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THE USE OF OPTIMAL CONTROL TECHNIQUES TO MEASURE ECONOMIC PERFORMANCE*

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1. INTRODUCTION

It is a common practice in political discussions in the United States to hold a presidential administration accountable for the state of the economy that existed during its four-year period in office. Administrations are generally blamed for high unemployment rates, low real growth, and high inflation rates during their years in office and praised for the opposite. Although at first glance this seems a natural way of evaluating the economic performances of administrations, there are at least two serious problems with it. The first is that this kind of evaluation does not take into account possible differences in the degree of difficulty of controlling the economy in different periods. The economy may be more difficult to control for one administration than for another either because of more unfavorable values of noncontrolled exogenous variables for one than for another or because of a more unfavorable initial state of the economy for one than for another (or both).

The second problem with evaluating the economic performance of an administration on the basis of the state of the economy that existed during its four-year period in office is that it ignores the effects of an administration's policies on the state of the economy beyond the four-year period. If, for example, an administration strongly stimulates the economy in the year of the presidential election, in, say, the belief that this might improve the chances of its party staying in power, most of the inflationary effects of this policy may not be felt until the next four-year period. Any evaluation of performance that was concerned only with the administration's four-year period in office would not, of course, pick up these effects.

The purpose of this paper is to propose a measure of economic performance that takes into account both of these problems. The measure is based on the solutions of optimal control problems. It requires that a welfare or loss function be postulated and that the economy be represented by an econometric model. The welfare or loss function must be additive across time.

The measure is presented and discussed in Section 2, and then an illustration of its use is presented in Section 3. The illustration in Section 3 consists of ap-

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proximate estimates of the measure for two different loss functions for the five administrations between Eisenhower-I and Nixon-I. The main conclusions of this study are summarized in Section 4.

2. THE MEASURE OF PERFORMANCE

2.1. *The Definition of the Measure.* The measure is easy to define. Consider two consecutive administrations, called 1 and 2. The measure of performance of administration 1, denoted as M, is defined to be the following (low values of M are good):

(1) M = expected actual loss in administration 1's four-year period in office
− expected loss in the four-year period if administration 1 had behaved optimally
+ expected loss in the next four-year period given that administration 1 did not behave optimally, but assuming that administration 2 did
− expected loss in the next four-year period if both administrations 1 and 2 had behaved optimally.

The first two terms in (1) measure the expected loss that could have been avoided during administration 1's four-year period in office had it behaved optimally. The other two terms measure the potential expected loss to administration 2 from the fact that administration 1 did not behave optimally.

M takes into account both of the problems mentioned in the Introduction. If the economy is difficult to control for administration 1, then the second term in (1) will be large, which will then offset more than otherwise a large value of expected actual loss. The last two terms in (1) measure the effects of administration 1's policies on the economy beyond its own four-year period in office, these effects being measured under the assumption that the next administration behaves optimally.

Although the emphasis in this paper is on the four-year periods of presidential administrations, it should be clear that the definition of M can be easily modified to fit other periods. If, for example, one were concerned with evaluating only the first-quarter performance in a T-quarter horizon, then the first two terms in (1) would refer to the first quarter and the other two terms would refer to quarters 2 through T. In this case, "administration 1" would refer to the policy makers in quarter 1 and "administration 2" would refer to the policy makers in quarters 2 through T. Along this same line, it should be noted that although the definition of M ignores the potential expected loss to administrations 3 and beyond from the fact that administration 1 did not behave optimally, it can be easily modified to incorporate this loss. This can be done by just reinterpreting administration 2 in (1) to be all administrations beyond 1 that are within, say, a T-quarter horizon and replacing "next four-year period" in the last two terms in (1) by "quarters 17 through T." Since there is no important loss of generality from ignoring these possible modifications of M, the following discussion will be concerned only with M as defined in (1).

2.2. *The Computation of M.* In order to compute the expected optimal losses in the definition of M, it is necessary to solve some stochastic optimal control problems. This requires that a model of the economy be specified and that a welfare or loss function be postulated. The model will be written as:

$$\phi_g(y_t, q_{t-1}, z_t, x_t, \beta_g) = u_{gt}, \qquad (g = 1,\ldots, G), \tag{2}$$

where y_t is a G-component vector of the endogenous variables, q_{t-1} is a vector of all lagged endogenous variables (including any endogenous variables lagged more than one period), z_t is a vector of the non-control exogenous variables, x_t is a vector of the control variables, β_g is a vector of unknown coefficients in equation g, and u_{gt} is an error term. If equation g is an identity, then u_{gt} is zero for all t. The model is assumed to be quarterly. It will be useful for purposes of the following discussion to let θ_t represent the information on the coefficients and the non-control exogenous variables that is available as of the end of quarter t.

The value of loss in quarter t, denoted as L_t, is assumed to be a function of y_t, q_{t-1}, z_t, and x_t:

$$L_t = h(y_t, q_{t-1}, z_t, x_t). \tag{3}$$

The length of the control period for each problem is assumed to be T quarters, and the loss function for each problem is assumed to be the sum of L_t over the relevant T quarters. For a problem solved at the beginning of quarter i, the loss function, denoted as $\mathscr{L}_i$, is thus

$$\mathscr{L}_i = \sum_{t=i}^{T+i-1} L_t. \tag{4}$$

Assume now that quarter 1 is the first quarter of administration 1's four-year term. The stochastic control problem facing the administration at the beginning of quarter 1 is to choose the values of $x_1,\ldots, x_T$ so as to minimize the expected value of $\mathscr{L}_1$ in (4), given the model, q_0, and θ_0. For nonlinear models, only approximate solutions to this problem are available, and so the best that an administration can be expected to do if the model is nonlinear is to obtain an approximate solution. A number of approximate solutions have been proposed in the literature, but it is unnecessary for present purposes to consider this question in any detail.[2] An administration can merely be defined here as behaving optimally if it obtains one of the proposed approximate solutions. To avoid tedious repe-

[2] If it is assumed, as in this paper, that an administration can reoptimize each quarter, then for purposes of computing the optimal values for the first quarter of the control period, one need not obtain feedback equations. The values of $x_1,\ldots, x_T$ that approximately minimize the expected value of $\mathscr{L}_1$ in (4) can be obtained using standard algorithms for solving unconstrained nonlinear optimization problems. This procedure is discussed in Fair [1974b]. When this procedure is followed, the expected value of $\mathscr{L}_1$ can be obtained either by stochastic simulation or

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tition in the following discussion, values of the control variables obtained from approximate solutions will be referred to as "optimal " values rather than as "approximate optimal" values.

On the assumption that stochastic control problems can be approximately solved, it is possible to describe a procedure for computing the expected losses in (1). Let x_1^* denote the optimal value of x_1 obtained from the minimization of the expected value of $\mathscr{L}_1$, given q_0 and θ_0,[3] and let x_1^a denote the actual, historic value of x_1. Given x_1^a and x_1^*, it is possible to compute part of the first two terms in (1). The expected actual loss in quarter 1 is $E(L_1|x_1^a, q_0, \theta_0)$, which will be denoted as $\hat{L}_1^a$, and the expected optimal loss in quarter 1 is $E(L_1|x_1^*, q_0, \theta_0)$, which will be denoted as $\hat{L}_1^*$.[4] $\hat{L}_1^a - \hat{L}_1^*$ is the difference between the expected actual and optimal losses for quarter 1.

Now move on to the beginning of quarter 2. The expected actual loss in quarter 2 is straightforward to compute; it is merely $E(L_2|x_2^a, q_1, \theta_1)$, which will be denoted as $\hat{L}_2^a$. It is not, however, straightforward to compute the expected optimal loss in quarter 2, since these computations must be based on the assumption that the administration behaved optimally in quarter 1. The data that are available are, of course, only for the actual behavior of the administration. The procedure that can be followed in this case is the following. First, one has an estimate of the actual error terms for quarter 1. Second, one has available x_1^*.

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by setting the error terms in the model equal to their expected values. The first way is subject to sampling error, but is otherwise exact with respect to computing the expected value, whereas the second way is only approximate. The second way is, however, much cheaper than the first. If the second way is used, then the stochastic control problem is effectively converted into a series of deterministic control problems, one per quarter. It should also be noted that the only optimal values of the control variables that would actually be used in practice by this procedure would be the values for the first quarter, denoted, say, as x_1^*. The optimal values for the second quarter, for example, would not be used because in general different optimal values for the second quarter would be computed when the administration reoptimized at the beginning of the second quarter. When solving the control problem in this way, the length of the control period, T, need only be taken large enough to make the addition of an extra quarter to the length have a negligible influence on x_1^*. Another procedure for obtaining approximate solutions to stochastic control problems for nonlinear models is to obtain a linear approximation to the model and a quadratic approximation to the objective function and then solve the resulting linear-quadratic problem by standard methods. One can then iterate on the approximations. This procedure does allow feedback equations to be obtained. See Chow [1975, Chapter 12] for a discussion of this procedure as well as a good discussion as to why all the currently proposed procedures for nonlinear models are only approximate.

[3] At the beginning of quarter 1 the model can be reestimated through quarter 0 using the latest revised data. The latest data and information can also be used to revise previous projections of the non-control exogenous variables. θ_0 is meant to represent all the relevant information in this regard that is available at the beginning of quarter 1.

[4] There are two ways in which these expected values can be computed, one exact except for sampling error and one approximate. The exact way is by means of stochastic simulation, and the approximate way is to set the error terms equal to their expected values and solve the model only once. For linear models the latter way is, of course, also exact.

Therefore, one can solve the model for quarter 1 using the actual error terms for this quarter and x_1^*. This will produce values of the endogenous variables that one can take as estimates of what would have been observed in quarter 1 had the administration behaved optimally. Let q_1^* denote the vector of these values.[5] The stochastic control problem that would have faced administration 1 at the beginning of quarter 2 had it behaved optimally in quarter 1 is to choose the values of $x_2,\ldots, x_{T+1}$ so as to minimize the expected value of $\mathscr{L}_2$ in (4), given q_1^* and θ_1. Since an estimate of q_1^* is available, this problem can be solved. Let x_2^* denote the optimal value of x_2 obtained from this solution.[6] The expected optimal loss in quarter 2 is then $E(L_2|x_2^*, q_1^*, \theta_1)$, which will be denoted as $\hat{L}_2^*$. $\hat{L}_2^a - \hat{L}_2^*$ is the difference between the expected actual and optimal losses for quarter 2.

The process just described for quarter 2 can be repeated for quarters 3–16. For example, q_2^* can be computed given x_2^* and an estimate of the actual error terms for quarter 2.[7] Then x_3^* can be obtained from the minimization of the expected value of $\mathscr{L}_3$ in (4), given q_2^* and θ_2. Then $\hat{L}_3^a$ and $\hat{L}_3^*$ can be computed. Once this process is completed for the 16 quarters, the first two terms in (1) are merely $\sum_{t=1}^{16} \hat{L}_t^a$ and $\sum_{t=1}^{16} \hat{L}_t^*$, respectively.

Consider finally quarters 17–32. The third term in (1), the expected optimal loss in quarters 17–32 given the actual behavior of administration 1, can be computed in essentially the same way as described above for the second term. x_{17}^* can be obtained from the minimization of the expected value of $\mathscr{L}_{17}$ in (4), given q_{16} and θ_{16}. The expected loss in quarter 17 is then $E(L_{17}|x_{17}^*, q_{16}, \theta_{16})$. Given x_{17}^* and an estimate of the actual error terms for quarter 17, q_{17}^* can be

[5] If there are lagged control variables in the model, then these variables should also be considered to be in q_{t-1} in (2). In this case, then, x_1^* is included in q_1^*.

[6] Note that x_2^* is not in general equal to the optimal value of x_2 computed from the control problem solved at the beginning of quarter 1. See footnote 2. Note also that the length of the control period for the problem solved at the beginning of quarter 2 has been assumed to remain at T. This means that values of the control variables for quarter $T+1$ must now be determined in the solution of the overall problem. This is not, however, an important assumption. The only important consideration in this regard is to choose the length of the control period long enough to make the addition of an extra quarter to the length have a negligible influence on the optimal values for the first quarter of the control period (x_2^* in this case). Again see footnote 2.

Finally, it should be noted that for purposes of the computations, θ_1 must represent information that is available at the beginning of quarter 2 *given the actual behavior of the administration.* For example, the model must be reestimated through quarter 1 using the actual data rather than the data that would have existed had the administration behaved optimally. This is one unavoidable difference between what can be done here in computing optimal values for an administration and what an administration could actually have done had it behaved optimally.

[7] The estimated error terms in this case can be based on the model as reestimated through quarter 2. Also, if there are endogenous variables lagged, say, two quarters in the model, then the endogenous-variable values that were estimated for quarter 1 under the assumption that the administration behaved optimally in quarter 1 should be included in q_2^*. Similar considerations apply to quarters 3 and beyond and to lags longer than two quarters.

computed, which can then be used in the computation of x^*_{18}, and so on for the remaining 14 quarters. The third term in (1) is the sum of these 16 expected optimal losses. The computations for the fourth term in (1), the expected optimal loss in quarters 17–32 on the assumption that administration 1 behaved optimally, are again similar to those for the second term. In this case x^*_{17} is obtained from the minimization of the expected value of $\mathscr{L}_{17}$ in (4), given q^*_{16} and θ_{16}, where q^*_{16} is available from the computations for the second term. Except for the replacement of q_{16} by q^*_{16}, the computations for the fourth term are exactly the same as those for the third term. The fourth term is the sum of the 16 expected optimal losses computed by this procedure.

To summarize, the computations of the expected optimal losses in the definition of M are based on the solutions of a series of control problems. Each quarter a new control problem is solved, based on the available data up to that quarter. Since the overall loss function is additive across time, it is possible from this procedure to compute quarter by quarter the expected optimal loss, as well as the expected actual loss. The optimal values, $x^*_1, \ldots, x^*_{16}$, which are used in the computations of the expected optimal losses, are the best that one can expect an optimally behaving administration to use, given the model,[8] the available information each quarter, and the loss function.

3. AN ILLUSTRATION: APPROXIMATE ESTIMATES OF M FOR TWO LOSS FUNCTIONS AND FIVE ADMINISTRATIONS

In order to compute M as defined in (1) for a given administration, it is necessary to solve 48 control problems. Computing M for two loss functions and five administrations would thus require 480 solutions. Although it is not completely out of the question to solve this many problems,[9] it is beyond the scope of the present project. It is possible, however, to obtain approximate estimates of the 10 values of M by solving only 12 control problems, and this is the procedure followed here. It should thus be stressed that the estimates of M presented in this section are only approximations, even given the model and the loss function. In no sense are the results in this section meant to be a definitive evaluation of the economic performance of the five administrations; they are primarily illustrative. A more definitive evaluation must await further work. The five administrations considered are Eisenhower-I, Eisenhower-II, Kennedy-Johnson,

[8] The model that is used in any calculations of this kind should be a model that could have been specified at the time that each control problem is solved. In other words, a model that is used before a given date should not be based on information regarding the structure of the economy that became available after that data. In practice, of course, this is a hard distinction to make.

[9] The control problems that were solved in this study took on average about 8 minutes of computer time each on the IBM 370–158 at Yale. 480 problems would thus take about 64 hours. There are, however, a number of computers in existence that are at least 10 times faster than the IBM 370–158, and it would probably be feasible to solve the 480 problems on one of these machines.

Johnson, and Nixon-I.

3.1. *The Model, Loss Function, and Control Variable.* The econometric model that was used for the results is described in Fair [1976] and is based on the theoretical model in Fair [1974a]. The model is quarterly, consists of 84 equations, 26 of which are stochastic, and contains 78 exogenous variables. It is nonlinear and simultaneous. A general idea of the model and its properties can be obtained by reading the first section of Chapter 1 in Fair [1976]. There is one key feature of the model that has an important effect on the present results, and this feature will be discussed below. The coefficient estimates that were used for this work are the two-stage least squares estimates presented in Fair [1976]. These estimates are based on the sample period 1954I–1974II.

The basic loss function that was used targets a given level of real output and a zero rate of inflation each quarter. The loss in quarter t is:

$$L_t = \gamma \left| \frac{Y_t - Y_t^*}{Y_t^*} \right|^2 + (\% \Delta PF_t)^2, \qquad \gamma > 0, \tag{5}$$

where

Y_t = real output of the firm sector in quarter t (at a quarterly rate),
Y_t^* = target level of Y_t,

$$\left| \frac{Y_t - Y_t^*}{Y_t^*} \right|^2 = \begin{cases} \left(\dfrac{Y_t - Y_t^*}{Y_t^*}\right)^2 & \text{if} \quad Y_t < Y_t^*, \\ 0 & \text{if} \quad Y_t \geq Y_t^* \end{cases}$$

PF_t = the value of the key price deflator in the model in quarter t,
$\% \Delta PF_t = \left(\dfrac{PF_t}{PF_{t-1}}\right)^4 - 1$, (percentage change in PF_t at an annual rate).

The loss function penalizes rates of inflation that are both above and below the target value of zero, but it only penalizes values of Y_t that are below the target. The target values for real output are presented in the tables below, and their construction is explained in Chapter 10 in Fair [1976]. The values are meant to correspond to high levels of economic activity.

The parameter γ in (5) is the weight attached to the output target in the loss function. For the first set of estimates of M a value of γ of 1.0 was used, and for the second set a value of 0.1 was used. The weight attached to the output target was thus 10 times greater for the first set of estimates than for the second. The use of these two weights should provide a good indication of how sensitive the M estimates are to the use of fairly different loss functions.

One variable in the model was used as a control variable: the value of goods purchased by the government (in real terms), denoted as XG. This variable is one of the key fiscal-policy variables in the model. The key monetary-policy variable in the model is the value of government securities outstanding (in current-dollar terms), denoted as VBG. Monetary policy was assumed to be accommodating for the results in the sense that given XG, VBG was adjusted each quarter to achieve a given target bill rate for that quarter. The target bill-rate series is described in Fair [1976]. It has a positive trend between 1953I and 1970IV and is then flat (at 6.3 percent) from 1971I on. Although XG is the only fiscal-policy variable used, the following results would not be changed very much if more than one variable were used. Given that the objective function targets only real output and the rate of inflation, adding, say, a tax-rate variable as a control variable would have little effect on decreasing the loss from the minimum loss that can be achieved by using XG alone. The fiscal-policy variables are collinear in this sense.

3.2. *The Twelve Control Problems That Were Solved.* Before discussing how the approximate estimates of M were obtained from the solutions of the 12 control problems, the problems themselves must be explained. The results of solving the 12 problems are presented in Tables 1 through 6. The quarters in the tables are numbered consecutively beginning with 1953I, the first quarter of the first Eisenhower administration. The results in Table 1 are based on minimizing $\sum_{t=3}^{32} L_t$, where L_t is defined in (5). The control period in this case began with quarter 3 rather than with quarter 1 because of lack of enough earlier data. The control problem was converted into a deterministic control problem by setting the error terms in the model equal to their historic values. This problem was then solved by the procedure described in Fair [1974b] using a gradient algorithm. The same procedure was followed for the results in Tables 2 through 6, which are based on minimizing, respectively, $\sum_{t=17}^{48} L_t$, $\sum_{t=33}^{64} L_t$, $\sum_{t=49}^{80} L_t$, $\sum_{t=65}^{89} L_t$, and $\sum_{t=81}^{89} L_t$.

The optimal values of the three endogenous variables (Y, $\%\Delta PF$, and UR) presented in the tables were obtained from a dynamic simulation of the model using the optimal values of the control variable and the historic realization of the error terms. These values can be compared directly to the actual values of the endogenous variables because when the model is simulated using the actual values of the control variables and the historic realization of the error terms, the solution values of the endogenous variables are just the actual values.

The results in the tables are assumed to be an approximation to results that could have been achieved by the administrations had they behaved optimally, given the model and the loss function. The reasons the results are at best only approximate are the following:

1. Only one set of estimates of the model was used for all the results: the model was not reestimated each quarter.
2. Actual values of the exogenous variables were used for all the results: no attempt was made to estimate the information on the exogenous variables that was likely to be available at the beginning of each quarter.
3. Only one body of data was used: all problems of data revisions were

ignored.

4. The model was specified in 1975 and yet was used for periods prior to this.
5. The optimal control values for an administration were obtained from a solution of one deterministic control problem, using historic values of the error terms, rather than from a sequence of such problems, one a quarter, using expected values of the error term.
6. The lengths of the control periods were fairly short, i.e., T in (4) was fairly small for each problem.

Points 1–5 mean that the administrations are assumed to have had more information at their disposal than they actually had. Regarding point 5, for example, by solving the control problems in this way it is assumed that the administrations had knowledge of the future values of the error terms. This is clearly more information than they actually had. On the other hand, it should be remembered that in practice administrations can adjust to past error terms by reoptimizing each quarter, so that the use in this study of the historic values of the error terms (and solving only once) is not as restrictive as it might otherwise seem. If expected rather than historical values of the error terms had been used in this study (and the problem solved only once), this would have been assuming that the administrations had less information than they actually had.

3.3. *The Approximate Estimates of M.* It is now possible to explain how approximate estimates of M can be obtained from the results in the six tables. First, it should be mentioned that because of data limitations for Nixon-I, it was necessary, in order to make all the estimates comparable, to modify the third and fourth terms in (1) to be the loss in the next *two-year* period rather than in the next four-year period.

Consider now the estimate of M for Eisenhower-I. The first term in (1) can be estimated by taking the actual values of Y and $\% \Delta PF$ in Table 1 for each of the first 14 quarters, substituting them into (5) to compute a value of L_t for each quarter, and then summing the 14 values of L_t to estimate the first term. 14 rather than 16 quarters have to be used here because of data limitations. The second term in (1) can be estimated by doing the same thing for the optimal values of Y and $\% \Delta PF$ in Table 1 for the first 14 quarters. The third term in (1) (for two years now, rather than four) can be estimated by taking the optimal values of Y and $\% \Delta PF$ in *Table* 2 for each of the first 8 quarters (quarters 17 through 25), substituting them into (5) to compute a value of L_t for each quarter, and then summing the 8 values of L_t to estimate the third term. Finally, the fourth term in (1) can be estimated by doing the same thing for the optimal values of Y and $\% \Delta PF$ in Table 1 for quarters 17 through 24.

The estimate of M for Eisenhower-I thus requires the use of both Tables 1 and 2. The optimal values in Table 1 for quarters 3 through 16 are interpreted as being approximations to what Eisenhower-I could have achieved had it behaved optimally; the optimal values in Table 2 for quarters 17 through 24 are interpreted as being approximations to what Eisenhower-II could have achieved had

TABLE 1

CONTROL RESULTS FOR EISENHOWER-I

t Quarter	Actual Values						Optimal Values for $\gamma=1.0$					Optimal Values for $\gamma=0.1$				
	Y	100 $\%\Delta PF$	100 UR	$RBILL$	Target $RBILL$	Y^*	ΔXG	ΔVBG	Y	100 $\%\Delta PF$	100 UR	ΔXG	ΔVBG	Y	100 $\%\Delta PF$	100 UR
3 1953III	91.4	1.4	2.8	2.0	1.6	89.2	−1.6	−0.7	89.9	0.2	3.1	−1.8	−0.8	89.7	0.1	3.1
4 IV	90.0	0.1	3.7	1.5	1.6	90.0	0.4	−0.3	90.1	0.5	4.0	−0.4	−0.6	89.3	0.4	4.2
5 1954I	89.0	4.8	5.3	1.1	1.7	90.9	1.6	0.7	90.6	5.9	5.1	0.7	0.1	89.3	5.8	5.4
6 II	88.9	0.3	5.8	0.8	1.7	91.7	2.3	1.7	91.6	1.8	5.1	1.4	1.0	90.0	1.7	5.6
7 III	90.1	1.0	6.0	0.9	1.7	92.5	1.8	2.4	92.4	2.3	5.3	0.9	1.6	90.6	2.2	5.8
8 IV	92.3	2.0	5.4	1.0	1.8	93.4	1.0	2.7	93.4	3.0	5.1	0.2	1.9	91.5	2.8	5.6
9 1955I	95.4	0.6	4.7	1.3	1.8	94.2	−0.3	2.8	94.4	1.1	5.1	−1.0	1.9	92.5	1.0	5.6
10 II	97.3	−0.0	4.4	1.6	1.8	95.1	−0.3	3.1	95.2	−0.0	5.3	−1.0	2.3	93.3	−0.0	5.8
11 III	98.9	3.0	4.2	1.9	1.9	96.0	−0.8	3.5	95.8	2.7	5.4	−1.4	2.6	94.1	2.6	5.8
12 IV	99.9	4.1	4.2	2.3	1.9	96.9	−0.8	3.7	96.7	3.4	5.7	−1.3	2.8	95.1	3.3	6.0
13 1956I	99.3	3.1	4.1	2.4	2.0	97.7	0.5	4.7	97.5	2.5	5.2	−0.0	3.7	96.1	2.4	5.6
14 II	99.5	2.6	4.2	2.6	2.0	98.6	0.2	5.1	98.3	2.0	5.0	−0.2	4.0	96.9	1.8	5.3
15 III	99.3	3.9	4.2	2.6	2.0	99.6	0.6	5.7	98.9	3.4	4.6	0.1	4.4	97.7	3.2	4.9
16 IV	100.8	3.9	4.1	3.1	2.1	100.5	−0.6	5.2	99.8	3.0	4.5	−1.1	3.9	98.5	2.9	4.8
17 1957I	101.4	5.9	4.0	3.2	2.1	101.4	−0.4	5.0	100.9	5.0	4.2	−0.7	3.6	99.7	4.8	4.6
18 II	101.3	1.5	4.1	3.2	2.2	102.4	0.6	5.0	102.2	0.8	4.0	0.3	3.6	101.1	0.7	4.3
19 III	101.7	2.4	4.2	3.4	2.2	103.4	0.4	4.6	103.2	1.8	3.8	0.1	3.2	102.2	1.7	4.1
20 IV	99.9	2.5	5.0	3.3	2.2	104.5	2.4	5.0	104.3	2.2	3.9	2.1	3.5	103.1	2.0	4.2
21 1958I	97.2	0.9	6.3	1.8	2.3	105.5	4.2	5.9	105.4	1.9	4.4	3.9	4.4	104.0	1.8	4.7
22 II	97.6	0.3	7.4	1.0	2.3	106.6	3.4	5.9	106.4	2.1	5.1	3.1	4.3	105.0	2.0	5.4
23 III	100.3	2.1	7.3	1.7	2.4	107.6	1.7	4.7	107.4	2.3	5.6	1.4	3.1	105.9	2.3	5.9
24 IV	103.0	2.2	6.4	2.8	2.4	108.7	1.1	3.5	108.5	2.0	5.4	0.8	1.9	107.0	2.0	5.6

25 1959I	104.7	2.5	5.8	2.8	2.5	109.8	1.0	2.8	109.6	2.7	5.2	0.8	1.3	108.0	2.7	5.5
26 II	107.6	1.5	5.1	3.0	2.5	110.9	0.1	2.1	110.8	1.5	4.9	−0.1	0.6	109.3	1.4	5.1
27 III	105.9	1.5	5.3	3.5	2.6	112.0	3.2	3.0	112.0	1.4	4.5	3.0	1.6	110.5	1.3	4.8
28 IV	107.4	0.7	5.6	4.3	2.6	113.1	1.3	2.2	113.1	0.4	4.5	1.1	0.8	111.5	0.3	4.8
29 1960I	109.8	0.6	5.2	3.9	2.7	114.2	−0.6	0.3	114.1	0.6	4.1	−0.9	−1.0	112.3	0.5	4.5
30 II	109.3	1.2	5.3	3.1	2.7	115.4	1.3	−0.1	115.1	1.6	4.1	1.1	−1.3	113.3	1.5	4.5
31 III	108.7	0.1	5.6	2.4	2.8	116.5	2.7	0.2	116.2	1.1	4.0	2.5	−1.0	114.6	0.9	4.4
32 IV	107.7	1.3	6.3	2.4	2.8	117.7	4.3	1.0	117.5	2.2	4.1	4.1	−0.2	115.9	2.0	4.5

Notes: Y=production of the firm sector (in 1958 dollars at a quarterly rate) in Fair [1976].
$\%\Delta PF$=percentage change (at an annual rate) in the price deflator PF in Fair [1976].
UR=civilian unemployment rate.
$RBILL$=three-month treasury bill rate.
Y^*=target value of Y.
ΔXG=difference between optimal and actual values of XG.
XG=purchases of goods of the government (in 1958 dollars at a quarterly rate).
ΔVBG=difference between optimal and actual values of VBG.
VBG=value of government securities outstanding (in current dollars).

TABLE 2
CONTROL RESULTS FOR EISENHOWER-II

t Quarter	Actual Values						Optimal Values for $\gamma=1.0$					Optimal Values for $\gamma=0.1$				
	Y	100 $\%\Delta PF$	100 UR	$RBILL$	Target $RBILL$	Y^*	ΔXG	ΔVBG	Y	100 $\%\Delta PF$	100 UR	ΔXG	ΔVBG	Y	100 $\%\Delta PF$	100 UR
17 1957I	101.4	5.9	4.0	3.2	2.1	101.4	−0.3	−0.5	101.3	4.9	4.0	−1.1	−0.9	100.4	4.8	4.1
18 II	101.3	1.5	4.1	3.2	2.2	102.4	0.4	−0.6	102.3	1.1	3.9	−0.1	−1.1	101.5	1.0	4.2
19 III	101.7	2.4	4.2	3.4	2.2	103.4	0.3	−1.2	103.3	2.0	3.7	−0.5	−1.9	102.2	1.8	4.1
20 IV	99.9	2.5	5.0	3.3	2.2	104.5	2.3	−0.9	104.3	2.3	3.9	1.5	−1.8	103.0	2.1	4.3
21 1958I	97.2	0.9	6.3	1.8	2.3	105.5	4.1	−0.1	105.3	2.0	4.4	3.4	−1.1	104.0	1.9	4.8
22 II	97.6	0.3	7.4	1.0	2.3	106.6	3.3	−0.3	106.4	2.1	5.1	2.5	−1.4	104.9	2.1	5.5
23 III	100.3	2.1	7.3	1.7	2.4	107.6	1.6	−1.6	107.4	2.4	5.7	0.8	−2.8	105.8	2.3	6.0
24 IV	103.0	2.2	6.4	2.8	2.4	108.7	0.9	−2.9	108.5	2.1	5.4	0.2	−4.3	106.9	2.0	5.8
25 1959I	104.7	2.5	5.8	2.8	2.5	109.8	0.9	−3.7	109.6	2.8	5.2	0.2	−5.2	108.1	2.7	5.6
26 II	107.6	1.5	5.1	3.0	2.5	110.9	−0.1	−4.6	110.7	1.5	4.9	−0.5	−6.2	109.6	1.4	5.2
27 III	105.9	1.5	5.3	3.5	2.6	112.0	3.0	−3.8	111.9	1.5	4.5	2.8	−5.4	111.1	1.4	4.8
28 IV	107.4	0.7	5.6	4.3	2.6	113.1	1.1	−4.9	113.0	0.4	4.5	1.0	−6.5	112.5	0.4	4.7
29 1960I	109.8	0.6	5.2	3.9	2.7	114.2	−0.7	−7.0	114.2	0.7	4.2	−1.0	−8.7	113.4	0.6	4.3
30 II	109.3	1.2	5.3	3.1	2.7	115.4	1.3	−7.4	115.3	1.7	4.1	0.9	−9.3	114.5	1.6	4.3
31 III	108.7	0.1	5.6	2.4	2.8	116.5	2.7	−7.2	116.4	1.1	4.0	2.3	−9.3	115.6	1.0	4.2
32 IV	107.7	1.3	6.3	2.4	2.8	117.7	4.1	−6.7	117.6	2.2	4.1	3.7	−8.9	116.7	2.1	4.4
33 1961I	107.3	0.7	6.8	2.4	2.9	118.9	4.5	−6.4	118.8	1.7	4.2	4.2	−8.8	118.0	1.6	4.5
34 II	109.9	0.7	7.0	2.3	3.0	120.1	1.5	−8.7	120.0	1.7	4.6	1.2	−11.1	119.1	1.7	4.9
35 III	112.0	−0.5	6.8	2.3	3.0	121.3	1.4	−10.3	121.1	0.4	5.3	1.1	−12.8	120.2	0.4	5.5
36 IV	114.3	2.0	6.2	2.5	3.1	122.5	1.0	−11.5	122.3	2.7	5.3	0.7	−14.2	121.3	2.7	5.6
37 1962I	116.1	0.5	5.6	2.7	3.1	123.7	1.5	−12.1	123.6	1.2	5.0	1.2	−14.9	122.6	1.1	5.2
38 II	118.0	1.3	5.5	2.7	3.2	124.9	1.2	−12.5	124.8	2.1	4.9	0.9	−15.4	123.9	2.0	5.1
39 III	119.2	0.2	5.6	2.9	3.3	126.2	1.2	−13.0	126.1	0.9	4.9	0.9	−16.0	125.0	0.9	5.2
40 IV	120.6	1.1	5.5	2.8	3.3	127.5	0.4	−14.3	127.3	1.9	5.0	0.0	−17.4	125.9	1.8	5.2
41 1963I	121.3	0.9	5.8	2.9	3.4	128.7	1.0	−15.1	128.5	1.6	5.4	0.6	−18.2	127.1	1.5	5.7
42 II	122.4	2.0	5.7	2.9	3.5	130.0	1.0	−15.9	129.8	2.7	5.3	0.6	−19.0	128.4	2.7	5.6
43 III	124.5	0.4	5.5	3.3	3.5	131.3	−0.1	−17.6	131.1	0.8	5.3	−0.4	−20.5	129.6	0.8	5.5
44 IV	126.3	1.7	5.6	3.5	3.6	132.6	−0.5	−19.3	132.4	2.0	5.5	−0.8	−22.2	130.8	2.0	5.8
45 1964I	128.4	1.4	5.5	3.5	3.7	134.0	−1.6	−21.8	133.7	1.7	5.8	−1.9	−24.6	131.9	1.7	6.0
46 II	130.2	1.4	5.3	3.5	3.8	135.3	−1.5	−23.8	135.1	1.7	5.6	−1.8	−26.3	133.2	1.6	5.9
47 III	131.8	1.7	5.0	3.5	3.8	136.6	−1.7	−25.7	136.4	1.9	5.6	−1.9	−28.1	134.5	1.9	5.9
48 IV	132.5	1.3	5.0	3.7	3.9	138.0	−0.8	−27.1	137.8	1.5	5.6	−0.9	−29.1	136.1	1.5	5.8

Notes: See notes to Table 1.

TABLE 3
CONTROL RESULTS FOR KENNEDY-JOHNSON

t Quarter	Actual Values						Optimal Values for $\gamma=1.0$					Optimal Values for $\gamma=0.1$				
	Y	100 $\%\Delta PF$	100 UR	$RBILL$	Target $RBILL$	Y^*	ΔXG	ΔVBG	Y	100 $\%\Delta PF$	100 UR	ΔXG	ΔVBG	Y	100 $\%\Delta PF$	100 UR
33 1961I	107.3	0.7	6.8	2.4	2.9	118.9	10.3	5.2	118.8	1.5	5.3	9.9	5.0	118.4	1.5	5.4
34 II	109.9	0.7	7.0	2.3	3.0	120.1	3.8	4.3	119.9	1.7	4.6	3.0	3.8	118.9	1.6	4.8
35 III	112.0	−0.5	6.8	2.3	3.0	121.3	3.5	3.9	121.1	0.4	4.8	2.6	3.1	119.7	0.3	5.1
36 IV	114.3	2.0	6.2	2.5	3.1	122.5	2.8	3.5	122.2	2.7	5.0	1.8	2.4	120.6	2.6	5.4
37 1962I	116.1	0.5	5.6	2.7	3.1	123.7	3.3	4.0	123.5	1.1	4.7	2.5	2.8	121.9	1.0	5.2
38 II	118.0	1.3	5.5	2.7	3.2	124.9	3.2	4.7	124.7	2.0	4.7	2.4	3.4	123.4	1.9	5.1
39 III	119.2	0.2	5.6	2.9	3.3	126.2	3.3	5.7	126.0	0.9	4.8	2.7	4.2	124.7	0.8	5.1
40 IV	120.6	1.1	5.5	2.8	3.3	127.5	2.8	6.1	127.2	1.8	4.8	2.0	4.4	125.7	1.7	5.1
41 1963I	121.3	0.9	5.8	2.9	3.4	128.7	3.4	7.0	128.5	1.5	5.2	2.7	5.2	126.9	1.4	5.5
42 II	122.4	2.0	5.7	2.9	3.5	130.0	3.4	7.9	129.8	2.7	5.0	2.7	5.9	128.2	2.6	5.3
43 III	124.5	0.4	5.5	3.3	3.5	131.3	2.4	8.1	131.1	0.8	4.9	1.8	6.0	129.5	0.7	5.2
44 IV	126.3	1.7	5.6	3.5	3.6	132.6	2.1	8.2	132.4	2.0	5.1	1.3	6.0	130.7	1.9	5.4
45 1964I	128.4	1.4	5.5	3.5	3.7	134.0	1.1	7.8	133.7	1.7	5.3	0.4	5.5	131.8	1.6	5.6
46 II	130.2	1.4	5.3	3.5	3.8	135.3	1.2	8.0	135.0	1.7	5.1	0.5	5.6	133.1	1.6	5.5
47 III	131.8	1.7	5.0	3.5	3.8	136.6	1.0	8.1	136.4	2.0	5.1	0.5	5.8	134.5	1.9	5.4
48 IV	132.5	1.3	5.0	3.7	3.9	138.0	1.9	8.9	137.8	1.6	5.0	1.4	6.6	136.1	1.5	5.2
49 1965I	135.9	1.1	4.9	3.9	4.0	139.4	−0.3	8.0	139.2	1.2	5.0	−0.8	5.8	137.5	1.1	5.2
50 II	137.8	2.0	4.7	3.9	4.1	140.8	0.1	8.0	140.6	2.1	5.0	−0.3	5.8	139.1	2.1	5.2
51 III	140.5	1.3	4.4	3.9	4.1	142.2	−1.1	7.3	142.0	1.4	4.8	−1.4	5.1	140.5	1.3	5.0
52 IV	143.6	1.7	4.1	4.2	4.2	143.6	−2.2	6.3	143.4	1.5	4.9	−2.6	4.0	141.7	1.4	5.1
53 1966I	146.6	2.6	3.9	4.6	4.3	144.9	−3.2	4.7	144.7	2.1	5.0	−3.8	2.4	142.8	2.0	5.2
54 II	147.7	3.8	3.8	4.6	4.4	146.2	−2.6	3.9	146.0	3.4	5.0	−3.2	1.5	144.0	3.3	5.3
55 III	148.7	2.6	3.8	5.0	4.5	147.5	−2.5	3.0	147.3	2.0	4.9	−3.2	0.5	145.1	1.9	5.2
56 IV	150.5	4.4	3.7	5.2	4.6	148.9	−3.3	1.2	148.6	3.8	4.8	−4.1	−1.3	146.3	3.7	5.1
57 1967I	149.7	3.7	3.8	4.5	4.7	150.2	−1.3	1.2	150.0	3.7	4.7	−1.9	−1.3	147.8	3.6	5.0
58 II	150.9	1.6	3.8	3.7	4.8	151.6	−1.9	0.6	151.3	2.0	4.5	−2.6	−1.9	149.1	1.9	4.8
59 III	152.6	3.5	3.8	4.3	4.9	153.0	−2.2	−0.8	152.7	3.3	4.5	−3.0	−3.3	150.2	3.2	4.8
60 IV	153.7	3.4	4.0	4.8	5.0	154.4	−2.0	−2.0	154.0	3.3	4.6	−2.7	−4.5	151.5	3.2	5.0
61 1968I	155.6	2.8	3.8	5.1	5.1	155.8	−2.8	−3.5	155.4	2.6	4.6	−3.5	−6.0	152.8	2.4	4.9
62 II	158.5	3.5	3.6	5.5	5.2	157.2	−4.2	−6.0	156.8	3.0	4.6	−5.1	−8.5	154.0	2.9	5.0
63 III	160.0	3.4	3.6	5.2	5.3	158.6	−4.0	−7.8	158.3	3.1	4.8	−4.7	−10.2	155.5	3.0	5.1
64 IV	161.1	4.3	3.4	5.6	5.4	160.0	−3.6	−9.7	159.7	3.7	4.7	−3.9	−11.6	157.4	3.6	5.0

Notes: See notes to Table 1.

TABLE 4

CONTROL RESULTS FOR JOHNSON

t Quarter	Actual Values						Optimal Values for $\gamma=1.0$					Optimal Values for $\gamma=0.1$				
	Y	100 $\%\Delta PF$	100 UR	$RBILL$	Target $RBILL$	Y^*	ΔXG	ΔVBG	Y	100 $\%\Delta PF$	100 UR	ΔXG	ΔVBG	Y	100 $\%\Delta PF$	100 UR
49 1965I	135.9	1.1	4.9	3.9	4.0	139.4	2.9	1.7	139.3	1.2	4.5	2.3	1.3	138.6	1.2	4.6
50 II	137.8	2.0	4.7	3.9	4.1	140.8	1.2	1.8	140.5	2.3	4.1	0.1	0.9	139.0	2.2	4.3
51 III	140.5	1.3	4.4	3.9	4.1	142.2	0.1	1.3	141.9	1.6	4.0	−1.0	0.2	140.0	1.4	4.3
52 IV	143.6	1.7	4.1	4.2	4.2	143.6	−1.0	0.4	143.2	1.6	4.1	−2.5	−1.3	140.6	1.4	4.6
53 1966I	146.6	2.6	3.9	4.6	4.3	144.9	−2.0	−0.7	144.4	2.2	4.3	−3.7	−2.9	141.1	2.0	4.8
54 II	147.7	3.8	3.8	4.6	4.4	146.2	−1.2	−1.0	145.6	3.5	4.4	−3.0	−3.5	142.1	3.3	5.0
55 III	148.7	2.6	3.8	5.0	4.5	147.5	−0.9	−1.2	147.0	2.1	4.3	−2.7	−4.1	143.3	1.9	4.9
56 IV	150.5	4.4	3.7	5.2	4.6	148.9	−1.6	−2.0	148.3	4.0	4.2	−3.4	−5.1	144.5	3.7	4.8
57 1967I	149.7	3.7	3.8	4.5	4.7	150.2	0.5	−1.1	149.8	3.9	4.0	−0.6	−4.3	146.8	3.7	4.5
58 II	150.9	1.6	3.8	3.7	4.8	151.6	0.2	−0.7	151.2	2.2	3.8	−0.8	−3.9	148.5	2.0	4.2
59 III	152.6	3.5	3.8	4.3	4.9	153.0	0.1	−0.8	152.6	3.6	3.8	−1.1	−4.3	149.9	3.4	4.2
60 IV	153.7	3.4	4.0	4.8	5.0	154.4	0.4	−0.6	154.0	3.5	3.9	−0.4	−4.2	151.8	3.4	4.2
61 1968I	155.6	2.8	3.8	5.1	5.1	155.8	−0.2	−0.8	155.5	2.8	3.8	−0.8	−4.5	153.6	2.7	4.0
62 II	158.5	3.5	3.6	5.5	5.2	157.2	−1.5	−1.9	156.9	3.3	3.8	−2.3	−5.7	155.0	3.2	4.0
63 III	160.0	3.4	3.6	5.2	5.3	158.6	−1.2	−2.2	158.3	3.3	3.9	−2.0	−6.3	156.3	3.2	4.2
64 IV	161.1	4.3	3.4	5.6	5.4	160.0	−0.7	−2.3	159.7	3.9	3.8	−1.6	−6.6	157.7	3.8	4.1
65 1969I	162.5	4.1	3.4	6.1	5.5	161.5	−1.0	−2.9	161.2	3.7	3.8	−1.9	−7.3	159.1	3.6	4.1
66 II	163.2	4.4	3.5	6.2	5.6	162.9	−0.5	−3.0	162.6	4.2	3.7	−1.5	−7.9	160.5	4.0	4.1
67 III	163.8	4.7	3.6	7.0	5.7	164.4	−0.1	−3.5	164.1	4.3	3.7	−1.3	−8.8	161.8	4.1	4.2
68 IV	162.9	4.4	3.6	7.3	5.8	165.9	1.8	−3.0	165.5	4.4	3.3	0.2	−8.9	162.8	4.0	3.9
69 1970I	161.9	4.5	4.2	7.3	5.9	167.4	3.2	−1.9	166.9	4.9	3.4	1.4	−8.6	163.7	4.5	4.0
70 II	161.9	4.5	4.8	6.8	6.0	168.9	3.5	−1.3	168.4	5.0	3.5	1.6	−8.5	164.9	4.7	4.1
71 III	163.2	3.2	5.2	6.4	6.2	170.5	2.6	−1.7	169.8	3.6	3.8	0.3	−9.6	165.4	3.5	4.5
72 IV	161.2	7.2	5.8	5.4	6.3	172.0	5.8	0.4	171.3	7.9	4.2	3.3	−8.2	166.2	7.8	5.0
73 1971I	165.6	3.6	6.0	3.9	6.3	173.6	2.1	−0.6	172.9	4.8	4.5	0.6	−8.8	168.2	4.7	5.2
74 II	166.8	4.1	5.9	4.2	6.3	175.1	4.1	0.4	174.6	4.8	4.8	3.0	−7.4	170.4	4.8	5.4
75 III	167.8	2.6	6.0	5.1	6.3	176.7	4.4	1.2	176.2	3.0	5.0	3.5	−5.9	172.1	3.0	5.4
76 IV	170.6	0.4	6.0	4.2	6.3	178.3	3.5	2.2	177.7	1.4	5.1	2.2	−4.5	173.3	1.4	5.5
77 1972I	173.6	5.3	5.8	3.4	6.3	179.9	3.6	4.2	179.3	6.7	5.3	2.4	−2.2	174.9	6.6	5.6
78 II	177.5	1.7	5.7	3.7	6.3	181.6	2.7	5.8	181.0	2.6	5.5	1.9	0.3	176.9	2.5	5.8
79 III	180.2	2.2	5.6	4.2	6.3	183.2	3.0	8.4	182.7	2.9	5.6	2.3	3.8	178.5	2.9	5.9
80 IV	184.0	3.0	5.3	4.9	6.3	184.9	1.7	10.4	184.4	3.5	5.6	1.4	6.9	180.4	3.4	5.8

Notes: See notes to Table 1.

TABLE 5

CONTROL RESULTS FOR NIXON-I

t Quarter	Actual Values						Optimal Values for $\gamma=1.0$					Optimal Values for $\gamma=0.1$				
	Y	100 $\%\Delta PF$	100 UR	$RBILL$	Target $RBILL$	Y^*	ΔXG	ΔVBG	Y	100 $\%\Delta PF$	100 UR	ΔXG	ΔVBG	Y	100 $\%\Delta PF$	100 UR
65 1969I	162.5	4.1	3.4	6.1	5.5	161.5	−1.2	−1.0	161.5	3.8	3.5	−1.0	−0.9	161.6	3.8	3.5
66 II	163.2	4.4	3.5	6.2	5.6	162.9	−0.4	−1.2	162.8	4.2	3.6	−1.2	−1.7	162.0	4.1	3.7
67 III	163.8	4.7	3.6	7.0	5.7	164.4	−0.0	−1.6	164.2	4.3	3.7	−1.1	−2.7	162.9	4.1	3.9
68 IV	162.9	4.4	3.6	7.3	5.8	165.9	1.9	−0.9	165.5	4.4	3.3	0.5	−2.6	163.8	4.1	3.6
69 1970I	161.9	4.5	4.2	7.3	5.9	167.4	3.3	0.3	167.0	4.9	3.3	1.8	−2.1	165.0	4.5	3.8
70 II	161.9	4.5	4.8	6.8	6.0	168.9	3.6	1.1	168.5	5.0	3.4	1.8	−2.0	166.1	4.7	3.9
71 III	163.2	3.2	5.2	6.4	6.2	170.5	2.6	0.6	169.8	3.6	3.7	0.3	−3.5	166.6	3.4	4.4
72 IV	161.2	7.2	5.8	5.4	6.3	172.0	5.6	2.6	171.1	7.9	4.2	3.2	−2.5	167.4	7.7	4.9
73 1971I	165.6	3.6	6.0	3.9	6.3	173.6	2.1	1.7	173.0	4.8	4.5	0.2	−3.7	169.4	4.7	5.2
74 II	166.8	4.1	5.9	4.2	6.3	175.1	4.3	2.9	174.8	4.8	4.8	2.7	−2.9	171.6	4.7	5.4
75 III	167.8	2.6	6.0	5.1	6.3	176.7	4.5	3.7	176.4	3.0	4.9	3.1	−2.2	173.4	3.0	5.4
76 IV	170.6	0.4	6.0	4.2	6.3	178.3	3.4	4.6	177.9	1.4	5.1	1.9	−1.6	174.9	1.4	5.5
77 1972I	173.6	5.3	5.8	3.4	6.3	179.9	3.5	6.5	179.4	6.7	5.3	2.1	−0.1	176.6	6.6	5.7
78 II	177.5	1.7	5.7	3.7	6.3	181.6	2.7	8.1	181.2	2.6	5.5	1.7	1.4	178.7	2.5	5.8
79 III	180.2	2.2	5.6	4.2	6.3	183.2	3.0	10.7	182.9	2.9	5.6	1.9	3.8	180.3	2.9	5.9
80 IV	184.0	3.0	5.3	4.9	6.3	184.9	1.7	12.6	184.5	3.4	5.6	0.4	5.3	181.7	3.4	6.0
81 1973I	188.7	3.3	5.0	5.6	6.3	186.5	−0.4	13.2	186.2	3.5	5.7	−2.1	5.4	182.6	3.4	6.1
82 II	189.6	5.7	4.9	6.6	6.3	188.2	1.3	15.9	187.8	5.5	5.7	−0.8	7.4	183.4	5.4	6.2
83 III	190.5	5.4	4.8	8.4	6.3	189.9	0.9	17.1	189.0	4.7	5.5	−2.3	7.4	183.0	4.5	6.1
84 IV	191.7	9.5	4.7	7.5	6.3	191.7	−0.3	18.1	189.6	9.3	5.3	−5.4	6.0	180.7	9.0	6.4

85 1974I	187.8	14.5	5.1	7.6	6.3	193.4	3.8	22.7	190.2	14.1	5.4	−2.6	7.8	178.4	13.8	6.8
86 II	186.9	15.5	5.1	8.3	6.3	195.1	3.6	25.2	191.3	15.0	4.8	−3.1	8.4	177.7	14.5	6.5
87 III	185.9	13.0	5.5	8.3	6.3	196.9	5.1	28.5	193.2	12.7	4.6	−0.6	11.3	179.9	12.1	6.4
88 IV	181.0	13.3	6.6	7.3	6.3	198.7	10.5	37.6	195.9	13.4	4.9	6.8	21.2	185.3	13.0	6.3
89 1975I	174.7	11.0	8.3	5.9	6.3	200.5	15.1	47.9	199.1	11.6	5.5	13.8	34.7	192.1	11.3	6.4

Notes: See notes to Table 1.

TABLE 6

CONTROL RESULTS FOR NIXON-FORD

t Quarter	Actual Values						Optimal Values for $\gamma=1.0$					Optimal Values for $\gamma=0.1$				
	Y	100 $\%\Delta PF$	100 UR	$RBILL$	Target $RBILL$	Y^*	ΔXG	ΔVBG	Y	100 $\%\Delta PF$	100 UR	ΔXG	ΔVBG	Y	100 $\%\Delta PF$	100 UR
81 1973I	188.7	3.3	5.0	5.6	6.3	186.5	−2.3	−1.6	186.2	3.5	5.2	−3.2	−2.3	185.3	3.5	5.3
82 II	189.6	5.7	4.9	6.6	6.3	188.2	−0.9	−2.0	187.5	5.4	5.3	−2.3	−3.7	185.6	5.3	5.5
83 III	190.5	5.4	4.8	8.4	6.3	189.9	−1.2	−3.4	188.6	4.7	5.1	−3.8	−6.8	185.0	4.5	5.6
84 IV	191.7	9.5	4.7	7.5	6.3	191.7	−2.1	−4.8	189.3	9.3	5.1	−6.9	−11.3	182.3	9.1	5.9
85 1974I	187.8	14.5	5.1	3.6	6.3	193.4	2.2	−2.5	189.9	14.2	5.1	−4.5	−13.0	179.2	13.9	6.4
86 II	186.9	15.5	5.1	8.3	6.3	195.1	2.4	−1.8	191.2	15.1	4.6	−5.1	−15.6	177.9	14.6	6.3
87 III	185.9	13.0	5.5	8.3	6.3	196.9	4.2	0.1	193.3	12.9	4.4	−2.3	−15.5	179.8	12.3	6.3
88 IV	181.0	13.3	6.6	7.3	6.3	198.7	9.8	7.5	196.1	13.7	4.5	5.6	−7.8	185.4	13.1	6.2
89 1975I	174.7	11.0	8.3	5.9	6.3	200.5	14.5	16.7	199.3	11.8	5.1	13.2	4.0	192.7	11.5	6.2

Notes: See notes to Table 1.

it behaved optimally, but Eisenhower-I did not; and the optimal values in Table 1 for quarters 17 through 24 are interpreted as being approximations to what Eisenhower-II could have achieved had both it and Eisenhower-I behaved optimally. The estimates of M for the other administrations are computed in an analogous way. The estimate of M for Nixon-I, for example, requires the use of both Tables 5 and 6.

The estimates of M for the five administrations and the two values of γ in the loss function are presented in Table 7. (An estimate of M will be denoted as $\hat{M}$.) Since the loss function is additive in output and the rate of inflation, it is possible to break $\hat{M}$ into two parts: a part due to the output performance, denoted as Q in the table, and a part due to the inflation-rate performance, denoted as P in the table. The values of $\hat{M}$ are the values in the last column of Table 7.

Two estimates of M are actually presented in the table for Nixon-I for each value of γ, and this requires some explanation before proceeding to a discussion of the results. The first estimate of M for Nixon-I for each value of γ is based on the results in Tables 5 and 6. The results in Tables 5 and 6, as in Tables 1 through 4, are based on the solutions of the optimal control problems in which the bill rate each quarter is constrained to be equal to a given target rate. The target rates are presented in Tables 1 through 6. The second estimate of M for each value of γ for Nixon-I is, on the other hand, based on the solutions of the optimal control problems in which the bill rate each quarter is constrained to be equal to the historic rate. In other words, for these problems the target bill rates were merely taken to be the historic rates. To conserve space, the results of solving the four extra optimal control problems that were needed to estimate the two extra values of M for Nixon-I are not presented here.

The reason for the two extra estimates of M for Nixon-I is as follows. First note from Table 5 that the bill rates in the last eight quarters of the Nixon-I administration (1971I–1972IV) are considerably less than the target rates of 6.3 percent. In the model the bill rate has, other things being equal, a positive effect on the rate of inflation. Consequently, constraining the optimum to correspond to the target bill rates causes the rate of inflation in these and later quarters to be higher than it would have been had, say, the target bill rates for the eight quarters been the actual rates. This constraint was severe enough to cause the estimate of M for $\gamma=0.1$ to be negative for Nixon-I. From columns $Q+P$ under a and b in Table 7 it can be seen that the actual loss for the period of the Nixon-I administration (2.820) is less than the optimal loss (3.093). The periods of the Nixon-I and Nixon-Ford administrations are the only periods in which there are substantial differences between the target bill rates and the actual rates, and so it was decided to reestimate M for Nixon-I under the assumption that the target rates are the actual rates. As can be seen in Table 7, the second estimates of M are only slightly larger than the first. For $\gamma=1.0$, $\hat{M}$ increased from 1.573 to 1.714, and for $\gamma=0.1$, $\hat{M}$ increased from -0.164 to 0.021. The second estimate of M for $\gamma=0.1$ is, however, positive.

TABLE 7

APPROXIMATE ESTIMATES OF M FOR TWO LOSS FUNCTIONS AND FIVE ADMINISTRATIONS

a=estimated expected actual loss in the administration's four-year period in office ($3\frac{1}{2}$ years for Eisenhower-I).
b=estimated expected loss in the four-year period ($3\frac{1}{2}$ years for Eisenhower-I) if the administration had behaved optimally.
c=estimated expected loss in the next two-year period given that the administration did not behave optimally, but assuming that the next administration did.
d=estimated expected loss in the next two-year period if both administrations had behaved optimally.
$\hat{M}=a-b+c-d$.
Q=output part of loss.
P=inflation-rate part of loss.

Administration	$\gamma=1.0$ *a*			*b*			*c*			*d*			$\hat{M}$		
	Q	P	$Q+P$	Q	P	$Q+P$	Q	P	$Q+P$	Q	P	$Q+P$	Q	P	$Q+P$
Eisenhower-I	0.225	1.036	1.261	0.013	1.035	1.048	0.002	0.534	0.536	0.005	0.510	0.515	0.209	0.025	0.234
Eisenhower-II	4.774	0.738	5.512	0.003	0.750	0.753	0.002	0.216	0.218	0.001	0.236	0.237	4.772	−0.033	4.739
Kennedy-Johnson	5.753	0.247	6.000	0.004	0.483	0.488	0.008	0.497	0.505	0.002	0.449	0.451	5.755	−0.188	5.567
Johnson	0.127	1.470	1.597	0.012	1.393	1.406	0.006	1.927	1.933	0.007	1.940	1.947	0.113	0.064	0.178
Nixon-I	1.972	2.623	4.595	0.011	3.209	3.220	0.146	9.328	9.474	0.135	9.141	9.276	1.972	−0.400	1.573
Nixon-I*	1.972	2.623	4.595	0.021	2.881	2.902	0.160	9.872	10.032	0.135	9.876	10.011	1.977	−0.262	1.714
	$\gamma=0.1$														
Eisenhower-I	0.022	1.036	1.058	0.043	0.959	1.002	0.015	0.498	0.513	0.017	0.473	0.489	−0.023	0.102	0.080
Eisenhower-II	0.477	0.738	1.215	0.022	0.695	0.717	0.012	0.195	0.207	0.007	0.224	0.230	0.461	0.014	0.475
Kennedy-Johnson	0.575	0.247	0.823	0.030	0.438	0.468	0.041	0.435	0.476	0.017	0.415	0.432	0.570	−0.191	0.399
Johnson	0.013	1.470	1.483	0.065	1.261	1.326	0.020	1.790	1.810	0.041	1.778	1.819	−0.074	0.221	0.147
Nixon-I	0.197	2.623	2.820	0.048	3.045	3.093	0.286	8.684	8.970	0.317	8.543	8.860	0.117	−0.281	−0.164
Nixon-I*	0.197	2.623	2.820	0.040	2.734	2.774	0.295	9.250	9.545	0.331	9.239	9.570	0.121	−0.100	0.021

Notes: *These calculations are based on the solutions of the optimal control problems in which the target bill rates are taken to be the historic rates.

All numbers in the table have been multiplied by 100. The numbers may not add because of rounding.

3.4. *A Discussion of the Results.* One important characteristic of the results in Tables 1–6 that should first be noted is that the output targets are generally much closer to being achieved than are the inflation targets. The model has the property that output can be increased to some reasonable target value (from a lower value) without having too serious an effect on the rate of inflation. It is not, however, generally possible to decrease the rate of inflation to, say, zero percent (from a higher rate) without having serious effects on the level of output. Consequently, when a loss function like (5) is minimized, with equal weights attached to the output and inflation targets, the optimum tends to correspond more closely to the output targets being achieved than it does to the inflation targets being achieved. Even when the weight on the output targets is only one-tenth of the weight on the inflation targets, it is still the case that the inflation targets of zero percent are generally not close to being achieved in the tables.

Consider now the rankings of the five administrations in Table 7. The results for both values of γ show that Eisenhower-II and Kennedy-Johnson did relatively poorly and that Eisenhower-I and Johnson did relatively well. Nixon-I was the best of the five for $\gamma=0.1$, but was only average for $\gamma=1.0$. Except for Nixon-I, the results in Table 7 indicate that the relative evaluation of the administrations is not very sensitive to the use of the two quite different weights on the output targets in the loss function.

In order to see better what lies behind the estimates of M, it will be useful to examine the results for Nixon-I and Kennedy-Johnson in more detail. Note first that for both values of γ, Nixon-I does not do well regarding the expected actual loss during its four-year period in office (column $Q+P$ under a). For $\gamma=0.1$, Nixon-I is in fact the worst of the five ($Q+P=2.820$). Most of this loss, however, is due to the inflation loss (e.g., $P=2.623$ for $\gamma=0.1$). Since the model has the property that it is expensive (in terms of lost output) to lower the rate of inflation, Nixon-I does not get penalized for the fact that there was a lot of inflation during its term in office. Most of the actual expected loss for Kennedy-Johnson, on the other hand, is due to the output loss. Since the model has the property that it is not expensive (in terms of extra inflation) to increase output to some high-activity level, Kennedy-Johnson gets penalized heavily for not doing so. It would thus be quite misleading in the present context to evaluate an administration on the basis of the expected actual loss during its term in office. The evaluation of an administration depends crucially on whether this loss is primarily output loss or inflation loss.

The value c–d in Table 7 measures for each administration the expected loss in the next two-year period from the fact that it did not behave optimally. These values are generally quite small and in a few cases are actually negative. The results thus indicate that none of the five administrations left its successor with a particularly bad state of the economy in the sense defined here. In particular, the results indicate that Johnson did not leave Nixon-I with a bad state of the economy, although it is commonly alleged that he did so. (Remember that by a bad state here is meant a state from which it is difficult for an optimally behaving administration to recover.)

Two final points about the results in Table 7. First, because the c–d estimates are small and because actual output loss is penalized much more heavily than actual inflation loss, a fairly close approximation to M in Table 7 is merely actual output loss (column Q under a in the table). This may, of course, not always be the case, but it is for the most part for the five administrations considered here. Second, note for Kennedy-Johnson that the results in Table 3 say that the administration should have increased government spending (XG) by a huge amount in the first quarter of its term (an increase in XG of 10.3 or 9.9 billion dollars at a quarterly rate). This may be a bit extreme to expect of an administration, but even if the administration had been allowed to spread this amount out over, say, four or six quarters with no cost, it still would have done poorly in the present context. The bad output performance of the Kennedy-Johnson administration, and hence the bad value of $\hat{M}$, is not due solely to the first few quarters of its term. Its output performance is fairly weak over the entire 16 quarters.

4. SUMMARY AND CONCLUSION

The measure of economic performance proposed in this paper takes into account the difficulty of controlling the economy and the problem of leaving one's successor with a bad state of the economy. The results of obtaining approximate estimates of this measure for the past five administrations show that Eisenhower-I and Johnson did well, that Eisenhower-II and Kennedy-Johnson did poorly, and that Nixon-I did well according to one loss function and average according to the other. The results also show that none of the five administrations left a particularly bad state of the economy to its successor (bad in the sense defined here). Finally, the results show that the measure of performance of an administration is not closely correlated with the expected actual loss during the administration's four-year period in office, but that it is closely correlated with the output part of this loss.

A key property of the econometric model used here is that it is expensive in terms of lost output to lower the rate of inflation, but that, conversely, it is not expensive in terms of extra inflation to raise the level of output. For a model with the opposite property, the results in Section 3 would be quite different. It does seem to be the case, however, that most models have the property of the present model,[10] which adds some support to the results in Section 3. Nevertheless, the sensitivity of the results in Section 3 to this property should be stressed. Given this property, the results in Section 3 are not very sensitive to the use of

[10] This property, for example, was much in evidence in the optimal control results for eight models (including the present model) that were presented at an AEA session, "Economic Fluctuations and Stabilization Policy 1965–75: Some Econometric Evidence," in Dallas on December 28, 1975.

quite different weights on the output targets in the loss function.

The estimates of M in Section 3 may also be sensitive to the approximations that were used, and this remains an important open question.

Finally, an obvious point. If an administration wants to carry out a certain policy and the Congress or Federal Reserve prevents it from doing so, it should not necessarily be blamed for what happened. For simplicity, this paper was written as if an administration should be completely praised or blamed for what happens, but the word "administration" could obviously be changed to, say, "government" or "policy makers." It is beyond the scope of this paper to allocate praise or blame to particular people or groups. If, however, the measure proposed in this paper does become widely used as a way of evaluating the economic performances of particular people or groups, I have a perfect name for it.

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