Evaluating the Reliability of Macro-economic Models
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CHAPTER 11

The Effects of Misspecification on Predictive Accuracy

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11.1 INTRODUCTION

This paper is concerned with the estimation of misspecification effects on predictive accuracy. In a recent study (Fair, 1980a) I have proposed a method for estimating the total uncertainty of a forecast from an econometric model: uncertainty due to (1) the error terms, (2) the coefficient estimates, (3) the exogenous variables, and (4) the possible misspecification of the model. The method allows one to estimate the uncertainty from each of these sources separately, and so it can be used to examine the effects of misspecification on predictive accuracy. The emphasis in this paper is on the misspecification effects. The method is reviewed in Section 11.2, and then misspecification effects for my model are examined in Section 11.3. Estimates of total predictive uncertainty for my model are presented in Section 11.4. Section 11.5 contains a brief conclusion.

The method can be applied to a model that is nonlinear in both variables and coefficients. The general model will be written as:

$$f_i(\mathbf{y}_t, \mathbf{x}_t, \boldsymbol{\alpha}_i) = u_{it}, \qquad i = 1, \ldots, n, \qquad t = 1, \ldots, T, \tag{1}$$

where $\mathbf{y}_t$ is an n-dimensional vector of endogenous variables, $\mathbf{x}_t$ is a vector of predetermined variables (including lagged endogenous variables), and $\boldsymbol{\alpha}_i$ is a vector of unknown coefficients. The first m equations are assumed to be stochastic, with the remaining u_{it} $(i = m + 1, \ldots, n)$ identically zero for all t. For expositional convenience the model will be assumed to be quarterly.

11.2 A REVIEW OF THE METHOD

The uncertainty of a model's forecast that is due to the error terms and coefficient estimates can be estimated by stochastic simulation, given an estimate of the distribution of the error terms and an estimate of the distribution of the coefficient estimates. Each 'trial' consists of draws of the error terms for each quarter of the forecast period and of the coefficients. Stochastic-simulation procedures are to some extent model specific, and for purposes of describing the method it is unnecessary to discuss the details of any particular procedure. The procedure that was followed for the results in this paper is discussed in Section 11.3. Let σ_{itk}^2 denote the variance of the forecast error for a k-quarter-ahead forecast of variable i from a simulation beginning in quarter t, and let $\tilde{\sigma}_{itk}^2$ denote the stochastic-simulation estimate of σ_{itk}^2.[1]

It is also possible to estimate by means of stochastic simulation the uncertainty of a model's forecast that is due to the uncertainty of the exogenous variables, given an assumption about the uncertainty of the exogenous variables themselves. There are two polar assumptions that can be made about the uncertainty of the exogenous variables. One is, of course, that there is no exogenous-variable uncertainty. The other is that the exogenous-variable forecasts are in some way as uncertain as the endogenous-variable forecasts. Under this second assumption one could, for example, estimate an autoregressive equation for each exogenous variable and add these equations to the model. This expanded model, which would have no exogenous variables, could then be used for the stochastic-simulation estimates of the variances. While the first assumption is clearly likely to underestimate exogenous-variable uncertainty in most applications, the second assumption is likely to overestimate it. This is particularly true for fiscal-policy variables in macro-econometric models, where government-budget data are usually quite useful for purposes of forecasting up to at least about eight quarters ahead. The best approximation is thus likely to lie somewhere in between these two assumptions.

The assumption that was made for the results in Fair (1980a) and for the results in Section 11.4 is in between the two polar assumptions. The procedure that was followed was to estimate an eighth-order autoregressive equation for each exogenous variable (including a constant and time in the equation) and then to take the estimated standard error from this regression as the estimate of the degree of uncertainty attached to forecasting the change in this variable for each quarter. This procedure ignores the uncertainty of the coefficient estimates in the autoregressive equations, which is one of the reasons it is not as extreme as the second polar assumption.

It is also unnecessary for purposes of describing the method to discuss the details of any particular assumption about exogenous-variable uncertainty. All that needs to be noted is that some assumptions must be made. If equations for the exogenous variables are not added to the model, but instead some in-between procedure is followed, then each stochastic-simulation trial consists of draws of error terms, coefficients, and exogenous-variable errors. If equations are added, then each trial consists of draws of error terms and coefficients from both the structural equations and the exogenous-variable equations. In either case, let $\hat{\tilde{\sigma}}_{itk}^2$

denote the stochastic-simulation estimate of the variance of the forecast error that takes into account exogenous-variable uncertainty as well as uncertainty from the error terms and coefficients.

Estimating the uncertainty from the possible misspecification of the model is the most difficult and costly part of the method. It requires successive re-estimation and stochastic simulation of the model. It is based on a comparison of estimated variances computed by means of stochastic simulation with estimated variances computed from outside-sample forecast errors.

Consider for now stochastic simulation with respect to the structural error terms and coefficients only (no exogenous-variable uncertainty). Assume that the *forecast period begins one quarter after the end of the estimation period*, and call this quarter t. As noted above, from this stochastic simulation one obtains an estimate of the variance of the forecast error, $\tilde{\sigma}^2_{itk}$. One also obtains from this simulation an estimate of the *expected value* of the k-quarter-ahead forecast of variable i. Let $\bar{\bar{y}}_{itk}$ denote this estimate. The difference between this estimate and the actual value, y_{it+k-1}, is the mean forecast error:[2]

$$\hat{\varepsilon}_{itk} = y_{it+k-1} - \bar{\bar{y}}_{itk}. \tag{2}$$

If it is assumed that $\bar{\bar{y}}_{itk}$ exactly equals the true expected value, $\bar{y}_{itk}$, then $\hat{\varepsilon}_{itk}$ in (2) is a sample draw from a distribution with a known mean of zero and variance σ^2_{itk}. The square of this error, $\hat{\varepsilon}^2_{itk}$, is thus under this assumption an unbiased estimate of σ^2_{itk}. One thus has two estimates of σ^2_{itk}: one computed from the mean forecast error and one computed by stochastic simulation. Let d_{itk} denote the difference between these two estimates:

$$d_{itk} = \hat{\varepsilon}^2_{itk} - \tilde{\sigma}^2_{itk}. \tag{3}$$

If it is further assumed that $\tilde{\sigma}^2_{itk}$ exactly equals the true value, then d_{itk} is the difference between the estimated variance based on the mean forecast error and the true variance. Therefore, under the two assumptions of no error in the stochastic-simulation estimates, the expected value of d_{itk} is zero.

The assumption of no stochastic-simulation error, i.e. $\bar{\bar{y}}_{itk} = \bar{y}_{itk}$ and $\tilde{\sigma}^2_{itk} = \sigma^2_{itk}$, is obviously only approximately correct at best. Even with an infinite number of draws the assumption would not be correct because the draws are from estimated rather than known distributions. It does seem, however, that the error introduced by this assumption is likely to be small relative to the error introduced by the fact that some assumption must be made about the mean of the distribution of d_{itk}. Because of this, nothing more will be said about stochastic-simulation error. The emphasis instead is on the possible assumptions about the mean of the distribution of d_{itk}, given the assumption of no stochastic-simulation error.

The procedure just described uses a given estimation period and a given simulation period. Assume for sake of an example that one has data from quarter 1 through 100. The model can then be estimated through, say, quarter 70, with the forecast period beginning with quarter 71. Stochastic simulation for the forecast period will yield for each i and k a value of d_{i71k} in (3). The model can then be re-estimated through quarter 71, with the forecast period now beginning with quarter 72. Stochastic simulation for this forecast period will yield for each i and k a value of d_{i72k} in (3). This process can be repeated through the estimation period ending with quarter 99. For the one-period-ahead forecast ($k = 1$) the procedure will yield, for each variable i, 30 values of d_{it1} ($t = 71, \ldots, 100$); for the two-period-ahead forecast ($k = 2$) it will yield 29 values of d_{it2} ($t = 72, \ldots, 100$); and so on. If the assumption of no simulation error holds for all t, then the expected value of d_{itk} is zero for all t.

The discussion so far is based on the assumption that the model is correctly specified. Misspecification has two effects on d_{itk} in (3). First, if the model is misspecified, the estimated covariance matrices that are used for the stochastic simulation will not in general be unbiased estimates of the true covariance matrices. The estimated variances computed by means of stochastic simulation will thus in general be biased. Secondly, the estimated variances computed from the forecast errors will in general be biased estimates of the true variances. Since misspecification affects both estimates, the effect on d_{itk} is ambiguous. It is possible for misspecification to affect the two estimates in the same way and thus leave the expected value of the difference between them equal to zero. In general, however, this does not seem likely, and so one would not expect the expected value of d_{itk} to be zero for a misspecified model. The expected value may be negative rather than positive for a misspecified model, although in general it seems more likely that it will be positive. Because of the possibility of data mining, misspecification seems more likely to have a larger positive effect on the outside-sample forecast errors than on the (within-sample) estimated covariance matrices.

An examination of how the d_{itk} values change over time (for a given i and k) may reveal information about the strengths and weaknesses of the model that one would otherwise not have. This information may then be useful in future work on the model. The individual values may thus be of interest in their own right, besides their possible use in estimating total predictive uncertainty.

For the total uncertainty estimates some assumption has to be made about how misspecification affects the expected value of d_{itk}. For the results in Fair (1980a) it was assumed that the expected value of d_{itk} is constant across time: for a given i and k, misspecification was assumed to affect the mean of the distribution of d_{itk} in the same way for all t. Other possible assumptions are, of course, possible. One could, for example, assume that the mean of the distribution is a function of other variables. (A simple assumption in this respect is that the mean follows a linear time trend.) Given this assumption, the mean can be then estimated from a regression of d_{itk} on the variables. For the assumption of a constant mean, this regression is merely a regression on a constant (i.e. the estimated constant term is merely the mean of the d_{itk} values).[3] The predicted value from this regression for period t, denoted $\hat{d}_{itk}$, is then the estimated mean for period t.

An estimate of the total variance of the forecast error, denoted $\hat{\sigma}^2_{itk}$, is the sum of $\tilde{\sigma}^2_{itk}$ — the stochastic-simulation estimate of the variance due to the error terms, coefficient estimates, and exogenous variables — and $\hat{d}_{itk}$:

$$\hat{\sigma}^2_{itk} = \tilde{\sigma}^2_{itk} + \hat{d}_{itk}. \tag{4}$$

Since the procedure in arriving at $\hat{\sigma}^2_{itk}$ takes into account the four main sources of uncertainty of a forecast, the values of $\hat{\sigma}^2_{itk}$ can be compared across models for a given i, k, and t. If, for example, one model has consistently smaller values of $\hat{\sigma}^2_{itk}$ than another, this would be fairly strong evidence for concluding that it is a more accurate model, i.e. a better approximation to the true structure.

This completes the outline of the method. It may be useful to review the main steps involved in computing $\hat{\sigma}^2_{itk}$ in (4). Assume that data are available for quarters 1 through T and that one is interested in estimating the uncertainty of an eight-quarter-ahead forecast that began in quarter $T+1$ (i.e. in computing $\hat{\sigma}^2_{itk}$ for $t = T+1$ and $k = 1,\ldots,8$). Given a base set of values for the exogenous variables for quarters $T+1$ through $T+8$, one can compute $\tilde{\sigma}^2_{itk}$ for $t = T+1$ and $k = 1,\ldots,8$ by means of stochastic simulation. Each trial consists of one eight-quarter dynamic simulation and requires draws of the error terms, coefficients, and exogenous-variable errors. These draws are based on the estimate of the model through quarter T. This is the relatively inexpensive part of the method. The expensive part consists of the successive re-estimation and stochastic simulation of the model that are needed in computing the d_{itk} values. In the above example, the model would be estimated 30 times and stochastically simulated 30 times in computing the d_{itk} values. After these values are computed for, say, quarters $T-r$ through T, then $\hat{d}_{itk}$ can be computed for $t = T+1$ and $k = 1,\ldots,8$ using whatever assumption has been made about the distribution of d_{itk}. This then allows $\hat{\sigma}^2_{itk}$ in (4) to be computed for $t = T+1$ and $k = 1,\ldots,8$.

In the successive re-estimation of the model, the first quarter of the estimation period may or may not be increased by one each time. The criterion that one should use in deciding this is to pick the procedure that seems likely to correspond to the chosen assumption about the distribution of d_{itk} being the best approximation to the truth. It is also possible to take the distance between the last quarter of the estimation period and the first quarter of the forecast period to be other than one, as was done above.

It is important to note that the above estimate of the mean of the d_{itk} distribution is not in general efficient because in general the error term in the d_{itk} regression is heteroskedastic. Even under the assumption of no misspecification, the variance of d_{itk} is not constant across time. Without further assumptions about the distribution of $\hat{\varepsilon}_{itk}$ in (2), there is little that can be done about improving the efficiency of the estimates. Litterman (1979) for some of his results assumes that $\hat{\varepsilon}_{itk}$ is $N(0, \sigma^2_{itk})$, and this allows him to estimate the mean of the d_{itk} distribution by maximum likelihood (pp. 63–64). The problem with this approach is that the assumption that $\hat{\varepsilon}_{itk}$ is normally distributed is not in general correct, and so it is not clear that Litterman's approach does in fact lead to more efficient estimates. This is still an open question.

11.3 CALCULATION AND ANALYSIS OF THE d_{itk} VALUES

I have used the method discussed in Section 11.2 to calculate d_{itk} values for my model (Fair, 1976, 1980b) for a forecast horizon of eight quarters. The model was estimated and stochastically simulated 44 times for these results. It may help in understanding the method to describe in some detail the steps that were involved in these calculations.

The model consists of 97 equations, 29 of which are stochastic, and has 183 unknown coefficients to estimate, including 12 first-order serial correlation coefficients. It is nonlinear in both variables and coefficients. The data base that was used for this study consists of observations from 1952 I through 1980 II, although the observations for 1980 II were preliminary and with one exception noted in Section 11.4 were not used for the results.

The first of the 44 estimation periods was 1954 I–1968 IV (60 observations). The coefficients were estimated by two-stage least squares (2SLS), and the covariance matrix of the coefficient estimates was computed. Let $\hat{\alpha}$ denote the vector of coefficient estimates, and let $\hat{V}$ denote the estimated covariance matrix.[4] Given the coefficient estimates, the covariance matrix of the error terms was estimated as $(1/60)EE'$, where E is the 29×60 matrix of values of the estimated error terms. Let $\hat{\Sigma}$ denote this matrix. Given these estimates, stochastic simulation was then performed for the 1969 II–1971 I period (8 quarters).[5] Both the distribution of the error terms and the distribution of the coefficient estimates were assumed to be normal. Each trial consisted of a draw of the 29 error terms for each of the 8 quarters from the $N(0, \hat{\Sigma})$ distribution and of a draw of the coefficients from the $N(\hat{\alpha}, \hat{V})$ distribution.[6] Each trial is a dynamic eight-quarter simulation. The actual values of the exogenous variables were used for these simulations. Assume that the quarters are numbered consecutively beginning with 1952 I, so that 1969 II, the first quarter of the simulation period, is quarter number 70. Let $\tilde{y}^j_{i70k}$ denote the value on the jth trial of the k-quarter-ahead prediction of variable i from the simulation beginning in quarter 70. For J trials, the estimate of the expected value of this variable, denoted $\bar{\tilde{y}}_{i70k}$, is:

$$\bar{\tilde{y}}_{i70k} = \frac{1}{J} \sum_{j=1}^{J} \tilde{y}^j_{i70k}. \tag{5}$$

The estimate of the variance of the forecast error, $\tilde{\sigma}^2_{i70k}$, is:

$$\tilde{\sigma}^2_{i70k} = \frac{1}{J} \sum_{j=1}^{J} (\tilde{y}^j_{i70k} - \bar{\tilde{y}}_{i70k})^2. \tag{6}$$

The number of trials used for these estimates was 25.[7] Values of $100(\tilde{\sigma}_{i70k}/\bar{\tilde{y}}_{i70k})$, $k = 1, \ldots, 8$, for real GNP are presented in the first half of row 1 in Table 11.1. These values are the stochastic-simulation estimates of the standard errors of the forecast, expressed as a percentage of the forecast mean. Given $\bar{\tilde{y}}_{i70k}$, the mean forecast errors were next computed using (2). Values of $100(|\hat{\varepsilon}_{i70k}|/\bar{\tilde{y}}_{i70k})$, $k = 1, \ldots, 8$, for real GNP are presented in the second half of row 1 in Table 11.1. These ratios are the estimates of the standard errors of the forecast based on the actual outside-sample forecast errors, again expressed as a percentage of the forecast mean.

The second estimation period was 1954 I–1969 I (61 observations), which differs from the first period by the addition of one quarter at the end. Note that the first quarter of the period was left unchanged. The coefficients were estimated by 2SLS for the period, and $\hat{V}$ and $\hat{\Sigma}$ were computed. Stochastic simulation was then performed for the 1969 III–1971 II period, yielding (for each variable i and $k = 1, \ldots, 8$) values of $\bar{\tilde{y}}_{i71k}$ and $\tilde{\sigma}^2_{i71k}$. Given $\bar{\tilde{y}}_{i71k}$, the mean forecast errors were computed using (2). Values of $100(\tilde{\sigma}_{i71k}/\bar{\tilde{y}}_{i71k})$ and $100(|\hat{\varepsilon}_{i71k}|/\bar{\tilde{y}}_{i71k})$, $k = 1, \ldots, 8$, for real GNP are presented in row 2 in Table 11.1.

The above process was repeated for the remaining 42 estimation periods. Since only data through 1980 I were used for the present results, the length of the simulation periods for the last seven sets of estimates was less than eight, as can be seen in Table 11.1. The last estimation period was 1954 I–1979 III (103 observations), and for this set of estimates the simulation period was merely one quarter, 1980 I, which is quarter number 113.

The results in Table 11.1 show that the stochastic-simulation estimates of the standard errors vary considerably across prediction periods (for a fixed k). Part of this variability can be explained by sampling error, since only 25 trials were used for each stochastic simulation. (Cost considerations prevented more trials from being performed.) This does not, however, appear to be the main cause of the variability. I re-ran a few of the 44 stochastic simulations using 250 trials, and in general the variability between the estimates based on 25 trials and those based on 250 trials was small relative to the variability across prediction periods. It thus appears that there is considerable variability of forecast-error variances across time (for a fixed k), at least for my model. This variability is due to different covariance matrices, different initial conditions (i.e. different lagged values of the endogenous and exogenous variables), and different values of the exogenous variables for the prediction periods.

It is possible from the results for the 44 periods to compute values of d_{itk} in (3) for each of the 97 endogenous variables for $k = 1, \ldots, 8$. For $k = 1$, 44 values of d_{itk} can be computed ($t = 70, \ldots, 113$). For $k = 2$, 43 values can be computed ($t = 71, \ldots, 113$), and so on through $k = 8$ ($t = 77, \ldots, 113$).

It is possible, as discussed in Section 11.2, to try to estimate equations explaining d_{itk} (for a fixed i and k). The explanatory variables in these equations would be variables that one felt had an effect on the degree of misspecification of

Table 11.1 Estimated standard errors for 44 estimation periods for real GNP. (Each simulation period begins two quarters after the end *of the estimation period*)

	Estimation period ending in	$100(\hat{\sigma}_{itk}/\hat{\bar{Y}}_{itk})$								$100(\lvert\hat{\varepsilon}_{itk}\rvert/\hat{\bar{Y}}_{itk})$							
	k:	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8
1	1968 IV	0.66	0.94	1.41	1.63	2.10	2.95	4.38	5.49	0.24	1.69	2.69	3.36	3.46	3.28	1.95	2.27
2	1969 I	0.77	1.23	1.71	2.01	2.31	3.28	4.04	4.48	0.94	1.40	1.92	1.78	1.34	0.17	0.56	0.32
3	II	0.68	0.93	1.13	1.34	1.82	2.03	2.68	3.29	0.30	1.02	1.37	1.66	0.55	1.61	1.56	1.28
4	III	0.70	0.94	1.10	1.34	1.58	1.78	2.21	2.34	0.61	0.49	0.81	0.24	1.01	1.33	1.35	1.39
5	IV	0.66	1.17	1.75	2.15	2.63	2.97	3.45	3.76	0.25	0.57	0.24	1.19	1.62	1.88	2.61	3.19
6	1970 I	0.80	1.07	1.01	1.33	1.59	1.80	1.85	2.19	1.10	0.49	1.91	2.10	2.26	2.66	2.95	4.24
7	II	0.68	0.82	1.18	1.86	2.23	2.78	3.05	3.40	0.85	0.58	1.49	1.66	2.52	3.38	5.03	5.18
8	III	0.69	1.21	1.75	2.07	2.41	2.44	2.35	2.43	1.58	2.49	2.74	3.29	3.22	4.34	4.15	3.72
9	IV	0.43	0.78	1.29	1.57	1.94	2.11	2.25	2.03	0.56	1.00	0.43	0.11	1.38	1.76	1.95	2.03
10	1971 I	0.70	1.30	1.67	1.75	2.12	2.52	2.90	3.06	0.22	0.16	0.59	1.91	2.06	2.10	2.45	1.38
11	II	0.74	1.36	1.82	2.13	2.32	2.41	2.90	3.31	0.60	1.49	3.17	3.82	4.47	5.10	3.98	3.80
12	III	0.76	0.97	1.19	1.59	1.91	1.98	2.21	2.69	0.42	1.96	2.56	3.26	3.94	3.18	3.57	4.05
13	IV	0.75	1.11	1.69	1.86	2.04	2.19	2.32	2.37	1.09	1.13	1.83	2.49	1.43	1.82	2.36	0.80
14	1972 I	0.52	0.72	1.06	1.38	1.40	1.71	2.18	2.98	0.26	0.71	1.43	0.83	1.20	1.69	0.20	0.99
15	II	0.58	0.91	1.17	1.31	1.24	1.40	1.82	2.27	0.35	1.04	0.16	0.43	0.66	1.25	0.87	0.77
16	III	0.51	0.70	1.06	1.37	1.91	2.39	3.12	4.29	0.54	0.40	0.34	0.01	1.52	1.23	0.97	1.80
17	IV	0.65	0.95	1.29	1.61	2.29	3.12	3.64	4.66	1.12	0.79	0.27	1.60	1.21	0.73	1.68	3.33
18	1973 I	0.53	0.87	1.25	1.66	2.33	3.02	4.17	5.37	0.32	0.53	2.50	2.15	1.91	2.51	3.67	1.26
19	II	0.48	0.88	1.45	2.25	2.67	3.16	3.41	3.47	0.20	1.73	1.20	1.24	2.35	4.30	2.80	1.99

20	III	0.66	1.12	1.59	1.99	2.13	2.45	2.87	3.29	1.54	1.35	1.05	2.31	4.45	2.85	2.10	1.84
21	IV	0.61	1.22	1.74	2.24	2.82	3.46	3.97	4.99	0.15	0.42	1.80	4.22	2.23	0.86	0.28	2.48
22	1974 I	0.77	1.13	1.38	1.87	1.95	2.20	2.25	2.86	0.08	1.06	2.79	1.00	0.41	0.73	3.11	3.79
23	II	0.87	1.36	2.06	2.58	2.88	3.09	3.97	4.72	0.12	1.30	1.53	3.28	3.86	7.26	9.12	9.91
24	III	0.72	1.15	1.94	2.30	2.46	3.04	3.38	3.42	1.58	0.78	2.55	3.62	6.34	7.47	7.59	7.81
25	IV	0.83	1.66	2.11	2.43	2.46	2.45	2.69	3.01	1.23	1.77	1.70	3.79	4.46	4.79	5.56	7.86
26	1975 I	0.90	1.49	2.02	2.41	3.07	3.17	3.53	3.62	0.23	0.93	0.75	1.14	1.13	1.76	3.03	2.72
27	II	1.01	1.51	2.18	2.18	2.05	2.19	2.22	2.28	0.64	0.88	0.72	0.05	0.40	0.57	0.02	0.09
28	III	0.69	1.07	1.44	1.84	2.12	2.32	2.32	2.48	1.57	1.55	1.14	1.07	2.36	1.75	1.76	0.96
29	IV	0.68	0.98	1.28	1.57	1.70	1.82	2.09	2.16	0.28	0.73	0.45	1.07	0.49	0.49	0.03	0.36
30	1976 I	0.74	1.07	1.20	1.64	1.56	1.57	1.43	1.54	0.71	1.08	0.18	0.94	0.74	1.25	1.53	0.51
31	II	0.80	1.27	1.40	1.40	1.63	1.86	1.80	2.02	0.01	1.17	0.53	0.79	0.01	0.77	0.19	0.53
32	III	0.56	0.89	1.30	1.48	1.59	1.85	1.74	1.71	1.27	0.67	0.79	0.37	1.08	0.37	0.75	0.70
33	IV	0.88	1.12	1.34	1.35	1.75	1.82	1.76	1.82	0.55	0.45	1.06	1.38	0.56	0.67	0.29	0.40
34	1977 I	0.74	1.49	1.57	1.45	1.54	1.56	1.23	1.25	0.06	0.78	1.05	0.26	0.78	0.75	1.04	1.83
35	II	0.68	0.89	1.17	1.46	1.70	1.79	2.05	2.37	0.19	0.55	0.54	0.21	0.44	0.29	0.56	0.83
36	III	0.57	0.89	1.20	1.56	1.67	1.87	1.92	1.99	0.36	0.47	0.28	0.61	0.34	0.14	0.40	1.09
37	IV	0.79	0.92	0.92	1.05	1.29	1.49	1.33	1.51	1.08	0.57	0.59	0.64	0.08	0.18	0.59	0.56
38	1978 I	0.59	1.10	1.04	1.31	1.44	1.54	1.40	—	0.06	0.45	0.38	0.32	0.29	0.93	0.93	—
39	II	0.49	0.92	0.99	1.16	1.46	1.44	—	—	0.69	0.50	0.13	0.23	0.53	0.64	—	—
40	III	0.62	0.67	1.06	1.09	1.29	—	—	—	0.28	0.31	0.06	0.40	0.28	—	—	—
41	IV	0.76	1.11	1.45	1.39	—	—	—	—	0.36	0.08	0.23	0.18	—	—	—	—
42	1979 I	0.47	0.96	1.18	—	—	—	—	—	0.02	0.49	0.20	—	—	—	—	—
43	II	0.70	0.93	—	—	—	—	—	—	0.48	0.28	—	—	—	—	—	—
44	III	0.63	—	—	—	—	—	—	—	0.72	—	—	—	—	—	—	—

the model. If, for example, one felt that the misspecification of the model was greater during periods of rapidly rising import prices, some variable measuring import prices or the rate of change of import prices could be tried as an explanatory variable. Another possibility would be to use as an explanatory variable a dummy variable that took on, say, a value of one in the quarters in which misspecification was considered greater and zero otherwise. If one felt that the degree of misspecification was changing smoothly over time, then a time trend could be tried as an explanatory variable to test this. The ordinary least squares estimates of these equations would not be efficient even if the correct explanatory variables were used because, as discussed in Section 11.2, the error terms in the equations are heteroskedastic. Nevertheless, the estimates might reveal useful information regarding the misspecification of the model.

No regressions of the above kind were in fact run for the results in this paper. Instead, values of d_{itk} were plotted and examined in a casual way to see if any systematic tendencies could be noted. Given the relative small number of observations (between 37 and 44) and the fact that there is little past experience examining values of this kind, it seemed best at this stage to take a less formal approach. For the plots for variables with trends, like real GNP, d_{itk} was normalized by the square of the forecast mean before being plotted. In other words, $d_{itk}/\bar{\bar{y}}_{itk}^2$ was plotted instead of d_{itk}. (Remember that d_{itk} is in units of the variable squared.) With respect to the values in Table 11.1, $d_{itk}/\bar{\bar{y}}_{itk}^2$ is for a given k the difference between the square of the value in the first half of a particular row and the square of the value in the second half of the row.

The plot for real GNP for the one-quarter-ahead forecast ($k = 1$) is presented in Figure 11.1. It is clear from this plot that there is no obvious pattern. The values are not obviously larger on average for one subperiod than for another, and there is no obvious trend in either direction. The four largest values are for 1971 I, 1974 I, 1975 I, and 1976 I. This can also be seen from the results in Table 11.1. The estimation period ended in 1970 III (row 8) for the simulation period beginning in 1971 I and, for this set of estimates, the absolute value of the one-quarter-ahead forecast error is large (1.58 percent). Likewise, the one-quarter-ahead errors in rows 20, 24, and 28 are large (1.54, 1.58, and 1.57 percent, respectively). 1971 I is a quarter following an automobile strike, and this probably accounts for the large value for this quarter. Although the other three quarters are in a period of rapidly rising U.S. import prices (due to rapidly rising oil and food prices and the weakness of the dollar), the fact that the values for these three quarters are not surrounded by other large values considerably lessens the evidence in favour of the hypothesis that the misspecification of the model was greater during this period.

Similar 'nonsystematic' plots were obtained for other variables in the model. Plots are presented in Figures 11.2–11.6 for five other variables (for $k = 1$): the GNP deflator, the unemployment rate, the wage rate, the bill rate, and the money supply. As can be seen, these plots show no real systematic tendencies. A partial

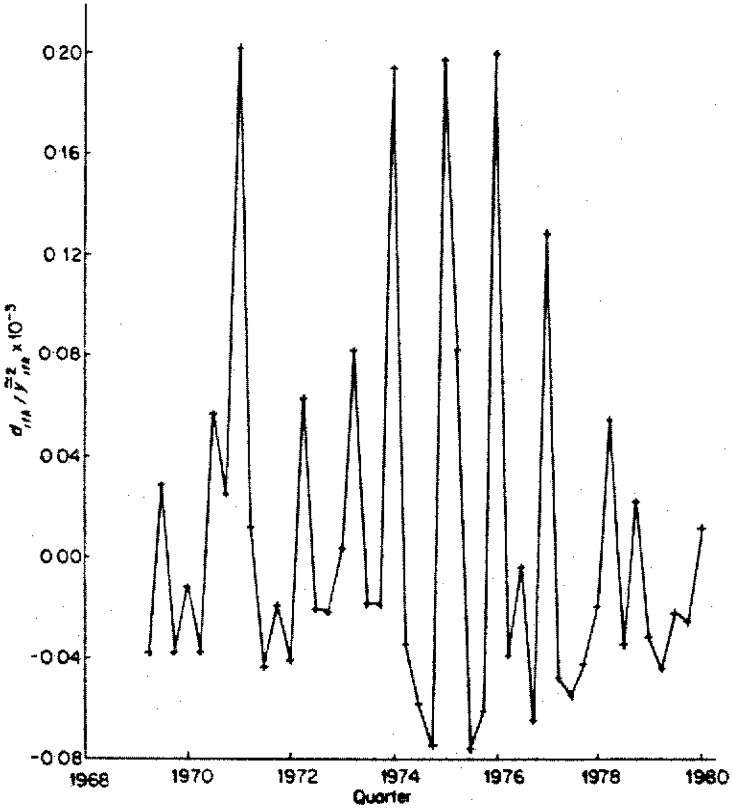

Figure 11.1 Real GNP one quarter ahead ($k = 1$). 44 observations: 1969 II–1980 I

exception to this is the plot for the bill rate, where the values for the last two quarters are large. If there was a substantial change in U.S. monetary policy following the October 6, 1979, announcement of the Federal Reserve, then one would expect the bill-rate equation in the model, which is meant to explain the behaviour of the Fed, to be misspecified more in 1979 IV and 1980 I than otherwise. This appears to be the case from the plot in Figure 11.5. This issue of the possible change in Fed behaviour is discussed in detail in Fair (1981).

Plots of d_{itk} or $d_{itk}/\bar{\bar{y}}^{2}_{itk}$ for values of k greater than one also showed no systematic tendencies except for a tendency for some of the series to be serially correlated for values of k greater than about four or five. This can be explained as follows. If, say, quarter 85 is a difficult quarter to predict, perhaps because of a large unexplained shock in the quarter, then a dynamic simulation that runs through this quarter may also do poorly in predicting quarters 86 and beyond. In other words, the simulation may get thrown off by the bad prediction in quarter 85. This means, for example, that five-quarter-ahead forecasts for quarters 85, 86,

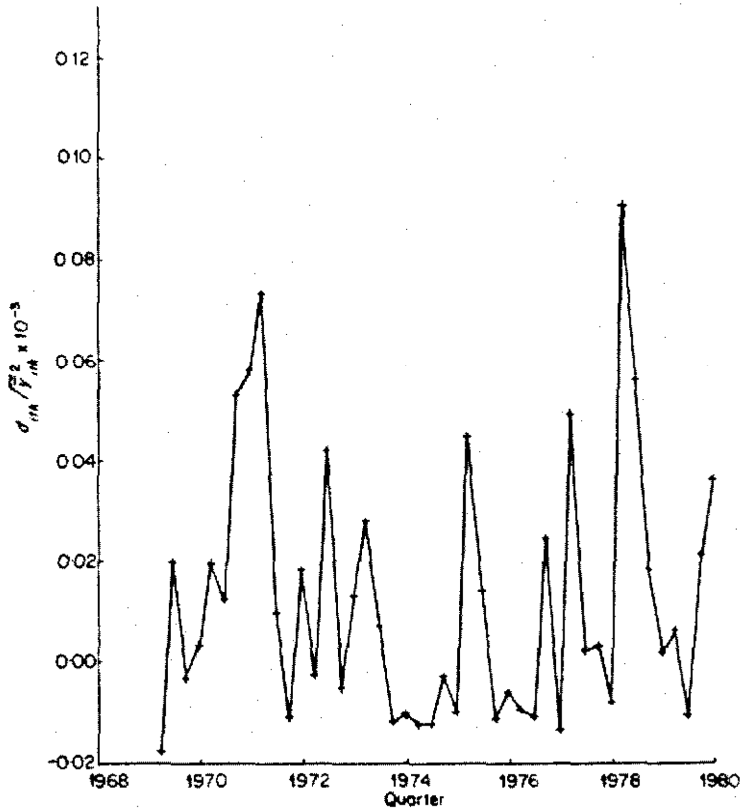

Figure 11.2 GNP deflator one quarter ahead ($k = 1$). 44 observations: 1969 II–1980 I

87, 88, and 89 may all be on average poor, thus implying large values for $\tilde{\varepsilon}_{itk}^2$ ($k = 5$ and $t = 85, \ldots, 89$). The shock in quarter 85 will have no effect on the stochastic-simulation estimates of the variances, since these are not based on the actual data for the endogenous variables for this quarter, and so the large values of the outside-sample errors imply large values of d_{itk}. In this way, serial correlation may be introduced into the d_{itk} series for values of k greater than one.

My impression from examining all the plots is that the misspecification of the model does not appear to have changed over time or to have been different in any subperiods. To give one final example of these plots, $d_{itk}/\bar{\hat{y}}_{itk}^2$ for real GNP for $k = 8$ is plotted in Figure 11.7. The values for 1976 III, 1976 IV, and 1977 I are large, but otherwise there is nothing unusual. The values for these three quarters are from simulations beginning in 1974 IV, 1975 I, and 1975 II, the period of rapidly rising U.S. import prices. As can be seen from rows 23, 24, and 25 in Table 11.1, the outside-sample errors for these three simulations are generally large. This pattern does not persist beyond row 25 (i.e. beyond 1977 I in Figure 11.7), and so one would probably not conclude from Figure 11.7 that the misspecification of the model changed in this period.

The fact that the misspecification of the model does not appear to have changed over time is not in itself encouraging regarding the accuracy of the model. The misspecification may in fact be quite large, even though unchanging, and have a large effect on total forecasting uncertainty. What is encouraging about the results is that the assumption of a constant mean for the d_{itk} distribution (for a fixed i and k) seems to be a reasonable approximation.[8] Given this assumption, it is possible to estimate the effect of misspecification on total uncertainty, and this is done in the next section.

11.4 ESTIMATION OF TOTAL UNCERTAINTY

The estimates of total predictive uncertainty for the period 1980 III–1982 IV are presented in Table 11.2 for six variables. The steps that were involved in obtaining

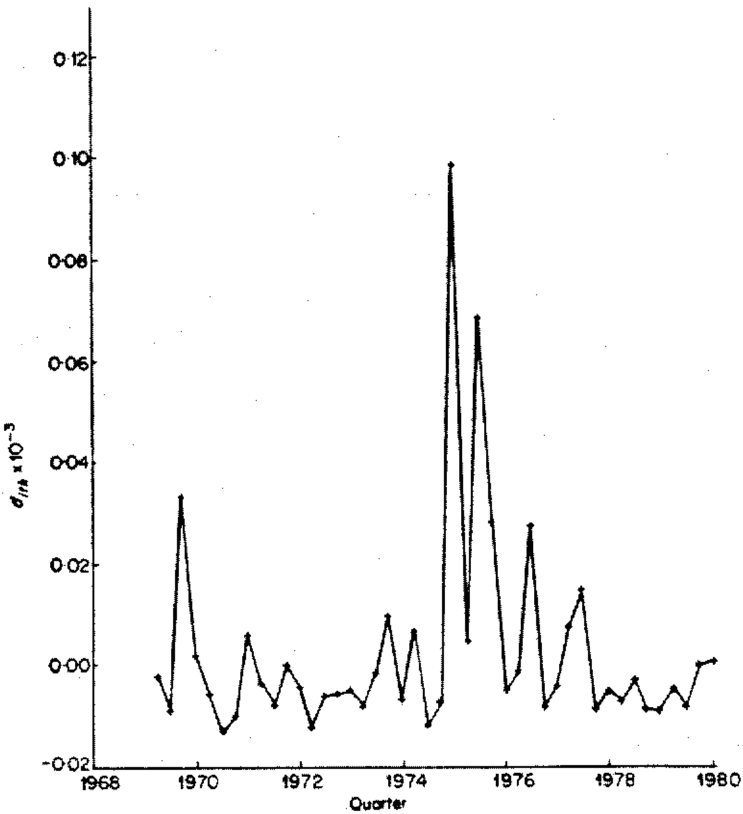

Figure 11.3 Unemployment rate one quarter ahead ($k = 1$). 44 observations: 1969 II–1980 I

these results will first be explained, and then the results themselves will be briefly discussed.

Remember that the data base for this study consists of observations from 1952 I through 1980 II, although the data for 1980 II are preliminary. The last of the 44 estimation periods used for the results in Section *III* was 1954 I–1979 III. For the results in Table 11.2 the model was re-estimated through 1980 I, i.e. for the 1954 I–1980 I period.[9] The simulation period was chosen to be 1980 III–1982 IV. The values for 1980 II were used as initial conditions for the simulations, although, as just noted, they were not used in the estimation. The values of the exogenous variables for the simulation period had to be guessed. The values that were used for this purpose were values that I used in August 1980 to make a forecast with the model. For future reference, these values will be called 'base' values.

Given the estimates of the coefficients and of the covariance matrices and given the exogenous-variable values, the uncertainty from the error terms and coefficient estimates was computed by means of stochastic simulation. These

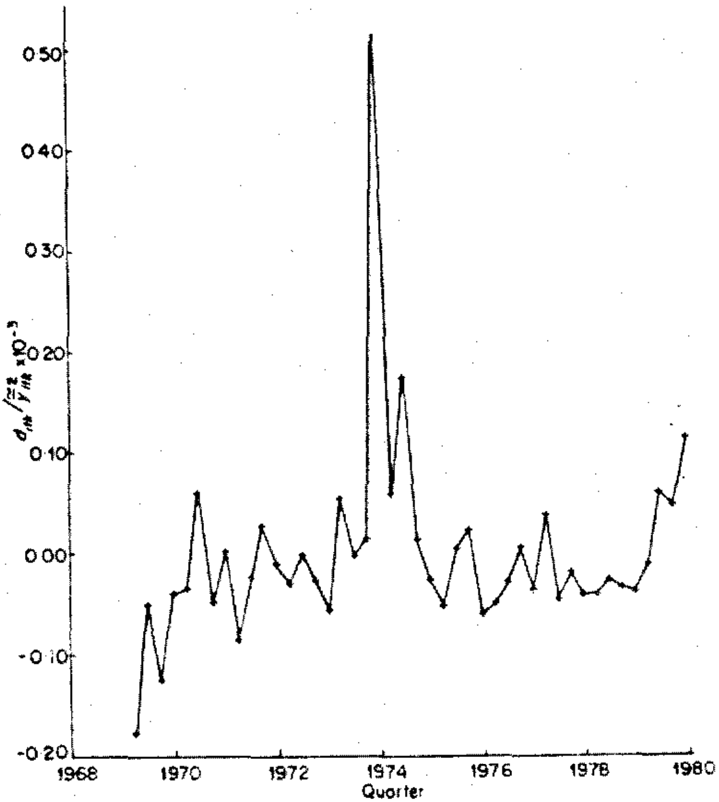

Figure 11.4 Wage rate one quarter ahead ($k = 1$). 44 observations: 1969 II–1980 I

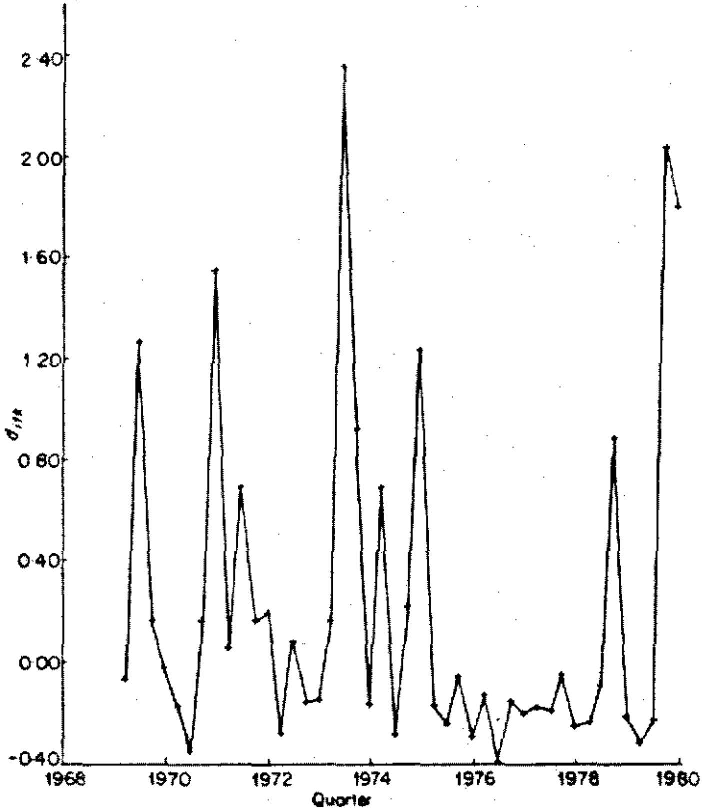

Figure 11.5 Bill rate one quarter ahead ($k = 1$). 44 observations: 1969 II–1980 I

results are presented in the *a* and *b* rows in Table 11.2. For the *a*-row results, only draws from the distribution of the error terms were made, whereas for the *b*-row results draws from both the distribution of the error terms and the distribution of the coefficient estimates were made. The number of trials used for each of these two stochastic simulations was 250.

As noted in Section 11.2, for the estimates of exogenous-variable uncertainty an eighth-order autoregressive equation for each exogenous variable (including a constant and time in the equation) was estimated. This was done for the 60 nontrivial exogenous variables in the model. The estimation period was 1954 II–1980 I. For each exogenous variable the estimated standard error from this regression was used for the estimate of the degree of uncertainty attached to forecasting the change in the variable for each quarter. Given these estimates and given the base values of the exogenous variables, alternative values of the exogenous variables can be drawn for the stochastic-simulation trials.[10] The results of stochastic simulation with respect to the error terms, coefficients, and

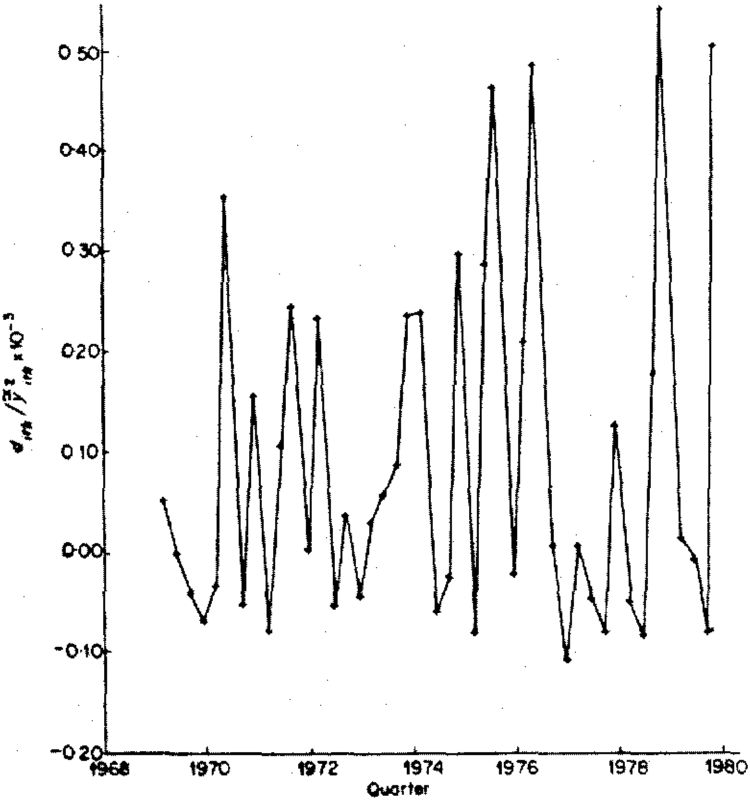

Figure 11.6 Money supply one quarter ahead ($k = 1$). 44 observations: 1969 II–1980 I

exogenous-variable values are presented in the c rows in Table 11.2. These results were also based on 250 trials. Using the notation in Section 11.2, the c-row values are values of $\hat{\tilde{\sigma}}_{itk}$ or $\hat{\tilde{\sigma}}_{itk}/\bar{\tilde{y}}_{itk}$.

The final step is to estimate for each i and k the mean of the d_{itk} distribution, $\hat{d}_{itk}$, and then use equation (4) to compute the estimate of the total variance of the forecast error, $\hat{\sigma}^2_{itk}$. Given the assumption that the mean of the d_{itk} is constant, $\hat{d}_{itk}$ is for each i and k merely the mean of the d_{itk} values.[11] From the results in Section 11.3, these means can be computed for each i and k. For $k = 1$, there are 44 observations; for $k = 2$, there are 43 observations; and so on. These means were calculated, and $\hat{\sigma}^2_{itk}$ in (4) was computed. The values in the d rows in Table 11.2 are the square roots of $\hat{\sigma}^2_{itk}$.[12] Examining the differences between the d- and c-row values in Table 11.2 is a way of examining the effects of misspecifications on the predictive accuracy of the model.

The results in Table 11.2 are fairly self explanatory, although a few features should be mentioned. Note first that some of the c-row values are less than the

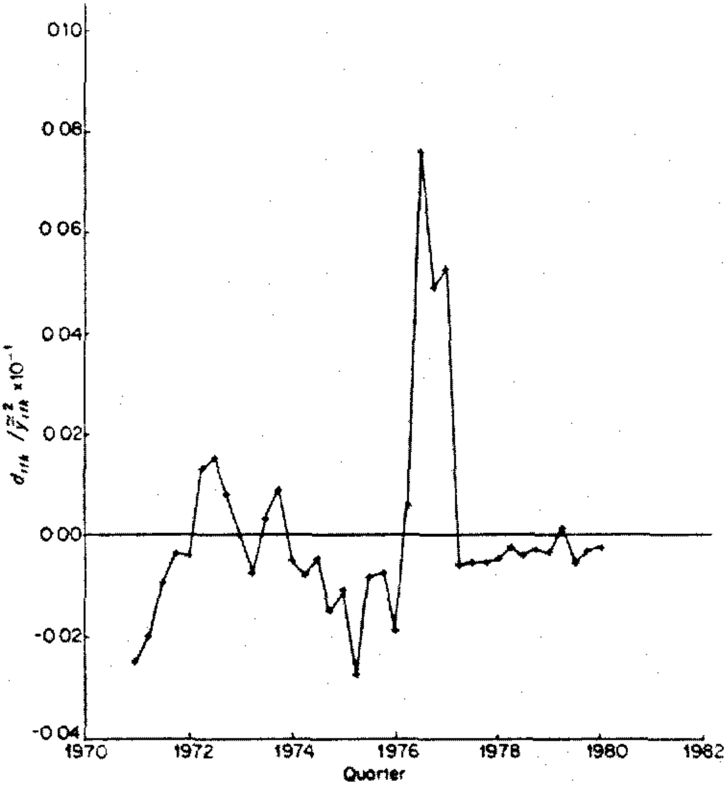

Figure 11.7 Real GNP eight quarters ahead ($k = 8$). 37 observations: 1971 I–1980 I

corresponding *b*-row values. Although some of this may be due to sampling error, there is no requirement that each *c*-row value be larger than the corresponding *b*-row value. In the present model there is a tendency for more variability of prices to be associated with less variability of real GNP and related real variables. One of the most important and uncertain exogenous variables in the model is the price of imports, and this variable has its major impact on prices. Adding exogenous-variable uncertainty thus increases the variability of prices, and leads, as in row *c* in Table 11.2 for the GNP deflator, to increased predictive uncertainty for prices. This increased variability in prices leads, other things being equal, to decreased variability (and thus decreased predictive uncertainty) in real GNP and related real variables. There are also undoubtedly other offset effects like this. The net result of these and other effects may be that for some variables adding exogenous-variable uncertainty lessens predictive uncertainty.

Note also that some of the *d*-row values in Table 11.2 are less than the corresponding *c*-row values. As discussed in Section 11.2, the mean of the d_{itk}

Table 11.2 Estimated standard errors of forecasts for 1980 III–1982 IV

a = uncertainty due to error terms.

b = uncertainty due to error terms and coefficient estimates — $\tilde{\sigma}_{itk}$ or $\tilde{\sigma}_{itk}/\bar{\bar{y}}_{itk}$.

c = uncertainty due to error terms, coefficient estimates, and exogenous variables — $\tilde{\tilde{\sigma}}_{itk}$ or $\tilde{\tilde{\sigma}}_{itk}/\bar{\bar{y}}_{itk}$.

d = uncertainty due to error terms, coefficient estimates, exogenous variables, and the possible misspecification of the model — $\hat{\sigma}_{itk}$ or $\hat{\sigma}_{itk}/\bar{\bar{y}}_{itk}$.

		1980		1981				1982			
		III	IV	I	II	III	IV	I	II	III	IV
Real GNP	a	0.59	0.96	1.08	1.20	1.44	1.52	1.54	1.55	1.48	1.48
	b	0.70	1.11	1.43	1.77	1.99	2.15	2.25	2.35	2.46	2.48
	c	0.77	1.11	1.43	1.69	2.02	2.34	2.63	2.68	2.78	2.93
	d	0.82	1.06	1.42	1.37	2.27	2.69	2.89	2.78		
GNP deflator	a	0.32	0.43	0.51	0.58	0.60	0.64	0.72	0.76	0.82	0.85
	b	0.32	0.46	0.67	0.84	1.03	1.22	1.43	1.61	1.77	1.94
	c	0.46	0.67	0.89	1.09	1.26	1.45	1.64	1.84	2.00	2.19
	d	0.58	1.03	1.53	2.06	2.56	3.06	3.58	4.04		
Unemployment rate	a	0.26	0.40	0.52	0.58	0.66	0.74	0.82	0.85	0.89	0.92
	b	0.33	0.54	0.75	0.91	1.08	1.13	1.18	1.24	1.28	1.36
	c	0.33	0.52	0.67	0.79	0.96	1.08	1.20	1.32	1.39	1.50
	d	0.37	0.53	0.61	0.67	0.81	0.87	0.91	0.91		
Wage rate	a	0.58	0.78	0.99	1.08	1.19	1.30	1.36	1.42	1.44	1.51
	b	0.65	1.05	1.40	1.73	2.06	2.33	2.63	2.86	3.12	3.38
	c	0.71	1.08	1.46	1.78	2.04	2.38	2.66	2.94	3.27	3.49
	d	0.71	1.23	1.80	2.37	3.03	3.67	4.33	4.92		
Bill rate	a	0.46	0.67	0.82	0.93	0.97	1.00	1.03	1.10	1.20	1.23
	b	0.55	0.84	1.07	1.20	1.38	1.52	1.64	1.82	1.95	2.06
	c	0.54	0.80	0.99	1.14	1.29	1.40	1.51	1.63	1.72	1.79
	d	0.72	1.08	1.18	1.31	1.49	1.61	1.69	1.75		
Money supply	a	0.83	1.09	1.31	1.46	1.62	1.79	1.86	1.95	2.04	2.06
	b	0.94	1.28	1.63	2.05	2.29	2.53	2.77	3.12	3.33	3.63
	c	0.92	1.33	1.67	1.99	2.39	2.71	3.13	3.47	3.86	4.28
	d	1.32	2.03	2.64	3.30	4.10	4.70	5.41	6.21		

Estimation period for a, b, c calculations: 1954 I–1980 I.

d-row values based on 44 sets of estimates of the model.

Units are percentage points.

The standard errors are divided by $\bar{\bar{y}}_{itk}$ except for the unemployment rate and the bill rate.

distribution may be negative for a misspecified model, although in most cases this does not seem likely. There are a few cases in Table 11.2 where this is true, although in general the estimated means are positive.

The results in Table 11.2 are an update of the results in Table 2 in Fair (1980a). The previous results were based on 35 sets of estimates of the model, as opposed to 44 here. The new results are quite close to the old; there appear to be no major changes in the predictive uncertainty of the model by the addition of 9 more quarters. For a user of the model the results in Table 11.2 can be used to gauge how much confidence to place on any given forecast from the model.

It should finally be noted that because the *d*-row values account for all four sources of uncertainty, they can be used to make accuracy comparisons across models. I have used the method for this purpose in Fair (1979), where four models are compared. The hope is that over time the method can be used to eliminate less accurate models.

11.5 CONCLUSION

The emphasis in this paper has been on estimating the effects of misspecification on predictive accuracy. For a correctly specified model the expected value of the difference between a forecast-error variance estimated by stochastic simulation and by an outside-sample forecast error is zero (ignoring simulation error). For a misspecified model the expected value is unlikely to be zero, and so examining the differences between these two estimates is a way of examining the effects of misspecification. The examination of the differences for my model in Section 11.3 did not reveal any evidence that the degree of misspecification of the model was changing over time or was different for different subperiods. This implies that the assumption that the mean of the distribution of the differences (for a given variable and horizon of the forecast) is constant across time may be a fairly good approximation. Given this assumption, it is possible to estimate the effects of misspecification on predictive accuracy, and this was done in Section 11.4.

The approach taken in this paper is one of estimation rather than hypothesis testing. The implicit premise is that misspecification exists and so must be accounted for in some way. If more were known about the distribution of the differences, it might be possible to test the hypothesis of no misspecification. As noted in Section 11.2, Litterman (1979) has made some progress in this area, although his results are based on a strong assumption about the distribution of the outside-sample forecast errors. It is clearly an open question as to how much can be learned about the distribution of the differences. If, however, most models are misspecified, as I believe is likely to be the case, then testing the hypothesis of no misspecification is of less importance than trying to estimate the effects of misspecification. Given an assumption about the mean of the distribution of the differences, the method discussed in this paper does allow this to be done.

NOTES

[1] *Note that it is implicitly assumed here that the variances of the forecast errors exist.* For some estimation techniques this is not always the case. If in a given application the variances do not exist, then one should estimate other measures of dispersion of the distribution, such as the interquartile range or mean absolute deviation.

[2] Note that t denotes the first quarter of the simulation, so that $\bar{\tilde{y}}_{itk}$ is the estimated expected value for quarter $t + k - 1$.

[3] For the results in Fair (1980a) a slightly different assumption than that of a constant mean was made for variables with trends. For these variables it was assumed that the mean of d_{itk} is proportional to $\bar{\tilde{y}}^2_{itk}$, i.e. that the mean of $d_{itk}/\bar{\tilde{y}}^2_{itk}$ is constant across time.

[4] *There are a few dummy variables in the model that are not relevant for the early* estimation periods, which means that there are slightly fewer than 183 coefficients to estimate for the early periods. For the first estimation period, for example, there are 170 coefficients to estimate. Although the covariance matrix of the 2SLS coefficient estimates is not block diagonal (see Fair and Parke, 1980, p. 273), for purposes of the results in this section only the diagonal blocks of $\hat{V}$ were computed. It is fairly expensive to compute the off-diagonal blocks, and this would have to have been done 44 times. Instead, the off-diagonal blocks were taken to be zero.

[5] Note that there is a one quarter gap between the end of the estimation period and the beginning of the simulation period. In practice the data for the most recent quarter are usually preliminary, and in my work I use these data as initial conditions for a forecast but not as observations for estimation. The procedure in this section is consistent with this practice.

[6] The draws were performed as follows. Let $\mathbf{u}_t^*$ denote a particular draw of the 29 error terms for quarter t from the $N(0, \hat{\Sigma})$ distribution. $\hat{\Sigma}$ was factored into PP', and then $\mathbf{u}_t^*$ was computed as $P\mathbf{e}$, where $\mathbf{e}$ is a 29×1 vector of standard normal draws. Since $E\mathbf{ee}' = I$, then $E\mathbf{u}_t^*\mathbf{u}_t^{*\prime} = EP\mathbf{ee}'P' = \hat{\Sigma}$, which is as desired. $\mathbf{u}_t^*$ was drawn for each of the eight quarters. Let $\boldsymbol{\alpha}^*$ denote a particular draw of the coefficients from the $N(\hat{\boldsymbol{\alpha}}, \hat{V})$ distribution. $\hat{V}$ was factored into PP', and then $\boldsymbol{\alpha}^*$ was computed as $\hat{\boldsymbol{\alpha}} + P\mathbf{e}$, where $\mathbf{e}$ is a vector of standard normal draws with dimension equal to the number of coefficients. Since $E\mathbf{ee}' = I$, then $E(\boldsymbol{\alpha}^* - \hat{\boldsymbol{\alpha}})(\hat{\boldsymbol{\alpha}} - \hat{\boldsymbol{\alpha}})' = EP\mathbf{ee}'P' = \hat{V}$, which is as desired.

[7] The number of trials for the other 43 stochastic simulations was also 25. For a few of the 25×44 trials the solution algorithm failed, and in these cases the trials were merely skipped. This procedure is likely to introduce a downward bias in the stochastic-simulation estimates of the variances, since the algorithm presumably failed because of an extreme draw. In most, if not all, of these cases it is likely that a solution could have been obtained had the solution algorithm been appropriately damped. The number of failures was fairly small, and so this bias is not likely to be very large.

[8] For variables with trends the assumption is that the mean of d_{itk} is proportional to $\bar{\tilde{y}}^2_{itk}$. See Note 3.

[9] For this set of estimates the covariance matrix of the 2SLS coefficient estimates was not taken to be block diagonal, but was instead estimated using formula (4), p. 273 in Fair and Parke (1980).

[10] See Fair (1980a), pp. 374–375, for a detailed discussion of the procedure that was used to draw the exogenous-variable values.

[11] In this case no t subscript is really needed for $\hat{d}_{itk}$, since it is not a function of t.

[12] For variables with trends the mean of the $d_{itk}/\bar{\tilde{y}}^2_{itk}$ values was calculated (for a given i and k) and this mean was added to $\tilde{\sigma}^2_{itk}/\bar{\tilde{y}}^2_{itk}$ to yield a value of $\hat{\sigma}^2_{itk}/\bar{\tilde{y}}^2_{itk}$. The d-row values in Table 11.2 for these variables are the square roots of $\hat{\sigma}^2_{itk}/\bar{\tilde{y}}^2_{itk}$.

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