4	.0016
$AG2$	1.4067	4.58	T	0.07	1	.7983
$AG3$	-.3364	-1.99	Leads +1	9.16	1	.0025
$\log(CN/POP)_{-1}$	.6203	14.53	Leads +4	12.28	4	.0154
$\Delta\log(CN/POP)_{-1}$	.1374	2.17	Leads +8	11.54	2	.0031
$\log(AA/POP)_{-1}$	.0509	4.51	p_4^e	1.15	1	.2841
$\log[YD/(POP \cdot PH)]$	.2383	8.17	p_8^e	0.04	1	.8516
RMA	-.0019	-3.78	Other[a]	9.93	4	.0416
			Spread	10.02	4	.0400
SE	.00557					
R^2	.997					
DW	1.87					

$\chi^2(AGE) = 44.68$ (df = 3, p-value = .0000)

Stability Test:

AP	T_1	T_2	λ	Break
14.28*	1970:1	1979:4	2.75	1973:2

Estimation period is 1954:1–1993:2.
[a] $\log(WA/PH)$, $\log(PCN/PH)$, Z, $\log[YNL/(POP \cdot PH)]$.

The equation passes the lags test and the T test, but it fails the RHO=4 test. The variable for which led values were tried is again disposable income, and leads +1 and +8 are significant. The inflation expectations variables are not significant. The additional variables, representing the wage, price, nonlabor income, and labor constraint variables are not significant at the one percent level. Likewise, the spread values are not significant at the one percent level. The equation fails the stability test. The AP value is 14.28, which compares to the one percent critical value (for 9 coefficients) of 11.20. The maximum χ^2 value occurs for 1973:2.

Equation 3. CD, consumer expenditures: durables

Equation 3 is in real, per capital terms. One of the explanatory variables is the lagged stock of durable goods, and the justification for including this variable is as follows. Let KD^{**} denote the stock of durable goods that would be desired if there were no adjustment costs of any kind. If durable consumption is proportional to the stock of durables, then the determinants of consumption can be assumed to be the determinants of KD^{**}:

$$KD^{**} = f(\ldots) \tag{5.5}$$

where the arguments of f are the determinants of consumption. Two types of partial adjustment are then postulated. The first is an adjustment of the durable stock:

$$KD^{*} - KD_{-1} = \lambda(KD^{**} - KD_{-1}) \tag{5.6}$$

where KD^{*} is the stock of durable goods that would be desired if there were no costs of adjusting gross investment. Given KD^{*}, desired gross investment in durable goods is

$$CD^{*} = KD^{*} - (1 - DELD)KD_{-1} \tag{5.7}$$

where $DELD$ is the depreciation rate. By definition $CD = KD - (1 - DELD)KD_{-1}$, and equation 5.7 is merely the same equation for the desired values. The second type of adjustment is an adjustment of gross investment to its desired value:

$$CD - CD_{-1} = \gamma(CD^{*} - CD_{-1}) \tag{5.8}$$

Combining equations 5.5–5.8 yields:

$$CD = (1 - \gamma)CD_{-1} + \gamma(DELD - \lambda)KD_{-1} + \gamma\lambda f(\ldots) \tag{5.9}$$

The specification of the two types of adjustment is a way of adding to the durable expenditure equation both the lagged dependent variable and the lagged stock of durables. Otherwise, the explanatory variables are the same as they are in the other expenditure equations.[7]

The disposable income and interest rate variables are significant in Table 5.3. The coefficient of the lagged dependent variable is .5746, and so γ above is .4254. As discussed in Chapter 3, the depreciation rate, $DELD$, is equal to .049511. Given these two values and given the coefficient of the lagged stock variable in Table 5.3 of $-.0106$, the implied value of λ is .074. This implies an adjustment of the durable stock to its desired value of 7.4 percent per quarter.

[7]Note in equation 3 that CD is divided by POP and CD_{-1} and KD_{-1} are divided by POP_{-1}, where POP is population. If equations 5.5–5.8 are defined in per capita terms, where the current values are divided by POP and the lagged values are divided by POP_{-1}, then the present per capita treatment of equation 5.9 follows. The only problem with this is that the definition used to justify 5.7 does not hold if the lagged stock is divided by POP_{-1}. All variables must be divided by the same population variable for the definition to hold. This is, however, a minor problem, and it has been ignored here. The same holds for equation 4.

Table 5.3
Equation 3
LHS Variable is CD/POP

Equation			χ^2 Tests			
RHS Variable	Est.	t-stat.	Test	χ^2	df	p-value
cnst	-.5903	-3.24	Lags	2.49	3	.4769
$AG1$	.7377	2.25	$RHO = 4$	15.74	4	.0034
$AG2$	.2590	0.28	T	22.34	1	.0000
$AG3$	-1.0850	-2.65	Leads +1	18.35	1	.0000
$(CD/POP)_{-1}$	.5746	9.05	Leads +4	21.67	4	.0002
$(KD/POP)_{-1}$	-.0106	-1.78	Leads +8	21.96	2	.0000
$YD/(POP \cdot PH)$	.1709	6.93	$p_4^e \cdot CDA$	3.40	1	.0650
$RMA \cdot CDA$	-.0063	-3.14	$p_8^e \cdot CDA$	3.28	1	.0701
			Other[a]	14.97	5	.0105
			Spread	20.43	4	.0004
SE	.01105					
R^2	.993					
DW	2.00					

$\chi^2(AGE) = 35.12$ (df = 3, p-value = .0000)

Stability Test:

AP	T_1	T_2	λ	Break
28.57*	1970:1	1979:4	2.75	1977:1

Estimation period is 1954:1–1993:2.
[a] $(AA/POP)_{-1}$, WA/PH, PCD/PH, Z, $YNL/(POP \cdot PH)$.

The age variables are jointly highly significant. The age coefficients show people 26–55 spending more, other things being equal, than the others. The pattern here is thus different than the pattern for service and nondurable consumption.

Regarding the tests, equation 3 passes the lags test, but it fails the RHO=4 and T tests. The variable for which led values were tried is disposable income, and the led values are significant. The inflation expectations variables are not significant. The other variables, which are the asset, wage, price, nonlabor income, and labor constraint variables, are significant at the five percent level but not quite at the one percent level. The spread values are highly significant. The equation fails the stability test by a wide margin.

Equation 4. IHH, residential investment—h

The same partial adjustment model is used for housing investment than was used above for durable expenditures, which adds both the lagged dependent variable and the lagged stock of housing to the housing investment equation. For example, the coefficient of the lagged housing stock variable, KH_{-1},

Table 5.4
Equation 4
LHS Variable is IHH/POP

Equation			χ^2 Tests			
RHS Variable	Est.	t-stat.	Test	χ^2	df	p-value
cnst	1.8493	3.01	Lags	1.61	4	.8063
$(IHH/POP)_{-1}$	.5322	9.59	$RHO = 4$	0.19	2	.9101
$(KH/POP)_{-1}$	-.0809	-5.15	T	0.00	1	.9869
$(AA/POP)_{-1}$	.0026	2.92	Leads +1	3.53	1	.0603
$YD/(POP \cdot PH)$	.1124	4.06	Leads +4	7.78	4	.1001
$RMA_{-1} \cdot IHHA$	-.0267	-4.81	Leads +8	2.97	2	.2267
$RHO1$	.6394	7.55	$p^e_{4-1} \cdot IHHA$	0.39	1	.5349
$RHO2$	.3519	4.17	$p^e_{8-1} \cdot IHHA$	0.02	1	.8797
			Other[a]	11.85	4	.0185
			Spread	1.46	4	.8334
SE	.00855					
R^2	.957					
DW	1.99					

$\chi^2(AGE) = 0.94$ (df = 3, p-value = .8151)

Stability Test:

AP	T_1	T_2	λ	Break
3.47	1970:1	1979:4	2.75	1974:1

Estimation period is 1954:1–1993:2.
[a] $(WA/PH)_{-1}$, $(PIH/PH)_{-1}$, Z_{-1}, $[YNL/(POP \cdot PH)]_{-1}$.

is $\gamma(DELH - \lambda)$, where $DELH$ is the depreciation rate of the housing stock. The equation is estimated under the assumption of a second order autoregressive error term.

The asset, income, and interest rate variables are significant in Table 5.4, as are the lagged dependent variable and the lagged housing stock variable. The coefficient of the lagged dependent variable is .5322, and so γ is .4678. As discussed in Chapter 3, the depreciation rate for the housing stock, $DELH$, is .006716. Given these two values and given the coefficient of the lagged housing stock variable of $-.0809$, the implied value of λ is .180. The estimated adjustment speed of the housing stock to its desired value is thus greater than the estimated adjustment speed of the durable goods stock. This is not necessarily what one would expect, and it may suggest that the estimated speed for the durable goods stock is too low.

The χ^2 test for the age variables shows that the age variables are not jointly significant. This is the reason they were not included in the final specification of the equation. Equation 4 passes the lags, RHO=4, and T tests. The variable for which led values were tried is disposable income, and the led values are not significant. The inflation expectations variables are not significant; the "other" variables are not significant; and the spread values are not close to being significant. Equation 4 thus passes all the χ^2 tests, the only expenditure equation of the four to do so. It also passes the stability test, again the only expenditure equation to do so.

The next four equations of the household sector are the labor supply equations, which will now be discussed.

Equation 5. $L1$, labor force—men 25–54[8]

One would expect from the theory of household behavior for labor supply to depend, among other things, on the after tax wage rate, the price level, and wealth. In addition, if the labor constraint is at times binding on households, one would expect a labor constraint variable like Z to affect labor supply through the discouraged worker effect.

Equation 5 explains the labor force participation rate of men 25–54. It is in log form and includes as explanatory variables the real wage (WA/PH), the labor constraint variable (Z), a time trend, and the lagged dependent variable. The coefficient estimate for the real wage is negative, implying that the income effect dominates the substitution effect for men 25–54, although the estimate is not significant. The coefficient estimate of the labor constraint variable is positive, as expected, but it is also not significant. The coefficient estimate for the time trend is negative and significant. There is a slight negative trend in the labor force participation of men 25–54 that does not seem to be explained by other variables, and so the time trend was included in the equation.

Equation 5 passes the lags test, but fails the RHO=4 test. The variable for which led values were tried is the real wage [$\log(WA/PH)$], and the led values are not significant. Another test reported in Table 5.5 has $\log PH$ added as an explanatory variable. This is a test of the use of the real wage in the equation. If $\log PH$ is significant, this is a rejection of the hypothesis that the

[8] In Section II in Fair and Dominguez (1991) the age distribution data discussed above were used to examine some of Easterlin's (1987) ideas regarding the effects of cohort size on wage rates. This was done in the context of specifying equations for $L1$ and $L2$. I now have, however, (for reasons that will not be pursued here) some reservations about the appropriateness of the specifications that were used, and in the present work the age distribution data have not been used in the specification of equations 5 and 6. This is an area of future research. I am indebted to Diane Macunovich for helpful discussions in this area.

Table 5.5
Equation 5
LHS Variable is $\log(L1/POP1)$

Equation			χ^2 Tests			
RHS Variable	Est.	t-stat.	Test	χ^2	df	p-value
cnst	-.0060	-3.11	Lags	3.35	3	.3401
$\log(L1/POP1)_{-1}$	.7763	15.63	$RHO = 4$	39.86	4	.0000
$\log(WA/PH)$	-.0036	-1.30	Leads +1	1.59	1	.2067
Z	.0139	1.50	Leads +4	10.11	4	.0386
T	-.0001	-3.73	Leads +8	1.14	2	.5667
			$\log PH$	6.36	1	.0117
			Other[a]	5.93	2	.0515
SE	.00196					
R^2	.984					
DW	2.22					
Stability Test:						
AP	T_1	T_2	λ	Break		
17.34*	1970:1	1979:4	2.75	1970:3		

Estimation period is 1954:1–1993:2.
[a] $\log(AA/POP)_{-1}$, $\log[YNL/(POP \cdot PH)]_{-1}$.

coefficient of $\log WA$ is equal to the negative of the coefficient of $\log PH$, which is implied by the use of the real wage. As can be seen, $\log PH$ is significant at the five percent level but not the one percent level. The final χ^2 test in Table 5.5 has asset and nonlabor income variables added to the equation. These variables are not significant at the five percent level. Equation 5 fails the stability test, with the maximum χ^2 value occurring for 1970:3.

Equation 6. $L2$, labor force—women 25–54

Equation 6 explains the labor force participation rate of women 25–54. It is also in log form and includes as explanatory variables the real wage, the labor constraint variable, and the lagged dependent variable. The coefficient estimate for the real wage is positive, implying that the substitution effect dominates the income effect for women 25–54. This is contrary to the case for men 25–54, where the income effect dominates. The coefficient estimate for the labor constraint is positive but not significant. The coefficient estimate for the lagged dependent variable is quite high (.9872).

Regarding the tests, the equation passes the lags test, the RHO=4 test, and the T test. The variable for which led values were tried is the real wage ($\log(WA/PH)$), and the led values are not significant. The equation thus

Table 5.6
Equation 6
LHS Variable is $\log(L2/POP2)$

Equation			χ^2 Tests			
RHS Variable	Est.	t-stat.	Test	χ^2	df	p-value
cnst	.0022	1.26	Lags	9.29	3	.0257
$\log(L2/POP2)_{-1}$	.9872	192.74	$RHO = 4$	5.77	4	.2170
$\log(WA/PH)$	.0177	2.43	T	0.49	1	.4816
Z	.0403	1.51	Leads +1	2.97	1	.0849
			Leads +4	7.21	4	.1251
			Leads +8	2.01	2	.3652
			$\log PH$	11.27	1	.0008
			Other[a]	12.15	2	.0023
SE	.00615					
R^2	.999					
DW	2.15					
Stability Test:						
AP	T_1	T_2	λ	Break		
11.20*	1970:1	1979:4	2.75	1973:4		

Estimation period is 1954:1–1993:2.
[a] $\log(AA/POP)_{-1}$, $\log[YNL/(POP \cdot PH)]_{-1}$.

does well on these tests. However, when $\log PH$ is added to the equation, it is highly significant, thus rejecting the real wage constraint. Although not shown in the table, when $\log PH$ is added to the equation, the coefficient for $\log WA$ is .0610 and the coefficient for $\log PH$ is $-.0087$. (The coefficient estimate for the lagged dependent variable is noticeably smaller—.828—when $\log PH$ is added.) It thus appears that it is primarily the nominal wage that is affecting participation. This is, of course, contrary to what one expects from most theories, and it certainly does not seem sensible that in the long run participation rises simply from an overall rise in prices and wages. Therefore, the real wage constraint was imposed on the equation, even though it is strongly rejected by the data.

For a final χ^2 test, asset and nonlabor income variables were added to the equation. These variables are significant, but (although not shown in the table) the coefficient estimates were of the wrong expected sign. One expects the level of assets and nonlabor income to have a negative effect on participation, but the coefficient estimates were positive.

The equation fails the stability test. The AP value is 11.20, which compares to the critical one percent value for 4 coefficients of 7.00.

Table 5.7
Equation 7
LHS Variable is $\log(L3/POP3)$

Equation			χ^2 Tests			
RHS Variable	Est.	t-stat.	Test	χ^2	df	p-value
cnst	.0162	0.66	Lags	3.29	4	.5110
$\log(L3/POP3)_{-1}$	.8896	24.73	$RHO = 4$	0.94	4	.9180
$\log(WA/PH)$	.0477	3.58	Leads +1	2.48	1	.1153
Z	.0663	2.36	Leads +4	14.58	4	.0057
$\log(AA/POP)_{-1}$	-.0158	-1.74	Leads +8	9.78	2	.0075
T	-.0002	-3.63	log PH	0.11	1	.7428
			Other[a]	3.91	1	.0481
SE	.00533					
R^2	.981					
DW	1.88					
Stability Test:						
AP	T_1	T_2	λ	Break		
3.80	1970:1	1979:4	2.75	1979:2		

Estimation period is 1954:1–1993:2.
[a] $\log[YNL/(POP \cdot PH)]_{-1}$.

Equation 7. $L3$, labor force—all others 16+

Equation 7 explains the labor force participation rate of all others 16+. It is also in log form and includes as explanatory variables the real wage, the labor constraint variable, an asset variable, the time trend, and the lagged dependent variable. The coefficient estimate for the real wage is positive, implying that the substitution effect dominates the income effect for all others 16+. The asset variable has a negative coefficient estimate and the labor market tightness variable has a positive one, as expected. The coefficient estimate for the time trend is negative and significant, and so, like $L1$, $L3$ appears to have a negative trend that is not explained by other variables.

Equation 7 passes the lags test and the RHO=4 test. The variable for which led values were tried is the real wage, and the values led 4 and 8 are significant. When log PH is added to the equation, it is insignificant. The "other" variable that is added is the lagged value of nonlabor income, and it is not significant at the one percent level. The equation passes the stability test.

Equation 8. LM, number of moonlighters

Equation 8 determines the number of moonlighters. It is in log form and includes as explanatory variables the real wage, the labor constraint variable,

Table 5.8
Equation 8
LHS Variable is $\log(LM/POP)$

Equation			χ^2 Tests			
RHS Variable	Est.	t-stat.	Test	χ^2	df	p-value
cnst	-.4584	-3.92	Lags	4.00	3	.2618
$\log(LM/POP)_{-1}$	.8634	25.97	$RHO = 4$	2.54	4	.6380
$\log(WA/PH)$	.0185	0.62	T	0.02	1	.8919
Z	1.0396	3.87	Leads +1	0.05	1	.8164
			Leads +4	7.47	4	.1128
			Leads +8	3.04	2	.2185
			log PH	0.07	1	.7920
			Other[a]	6.42	2	.0403
SE	.05647					
R^2	.858					
DW	1.98					
Stability Test:						
AP	T_1	T_2	λ	Break		
3.32	1970:1	1979:4	2.75	1973:2		

Estimation period is 1954:1–1993:2.
[a] $\log(AA/POP)_{-1}$, $\log[YNL/(POP \cdot PH)]_{-1}$.

and the lagged dependent variable. The coefficient estimate for the real wage is positive, suggesting that the substitution effect dominates for moonlighters, although the variable is not significant. The coefficient estimate for the labor constraint variable is positive and significant. The larger is the labor constraint, the fewer are the number of people holding two jobs.

Equation 8 does brilliantly in the tests. It passes the lags test, the RHO=4 test, and the T test. The variable for which led values were tried is the real wage, and the led values are not significant. When log PH is added to the equation, it is not significant, and so the real wage constraint is supported. The "other" variables that were added are the lagged value of wealth and the lagged value of nonlabor income, and they are not significant at the one percent level. Finally, the equation passes the stability test.

This completes the discussion of the household expenditure and labor supply equations. A summary of some of the general results across the equations is in Section 5.10.

5.3 Money Demand Equations[9]

In the theoretical model a household's demand for money depends on the level of transactions, the interest rate, and the household's wage rate. High wage rate households spend less time taking care of money holdings than do low wage rate households and thus on average hold more money. With aggregate data it is not possible to estimate this wage rate effect on the demand for money, and in the empirical work the demand for money has simply been taken to be a function of the interest rate and a transactions variable. However, the age distribution variables have been added to the household money demand equation, and, as discussed below, this may pick up a wage rate effect.

The model contains three demand for money equations: one for the household sector, one for the firm sector, and a demand for currency equation. Before presenting these equations it will be useful to discuss how the dynamics were handled. The key question about the dynamics is whether the adjustment of actual to desired values is in nominal or real terms.

Let M_t^*/P_t denote the desired level of real money balances, let y_t denote a measure of real transactions, and let r_t denote a short term interest rate. Assume that the equation determining desired money balances is in log form and write

$$\log(M_t^*/P_t) = \alpha + \beta \log y_t + \gamma r_t \tag{5.10}$$

Note that the log form has not been used for the interest rate. Interest rates can at times be quite low, and it may not be sensible to take the log of the interest rate. If, for example, the interest rate rises from .02 to .03, the log of the rate rises from -3.91 to -3.51, a change of .40. If, on the other hand, the interest rate rises from .10 to .11, the log of the rate rises from -2.30 to -2.21, a change of only .09. One does not necessarily expect a one percentage point rise in the interest rate to have four times the effect on the log of desired money holdings when the change is from a base of .02 rather than .10. In practice the results of estimating money demand equations do not seem to be very sensitive to whether the level or the log of the interest rate is used. For the work in this book the level of the interest rate has been used.

If the adjustment of actual to desired money holdings is in real terms, the

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[9]The material in this section on the test of real versus nominal adjustment is taken from Fair (1987). The use of the age distribution variables in equation 9 is taken from Fair and Dominguez (1991).

adjustment equation is

$$\log(M_t/P_t) - \log(M_{t-1}/P_{t-1}) = \lambda[\log(M_t^*/P_t) - \log(M_{t-1}/P_{t-1})] + \epsilon_t \tag{5.11}$$

If the adjustment is in nominal terms, the adjustment equation is

$$\log M_t - \log M_{t-1} = \lambda(\log M_t^* - \log M_{t-1}) + \mu_t \tag{5.12}$$

Combining 5.10 and 5.11 yields

$$\log(M_t/P_t) = \lambda\alpha + \lambda\beta \log y_t + \lambda\gamma r_t + (1-\lambda)\log(M_{t-1}/P_{t-1}) + \epsilon_t \tag{5.13}$$

Combining 5.10 and 5.12 yields

$$\log(M_t/P_t) = \lambda\alpha + \lambda\beta \log y_t + \lambda\gamma r_t + (1-\lambda)\log(M_{t-1}/P_t) + \mu_t \tag{5.14}$$

Equations 5.13 and 5.14 differ in the lagged money term. In 5.13, which is the real adjustment specification, M_{t-1} is divided by P_{t-1}, whereas in 5.14, which is the nominal adjustment specification, M_{t-1} is divided by P_t.

A test of the two hypotheses is simply to put both lagged money variables in the equation and see which one dominates. If the real adjustment specification is correct, $\log(M_{t-1}/P_{t-1})$ should be significant and $\log(M_{t-1}/P_t)$ should not, and vice versa if the nominal adjustment specification is correct. This test may, of course, be inconclusive in that both terms may be significant or insignificant, but I have found that this is rarely the case. This test was performed on the three demand for money equations, and in each case the nominal adjustment specification won. The nominal adjustment specification has thus been used in the model.[10]

Equation 9. MH, demand deposits and currency—h

Equation 9 is the demand for money equation of the household sector. It is in per capita terms and is in log form. Disposable income is used as the transactions variable, and the after tax three month Treasury bill rate is used

[10] The nominal adjustment hypothesis is also supported in Fair (1987), where demand for money equations were estimated for 27 countries. Three equations were estimated for the United States (versions of equations 9, 17, and 26) and one for each of the other 26 countries. Of the 29 estimated equations, the nominal adjustment dominated for 25, the real adjustment dominated for 3, and there was 1 tie. The nominal adjustment hypothesis is also supported in Chapter 6. Of the 19 countries for which the demand for money equation (equation 6) is estimated, the nominal adjustment hypothesis dominates in 13.

Table 5.9
Equation 9
LHS Variable is $\log[MH/(POP \cdot PH)]$

Equation			χ^2 Tests			
RHS Variable	Est.	t-stat.	Test	χ^2	df	p-value
cnst	-.2929	-1.59	$\log(\frac{MH}{POP \cdot PH})_{-1}$	0.05	1	.8164
$AG1$	.6533	1.78	Lags	5.91	3	.1159
$AG2$	-.5728	-0.65	$RHO = 4$	19.66	3	.0002
$AG3$	-.7462	-1.22	T	0.06	1	.8141
$\log \frac{MH_{-1}}{POP_{-1} \cdot PH}$	.8962	22.88				
$\log[YD/(POP \cdot PH)]$	.0796	1.72				
RSA	-.0035	-3.17				
$RHO1$	-.2677	-3.23				
SE	.02318					
R^2	.902					
DW	1.94					

$\chi^2(AGE) = 3.87$ (df = 3, p-value = .2763)

Stability Test:

AP	T_1	T_2	λ	Break
16.67*	1970:1	1979:4	2.75	1975:3

Estimation period is 1954:1–1993:2.

as the interest rate. The age distribution variables are included in the equation to pick up possible differences in the demand for money by age. The equation is estimated under the assumption of a first order autoregressive error term.

The short run income elasticity of the demand for money in Table 5.9 is .0796, and the long run elasticity is $.767 = .0796/(1.0 - .8962)$. The coefficients for the age variables show that people 26–55 hold more money, other things being equal, than do others, which is as expected if people 26–55 have on average higher wage rates than the others. The age variables are not, however, jointly significant at the five percent level, and so not much confidence should be placed on this result.[11]

The test results show that the lagged dependent variable that pertains to the real adjustment specification—$\log[MH/(POP \cdot PH)]_{-1}$—is insignificant. As discussed above, this supports the nominal adjustment hypothesis. Equation 9 passes the lags and T tests, but it fails the RHO=4 and stability tests. For the stability test the largest χ^2 value occurred for 1975:3.

[11] A similar result was obtained in Fair and Dominguez (1991), Table 3. The sign pattern was as expected, but the χ^2 value of 4.92 was less than the five percent critical value.

Table 5.17
Equation 17
LHS Variable is $\log(MF/PF)$

Equation			χ^2 Tests			
RHS Variable	Est.	t-stat.	Test	χ^2	df	p-value
cnst	.1784	2.01	$\log(MF/PF)_{-1}$	0.57	1	.4511
$\log(MF_{-1}/PF)$	.9038	30.89	Lags	7.74	3	.0518
$\log(X - FA)$	.0552	3.00	$RHO = 4$	25.79	4	.0000
$RS(1 - D2G - D2S)$	-.0073	-2.42	T	3.41	1	.0648
SE	.03776					
R^2	.956					
DW	2.21					
Stability Test:						
AP	T_1	T_2	λ	Break		
1.77	1970:1	1979:4	2.75	1974:4		

Estimation period is 1954:1–1993:2.

Equation 17. MF, demand deposits and currency—f

Equation 17 is the demand for money equation of the firm sector. The results for this equation are presented in Table 5.17. The equation is in log form. The transactions variable is the level of nonfarm firm sales, $X - FA$, and the interest rate variable is the after tax three month Treasury bill rate. The tax rates used in this equation are the corporate tax rates, $D2G$ and $D2S$, not the personal tax rates used for RSA in equation 9.

All the variables in the equation are significant. Again, the test results show that the lagged dependent variable that pertains to the real adjustment specification [$\log(MF/PF)_{-1}$] is insignificant. The equation passes the lags test, the T test, and the stability test. It fails the RHO=4 test.

Equation 26. CUR, currency held outside banks

Equation 26 is the demand for currency equation. It is in per capita terms and is in log form. The transactions variable that is used is the level of nonfarm firm sales. The interest rate variable used is RSA, and the equation is estimated under the assumption of a first order autoregressive error term.

The results are presented in Table 5.26. All the variables in the equation are significant. The test results show that the lagged dependent variable that pertains to the real adjustment specification—$\log[CUR/(POP \cdot PF)]_{-1}$—is insignificant at the one percent level. The equation passes all the tests.

Table 5.26
Equation 26
LHS Variable is $\log[CUR/(POP \cdot PF)]$

Equation			χ^2 Tests			
RHS Variable	Est.	t-stat.	Test	χ^2	df	p-value
cnst	-.0529	-7.67	$\log(\frac{CUR}{POP \cdot PF})_{-1}$	6.25	1	.0124
$\log \frac{CUR_{-1}}{POP_{-1} \cdot PF}$	.9572	86.21	Lags	5.02	3	.1702
$\log[(X - FA)/POP]$	.0499	7.95	$RHO = 4$	6.06	3	.1085
RSA	-.0009	-2.10	T	0.67	1	.4122
$RHO1$	-.3262	-4.32				
SE	.00966					
R^2	.989					
DW	2.02					
Stability Test:						
AP	T_1	T_2	λ	Break		
5.91	1970:1	1979:4	2.75	1977:3		

Estimation period is 1954:1–1993:2.

5.4 The Main Firm Sector Equations

In the maximization problem of a firm in the theoretical model there are five main decision variables: the firm's price, production, investment, demand for employment, and wage rate. These five decision variables are determined jointly in that they are the result of solving one maximization problem. The variables that affect this solution include 1) the initial stocks of excess capital, excess labor, and inventories, 2) the current and expected future values of the interest rate, 3) the current and expected future demand schedules for the firm's output, 4) the current and expected future supply schedules of labor facing the firm, and 5) the firm's expectations of other firms' future price and wage decisions.

In the econometric model seven variables were chosen to represent the five decisions: 1) the price level of the firm sector, PF, 2) production, Y, 3) investment in nonresidential plant and equipment, IKF, 4) the number of jobs in the firm sector, JF, 5) the average number of hours paid per job, HF, 6) the average number of overtime hours paid per job, HO, and 7) the wage rate of the firm sector, WF. Each of these variables is determined by a stochastic equation, and these are the main stochastic equations of the firm sector.

Moving from the theoretical model of firm behavior to the econometric specifications is not straightforward, and a number of approximations have to be made. One of the key approximations is that the econometric specifications in effect assume that the five decisions of a firm are made sequentially rather than jointly. The sequence is from the price decision, to the production decision, to the investment and employment decisions, and to the wage rate decision. In this way of looking at the problem, the firm first chooses its optimal price path. This path implies a certain expected sales path, from which the optimal production path is chosen. Given the optimal production path, the optimal paths of investment and employment are chosen. Finally, given the optimal employment path, the optimal wage path is chosen.

Equation 10. PF, price deflator for $X - FA$[12]

Equation 10 is the key price equation in the model, and the results for this equation are in Table 5.10. The equation is in log form. The price level is a function of the lagged price level, the wage rate inclusive of the employer social security tax rate, the price of imports, and a demand pressure variable. The equation is estimated under the assumption of a first order autoregressive error term. The lagged price level is meant to pick up expectational effects, and the wage rate and import price variables are meant to pick up cost effects. The demand pressure variable, $\log[(YS_{-1} - Y_{-1})/YS_{-1} + .04]$, is the log of the percentage gap between potential and actual output plus .04. (Remember that YS is the potential value of Y.) This functional form implies that as actual output approaches four percent more than potential, the demand pressure variable approaches minus infinity, which implies that the price level approaches plus infinity. This functional form effectively prevents actual output from ever exceeding potential output by more than four percent. The demand pressure variable is lagged one quarter in equation 10 because this gave slightly better results than did the use of the variable unlagged.

An important feature of the price equation is that the price *level* is explained by the equation, not the price *change*. This treatment is contrary to the standard Phillips-curve treatment, where the price (or wage) change is explained by the equation. Given the theory outlined in Chapter 2, the natural decision variables of a firm would seem to be the levels of prices and wages. For example, the market share equations in the theoretical model have a firm's market share as a function of the ratio of the firm's price to the average price of other firms. These are price levels, and the objective of the firm is to choose the price level path (along with the paths of the other decision variables) that maximizes the

[12] The material on the level versus change specification in this section is taken from Fair (1993a).

Table 5.10
Equation 10
LHS Variable is $\log PF$

Equation			χ^2 Tests			
RHS Variable	Est.	t-stat.	Test	χ^2	df	p-value
$\log PF_{-1}$	.9194	155.61	Level form[a]	3.94	2	.1395
$\log[WF(1+D5G)]$	.0294	7.68	Lags	8.09	4	.0885
cnst	.1142	6.97	$RHO = 4$	10.67	3	.0137
$\log PIM$	.0361	12.12	T	0.04	1	.8499
$\log[(\frac{YS-Y}{YS})_{-1} + .04]$	-.0051	-3.78	Leads +1	0.06	1	.8150
$RHO1$	.1418	1.83	Leads +4	6.39	4	.1719
			Leads +8	2.98	2	.2253
			UR_{-1}	17.46	1	.0000
			$(\frac{YS-Y}{YS})_{-1}$	6.67	1	.0098
			Change form[b]	92.30	3	.0000
SE	.00400					
R^2	.999					
DW	1.96					
Stability Test:						
AP	T_1	T_2	λ	Break		
12.28*	1970:1	1979:4	2.75	1972:2		

Estimation period is 1954:1–1993:2.
[a] $\log[WF(1+D5G)]_{-1}$ and $\log PIM_{-1}$ added to the equation.
[b] $\log PF_{-1}$, $\log[WF(1+D5G)]_{-1}$, and $\log PIM_{-1}$ added to the equation with $\Delta \log PF$ on the left hand side and a constant, $\Delta \log[WF(1+D5G)]$, $\Delta \log PIM$, and $\log[((YS-Y)/YS)_{-1} + .04]$ on the right hand side (with the RHO1 assumption).

multiperiod objective function. A firm decides what its price *level* should be relative to the price *levels* of other firms.

Fortunately, it is possible to test whether the price level or price change specification is better. Let p denote the log of the price level, let w denote the log of the wage rate, and let D denote the level of some demand pressure variable. The price equation in level form is

$$p = \beta_0 + \beta_1 p_{-1} + \beta_2 w + \beta_3 D \tag{5.15}$$

and the equation in change form is

$$\Delta p = \eta_0 + \eta_1 \Delta w + \eta_2 D \tag{5.16}$$

The key difference between 5.15 and 5.16 is that D and not ΔD is in 5.16. If β_1 in 5.15 is less than one, a permanent change in D results in a permanent change in the level of P but not in the change in P. In 5.16, on the other hand, a permanent change in D results in a permanent change in the change in P. The constant term η_0 in 5.16 accounts for any trend in the level of P not captured by the other variables.

It is not possible to nest 5.15 into 5.16 or vice versa, but they can each be nested in a more general equation. This equation is

$$p = \delta_0 + \delta_1 p_{-1} + \delta_2 w + \delta_3 w_{-1} + \delta_4 D \tag{5.17}$$

The restriction in 5.17 implied by the level specification in 5.15 is $\delta_3 = 0$. The restrictions in 5.17 implied by the change specification in 5.16 are $\delta_1 = 1$ and $\delta_2 = -\delta_3$. These restrictions can be tested. If both sets of restrictions are accepted, then the test has not discriminated between the two specifications. If neither set is accepted, then neither specification is supported by the data. Otherwise, one specification will be selected over the other.

Equation 10 in the model is like equation 5.15 above except that the price of imports is also an explanatory variable. This is a variable like w in that it is assumed to pick up cost effects. The results in Table 5.10 show that all the explanatory variables are significant.

The test results are as follows. First, to test the level specification, the lagged values of the wage rate and price of imports—$\log[WF(1 + D5G)]_{-1}$ and $\log PIM_{-1}$—are added. As can be seen, these two variables are not significant, and so the level specification is supported over the more general specification. The change specification, on the other hand, is not supported, as can be seen in the last χ^2 test in Table 5.10. When the lagged values of the price level, the wage rate, and the price of imports are added to the change form of the price equation, they are highly significant, with a χ^2 value of 92.30. The change form is thus strongly rejected.[13]

Equation 10 passes the lags test, the RHO=4 test, and the T test. The variable for which led values were tried is the wage rate, and the led values are not significant.

The test results next show that the unemployment rate lagged once is significant when added to the equation. Although not shown in the table, the addition of the unemployment rate makes the demand pressure variable insignificant. The next test shows that the simple percentage gap variable

[13]The level versus change specification was also tested in Fair (1993a) for 40 disaggregate price equations. The results were somewhat mixed, but overall slightly favored the level specification. The disaggregate results thus provide some support for the current aggregate estimates. Also, as will be seen, the results in Chapter 6 of estimating price equations for different countries strongly support the level specification over the change specification. As discussed later in this chapter, the result that the level specification is supported over the change specification has important implications for the long run properties of the economy.

lagged once, $[(YS - Y)/YS]_{-1}$, is significant. Although not shown in the table, the addition of this variable also makes the demand pressure variable insignificant. The functional form chosen for the demand pressure variable is thus not supported by the data. The best results (e.g., best fit) are obtained when no nonlinearity is introduced. This situation is unsatisfactory from a theoretical perspective in that one expects that there is some degree of demand pressure beyond which prices rise faster for a further increase in demand pressure than they did before this degree was reached. The problem is that the U.S. economy has not experienced enough periods in which demand pressure was very high to allow one to estimate adequately how prices behave in these extreme periods. The large price increases in the 1970s were primarily cost driven, and so they are of no help for this problem. The way that this problem has been handled in the model is simply to use the nonlinear functional form described above even though simpler forms do somewhat better. This problem is, of course, an important area for future work, and it suggests that any policy experiments with the model that push the economy to very high levels of activity should be interpreted with considerable caution. The behavior of prices at very high activity levels has probably not been very accurately estimated.

Finally, the price equation fails the stability test. The largest χ^2 value occurs for 1972:2, near the beginning of the first oil price shock.

Equation 11. Y**, production—f**

The specification of the production equation is where the assumption that a firm's decisions are made sequentially begins to be used. The equation is based on the assumption that the firm sector first sets it price, then knows what its sales for the current period will be, and from this latter information decides on what its production for the current period will be.

In the theoretical model production is smoothed relative to sales. The reason for this is various costs of adjustment, which include costs of changing employment, costs of changing the capital stock, and costs of having the stock of inventories deviate from some proportion of sales. If a firm were only interested in minimizing inventory costs, it would produce according to the following equation (assuming that sales for the current period are known):

$$Y = X + \beta X - V_{-1} \tag{5.18}$$

where Y is the level of production, X is the level of sales, V_{-1} is the stock of inventories at the beginning of the period, and β is the inventory-sales ratio that minimizes inventory costs. Since by definition, $V - V_{-1} = Y - X$, producing according to 5.18 would ensure that $V = \beta X$. Because of the other adjustment costs, it is generally not optimal for a firm to produce according to 5.18. In the theoretical model there was no need to postulate explicitly how a firm's production plan deviated from 5.18 because its optimal production plan just resulted, along with the other optimal paths, from the direct solution of its maximization problem. For the empirical work, however, it is necessary to make further assumptions.

The estimated production equation is based on the following three assumptions:

$$V^* = \beta X \tag{5.19}$$

$$Y^* = X + \alpha(V^* - V_{-1}) \tag{5.20}$$

$$Y - Y_{-1} = \lambda(Y^* - Y_{-1}) \tag{5.21}$$

where * denotes a desired value. Equation 5.19 states that the desired stock of inventories is proportional to current sales. Equation 5.20 states that the desired level of production is equal to sales plus some fraction of the difference between the desired stock of inventories and the stock on hand at the end of the previous period. Equation 5.21 states that actual production partially adjusts to desired production each period. Combining the three equations yields

$$Y = (1 - \lambda)Y_{-1} + \lambda(1 + \alpha\beta)X - \lambda\alpha V_{-1} \tag{5.22}$$

Equation 11 in Table 5.11 is the estimated version of equation 5.22. The equation is estimated under the assumption of a third order autoregressive process of the error term. The implied value of λ is $.7074 = 1.0 - .2926$, which means that actual production adjusts 70.74 percent of the way to desired production in the current quarter. The implied value of α is .4727 = .3344/.7074, which means that desired production is equal to sales plus 47.27 percent of the desired change in inventories. The implied value of β is .7629, which means that the desired stock of inventories is estimated to equal 76.29 percent of the (quarterly) level of sales.

Equation 11 passes all of the tests except the stability test. The variable for which led values were used is the level of sales, X, and it is interesting that the led values are not significant.[14] The hypothesis that firms have rational expectations regarding future values of sales is rejected. Note also that the spread values are not significant.

The estimates of equation 11 are consistent with the view that firms smooth production relative to sales. The view that production is smoothed relative to

[14] Collinearity problems prevented Leads +4 from being calculated for equation 11.

Table 5.11
Equation 11
LHS Variable is Y

Equation			χ^2 Tests			
RHS Variable	Est.	t-stat.	Test	χ^2	df	p-value
cnst	28.0418	1.85	Lags	1.24	2	.5392
Y_{-1}	.2926	6.50	$RHO = 4$	0.25	1	.6189
X	.9625	19.84	T	0.06	1	.8033
V_{-1}	-.3344	-8.26	Leads +1	4.58	1	.0323
$RHO1$	.3906	4.69	Leads +8	1.49	2	.4753
$RHO2$	.2788	3.41	Spread	5.45	4	.2442
$RHO3$	.2585	3.25				
SE	3.05348					
R^2	.999					
DW	1.98					
Stability Test:						
AP	T_1	T_2	λ	Break		
13.79*	1970:1	1979:4	2.75	1979:3		

Estimation period is 1954:1–1993:2.

sales has been challenged by Blinder (1981) and others, and this work has in turn been challenged in Fair (1989) as being based on faulty data. The results in Fair (1989) using physical units data for specific industries suggests that production is smoothed relative to sales. The results using the physical units data thus provide some support for the current aggregate estimates.

Equation 12. IKF, nonresidential fixed investment—f

Equation 12 explains nonresidential fixed investment of the firm sector. It is based on the assumption that the production decision has already been made. In the theoretical model, because of costs of changing the capital stock, it may sometimes be optimal for a firm to hold excess capital. If there were no such costs, investment each period would merely be the amount needed to have enough capital to produce the output of the period. In the theoretical model there was no need to postulate explicitly how investment deviates from this amount, but for the empirical work this must be done.

The estimated investment equation is based on the following three equations:

$$(KK - KK_{-1})^* = \alpha_0(KK_{-1} - KKMIN_{-1}) + \alpha_1\Delta Y + \alpha_2\Delta Y_{-1} + \alpha_3\Delta Y_{-2} + \alpha_4\Delta Y_{-3} + \alpha_5\Delta Y_{-4} \tag{5.23}$$

$$IKF^* = (KK - KK_{-1})^* + DELK \cdot KK_{-1} \tag{5.24}$$

$$\Delta IKF = \lambda(IKF^* - IKF_{-1}) \tag{5.25}$$

where again * denotes a desired value. IKF is gross investment of the firm sector, KK is the capital stock, and $KKMIN$ is the minimum amount of capital needed to produce the output of the period. $(KK - KK_{-1})^*$ is desired net investment, and IKF^* is desired gross investment. Equation 5.23 states that desired net investment is a function of the amount of excess capital on hand and of five change in output terms. If output has not changed for four periods and if there is no excess capital, then desired net investment is zero. The change in output terms are meant to be proxies for expected future output changes. Equation 5.24 relates desired gross investment to desired net investment. $DELK \cdot KK_{-1}$ is the depreciation of the capital stock during period $t - 1$. By definition, $IKF = KK - KK_{-1} + DELK \cdot KK_{-1}$, and 5.24 is merely this same equation for the desired values. Equation 5.25 is a partial adjustment equation relating the desired change in gross investment to the actual change. It is meant to approximate cost of adjustment effects. Combining 5.23–5.25 yields

$$\begin{aligned}\Delta IKF = {} & \lambda\alpha_0(KK_{-1} - KKMIN_{-1}) + \lambda\alpha_1\Delta Y + \lambda\alpha_2\Delta Y_{-1} \\ & + \lambda\alpha_3\Delta Y_{-2} + \lambda\alpha_4\Delta Y_{-3} + \lambda\alpha_5\Delta Y_{-4} \\ & - \lambda(IKF_{-1} - DELK \cdot KK_{-1})\end{aligned} \tag{5.26}$$

Equation 12 in Table 5.12 is the estimated version of 5.26 with two additions. The two additional variables in Table 5.12 are cost of capital variables: an investment tax credit dummy variable, $TXCR$, and the real bond rate lagged three quarters, RB'_{-3}.[15] Both of these variables are multiplied by $IKFA$, which is a variable constructed by peak to peak interpolations of IKF. Since IKF has a trend and $TXCR$ and RB' do not, one would expect a given change in $TXCR$ or RB' to have an effect on IKF that increases over time, and multiplying both variables by $IKFA$ is a way of accounting for this. $IKFA$ is exogenous; it is merely a scale variable.

How can the use of the cost of capital variables be justified? In the theoretical model the cost of capital affects investment by affecting the kinds of machines that are purchased. If the cost of capital falls, more capital intensive

[15] RB' is equal to the after tax bond rate, $RB(1 - D2G - D2S)$, minus p_4^e, one of the measures of inflation expectations. Remember from Section 5.1 that p_4^e equals $100[(PD/PD_{-4}) - 1]$.

Table 5.12
Equation 12
LHS Variable is ΔIKF

Equation			χ^2 Tests			
RHS Variable	Est.	t-stat.	Test	χ^2	df	p-value
$(KK - KKMIN)_{-1}$	-.0013	-0.51	Lags	19.19	5	.0018
$IKF_{-1} - DELK \cdot KK_{-1}$	-.0396	-2.99	$RHO = 4$	7.31	4	.1203
ΔY	.0616	2.80	T	2.24	1	.1341
ΔY_{-1}	.0660	3.83	Leads +1	0.20	1	.6559
ΔY_{-2}	.0308	1.83	Leads +4	6.59	4	.1592
ΔY_{-3}	.0515	3.04	Leads +8	6.30	2	.0428
ΔY_{-4}	.0346	2.00	cnst	0.14	1	.7039
$TXCR \cdot IKFA$	.0013	0.45	Spread	1.19	4	.8792
$RB'_{-3} \cdot IKFA$	-.0016	-2.52				
SE	1.23863					
R^2	.436					
DW	1.99					
Stability Test:						
AP	T_1	T_2	λ	Break		
2.76	1970:1	1979:4	2.75	1978:2		

Estimation period is 1954:1–1993:2.

machines are purchased and investment expenditures increase. For the empirical work, data are not available by types of machines, and approximations have to be made. The key approximation is the postulation of the production function 3.1 in Chapter 3. This production function is one of fixed proportions in the short run. Technical change and changes in the cost of capital relative to the cost of labor affect over time the λ and μ coefficients in the equation, and these are accounted for through the peak to peak interpolations discussed in Chapter 3. $KKMIN$ in equation 93 in the model is determined using MUH, the peak to peak interpolation of $\mu \cdot \bar{H}$.

If, as seems quite likely, the effects of cost of capital changes on firms' decisions are not completely captured through the peak to peak interpolations, then adding cost of capital variables to equation 5.23 (and thus equation 5.26) may be warranted. For example, when the cost of capital falls, $KKMIN$ may underestimate the desired amount of capital, and at least part of this error may be picked up by adding cost of capital variables to the equation.

The estimate of λ in equation 12 is .0396, which says that gross investment adjusts 3.96 percent to its desired value each quarter. The implied value of α_0 is $-.0328 = -.0013/.0396$, which says that 3.28 percent of the amount of excess capital on hand is desired to be eliminated each quarter. The change in output terms have t-statistics greater than or equal to two except for the change lagged twice, which has a t-statistic of 1.83. The tax credit variable has a t-statistic of 0.45, and the real bond rate lagged three times has a t-statistic of -2.52. The tax credit variable is thus not significant, although its coefficient estimate is of the expected sign, and the bond rate is significant. I have found it very difficult over the years to obtain significant cost of capital effects in equation 12, and the current results are probably the best that I have ever done. Even here the lag of three quarters for the bond rate seems a little long, but shorter lags gave poorer results. The results may thus be spurious and merely the result of data mining, but they are retained because it is embarrassing not to have cost of capital effects in the investment equation.

Equation 12 fails the lags test, but it passes all the others, including the stability test. The variable used for the led values was the change in output, and it is interesting to see that the future output changes are not significant. This is evidence against the hypothesis that firms have rational expectations with respect to future values of output. Note also in Table 5.12 that the constant term is not significant. According to equation 5.26 there should be no constant term in the equation, and the results bear this out. The χ^2 test for the addition of the constant term is not significant. The spread values are also not significant.

Equation 13. JF, number of jobs—f

The employment equation 13 and the hours equation 14 are similar in spirit to the investment equation 12. They are also based on the assumption that the production decision is made first. Because of adjustment costs, it is sometimes optimal in the theoretical model for firms to hold excess labor. Were it not for the costs of changing employment, the optimal level of employment would merely be the amount needed to produce the output of the period. In the theoretical model there was no need to postulate explicitly how employment deviates from this amount, but this must be done for the empirical work.

The estimated employment equation is based on the following three equations:

$$\Delta \log JF = \alpha_0 \log(JF_{-1}/JF^*_{-1}) + \alpha_1 \Delta \log Y \tag{5.27}$$

$$JF^*_{-1} = JHMIN_{-1}/HF^*_{-1} \tag{5.28}$$

$$HF^*_{-1} = \bar{H}e^{\delta t} \tag{5.29}$$

where $JHMIN$ is the number of worker hours required to produce the output of the period, HF^* is the average number of hours per job that the firm would like to be worked if there were no adjustment costs, and JF^* is the number of workers the firm would like to employ if there were no adjustment costs. The term $\log(JF_{-1}/JF^*_{-1})$ in 5.27 will be referred to as the "amount of excess labor on hand." Equation 5.27 states that the change in employment is a function of the amount of excess labor on hand and the change in output (all changes are in logs). If there is no change in output and if there is no excess labor on hand, the change in employment is zero. Equation 5.28 defines the desired number of jobs, which is simply the required number of worker hours divided by the desired number of hours worked per job. Equation 5.29 postulates that the desired number of hours worked is a smoothly trending variable, where $\bar{H}$ and δ are constants. Combining 5.27–5.29 yields

$$\begin{aligned}\Delta \log JF = \alpha_0 \log \bar{H} + \alpha_0 \log(JF_{-1}/JHMIN_{-1}) \\ + \alpha_0 \delta t + \alpha_1 \Delta \log Y \qquad (5.30)\end{aligned}$$

Equation 13 in Table 5.13 is the estimated version of equation 5.30 with two additions. The first addition is the use of the lagged dependent variable, $\Delta \log JF_{-1}$. This was added to pick up dynamic effects that did not seem to be captured by the original specification.

The second addition is accounting for what seemed to be a structural break in the mid 1970s. When testing the equation for structural stability, there was evidence of a structural break in the middle of the sample period, with the largest χ^2 value occurring in 1977:2. Contrary to the case for most equations that fail the stability test, the results for equation 13 suggested that the break could be modeled fairly simply. In particular, the coefficient of the change in output did not appear to change, but the others did. This was modeled by creating a dummy variable, $DD772$, that is one from 1977:2 on and zero otherwise and adding to the equation all the explanatory variables in the equation (except the change in output) multiplied by $DD772$ as additional explanatory variables.

The results in Table 5.13 show that the estimate of α_0, the coefficient of the excess labor variable, is $-.0867$ for the period before 1977:2 and $-.1843 = -.0867 - .0976$ after that. This means that in the latter period 18.43 percent of the amount of excess labor on hand is eliminated each quarter, up substantially from the earlier period.

Equation 13 does not pass the lags test, where the χ^2 value is quite large. Experimenting with various specifications of this equation reveals that it is very fragile with respect to adding lagged values in that adding these values changes the values of the other coefficient estimates substantially and in ways that do not seem sensible. The equation also fails the RHO=4 test. The variable

Table 5.13
Equation 13
LHS Variable is $\Delta \log JF$

Equation			χ^2 Tests			
RHS Variable	Est.	t-stat.	Test	χ^2	df	p-value
cnst	-.5418	-3.66	Lags	38.03	5	.0000
$DD772$	-.5775	-1.69	$RHO = 4$	14.07	4	.0071
$\log(\frac{JF}{JHMIN})_{-1}$	-.0867	-3.65	Leads +1	4.46	1	.0348
$DD772 \cdot \log(\frac{JF}{JHMIN})_{-1}$	-.0976	-1.75	Leads +4	20.18	4	.0005
$\Delta \log JF_{-1}$	.4233	7.48	Leads +8	6.31	2	.0427
$DD772 \cdot \Delta \log JF_{-1}$	-.2634	-1.93				
T	.0001	3.56				
$DD772 \cdot T$	-.0001	-2.28				
$\Delta \log Y$	.3037	9.34				
SE	.00329					
R^2	.755					
DW	2.07					

Estimation period is 1954:1–1993:2.

for which led values were tried is the change in output variable, and the values led four quarters (but not one and eight) are significant at the one percent level.

Equation 14. HF, average number of hours paid per job—f

The hours equation is based on equations 5.28 and 5.29 and the following equation:

$$\Delta \log HF = \lambda \log(HF_{-1}/HF^*_{-1}) + \alpha_0 \log(JF_{-1}/JF^*_{-1}) + \alpha_1 \Delta \log Y \tag{5.31}$$

The first term on the right hand side of 5.31 is the (logarithmic) difference between the actual number of hours paid for in the previous period and the desired number. The reason for the inclusion of this term in the hours equation but not in the employment equation is that, unlike JF, HF fluctuates around a slowly trending level of hours. This restriction is captured by the first term in 5.31. The other two terms are the amount of excess labor on hand and the current change in output. Both of these terms affect the employment decision, and they should also affect the hours decision since the two are closely related. Combining 5.28, 5.29, and 5.31 yields

$$\Delta \log HF = (\alpha_0 - \lambda) \log \bar{H} + \lambda \log HF_{-1} + \alpha_0 \log(JF_{-1}/JHMIN_{-1}) + (\alpha_0 - \lambda)\delta t + \alpha_1 \Delta \log Y \tag{5.32}$$

Table 5.14
Equation 14
LHS Variable is $\Delta \log HF$

Equation			χ^2 Tests			
RHS Variable	Est.	t-stat.	Test	χ^2	df	p-value
cnst	.7219	4.64	Lags	3.11	4	.5394
$DD772$	.3034	2.10	$RHO = 4$	3.40	3	.3345
$\log HF_{-1}$	-.1837	-5.19	Leads +1	2.27	1	.1323
$\log(\frac{JF}{JHMIN})_{-1}$	-.0681	-3.50	Leads +4	2.07	4	.7236
$DD772 \cdot \log(\frac{JF}{JHMIN})_{-1}$	.0521	2.22	Leads +8	3.32	2	.1899
T	-.0002	-5.03				
$DD772 \cdot T$	.0001	4.02				
$\Delta \log Y$	.1694	7.30				
$RHO1$	-.2402	-2.85				
SE	.00257					
R^2	.466					
DW	1.97					

Estimation period is 1954:1–1993:2.

Equation 14 in Table 5.14 is the estimated version of 5.32 with the addition of the terms multiplied by $DD772$ to pick up the structural break. The equation is estimated under the assumption of a first order autoregressive error term. The estimated value of λ is $-.1837$, which means that, other things being equal, actual hours are adjusted toward desired hours by 18.37 percent per quarter. The excess labor variable is significant in the equation, as are the time trend and the change in output.

Equation 14 passes the lags and RHO=4 tests. The variable for which led values were tried is the change in output variable, and the led values are not significant. This is contrary to the case for equation 13, where the values led four quarters are significant.

Equation 15. HO, average number of overtime hours paid per job—f

Equation 15 explains overtime hours, HO. One would expect HO to be close to zero for low values of total hours, HF, and to increase roughly one for one for high values of HF. An approximation to this relationship is

$$HO = e^{\alpha_1 + \alpha_2 HF} \tag{5.33}$$

which in log form is

$$\log HO = \alpha_1 + \alpha_2 HF \tag{5.34}$$

Table 5.15
Equation 15
LHS Variable is log HO

Equation			χ^2 Tests			
RHS Variable	Est.	t-stat.	Test	χ^2	df	p-value
cnst	3.8834	78.31	Lags	4.99	2	.0825
HFF	.0201	8.08	$RHO = 4$	4.00	3	.2611
HFF_{-1}	.0122	4.91	T	3.92	1	.0476
$RHO1$	.9159	25.83				
SE	.04761					
R^2	.930					
DW	1.76					

Stability Test:

AP	T_1	T_2	λ	Break
1.40	1970:1	1979:4	2.90	1975:3

Estimation period is 1956:1–1993:2.

Two modifications were made in going from equation 5.34 to equation 15 in Table 5.15. First, HF was detrended before being used in 5.34. HF has a negative trend over the sample period, although the trend appears somewhat irregular. To account for the irregular trend, a variable HFS was constructed from peak to peak interpolations of HF, and then $HF - HFS$, which is denoted HFF in the model, was included in equation 15. (The peak quarters used for the interpolation are presented in Table A.6.) HFF is defined by equation 100 in Table A.3. It is the deviation of HF from its peak to peak interpolations. Second, both HFF and HFF_{-1} were included in the equation, which appeared to capture the dynamics better. The equation is estimated under the assumption of a first order autoregressive error term.

The coefficient estimates are significant in equation 15. The equation passes the lags, RHO=4, and T tests. It also passes the stability test. The equation thus seems to be a reasonable approximation to the way that HO is determined, although the estimate of the autoregressive coefficient of the error term is quite high.

Equation 16. WF, average hourly earnings excluding overtime—f

Equation 16 is the wage rate equation. It is in log form. In the final specification, WF was simply taken to be a function of a constant, time, the current value of the price level, and the first four lagged values of the price level and the wage rate. Labor market tightness variables like the unemployment rate were not significant in the equation. The time trend is added to account for trend changes in the wage rate relative to the price level. Its inclusion is important, since it along with some of the lags identifies the price equation, equation 10. Equation 16 is estimated under the assumption of a first order autoregressive error.

Constraints were imposed on the coefficients in the wage equation to ensure that the determination of the real wage implied by equations 10 and 16 is sensible. Let $p = \log PF$ and $w = \log WF$. The relevant parts of the price and wage equations regarding the constraints are

$$p = \beta_1 p_{-1} + \beta_2 w + \dots \tag{5.35}$$

$$\begin{aligned} w = \gamma_1 w_{-1} + \gamma_2 p + \gamma_3 p_{-1} + \gamma_4 w_{-2} + \gamma_5 p_{-2} + \gamma_6 w_{-3} \\ + \gamma_7 p_{-3} + \gamma_8 w_{-4} + \gamma_9 p_{-4} + \dots \end{aligned} \tag{5.36}$$

The implied real wage equation from these two equations should not have $w - p$ as a function of either w or p separately, since one does not expect the real wage to grow simply because the level of w and p are growing. The desired form of the real wage equation is thus

$$\begin{aligned} w - p = \delta_1(w_{-1} - p_{-1}) + \delta_2(w_{-2} - p_{-2}) + \delta_3(w_{-3} - p_{-3}) \\ + \delta_4(w_{-4} - p_{-4}) + \dots \end{aligned} \tag{5.37}$$

which says that the real wage is a function of its own lagged values plus other terms. The real wage in 5.37 is *not* a function of the level of w or p separately. The constraints on the coefficients in equations 5.35 and 5.36 that impose this restriction are:

$$\gamma_3 = [\beta_1/(1 - \beta_2)](1 - \gamma_2) - \gamma_1$$

$$\gamma_5 = -\gamma_4$$

$$\gamma_7 = -\gamma_6$$

$$\gamma_9 = -\gamma_8$$

When using 2SLS or 2SLAD, these constraints were imposed by first estimating the price equation to get estimates of β_1 and β_2 and then using these estimates to impose the constraint on γ_3 in the wage equation. No sequential procedure is needed to impose the constraints when using 3SLS and FIML, since all the equations are estimated together.

The results for equation 16 (using 2SLS) are presented in Table 5.16. The wage rate lagged four times is significant, and this is the reason for the use

Table 5.16
Equation 16
LHS Variable is $\log WF$

Equation			χ^2 Tests			
RHS Variable	Est.	t-stat.	Test	χ^2	df	p-value
$\log WF_{-1}$	.6637	5.92	Real Wage Restr.[b]	6.54	4	.1625
$\log PF$	.2843	2.49	Lags	1.35	1	.2444
$\log WF_{-2}$	-.0038	-0.04	$RHO = 4$	7.34	3	.0619
$\log WF_{-3}$	.1506	1.96	UR_{-1}	0.00	1	.9863
$\log WF_{-4}$	.1757	2.18				
cnst	-.1180	-1.90				
T	.0005	5.05				
$RHO1$	.3269	2.33				
$\log PF_{-1}$ [a]	.0142	–				
$\log PF_{-2}$ [a]	.0038	–				
$\log PF_{-3}$ [a]	-.1506	–				
$\log PF_{-4}$ [a]	-.1757	–				
SE	.00628					
R^2	.999					
DW	1.91					
Stability Test:						
AP	T_1	T_2	λ	Break		
6.13	1970:1	1979:4	2.75	1972:3		

Estimation period is 1954:1–1993:2
[a]Coefficient constrained; see the discussion in the text.
[b]Equation estimated with no restrictions on the coefficients.

of four lags even though lag 2 is not significant. The time trend is highly significant, which is picking up a trend in the real wage.

The χ^2 test results show that the real wage restrictions discussed above are not rejected by the data. The equation also passes the lags and RHO=4 tests. The final χ^2 test in the table has the unemployment rate lagged once added as an explanatory variable, and it is not significant. As noted above, no demand pressure variables were found to be significant in the wage equation. Finally, the equation passes the stability test.

5.5 Other Firm Sector Equations

Equation 18. DF, dividends paid—f

Let π denote after tax profits. If in the long run firms desire to pay out all of their after tax profits in dividends, then one can write $DF^* = \pi$, where DF^*

Table 5.18
Equation 18
LHS Variable is $\Delta \log DF$

Equation			χ^2 Tests			
RHS Variable	Est.	t-stat.	Test	χ^2	df	p-value
$\log \frac{PIEF-TFG-TFS}{DF_{-1}}$	.0251	9.30	Restriction	0.14	1	.7065
			Lags	10.29	2	.0058
			$RHO = 4$	31.66	4	.0000
			T	0.42	1	.5175
			cnst	1.25	1	.2636
SE	.02616					
R^2	.080					
DW	1.48					
Stability Test:						
AP	T_1	T_2	λ	Break		
1.65	1970:1	1979:4	2.75	1976:1		

Estimation period is 1954:1–1993:2.

is the long run desired value of dividends for profit level π. If it is assumed that actual dividends are partially adjusted to desired dividends each period as

$$DF/DF_{-1} = (DF^*/DF_{-1})^{\lambda} \tag{5.38}$$

then the equation to be estimated is

$$\log(DF/DF_{-1}) = \lambda \log(\pi/DF_{-1}) \tag{5.39}$$

Equation 18 in Table 5.18 is the estimated version of equation 5.39. The level of after tax profits in the notation of the model is $PIEF - TFG - TFS$. The estimate of λ is .0251, which implies a fairly slow adjustment of actual to desired dividends.

Because of the assumption that $DF^* = \pi$, the coefficient of $\log(PIEF - TFG - TFS)$ is restricted to be the negative of the coefficient of $\log DF_{-1}$ in equation 18. If instead $DF^* = \pi^{\gamma}$, where γ is not equal to one, then the restriction does not hold. The first test in Table 5.18 is a test of the restriction (i.e., a test that $\gamma = 1$), and the test is passed. The equation fails the lags and RHO=4 tests, and it passes the T and stability tests. The test results also show that the constant term is not significant. The above specification does not call for a constant term, and this is supported by the data.

Table 5.20
Equation 20
LHS Variable is IVA

Equation			χ^2 Tests			
RHS Variable	Est.	t-stat.	Test	χ^2	df	p-value
cnst	-1.2649	-1.75	Lags	4.05	2	.1316
$(PX - PX_{-1})V_{-1}$	-.2757	-3.04	$RHO = 4$	17.49	3	.0006
$RHO1$	.7687	14.37	T	0.33	1	.5664
SE	1.80039					
R^2	.711					
DW	2.03					
Stability Test:						
AP	T_1	T_2	λ	Break		
3.59	1970:1	1979:4	2.75	1974:4		

Estimation period is 1954:1–1993:2.

Equation 20. IVA, inventory valuation adjustment

In theory $IVA = -(P - P_{-1})V_{-1}$, where P is the price of the good and V is the stock of inventories of the good. Equation 20 in Table 5.20 is meant to approximate this. IVA is regressed on a constant and $(PX - PX_{-1})V_{-1}$, where PX is the price deflator for the sales of the firm sector. The equation is estimated under the assumption of a first order autoregressive error term. As an approximation, the equation seems fairly good. It passes all but RHO=4 test, including the stability test.

Equation 21. CCF, capital consumption—f

In practice capital consumption allowances of a firm depend on tax laws and on current and past values of its investment. Equation 21 in Table 5.21 is an attempt to approximate this for the firm sector. $PIK \cdot IKF$ is the current value of investment. The use of the lagged dependent variable in the equation is meant to approximate the dependence of capital consumption allowances on past values of investment. This specification implies that the lag structure is geometrically declining. The restriction is also imposed that the sum of the lag coefficients is one, which means that capital consumption allowances are assumed to be taken on all investment in the long run.

There are two periods, 1981:1–1982:4 and 1983:1–1983:4, in which CCF is noticeably higher than would be predicted by the equation with only $\log[(PIK \cdot IKF)/CCF_{-1}]$ in it, and two dummy variables, $D811824$ and

Table 5.21
Equation 21
LHS Variable is $\Delta \log CCF$

Equation			χ^2 Tests			
RHS Variable	Est.	t-stat.	Test	χ^2	df	p-value
$\log[(PIK \cdot IKF)/CCF_{-1}]$	.0568	16.43	Restriction	4.22	1	.0398
$D811824$	.0174	3.15	Lags	5.89	2	.0525
$D831834$	.0346	4.59	$RHO = 4$	6.02	4	.1976
			T	4.50	1	.0339
			cnst	3.04	1	.0812
SE	.01505					
R^2	.271					
DW	1.87					

Estimation period is 1954:1–1993:2.

$D831834$, have been added to the equation to account for this. This is, of course, a crude procedure, but the equation itself is only a rough approximation to the way that capital consumption allowances are actually determined each period. Tax law changes have effects on CCF that are not captured in the equation.

Regarding the use of the two dummy variables, if CCF is larger than usual in the two subperiods, which the coefficient estimates for the two dummy variables suggest, then one might expect CCF to be lower at some later point (since capital consumption allowances can be taken on only 100 percent of investment in the long run). No attempt, however, was made to try to account for this in equation 21.

The coefficient estimate of .0568 in Table 5.21 says that capital consumption allowances are taken on 5.68 percent of new investment in the current quarter, then 5.36 percent [$.0568(1 - .0568)$] of this investment in the next quarter, then 5.05 percent [$.0568(1 - .0568)^2$] in the next quarter, and so on.

The first χ^2 test in Table 5.21 is a test of the restriction that the sum of the lag coefficients is one. This is done by adding $\log CCF_{-1}$ to the equation. The results show that the restriction is not rejected at the one percent level. The equation passes the lags, RHO=4, and T tests. The results of the last χ^2 test in the table show that the constant term is not significant in the equation. This is as expected since the above specification does not call for a constant term. The stability test was not performed because of the use of the dummy variables.

Table 5.22
Equation 22
LHS Variable is BO/BR

Equation			χ^2 Tests			
RHS Variable	Est.	t-stat.	Test	χ^2	df	p-value
cnst	.0034	0.99	Lags	6.01	3	.1111
$(BO/BR)_{-1}$	.3170	4.23	$RHO = 4$	26.69	4	.0000
RS	.0062	2.05	T	0.80	1	.3698
RD	-.0039	-1.37				
SE	.01989					
R^2	.332					
DW	2.06					
Stability Test:						
AP	T_1	T_2	λ	Break		
6.19	1970:1	1979:4	2.75	1972:4		

Estimation period is 1954:1–1993:2.

5.6 Financial Sector Equations

The stochastic equations for the financial sector consist of an equation explaining member bank borrowing from the Federal Reserve, two term structure equations, and an equation explaining the change in stock prices.

Equation 22. BO, bank borrowing from the Fed

The variable BO/BR is the ratio of borrowed reserves to total reserves. This ratio is assumed to be a positive function of the three month Treasury bill rate, RS, and a negative function of the discount rate, RD. The estimated equation also includes a constant term and the lagged dependent variable.

The coefficient estimates of RS and RD in Table 5.22 are positive and negative, respectively, as expected. The equation passes the lags, T, and stability tests, and it fails the RHO=4 test.

Equation 23. RB, bond rate; Equation 24. RM, mortgage rate

The expectations theory of the term structure of interest rates states that long term rates are a function of the current and expected future short term rates. The two long term interest rates in the model are the bond rate, RB, and the mortgage rate, RM. These rates are assumed to be determined according to the expectations theory, where the current and past values of the short term

Table 5.23
Equation 23
LHS Variable is $RB - RS_{-2}$

Equation			χ^2 Tests			
RHS Variable	Est.	t-stat.	Test	χ^2	df	p-value
cnst	.2438	5.34	Restriction	1.89	1	.1688
$RB_{-1} - RS_{-2}$	.8813	42.95	Lags	2.51	2	.2856
$RS - RS_{-2}$	.2963	8.78	$RHO = 4$	7.93	3	.0474
$RS_{-1} - RS_{-2}$	-.2180	-5.04	T	4.62	1	.0316
$RHO1$	.2019	2.47	Leads +1	0.33	1	.5658
			Leads +4	2.23	4	.6939
			p_4^e	3.33	1	.0682
			p_8^e	3.85	1	.0499
SE	.25359					
R^2	.958					
DW	2.04					
Stability Test:						
AP	T_1	T_2	λ	Break		
6.25	1970:1	1979:4	2.75	1979:4		

Estimation period is 1954:1–1993:2.

interest rate (the three month bill rate, RS) are used as proxies for expected future values. Equations 23 and 24 are the two estimated equations. The lagged dependent variable is used in each of these equations, which implies a fairly complicated lag structure relating each long term rate to the past values of the short term rate. In addition, a constraint has been imposed on the coefficient estimates. The sum of the coefficients of the current and lagged values of the short term rate has been constrained to be equal to one minus the coefficient of the lagged long term rate. This means that, for example, a sustained one percentage point increase in the short term rate eventually results in a one percentage point increase in the long term rate. (This restriction is imposed by subtracting RS_{-2} from each of the other interest rates in the equations.) Equation 23 (but not 24) is estimated under the assumption of a first order autoregressive error term.

The results for equations 23 and 24 are presented in Tables 5.23 and 5.24, respectively. The short rates are significant except for RS_{-1} in equation 24. The test results show that the coefficient restriction is not rejected for either equation. Both equations pass the lags, RHO=4, T, and stability tests. The results for both term structure equations are thus strong. My experience with these equations over the years is that they are quite stable and reliable. During most periods they provide a very accurate link from short rates to long rates.

Table 5.24
Equation 24
LHS Variable is $RM - RS_{-2}$

Equation			χ^2 Tests			
RHS Variable	Est.	t-stat.	Test	χ^2	df	p-value
cnst	.4749	6.03	Restriction	2.71	1	.0995
$RM_{-1} - RS_{-2}$	.8418	33.24	Lags	2.92	2	.2322
$RS - RS_{-2}$	.2597	5.72	$RHO = 4$	4.79	4	.3093
$RS_{-1} - RS_{-2}$	-.0169	-0.27	T	2.65	1	.1039
			Leads +1	2.12	1	.1457
			Leads +4	12.70	4	.0128
			Leads +8	7.20	2	.0273
			p_4^e	1.77	1	.1835
			p_8^e	2.29	1	.1298
SE	.34387					
R^2	.897					
DW	2.02					
Stability Test:						
AP	T_1	T_2	λ	Break		
5.72	1970:1	1979:4	2.75	1979:4		

Estimation period is 1954:1–1993:2.

The variable for which led values were tried was the short term interest rate (RS), and the χ^2 tests show that the led values are not significant at the one percent level.[16] This is thus at least slight evidence against the bond market having rational expectations with respect to the short term interest rate. The test results also show that the inflation expectations variables, p_4^e and p_8^e, are not significant in the equations.

Equation 25. CG, capital gains or losses on corporate stocks held by h

The variable CG is the change in the market value of stocks held by the household sector. In the theoretical model the aggregate value of stocks is determined as the present discounted value of expected future after tax cash flow, the discount rates being the current and expected future short term interest rates. The theoretical model thus implies that CG should be a function of changes in expected future after tax cash flow and of changes in the current and expected future interest rates. In the empirical work the change in the bond rate, ΔRB, was used as a proxy for changes in expected future interest rates, and the change in after tax cash flow, $\Delta(CF - TFG - TFS)$, was used

[16] Collinearity problems prevented Leads +8 from being calculated for equation 23.

Table 5.25
Equation 25
LHS Variable is CG

Equation			χ^2 Tests			
RHS Variable	Est.	t-stat.	Test	χ^2	df	p-value
cnst	23.1530	3.22	Lags	2.26	3	.5204
ΔRB	-68.1158	-2.47	$RHO = 4$	19.63	4	.0006
$\Delta(CF - TFG - TFS)$	1.5451	0.60	T	4.79	1	.0286
			Leads +1	0.12	2	.9413
			Leads +4	9.31	8	.3172
			Leads +8	13.92	4	.0076
			ΔRS	0.73	1	.3942
SE	89.27602					
R^2	.147					
DW	1.96					
Stability Test:						
AP	T_1	T_2	λ	Break		
2.22	1970:1	1979:4	2.75	1979:1		

Estimation period is 1954:1–1993:2.

as a proxy for changes in expected future after tax cash flow. Equation 25 in Table 5.25 is the estimated equation.

The fit of equation 25 is not very good, and the cash flow variable is not significant. The change in the bond rate is significant, however, which provides some link from interest rates to stock prices in the model. The equation passes the lags, T, and stability tests, and it fails the RHO=4 test. The variables for which led values were tried are the change in the bond rate and the change in after tax cash flow. The values led one and four quarters are not significant, but the values led eight quarters are. This is thus slight evidence in favor of there being rational expectations in the stock market. For the final χ^2 test ΔRS, the change in the short term rate, was added under the view that it might also be a proxy for expected future interest rate changes, and it is not significant.

5.7 The Import Equation

Equation 27. IM, Imports

The import equation is in per capita terms and is in log form. The explanatory variables are 1) per capita real disposable income, 2) the private, nonfarm price deflator (a price deflator for domestically produced goods) relative to the import price deflator, 3) the long term after tax interest rate lagged one

Table 5.27
Equation 27
LHS Variable is $\log(IM/POP)$

Equation			χ^2 Tests			
RHS Variable	Est.	t-stat.	Test	χ^2	df	p-value
cnst	-.4533	-4.00	Lags	9.21	4	.0561
$\log(IM/POP)_{-1}$	.8716	26.77	$RHO = 4$	24.08	4	.0001
$\log[YD/(POP \cdot PH)]$	.3172	4.19	T	9.12	1	.0025
$\log(PF/PIM)$	.0365	1.50	Leads +1	0.58	1	.4447
RMA_{-1}	-.0027	-1.32	Leads +4	10.90	4	.0277
$D691$	-.1183	-3.64	Leads +8	0.17	2	.9171
$D692$	.1478	4.52	p_4^e	22.44	1	.0000
$D714$	-.0871	-2.68	p_8^e	14.99	1	.0001
$D721$	.0943	2.91	$\log PF$	24.49	1	.0000
			$\log[(X - FA)/POP]$	2.94	1	.0863
			Other[a]	30.55	4	.0000
			Spread	35.07	4	.0000
SE	.03204					
R^2	.995					
DW	1.80					

Estimation period is 1954:1–1993:2.
[a] $\log(AA/POP)_{-1}$, $\log(WA/PH)$, Z, $\log[YNL/(POP \cdot PH)]$.

quarter, 4) the lagged dependent variable, and 5) four dummy variables to account for two dock strikes.

The results are in Table 5.27. The short run income elasticity of imports is .3172, and the long run elasticity is 2.47 [$.3127/(1 - .8716)$], both fairly high. The coefficient estimate for the relative price term is positive as expected, although it is not significant. The coefficient estimate for the long term interest rate is negative as expected, but it also is not significant.

Many χ^2 tests were performed for the import equation. It passes the lags test, but fails the RHO=4 and T tests. The variable for which led values were tried is disposable income, and the led values are not significant. The inflationary expectations variables, p_4^e and p_8^e, are highly significant, but (although not shown in the table) their coefficient estimates are of the wrong expected sign.

The next test in the table adds $\log PF$ to the equation, which is a test of the restriction that the coefficient of $\log PF$ is equal to the negative of the coefficient of $\log PIM$. The $\log PF$ variable is highly significant, and so the restriction is rejected. Although not shown in the table, when $\log PF$ is added to the equation, the coefficient for $\log PIM$ is $-.0890$ and the coefficient for $\log PF$ is .1844. The results thus suggest that the level of imports responds more to the domestic price deflator than to the import price deflator. As was the case for equation 6, this is contrary to what one expects from theory. It does not seem sensible that in the long run the level of imports rises simply from an overall rise in prices. Therefore, the relative price constraint was imposed on the equation, even though it is strongly rejected by the data.

The next test adds the level of per capita nonfarm firm sales—log$[(X - FA)/POP]$—to the equation to see if it better explains imports than does disposable income. The χ^2 values is not significant, and so on this score the sales variable does not have independent explanatory power. On the other hand (not shown in the table), the t-statistic on the sales variable was higher (1.77) than the t-statistic on the disposable income variable (0.35). The sales variable thus dominates the disposable income variable in this sense. The consequences of using the sales variable in place of the disposable income variable are examined in Section 11.3.4.

The "other" variables that were added for the next test, which are asset, real wage, labor constraint, and nonlabor income variables, are highly significant, but (not shown in the table) they all have coefficient estimates of the wrong expected sign. Finally, the spread values are highly significant. The stability test was not performed for the import equation because of the use of the dummy variables.

Experimenting with the import equation reveals that it does much better in the tests if the relative price restriction is not imposed. In other words, when log PF is added, the equation does much better. So in summary, the import equation passes the lags and leads tests, but it fails the others. It is clearly an equation in which future research is needed.

5.8 Government Sector Equations

There is one stochastic equation for the state and local government sector, explaining unemployment insurance benefits, UB. There are two stochastic equations for the federal government sector, one explaining interest payments, $INTG$, and one explaining the three month Treasury bill rate, RS. The equation explaining RS is interpreted as an interest rate reaction function of the Federal Reserve. The equations for UB and RS are discussed in this section, and the equation for $INTG$ is discussed in the next section.

Table 5.28
Equation 28
LHS Variable is $\log UB$

Equation			χ^2 Tests			
RHS Variable	Est.	t-stat.	Test	χ^2	df	p-value
cnst	.1220	0.27	Lags	8.68	3	.0339
$\log UB_{-1}$	.2349	3.57	$RHO = 4$	5.89	3	.1168
$\log U$	1.2565	10.71	T	1.84	1	.1751
$\log WF$	.3459	4.57				
$RHO1$	.8164	14.71				
SE	.06586					
R^2	.995					
DW	2.19					
Stability Test:						
AP	T_1	T_2	λ	Break		
12.25*	1970:1	1979:4	2.75	1975:1		

Estimation period is 1954:1–1993:2.

Equation 28. UB, unemployment insurance benefits

Equation 28 is in log form and contains as explanatory variables the level of unemployment, the nominal wage rate, and the lagged dependent variable. The inclusion of the nominal wage rate is designed to pick up the effects of increases in wages and prices on legislated benefits per unemployed worker. The equation is estimated under the assumption of a first order autoregressive error term.

The results in Table 5.28 show that the coefficient estimates are significant except for the estimate of the constant term. The equation passes the lags, RHO=4, and T tests, and it fails the stability test.

Equation 30. RS, three month Treasury bill rate

A key question in any macro model is what one assumes about monetary policy. In the theoretical model monetary policy is determined by an interest rate reaction function, and in the empirical work an equation like this is estimated. This equation is interpreted as an equation explaining the behavior of the Federal Reserve (Fed).

In one respect, trying to explain Fed behavior is more difficult than, say, trying to explain the behavior of the household or firm sectors. Since the Fed is run by a relatively small number of people, there can be fairly abrupt changes in behavior if the people with influence change their minds or are replaced by others with different views. Abrupt changes are less likely to happen for the household and firm sectors because of the large number of decision makers in each sector. Having said this, however, only one abrupt change in behavior appeared evident in the data, which was between 1979:4 and 1982:3, and, as will be seen, even this change appears capable of being modeled.

Equation 30 is the estimated interest rate reaction function It has on the left hand side RS. This treatment is based on the assumption that the Fed has a target bill rate each quarter and achieves this target through manipulation of its policy instruments. The right hand side variables in this equation are variables that seem likely to affect the target rate. The variables that were chosen are 1) the rate of inflation, 2) the degree of labor market tightness, 3) the percentage change in real GDP, and 4) the percentage change in the money supply lagged one quarter. What seemed to happen between 1979:4 and 1982:3 was that the size of the coefficient of the lagged money supply growth increased substantially. This was modeled by adding the variable $D794823 \cdot PCM1_{-1}$ to the equation, where $D794823$ is a dummy variable that is 1 between 1979:4 and 1982:3 and 0 otherwise. The estimated equation also includes the lagged dependent variable and two lagged bill rate changes to pick up the dynamics.

The signs of the coefficient estimates in Table 5.30 are as expected, and the equation passes all of the tests. The results thus seem good for this equation. The stability test was not run because of the use of the dummy variable. The variables for which led values were tried are the inflation variable, the labor market tightness variable, and the percentage change in real GDP, and the led values are not significant.

Equation 30 is a "leaning against the wind" equation in the sense that the Fed is predicted to allow the bill rate to rise in response to increases in inflation, labor market tightness, real growth, and money supply growth. As just noted, the results show that the weight given to money supply growth in the setting of the bill rate target was much greater in the 1979:4–1982:3 period than either before or after.

Table 5.30
Equation 30
LHS Variable is RS

Equation			χ^2 Tests			
RHS Variable	Est.	t-stat.	Test	χ^2	df	p-value
cnst	-15.5116	-5.87	Lags	9.34	6	.1552
RS_{-1}	.8923	47.76	$RHO = 4$	3.01	4	.5565
$100[(PD/PD_{-1})^4 - 1]$	.0684	3.50	T	0.07	1	.7861
JJS	15.7411	5.91	Leads +1	1.96	3	.5806
$PCGDPR$	.0777	5.14	Leads +4	10.07	12	.6098
$PCM1_{-1}$	.0196	3.08	Leads +8	5.28	6	.5085
$D794823 \cdot PCM1_{-1}$	.2245	9.26	p_4^e	0.44	1	.5074
ΔRS_{-1}	.2033	3.36	p_8^e	0.43	1	.5096
ΔRS_{-2}	-.2964	-5.25				
SE	.50945					
R^2	.970					
DW	2.01					

Estimation period is 1954:1–1993:2.

5.9 Interest Payments Equations

Equation 19. $INTF$, interest payments—f; Equation 29. $INTG$, interest payments—g

$INTF$ is the level of net interest payments of the firm sector, and $INTG$ is the same for the federal government. Data on both of these variables are NIPA data. AF is the level of net financial assets of the firm sector, and AG is the same for the federal government. Data on both of these variables are FFA data. AF and AG are negative because the firm sector and the federal government are net debtors, and they consist of both short term and long term securities.

The current level of interest payments depends on the amount of existing securities issued at each date in the past and on the relevant interest rate prevailing at each date. The link from AF to $INTF$ (and from AG to $INTG$) is thus complicated. It depends on past issues and the interest rates paid on these issues. A number of approximations have to be made in trying to model this link, and the following is a discussion of the procedure used here.

Consider the federal government variables first. The difference $|AG| - |AG_{-1}|$ is the net change in the value of securities of the federal government between the end of the previous quarter and the end of the current quarter. The value of new securities issued by the federal government during the current quarter is this difference *plus* the value of old securities that came due during the current quarter. It is first assumed that the government issues two kinds of securities, a "short term" security, where the short term is defined to be one quarter, and a "long term" security, which is taken to be of length k quarters, where k is to be estimated. It is next assumed that λ percent of the net change in the value of securities in a quarter (i.e., of $|AG| - |AG_{-1}|$) consist of long term issues, with the rest consisting of short term issues. In addition, it is assumed that the long term securities that expire during the quarter are replaced with new long term securities. Let BG denote the value of long term securities issued during the current quarter by the federal government. Then the above assumptions imply that :

$$BG = \lambda(|AG| - |AG_{-1}|) + BG_{-k} \tag{5.40}$$

λ is assumed to remain constant over time.

It is next assumed that the government pays an interest rate RS on its short term securities, where RS is the three month Treasury bill rate, and an interest rate $RB - \eta$ on its long term securities, where RB is the AAA bond rate and η is a constant parameter to be estimated. η is subtracted from RB because the government generally pays less than the AAA bond rate on its bonds. Given these assumptions, the interest payments of the federal government are:[17]

$$INTG = \sum_{i=-k}^{0} \frac{1}{400}(RB_i - \eta)BG_i + \frac{1}{400}RS(1 - \lambda)\,|AG| \tag{5.41}$$

The interest rates are divided by 400 in this equation because they are at annual rates in percentage points and they need to be at quarterly rates in percents. ($INTG$ is at a quarterly rate.) Given the above assumptions, the value of short term securities is $(1 - \lambda)\,|AG|$, and so RS multiplies this. The other securities have the relevant bond rate multiplying them. For example, BG_{-1} is the value of long term securities issued last quarter, and the relevant interest rate for these securities is $RB_{-1} - \eta$. $(RB_{-1} - \eta)BG_{-1}$ is thus part of $INTG$ until the securities expire after k quarters.

Using equations 5.40 and 5.41, the aim of the estimation work is to find values of k, λ, η that lead to a good fit, i.e., that lead to the predicted values of $INTG$ from equation 5.41 being close to the actual values. This work takes as given the actual values of RS, RB, and AG. The estimation period was 1952:1–1993:2, which is the period for which data on AG exist. The

[17] In the notation in this equation BG_0 is the same as BG. Similarly, for the firm sector BF_0 is the same as BF.

estimation procedure was as follows. First, given a value for k and a value for λ, the value of BG for 1952:1 was taken to be equal to $(1/k)\lambda\,|AG_{1952:1}|$. In addition, the $k-1$ values of BG before 1952:1 were taken to be equal to this value. Given these values, values of BG for 1952:2 through 1993:2 can be generated using equation 5.40 and the given values of k and λ. Second, if values of RB for the computations in equation 5.41 were needed before 1947:1, which is the first quarter for which data on RB exist, they were taken to be equal to the 1947:1 value. (Values before 1947:1 are needed if k is greater than 20 quarters.) Third, given the values of k and λ and the above computations and given a value for η, equation 5.41 can be used to obtain predicted values of $INTG$ for the 1952:1–1993:2 period, from which a root mean squared error (RMSE) can be computed. The entire procedure can then be repeated for a different set of values of k, λ, and η, and another RMSE computed.

A program was written to search over different sets of values of k, λ, and η and print out the RMSE for each set. The set that gave the smallest RMSE was $k = 10$, $\lambda = .72$, and $\eta = .5$, which produced a RMSE of .836. The objective function was, however, fairly flat over a number of values, and the set that was chosen for the model is $k = 16$, $\lambda = .66$, and $\eta = .4$, which produced a RMSE of .964. For the first set of values, a value of k of only 10 quarters seemed small, and so k was increased somewhat for the final set even though this resulted in some increase in RMSE.

Equation 5.40 above is equation 56 in the model, and equation 5.41 is equation 29. These two equations are presented in Table A.3 in Appendix A using the values of k, λ, and η chosen.

A similar procedure was followed for the interest payments of the firm sector, $INTF$, with two differences. First, η was taken to be zero, which means that firms are assumed to pay interest rate RB on their long term securities. Second, between 1981:3 and 1991:2 $INTF$ grew faster than seemed consistent with the values of the interest rates and AF. No values of k and λ could be found that gave sensible fits for this period. To account for this unexplained growth, a dummy variable, TI, was constructed that was 0 through 1981:2, 1 in 1981:3, 2 in 1981:4, . . . , 40 in 1991:2, and 40 after 1991:2. The term γTI was then added to the equivalent of equation 5.41 for the firm sector, where γ is a coefficient to be estimated. The searching procedure for the firm sector thus consisted in searching over values of k, λ, and γ.

The set of values that gave the smallest RMSE was $k = 52$, $\lambda = .43$, and $\gamma = .40$, which produced a RMSE of 1.063. Again, the objective function was fairly flat over a number of values, and the set that was chosen for the model is $k = 40$, $\lambda = .40$, and $\gamma = .41$, which produced a RMSE of 1.144. A value of k of 52 quarters seemed large, and so k was decreased somewhat for the final set even though this resulted in some increase in RMSE.

Equation 5.40 above for the firm sector is equation 55 in the model, and equation 5.41 for the firm sector is equation 19. These two equations are also presented in Table A.3 using the values of k, λ, and γ chosen.

Although the above specification is obviously only a rough approximation of the links from interest rates, AG, and AF to interest payments, it does tie changes in these variables to changes in interest payments in a way that is not likely to deviate substantially in the long run from the true relationship. In other words, interest payments change as interest rates change and as AG and AF change in a way that seems unlikely to drift too far from the truth.[18]

It will be seen in Section 11.7 that equation 29 has an effect on the effectiveness of monetary policy in the model. As the size of the federal government debt ($|AG|$) increases, the change in interest payments of the federal government for a given change in interest rates increases in absolute value. Since households hold much of the debt, the change in interest revenue of the household sector for a given change in interest rates is getting larger in absolute value as the size of the debt increases. This income effect on households is thus increasing over time and, as will be seen, is now offsetting more of the substitution effect of a change in interest rates than it did earlier.

Finally, although equations 19 and 29 have not been estimated in a usual way, they are still stochastic equations in the sense that the predicted values from the equations do not in general equal the actual values. (Equations 55 and 56 are, however, identities because BF and BG have simply been constructed using the equations.) With respect to the notation for the model in equation 4.1 in Chapter 4, both equations 19 and 29 have in general nonzero values of u_{it}. For 3SLS and FIML estimation and the stochastic simulation work below, where an estimate of the covariance matrix of all the errors in the model is needed, the error terms for equations 19 and 29 have been used after adjusting for heteroskedasticity. The variance of the error term in equation 19

[18]In previous specifications of equations 19 and 29 the interest payments variables were regressed on interest rates and the value of securities. The equations were usually in log form and usually included the lagged dependent variable. For example, one version of equation 29 had log $INTG$ regressed on a constant, log $INTG_{-1}$, $\log(-AG)$, log RS, and log RB. These types of equations provide a slightly better fit than the procedure discussed above, but they have poor dynamic properties. With no restrictions imposed, the predicted interest payments from the equations tend to drift away from sensible values, sensible in the sense of being consistent with the predicted values of interest rates and security issues.

was assumed to be proportional to $(AF + 10)^2$, and the variance of the error term in equation 29 was assumed to be proportional to AG^2. This means that equation 19 is divided through by $|AF + 10|$ and that equation 29 is divided through by $|AG|$ before computing the error terms to be used in estimating the covariance matrix.[19] This means that uncertainty from equations 19 and 29 is taken into account in 3SLS and FIML estimation and in the stochastic simulation work even though they are not estimated in a traditional way.

5.10 Additional Comments

The following is a discussion of some of the results that pertain to sets of equations.

1. The age variables are jointly significant at the five percent level in three of the four household expenditure equations, and the sign patterns are generally as expected. This is thus evidence that the U.S. age distribution has an effect on U.S. macroeconomic equations.[20]

2. The wealth variable is significant in two of the four household expenditure equations. Changes in stock prices thus affect expenditures in the model through their effect on household wealth.

3. At least some of the led values are significant in three of the four household expenditure equations and in one of the four labor supply equation. They are not significant at the one percent level in any of the other equations in which they were tried except for Leads +4 in the employment equation 13 and for Leads +8 in the capital gains equation 25. They are significant at the five percent level in eight other cases: 1) Leads +4 in equation 5, 2) Leads +1 in equation 11, 3) Leads +8 in equation 12, 4) Leads +1 in equation 13, 5) Leads +8 in equation 13, 6) Leads +4 in equation 24, 7) Leads +8 in equation 24, and 8) Leads +4 in equation 27. There is thus some evidence that the rational expectations assumption is helpful in explaining household behavior, but only slight evidence

[19] AF is close to zero for the first few quarters of the estimation period, and this is the reason for adding 10 to it.

[20] This same conclusion was also reached in Fair and Dominguez (1991). In Fair and Dominguez (1991), contrary to the case here, the age variables were also significant in the equation explaining IHH.

that it is helpful in explaining other behavior.[21] As noted previously, the economic consequences of the rational expectations assumption are examined in Section 11.6.

4. The evidence suggests that nominal interest rates rather than real interest rates affect household expenditures and imports. The inflation expectations variables are not significant in the four expenditure equations, and their coefficient estimates have the wrong expected sign in the import equation.

5. In all three of the money demand equations the nominal adjustment specification dominates the real adjustment specification. The nominal adjustment specification is equation 5.12.

6. All but 3 of 28 equations passed the lags test; all but 3 of 23 passed the T test; 18 of 28 passed the RHO=4 test; and 14 of 23 passed the stability test. The overall results thus suggest that the specifications of the equations are fairly accurate regarding dynamic and trend effects, but less so regarding the serial correlation properties of the error terms and stability. Given the number of equations that failed the stability test, it may be useful in future work to break some of the estimation periods in parts, but in general it seems that more observations are needed before this might be a sensible strategy.

7. The labor constraint variable (Z) is significant or close to significant in the four labor supply equations, suggesting that there is a discouraged worker effect in operation.

8. The excess labor variable is significant in the employment and hours equations, 13 and 14, but the excess capital variable is not significant in the investment equation 12.

9. Either the short term or long term interest rate is significant in the four household expenditure equations. Also, interest income is part of disposable personal income (YD), which is significant or nearly significant in the four equations. Therefore, an increase in interest rates has a negative effect on household expenditures through the interest rate variables

[21] This general conclusion is consistent with the results in Fair (1993b), Table 1, where led values were significant in three of the four household expenditure equations and in two of the four labor supply equations, but in almost none of the other equations.

and a positive effect through the disposable personal income variable. More will be said about this in Chapter 11.

10. There is a fairly small use of dummy variables in the equations. One appears in equations 13 and 14 to pick up a structural break; two appear in equation 21 to pick up an unexplained increase in capital consumption; four appear in equation 27 to pick up the effects of two dock strikes; one appears in equation 30 to pick up a shift of Fed behavior between 1979:4 and 1982:3; and one appears in equation 19 to pick up an unexplained increase in interest payments of the firm sector.

11. The level form of the price equation is not rejected, and the change form is strongly rejected. This result is consistent with the results of estimating highly disaggregate price equations in Fair (1993a), where the level form gave somewhat better results. The acceptance of the level form over the change form has important implications for the long run properties of the model. A permanent change in demand in the model does not have a permanent effect on the rate of inflation, only on the price level. The real wage constraint in the wage equation is not rejected, and so the data suggest that the real wage rate is not a function of the level of prices or nominal wage rates, which is as expected. On the other hand, the data have little to say about the behavior of prices in very high activity periods. One would expect there to be important nonlinearities in the behavior of prices as the economy moves into very high activity levels (and very low levels of unemployment), but this effect cannot be picked up in the data.

12. The spread values are highly significant in the consumption of durables and import equations. They are significant at the five but not one percent level in the consumption of nondurables equation. They are not significant in the housing investment equation 4, the production equation 11, and the investment equation 12. The evidence is thus mixed. If there is an effect of the spread values on the economy, it appears to come through the effects on household behavior rather than on firm behavior. This is not necessarily what one would expect from the discussion in Friedman and Kuttner (1993), where the stress is on the effects of the spread on investment behavior. This is perhaps an area for future research.

13. Four of the most serious negative test results are the highly significant time trends in equations 1 and 3, the significance of $\log PH$ in equation

6, and the significance of log PF in equation 27. Future work is needed on these equations.