Does the NAIRU Have the Right Dynamics?

By RAY C. FAIR*

The "NAIRU" view of the relationship between inflation and the unemployment rate is that there is a value of the unemployment rate (the NAIRU) below which the price level forever accelerates and above which the price level forever decelerates.[1] This view imposes two important restrictions on the dynamics of the price process. This can be seen by examining a simple version of the NAIRU equation:

$$(1)\quad \pi_t - \pi_{t-1} = \beta(u_t - u^*) + \gamma s_t + \varepsilon_t$$

$$\beta < 0 \quad \gamma > 0$$

where t is the time period, π_t is the rate of inflation, u_t is the unemployment rate, s_t is a cost shock variable, ε_t is an error term, and u^* is the NAIRU. If u_t equals u^* for all t, the rate of inflation will not change over time aside from the short-run effects of s_t and ε_t (assuming s_t and ε_t have zero means). Otherwise, the rate of inflation will increase over time (the price level will accelerate) if u_t is less than u^* for all t and will decrease over time (the price level will decelerate) if u_t is greater than u^* for all t.

Let p_t be the log of the price level for period t, and let π_t be measured as $p_t - p_{t-1}$. Using this notation, equation (1) can be written in terms of p rather than π:

$$(2)\quad p_t = 2p_{t-1} - p_{t-2} + \beta(u_t - u^*) + \gamma s_t + \varepsilon_t.$$

In other words, equation (1) can be written in terms of the current and past two price levels, with restrictions on the coefficients of the past two price levels. ("Price level" will be used to describe p even though p is actually the log of the price level.)

If equation (1) is correctly specified, adding p_{t-1} and p_{t-2} to it should not result in a significant increase in fit. Put another way, in equation (2) the joint hypothesis that the coefficient of p_{t-1} is 2 and the coefficient of p_{t-2} is -1 should not be rejected. In previous work (Fair, 1999) I have performed this test for a variety of specifications, and the results are generally not supportive of the NAIRU dynamics. The results of some of these tests are discussed in the following section.

I. Tests of the NAIRU Dynamics

To give the NAIRU specification the benefit of the doubt, a more general version than (1) is used as the base equation. This version is

$$(3)\quad \pi_t = \alpha + \sum_{i=1}^{12} \delta_i \pi_{t-i} + \sum_{i=0}^{3} \beta_i u_{t-i} + \sum_{i=0}^{3} \gamma_i s_{t-i} + \varepsilon_t \qquad \sum_{i=1}^{12} \delta_i = 1.$$

For the above specification, the NAIRU is $-\alpha/\Sigma_{i=0}^{3} \beta_i$. If the unemployment rate is always equal to this value, the inflation rate will be constant in the long run, aside from the short-run effects of s_t and ε_t. Using more than one lag for the variables lessens the chance that the results depend on a particular choice of lags.

Many estimates of equations like (3) use the GDP deflator as the measure of the price level. Other popular measures are the consumer price index (CPI) and the personal consumption deflator (PCD). Robert J. Gordon (1997), for example, uses all three. If, however, the aim is to measure prices set by U.S. firms, none of these measures seems very good. The GDP deflator includes prices of

* Cowles Foundation, Yale University, New Haven, CT 06520-8281 (e-mail: ray.fair@yale.edu; website: ⟨http://fairmodel.econ.yale.edu⟩). All the data used in this paper can be downloaded from the website.

[1] Recent studies using the NAIRU specification include Jeffrey C. Fuhrer (1995), George A. Akerlof et al. (1996), Robert J. Gordon (1997), and Douglas Staiger et al. (1997a, b).

government output and indirect business taxes, for example, which are clearly not decision variables of firms. The CPI and PCD are to some extent even worse, since they include import prices in addition to indirect business taxes.

The price variable used here is a business nonfarm price deflator, denoted PNF. Let YY be nominal business nonfarm output (National Income and Product Accounts [NIPA], table 1.7, line 3), let IBT be total indirect business taxes (NIPA, table 3.1, line 4), and let Y be business nonfarm output in 1992 dollars (NIPA, table 1.8, line 3). Then PNF is defined to be (YY − IBT)/Y. PNF is net of indirect business taxes, farm output, government output, and imports.

The civilian unemployment rate is used for the unemployment rate. The cost-shock variable s_t is taken to be the deviation of the log of the import price deflator from a trend line: $s_t = \text{pm}_t - \tau_0 - \tau_1 t$, where pm is the log of the import price deflator. The import price deflator is the ratio of nominal imports (NIPA, table 1.1, line 17) to imports in 1992 dollars (NIPA, table 1.2, line 17). The data are quarterly and were collected for the 1952:1–1998:1 period.

Given the assumption about s_t and the restriction that the δ_i's sum to 1, equation (3) is estimated in the following form:

$$(4)\quad \Delta\pi_t = \lambda_0 + \lambda_1 t + \sum_{i=1}^{11} \theta_i \Delta\pi_{t-i} + \sum_{i=1}^{3} \beta_i u_{t-i} + \sum_{i=1}^{3} \gamma_i \text{pm}_{t-i} + \varepsilon_t$$

where $\lambda_0 = \alpha + (\gamma_1 + \gamma_2 + \gamma_3)\tau_0 + (\gamma_1 + 2\gamma_2 + 3\gamma_3)\tau_1$ and $\lambda_1 = (\gamma_1 + \gamma_2 + \gamma_3)\tau_1$; α and τ_0 are not identified in equation (4), but for purposes of the tests this does not matter. If, however, one wanted to compute the NAIRU (i.e., $-\alpha/\Sigma_{i=1}^{3}\beta_i$), one would need a separate estimate of τ_0 in order to estimate α.[2] The procedure of interest here is to add p_{t-1} and p_{t-2} to equation (4) and test whether they are jointly significant.[3]

The estimation period for the tests is 1955:3–1998:1. When p_{t-1} and p_{t-2} are added to equation (4), their t statistics are −5.47 and 5.45, respectively, and the X^2 statistic for the hypothesis that the coefficients of both variables are zero is 26.28. The 5-percent critical χ^2 value for two degrees of freedom is 5.99 and the 1-percent critical value is 9.21. Thus, if the χ^2 distribution is a good approximation to the actual distribution, the two variables are highly significant, and thus the NAIRU dynamics are strongly rejected.

If equation (4) is in fact the way the price data are generated, the χ^2 distribution may not be a good approximation for the test because of possible unit-root problems. Fortunately, this can be checked by computing the "exact" distribution. This is done as follows. First, estimate equation (4), and record the coefficient estimates and the estimated variance of the error term. Call this the "base" equation. Assume that the error term is normally distributed with mean zero and variance equal to the estimated variance. Then:

(i) Draw a value of the error term for each quarter. Add these error terms to the base equation and solve it dynamically to generate new data for p. Given the new data for p and the data for u and pm (which

[2] The present specification assumes that the NAIRU is constant, although if the NAIRU had a trend, this would be absorbed in the estimate of the coefficient of the time trend in equation (4) and would change the interpretation of λ_1. In recent work, Gordon (1997) has argued that the NAIRU may be time-varying, and in future work it may be interesting to consider this case as well as other cases in which the NAIRU is postulated to change over time. There is, however, a danger in providing so much flexibility to the NAIRU that the theory becomes vacuous.

[3] It should be noted that this test is not a test of a particular expectations hypothesis. Say that expected inflation (denoted π_t^e) is equal to $\Sigma_{i=1}^{12}\rho_i\pi_{t-i}$ and that this variable enters equation (3) as $\alpha_1\pi_t^e$ in place of the second term on the right-hand side. Many years ago Thomas J. Sargent (1971) pointed out that estimating this type of equation cannot distinguish between the case where α_1 is 1 and the ρ_i's sum to less than 1 and the case where α_1 is less than 1 and the ρ_i's sum to 1. This paper is not concerned with discriminating between these two cases. The test here is simply to examine whether the specification in (3) is a good approximation of the data.

have not changed), compute and record the X^2 value.

(ii) Do step (i) 1,000 times, which gives 1,000 X^2 values. The distribution of these values is the "exact" distribution.

(iii) Sort the X^2 values by size, choose the value above which 5 percent of the values lie and the value above which 1 percent of the values lie. These are the 5-percent and 1-percent critical values, respectively.

These calculations were done; the 5-percent critical value was 18.10 and the 1-percent critical value was 24.91. These values are considerably larger than the critical values from the actual χ^2 distribution [which is as expected if equation (4) is the actual data-generating process], but they are still smaller than the computed value of 26.28. The two price variables are thus significant at the 99-percent confidence level, even using the alternative values.

This procedure treats u and pm as exogenous, and it may be that the estimated critical values are sensitive to this treatment. To check for this, the following two equations were postulated for u and pm:

$$(5)\quad \mathrm{pm}_t = a_1 + a_2 t + a_3 \mathrm{pm}_{t-1} + a_4 \mathrm{pm}_{t-2} + a_5 \mathrm{pm}_{t-3} + a_6 \mathrm{pm}_{t-4} + \nu_t$$

$$(6)\quad u_t = b_1 + b_2 t + b_3 u_{t-1} + b_4 u_{t-2} + b_5 u_{t-3} + b_6 u_{t-4} + b_7 \mathrm{pm}_{t-1} + b_8 \mathrm{pm}_{t-2} + b_9 \mathrm{pm}_{t-3} + b_{10} \mathrm{pm}_{t-4} + \eta_t.$$

These two equations along with equation (4) were taken to be the "model," and they were estimated along with equation (4) to get the "base" model. The error terms ε_t, ν_t, and η_t were then assumed to be multivariate normal with mean zero and covariance matrix equal to the estimated covariance matrix (obtained from the estimated error terms). Each trial then consisted of draws of the three error terms for each quarter and a dynamic simulation of the model to generate new data for p, pm, and u, from which the X^2 value was computed. The computed critical values were not very sensitive to this treatment of pm and u, and they actually fell slightly. The 5-percent value is 15.01 compared to 18.10 above, and the 1-percent value is 19.19 compared to 24.91 above.

To examine the sensitivity of the results to the use of 12 lags in equation (3), the test was done using 24 lags rather than 12. For this test, the estimation period began in 1958:3 rather than 1955:3. The X^2 value was 21.94 with computed 5- and 1-percent critical values of 12.41 and 17.30 (treating pm and u as endogenous for purposes of computing the critical values). The results are thus not sensitive to the use of more lags.

The results are somewhat sensitive to the use of other price measures. When the GDP deflator is used (with 12 lags), the X^2 value is 16.06 with computed 5- and 1-percent critical values of 12.48 and 17.55. In this case the two price variables are significant at the 95-percent confidence level but not the 99-percent level. When the overall CPI is used, the X^2 value is 10.24 with computed 5- and 1-percent critical values of 12.13 and 18.23. In this case the two price variables are not significant at even the 95-percent level. When the CPI excluding food and energy (the "core" CPI) is used, the X^2 value is 16.32 with computed 5- and 1-percent critical values of 14.05 and 18.86. In this case, as in the case using the GDP deflator, the two price variables are significant at the 95- but not the 99-percent level.

The choice of the price measure is thus somewhat important for purposes of the test. As argued above, the business nonfarm price deflator has the advantage over the GDP deflator of not including prices of government output and indirect business taxes. It has the advantage over the CPI of not including import prices and indirect business taxes. It thus seems that much less weight should be put on the results using the GDP deflator and the CPI, but even for these measures the two price variables are significant at the 95-percent level except for the overall CPI.[4]

[4] This general rejection of the NAIRU dynamics may help explain the results in Staiger et al. (1997b). Using a

II. Properties

How much difference does it make if p_{t-1} and p_{t-2} are added to equation (4)? If, say, the unemployment rate were permanently lowered by one percentage point, what would the two equations say are the price consequences? To answer this, the following experiment was performed using equation (4) and then equation (4) with p_{t-1} and p_{t-2} added. A dynamic simulation was run beginning in 1998:2 using the actual values of all the variables from 1998:1 back. The values of u from 1998:2 on were taken to be the actual value for 1998:1; pm was assumed to grow at a 2-percent annual rate from 1998:2 on. Call this simulation the "base" simulation. A second dynamic simulation was then run in which the only change was that the unemployment rate was decreased permanently by one percentage point from 1998:2 on. The difference between the predicted value of p from this simulation and that from the base simulation for a given quarter is the estimated effect of the change in u on p.[5]

For comparison purposes, one other result was obtained. Equation (4) was estimated with π_{t-1} ($=p_{t-1}-p_{t-2}$) added (not p_{t-1} and p_{t-2} separately). This equation is like an "old-fashioned" Phillips curve. When this version is estimated, the δ_i coefficients sum to 0.846.[6] The above experiment was also performed for this version.

TABLE 1—EFFECTS OF A ONE-PERCENTAGE-POINT FALL IN u

	Equation (4)		Equation (4), π_{t-1} added		Equation (4), p_{t-1} and p_{t-2} added	
Quarter	$\frac{P^{new}}{P^{base}}$	$\pi^{new}-\pi^{base}$	$\frac{P^{new}}{P^{base}}$	$\pi^{new}-\pi^{base}$	$\frac{P^{new}}{P^{base}}$	$\pi^{new}-\pi^{base}$
1	1.0000	0.00	1.0000	0.00	1.0000	0.00
2	1.0027	1.10	1.0025	0.99	1.0025	0.99
3	1.0036	0.32	1.0030	0.20	1.0024	−0.03
4	1.0052	0.66	1.0043	0.51	1.0038	0.57
5	1.0073	0.84	1.0059	0.65	1.0055	0.67
6	1.0094	0.83	1.0074	0.61	1.0069	0.56
7	1.0118	0.95	1.0092	0.70	1.0085	0.62
8	1.0142	0.93	1.0109	0.66	1.0098	0.51
9	1.0170	1.12	1.0129	0.81	1.0113	0.62
10	1.0198	1.10	1.0149	0.77	1.0125	0.48
11	1.0229	1.20	1.0170	0.84	1.0138	0.50
12	1.0259	1.20	1.0191	0.80	1.0148	0.41
40	1.2238	3.65	1.1174	1.59	1.0320	0.06
∞	∞	∞	∞	1.95	1.0330	0.00

Notes: P = price level, $p_t = 400 \log (P_t)$, $\pi_t = p_t - p_{t-1}$.

It should be stressed that this experiment is not meant to be realistic. For example, it is unlikely that the Fed would allow a permanent fall in u to take place as p rose. This experiment is simply meant to help illustrate how the long-run properties of the equations differ when the unemployment rate is held constant.

The results for the three experiments are presented in Table 1. Consider the very long-run properties first. For equation (4), the NAIRU specification, the new price level grows without bounds relative to the base price level, and the new inflation rate grows without bounds relative to the base inflation rate. For equation (4) with π_{t-1} added, the new price level grows without bounds relative to the base, but the inflation rate does not. It is 1.95 percentage points higher in the long run. For equation (4) with p_{t-1} and p_{t-2} added, the new price level is higher by 3.30 percent in the limit, and the new inflation rate is back to the base. The long-run properties are thus vastly different, as is, of course, obvious from the specifications. What is interesting, however, is that the effects are fairly close for the first few quarters. One would be hard pressed to choose among the equations on the basis of which short-run implications seem more "reasonable."

standard NAIRU specification, they find that estimated variances of NAIRU estimates are very large. This is not surprising if the NAIRU specification is misspecified, as the present results suggest.

[5] Because the equations are linear, it actually does not matter what values are used for pm as long as the same values are used for both simulations. Similarly, it does not matter what values are used for u as long as each value for the second simulation is one percentage point lower than the corresponding value for the base simulation.

[6] When π_{t-1} is added to equation (4) the X^2 value is 5.21 with computed (as in the above manner) 5- and 1-percent critical values of 9.18 and 14.11, respectively. Thus, π_{t-1} is not significant at even the 5-percent level, even though the summation seems substantially less than 1. When p_{t-1} is added to the equation with π_{t-1} already added, the X^2 value is 20.40 with computed 5- and 1-percent critical values of 13.56 and 18.21, respectively. Thus, p_{t-1} is highly significant when added to the equation without the summation restriction imposed.

III. An Alternative View

If the NAIRU specification is rejected, this changes the way one needs to think about the relationship between inflation and unemployment. One should not think that there is some unemployment rate below which the price level forever accelerates and above which it forever decelerates. On the other hand, equation (4) with p_{t-1} and p_{t-2} added is not a sensible alternative. This specification implies that a lowering of the unemployment rate has only a modest long-run effect on the price level regardless of how low the initial value of the unemployment rate is. For example, the results in Table 1 for this specification are independent of the initial value of the unemployment rate.

A weakness of all the above specifications (in my view) is the linearity assumption regarding the effects of u on p. It seems likely that there is a strongly nonlinear relationship between the price level and the unemployment rate at low levels of the unemployment rate. One possible specification, for example, would be to replace u with $1/(u - 0.02)$ in, say, equation (4) with p_{t-1} and p_{t-2} added. In this case as u approaches 0.02, the estimated effects on p become larger and larger. I have experimented with a variety of functional forms like this in estimating price equations to see whether the data can pick up a nonlinear relationship. Unfortunately, there are so few observations of very low unemployment rates that the data do not appear capable of discriminating among functional forms. A variety of functional forms, including the linear form, lead to very similar results.

The alternative view put forth here thus consists of two points, one supported by the data and one for which the data have little to say. The first point is that the NAIRU dynamics are not accurate, and the price process is better specified by not imposing the two NAIRU restrictions on it. The second point is that the relationship between the price level and the unemployment rate is nonlinear at low values of the unemployment rate.[7]

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[7] It is also my view that the price process is better modeled using structural price and wage equations than reduced-form price equations like in this paper. The results in Fair (1999) suggest that the structural approach leads to more accurate specifications.