Chapter Ten

Some Optimal Control Results

10.1 INTRODUCTION

Some results of obtaining optimal controls for the empirical model are presented in this chapter. It is now computationally feasible, as discussed in the next section, to obtain optimal controls for a model of the present size. Solving optimal control problems for a model is useful in the sense that one may gain insights into the properties of the model that one would not otherwise have obtained. It is also useful in allowing one to compare the historical record of the economy with the record that would have been achieved had some particular objective function been maximized instead.

In section 10.3 the results of solving six control problems are presented. Two problems are solved for the period of the Eisenhower Administrations, two for the period of the Kennedy-Johnson Administrations, and two for the period of the Nixon-Ford Administrations. The objective function for each problem targets, for each quarter, a given level of real output and a zero rate of inflation. The two problems for each period differ in the relative weights attached to the two targets. *XG* and *VBG* are used as the control variables for each problem.

The most important property of the model that is revealed from the work in this chapter is that the cost of increasing output (in terms of additional inflation generated) is generally much less than the cost of lowering the rate of inflation (in terms of lower output). The optima tend to correspond to the output targets being more closely met than the inflation targets. This property, if true of the real world, has important policy implications.

The optimal control problems that the government is assumed to be solving in this chapter should not be confused with the optimal control problems that the individual behavioral units are assumed to be solving in making their decisions. The government should be considered to be solving its control problem *subject to* the restriction that each behavioral unit in the economy takes as given the control values chosen by the government and solves its own control problem on the basis of these and other relevant values.

10.2 THE COMPUTATION OF THE OPTIMAL CONTROLS

The procedure that was used to solve the optimal control problems for the model is described in Fair [20]. This procedure is briefly as follows. Consider the model as represented by the equation system in (3.1) in Chapter Three:

$$\phi_g(y_{1t}, \ldots, y_{Gt}, x_{1t}, \ldots, x_{Nt}, \beta_g) = u_{gt}, \begin{matrix}(g = 1, \ldots, G)\\(t = 1, \ldots, T).\end{matrix} \tag{3.1}$$

Assume that the objective function, h, to be maximized is:

$$W = h(y_{11}, \ldots, y_{1T}, \ldots, y_{G1}, \ldots, y_{GT}, x_{11}, \ldots, x_{1T}, \ldots, x_{N1}, \ldots, x_{NT}). \tag{10.1}$$

Assume, finally, without loss of generality, that $x_{1t}(t = 1, \ldots, T)$ is the only control variable. Now, given a set of estimates of the β_g vectors and given values of the $x_{it}(i = 1, \ldots, N)$, the model in (3.1) can be solved numerically for the $y_{it}(i = 1, \ldots, G)$, after, say, all of the error terms have been set equal to zero.

Once the model has been solved for all T periods, the value of W in (10.1) can be computed. If lagged endogeneous variables are included among the x_{it} variables, they are merely updated in the usual way in the course of solving the model. Given a different set of values of the control variable, the model can be resolved and a new value of W computed. W can thus be considered to be an implicit function of the T control values:

$$W = \psi(x_{11}, \ldots, x_{1T}). \tag{10.2}$$

The optimal control problem set up in the above way is simply a standard nonlinear maximization problem, the problem of finding the T values of $x_{1t}(t = 1, 2, \ldots, T)$ for which W is at a maximum. Consequently, the maximization algorithms that were discussed in section 3.4 and that were used in the computation of the FIML estimates can be used to solve optimal control problems as well. All that one needs to do is to combine one of the algorithm programs with a program that solves the model. When using the maximization algorithms for this purpose, each function evaluation corresponds to solving the model once for T periods and then computing the value of W. If derivatives are needed for a particular algorithm, they can be computed numerically. Analytic derivatives are generally not available for this purpose because it is generally not possible to write the function ψ in (10.2) in analytic form. If there are two control variables, say x_{1t} and x_{2t}, then W in (10.2) is merely a function of both x_{1t} and $x_{2t}(t = 1, 2, \ldots, T)$.

The results in [20] indicate that it is possible to solve quite large control problems when they are set up in the above way. As mentioned in section 3.4, in one case a problem of 239 parameters was solved (four control variables for 60 periods, less one value that was known because the control variable entered the model with a lag of one period). Although the discussion so far has been in terms of solving deterministic control problems, some suggestions are also presented in [20] on how the above way of setting up the control problem might be used to solve stochastic control problems through the use of stochastic simulation. No attempt was made in this study, however, to solve any stochastic control problems.

The three control periods considered are 1953III–1960IV (30 quarters), 1961I–1968IV (32 quarters), and 1969I–1975I (25 quarters). The first period covers all the quarters of the two Eisenhower Administrations except for the first two quarters of the first Administration; the second period covers all of the quarters of the Kennedy-Johnson and Johnson Administrations; and the third period covers all of the quarters of the first Nixon Administration and the first nine quarters of the Nixon-Ford Administration. The first two quarters of the first Eisenhower Administration were not included in the first period because of a lack of enough earlier data.

The basic objective function that was used targets a given level of real output and a zero rate of inflation for each quarter. It is easiest to consider the objective function to be a loss function that is to be minimized. This loss function is:

$$L = \sum_{t=1}^{T} \left[\gamma \Big/ \frac{Y_t - Y_t^*}{Y_t^*} \Big/^2 + (\%\Delta PF_t)^2 \right], \gamma > 0, \tag{10.3}$$

where Y_t^* = target level of Y_t,

$$\Big/ \frac{Y_t - Y_t^*}{Y_t^*} \Big/^2 = \begin{cases} \left(\dfrac{Y_t - Y_t^*}{Y_t^*}\right)^2 & \text{if } \quad Y_t < Y_t^* \\ 0 & \text{if } Y_t \geqslant Y_t^*, \end{cases}$$

$$\%\Delta PF_t = \left(\frac{PF_t}{PF_{t-1}}\right)^4 - 1, \text{ (percentage change in } PF_t \text{ at an annual rate).}$$

The loss function penalizes rates of inflation that are both above and below the target value of zero, but it only penalizes values of Y_t that are below the target. A straight quadratic function in (10.3) would also penalize values of Y_t that are above the target. There is nothing in the present way of solving control problems that requires that the objective function be quadratic, and the specification in (10.3) seems more reasonable than a straight quadratic specification. There is also nothing in the present way of solving control problems that requires that the objective function be a sum over separate time periods, although the function in (10.3) is.

The target values for real output were computed as follows. Four quarters were first chosen as benchmark quarters: 1953IV, 1957I, 1965IV, and 1973IV. The unemployment rates in these four quarters were 3.7, 4.0, 4.1, and 4.7 percent, respectively. The four quarters are quarters in which there were high levels of economic activity. One may question whether the level of economic activity in 1973IV was as high as the levels in the other three benchmark quarters, but for present purposes it is assumed to be so.

The target value of output in each of these quarters was taken to be the actual value. The target values for the other quarters were then taken to lie on straight lines between the four benchmark values. The line between 1953IV and 1957I was extended backward to get a value for 1953III, and the line between 1965IV and 1973IV was extended forward to get values for 1974I–1975I. The target values are presented in Tables 10–1, 10–2, and 10–3, below. There are 20 quarters in the 1953III–1975I period in which the actual value of output is greater than the target value.

Two variables of the government were used as control variables, *XG* and *VBG*. In order to lessen computational costs, it turned out to be convenient to have *VBG* be adjusted each quarter so as to achieve a given target level of the bill rate. The target bill rate series is a series that has a positive trend between 1953II and 1970IV and then is flat (at 6.3 percent) from 1971I on. The values for the series between 1953II and 1970IV were taken to be the predicted values from the regression of log $RBILL_t$ on a constant and t for the 1952I–1970IV period. This is the same regression that is used in the construction of $RBILL_t^*$ in the model. (See Equation 79 in Table 2–2.)

The treatment of *VBG* in this way means that monetary policy is assumed to be accommodating in the sense of always achieving the given target level of the bill rate each quarter regardless of the value of *XG* chosen. Although *XG* is the only fiscal policy variable used, the following results would not be changed very much if more than one variable were used. Given that the objective function targets only real output and the rate of inflation, adding, say, a tax rate variable such as d_3 as a control variable would have little effect on decreasing the loss from the minimum loss that can be achieved by using *XG* alone. The fiscal policy variables are collinear in this sense.

As mentioned above, only deterministic control problems have been solved here. A standard procedure in solving deterministic control problems with a stochastic model is to set all the error terms in the model equal to their expected values, usually zero. An alternative procedure, however, is to set the error terms equal to their historic values, i.e., to their estimated values in the sample period, and this is the procedure followed here. Setting the error terms equal to their historic values means that when the model is solved using the actual values of the exogenous variables, the solution values of the endogenous variables are just the actual values.

In order to justify the procedure of setting the error terms equal to their historic values, consider how an administration would behave in practice if it could only solve deterministic control problems. Since an administration has plenty of time each quarter to reoptimize, it could solve a series of control problems, one each quarter, where each problem would be based on setting the future error terms equal to zero. The solution of each problem would result in optimal paths of the control variables, but only the values of the control variables for the first quarter for each problem would actually be carried out. As the administration reoptimized each quarter, it would adjust to the errors of the previous quarter by using in its solution the actual values of the endogenous variables of the previous quarter.

If more computer time had been available for this project, a series of control problems could have been solved for each of the three periods considered. All the problems would have been based on setting the future error terms equal to zero. The first problem would start in the first quarter and would take as given all the values of the endogenous variables up to, but not including, the first quarter. The optimal values of the endogenous and control variables for the first quarter that result from solving this problem would be recorded.

The second problem would start in the second quarter, would use as the first quarter value of each control variable the optimal value just recorded, and would use as the first quarter value of each endogenous variable the optimal value just recorded *plus* the historic value of the error term that pertained to the particular variable in question. The optimal values of the endogenous and control variables for the second quarter that result from solving the second problem would be recorded. This procedure would be repeated for the remaining problems. The recorded series of each control variable would then be taken to be the optimal series. These are series that an administration could have computed had it had the present model at its disposal and had it known all of the values of the noncontrolled exogenous variables.

Since it was not feasible to solve a series of problems for each of the three periods considered, some approximation to the set of solutions that would result from such an exercise had to be made. The procedure of setting the error terms equal to their historic values before solving assumes that an administration has more knowledge than it actually has. An administration clearly does not know all future values of the error terms. The procedure of setting the error terms equal to their expected values before solving (and solving only once), on the other hand, assumes that an administration has less knowledge than it actually has because it can continually adjust to past error terms by reoptimizing each quarter. The procedure of setting the error terms equal to their historic values was chosen on the grounds that it seemed likely to lead to a set of optimal values that more closely approximates the preferred set.

The control problems were solved using the gradient algorithm mentioned in section 3.4. The gradient algorithm turned out to be cheaper to use and more adept at decreasing the value of the loss function than was Powell's no-derivative algorithm. This is in contrast to the case for the FIML problem, where Powell's no-derivative algorithm worked better. All derivatives for the gradient algorithm were obtained numerically. For the first period of 30 quarters, there are 60 values to determine altogether, 30 for *XG* and 30 for *VBG*. The values for *VBG* are, however, quite easy to compute, since they are merely the ones necessary, given the values for *XG*, to have the bill rate be equal to its target value each quarter. For purposes of solving the control problems, *VBG* is effectively an endogenous variable and the bill rate is an exogenous variable. This means that there are really only the 30 values of *XG* that the algorithm has to determine for the first period. For the second period there are 32 values of *XG* to determine, and for the third period there are 25 values to determine.

For the algorithm the starting values of *XG* were not taken to be the historic values, as is commonly done. Instead, the values of *XG* that led to the output target's being met exactly were used as starting values. These values were obtained by treating *Y* as an exogenous variable (the values of this variable being equal to the target values) and *XG* as an endogenous variable and solving the model. For all three periods, the values of *XG* that led to the output target's being met exactly resulted in a smaller value of loss than did the historic values of *XG* and so were better starting points.

It was mentioned in section 3.5 that the time needed to solve the model once for an 82-quarter period is about ten seconds. This is for the version of the model in which the bill rate is taken to be endogenous. When the bill rate is taken to be exogenous, as for the work in this chapter, the model is somewhat easier to solve. The time needed to solve the model once for the 30-quarter period considered in this chapter, for example, is about two seconds, rather than about four seconds for the endogenous bill rate case.

The gradient algorithm converged in about five iterations for each problem. Each iteration corresponded to about 50 function evaluations—i.e., 50 solutions of the model for the 30-, 32-, or 25-quarter period. The gradient algorithm thus required about 250 function evaluations to converge, which at roughly two seconds per function evaluation is about eight minutes of computer time on the IBM 370–158 at Yale. It should be stressed that there is no guarantee that the algorithm actually found the true optimum in each case. Cost considerations prevented very much experimentation to see if the true optima had been found.

10.3 THE RESULTS

The results of solving the six control problems are presented in Tables 10–1, 10–2, and 10–3. For the first problem for each period a value of γ in (10.3) of 1.0 was used, and for the second problem for each period a value of γ of 0.1 was used. γ is the weight attached to the output target in the loss function. The weight attached to the output target is thus ten times greater for the first problem than for the second.

The following is a brief summary of the results in the three tables:

	Actual	Optimal for $\gamma = 1.0$	Optimal for $\gamma = 0.1$
Table 10–1 (Sum of Y^* over all 30 quarters = 3076.0)			
1. Sum of Y over all 30 quarters	2995.6	3071.3	3028.0
2. Average rate of inflation over the 30 quarters (annual rate)	1.92%	2.03%	1.92%
3. Average unemployment rate over the 30 quarters	5.07	4.68	5.01
Table 10–2 (Sum of Y^* over all 32 quarters = 4445.9)			
1. Sum of Y over all 32 quarters	4328.2	4438.1	4379.4
2. Average rate of inflation over the 32 quarters (annual rate)	1.94%	2.13%	2.04%
3. Average unemployment rate over the 32 quarters	4.86	4.87	5.16
Table 10–3 (Sum of Y^* over all 25 quarters = 4507.7)			
1. Sum of Y over all 25 quarters	4363.5	4482.8	4365.1
2. Average rate of inflation over the 25 quarters (annual rate)	5.97%	6.22%	6.04%
3. Average unemployment rate over the 25 quarters	5.22	4.70	5.35

The summary results for Table 10–1 show that for $\gamma = 0.1$ the optimal average rate of inflation over the 30 quarters is the same as the actual rate. The optimal amount of output for the 30 quarters is, however, larger than the actual amount, and the optimal average unemployment rate is lower

Table 10–1. Control Results for the Eisenhower Administrations

Quarter	*Actual Values*						*Optimal Values for* $\gamma = 1.0$					*Optimal Values for* $\gamma = 0.1$				
	Y	100 $\%\Delta PF$	100 UR	$RBILL$	Target $RBILL$	Y^*	ΔXG	ΔVBG	Y	100 $\%\Delta PF$	100 UR	ΔXG	ΔVBG	Y	100 $\%\Delta PF$	100 UR
1953III	91.4	1.4	2.8	2.0	1.6	89.2	−1.6	−0.7	89.9	0.2	3.1	−1.8	−0.8	89.7	0.1	3.1
IV	90.0	0.1	3.7	1.5	1.6	90.0	0.4	−0.3	90.1	0.5	4.0	−0.4	−0.6	89.3	0.4	4.2
1954I	89.0	4.8	5.3	1.1	1.7	90.9	1.6	0.7	90.6	5.9	5.1	0.7	0.1	89.3	5.8	5.4
II	88.9	0.3	5.8	0.8	1.7	91.7	2.3	1.7	91.6	1.8	5.1	1.4	1.0	90.0	1.7	5.6
III	90.1	1.0	6.0	0.9	1.7	92.5	1.8	2.4	92.4	2.3	5.3	0.9	1.6	90.6	2.2	5.8
IV	92.3	2.0	5.4	1.0	1.8	93.4	1.0	2.7	93.4	3.0	5.1	0.2	1.9	91.5	2.8	5.6
1955I	95.5	0.6	4.7	1.3	1.8	94.2	−0.3	2.8	94.4	1.1	5.1	−1.0	1.9	92.5	1.0	5.6
II	97.3	−0.0	4.4	1.6	1.8	95.1	−0.3	3.1	95.2	−0.0	5.3	−1.0	2.3	93.3	−0.0	5.8
III	98.9	3.0	4.2	1.9	1.9	96.0	−0.8	3.5	95.8	2.7	5.4	−1.4	2.6	94.1	2.6	5.8
IV	99.9	4.1	4.2	2.3	1.9	96.9	−0.8	3.7	96.7	3.4	5.7	−1.3	2.8	95.1	3.3	6.0
1956I	99.3	3.1	4.1	2.4	2.0	97.7	0.5	4.7	97.5	2.5	5.2	−0.0	3.7	96.1	2.4	5.6
II	99.5	2.6	4.2	2.6	2.0	98.6	0.2	5.1	98.3	2.0	5.0	−0.2	4.0	96.9	1.8	5.3
III	99.3	3.9	4.2	2.6	2.0	99.6	0.6	5.7	98.9	3.4	4.6	0.1	4.4	97.7	3.2	4.9
IV	100.8	3.9	4.1	3.1	2.1	100.5	−0.6	5.2	99.8	3.0	4.5	−1.1	3.9	98.5	2.9	4.8

1957I	101.4	5.9	4.0	3.2	2.1	101.4	−0.4	5.0	100.9	5.0	4.2	−0.7	3.6	99.7	4.8	4.6
II	101.3	1.5	4.1	3.2	2.2	102.4	0.6	5.0	102.2	0.8	4.0	0.3	3.6	101.1	0.7	4.3
III	101.7	2.4	4 2	3 4	2.2	103.4	0.4	4.6	103.2	1.8	3.8	0.1	3.2	102.2	1.7	4.1
IV	99.9	2.5	5.0	3.3	2.2	104.5	2.4	5.0	104.3	2.2	3.9	2.1	3.5	103.1	2.0	4.2
1958I	97.2	0.9	6.3	1.8	2.3	105.5	4.2	5.9	105.4	1.9	4.4	3.9	4.4	104.0	1.8	4.7
II	97.6	0.3	7.4	1.0	2.3	106.6	3.4	5.9	106.4	2.1	5.1	3.1	4.3	105.0	2.0	5.4
III	100.3	2.1	7.3	1.7	2.4	107.6	1.7	4.7	107.4	2.3	5.6	1.4	3.1	105.9	2.3	5.9
IV	103.0	2.2	6.4	2.8	2.4	108.7	1.1	3.5	108.5	2.0	5.4	0.8	1.9	107.0	2.0	5.6
1959I	104.7	2.5	5.8	2.8	2.5	109.8	1.0	2.8	109.6	2.7	5.2	0.8	1.3	108.0	2.7	5.5
II	107.6	1.5	5.1	3.0	2.5	110.9	0.1	2.1	110.8	1.5	4.9	−0.1	0.6	109.3	1.4	5.1
III	105.9	1.5	5.3	3.5	2.6	112.0	3.2	3.0	112.0	1.4	4.5	3.0	1.6	110.5	1.3	4.8
IV	107.4	0.7	5.6	4.3	2.6	113.1	1.3	2.2	113.1	0.4	4.5	1.1	0.8	111.5	0.3	4.8
1960I	109.8	0.6	5.2	3.9	2.7	114.2	−0.6	0.3	114.1	0.6	4.1	−0.9	−1.0	112.3	0.5	4.5
II	109.3	1.2	5.3	3.1	2.7	115.4	1.3	−0.1	115.1	1.6	4.1	1.1	−1.3	113.3	1.5	4.5
III	108.7	0.1	5.6	2.4	2.8	116.5	2.7	0.2	116.2	1.1	4.0	2.5	−1.0	114.6	0.9	4.4
IV	107.7	1.3	6.3	2.4	2.8	117.7	4.3	1.0	117.5	2.2	4.1	4.1	−0.2	115.9	2.0	4.5

Notes: ΔXG = difference between the optimal and actual values of *XG*.
ΔVBG = difference between the optimal and actual values of *VBG*.
For both problems the optimal bill rate series is the target *RBILL* series.

Table 10–2. Control Results for the Kennedy-Johnson Administrations

Quarter	Actual Values						Optimal Values for $\gamma = 1.0$					Optimal Values for $\gamma = 0.1$				
	Y	100 $\%\Delta PF$	100 UR	$RBILL$	Target $RBILL$	Y^*	ΔXG	ΔVBG	Y	100 $\%\Delta PF$	100 UR	ΔXG	ΔVBG	Y	100 $\%\Delta PF$	100 UR
1961 I	107.3	0.7	6.8	2.4	2.9	118.9	10.3	5.2	118.8	1.5	5.3	9.9	5.0	118.4	1.5	5.4
II	109.9	0.7	7.0	2.3	3.0	120.1	3.8	4.3	119.9	1.7	4.6	3.0	3.8	118.9	1.6	4.8
III	112.0	−0.5	6.8	2.3	3.0	121.3	3.5	3.9	121.1	0.4	4.8	2.6	3.1	119.7	0.3	5.1
IV	114.3	2.0	6.2	2.5	3.1	122.5	2.8	3.5	122.2	2.7	5.0	1.8	2.4	120.6	2.6	5.4
1962 I	116.1	0.5	5.6	2.7	3.1	123.7	3.3	4.0	123.5	1.1	4.7	2.5	2.8	121.9	1.0	5.2
II	118.0	1.3	5.5	2.7	3.2	124.9	3.2	4.7	124.7	2.0	4.7	2.4	3.4	123.4	1.9	5.1
III	119.2	0.2	5.6	2.9	3.3	126.2	3.3	5.7	126.0	0.9	4.8	2.7	4.2	124.7	0.8	5.1
IV	120.6	1.1	5.5	2.8	3.3	127.5	2.8	6.1	127.2	1.8	4.8	2.0	4.4	125.7	1.7	5.1
1963 I	121.3	0.9	5.8	2.9	3.4	128.7	3.4	7.0	128.5	1.5	5.2	2.7	5.2	126.9	1.4	5.5
II	122.4	2.0	5.7	2.9	3.5	130.0	3.4	7.9	129.8	2.7	5.0	2.7	5.9	128.2	2.6	5.3
III	124.5	0.4	5.5	3.3	3.5	131.3	2.4	8.1	131.1	0.8	4.9	1.8	6.0	129.5	0.7	5.2
IV	126.3	1.7	5.6	3.5	3.6	132.6	2.1	8.2	132.4	2.0	5.1	1.3	6.0	130.7	1.9	5.4
1964 I	128.4	1.4	5.5	3.5	3.7	134.0	1.1	7.8	133.7	1.7	5.3	0.4	5.5	131.8	1.6	5.6
II	130.2	1.4	5.3	3.5	3.8	135.3	1.2	8.0	135.0	1.7	5.1	0.5	5.6	133.1	1.6	5.5
III	131.8	1.7	5.0	3.5	3.8	136.6	1.0	8.1	136.4	2.0	5.1	0.5	5.8	134.5	1.9	5.4
IV	132.5	1.3	5.0	3.7	3.9	138.0	1.9	8.9	137.8	1.6	5.0	1.4	6.6	136.1	1.5	5.2

1965I	135.9	1.1	4.9	3.9	4.0	139.4	−0.3	8.0	139.2	1.2	5.0	−0.8	5.8	137.5	1.1	5.2
II	137.8	2.0	4.7	3.9	4.1	140.8	0.1	8.0	140.6	2.1	5.0	−0.3	5.8	139.1	2.1	5.2
III	140.5	1.3	4.4	3.9	4.1	142.2	−1.1	7.3	142.0	1.4	4.8	−1.4	5.1	140.5	1.3	5.0
IV	143.6	1.7	4.1	4.2	4.2	143.6	−2.2	6.3	143.4	1.5	4.9	−2.6	4.0	141.7	1.4	5.1
1966I	146.6	2.6	3.9	4.6	4.3	144.9	−3.2	4.7	144.7	2.1	5.0	−3.8	2.4	142.8	2.0	5.2
II	147.7	3.8	3.8	4.6	4.4	146.2	−2.6	3.9	146.0	3.4	5.0	−3.2	1.5	144.0	3.3	5.3
III	148.7	2.6	3.8	5.0	4.5	147.5	−2.5	3.0	147.3	2.0	4.9	−3.2	0.5	145.1	1.9	5.2
IV	150.5	4.4	3.7	5.2	4.6	148.9	−3.3	1.2	148.6	3.8	4.8	−4.1	−1.3	146.3	3.7	5.1
1967I	149.7	3.7	3.8	4.5	4.7	150.2	−1.3	1.2	150.0	3.7	4.7	−1.9	−1.3	147.8	3.6	5.0
II	150.9	1.6	3.8	3.7	4.8	151.6	−1.9	0.6	151.3	2.0	4.5	−2.6	−1.9	149.1	1.9	4.8
III	152.6	3.5	3.8	4.3	4.9	153.0	−2.2	−0.8	152.7	3.3	4.5	−3.0	−3.3	150.2	3.2	4.8
IV	153.7	3.4	4.0	4.8	5.0	154.4	−2.0	−2.0	154.0	3.3	4.6	−2.7	−4.5	151.5	3.2	5.0
1968I	155.6	2.8	3.8	5.1	5.1	155.8	−2.8	−3.5	155.4	2.6	4.6	−3.5	−6.0	152.8	2.4	4.9
II	158.5	3.5	3.6	5.5	5.2	157.2	−4.2	−6.0	156.8	3.0	4.6	−5.1	−8.5	154.0	2.9	5.0
III	160.0	3.4	3.6	5.2	5.3	158.6	−4.0	−7.8	158.3	3.1	4.8	−4.7	−10.2	155.5	3.0	5.1
IV	161.1	4.3	3.4	5.6	5.4	160.0	−3.6	−9.7	159.7	3.7	4.7	−3.9	−11.6	157.4	3.6	5.0

Notes: See notes to Table 10-1.

Table 10–3. Control Results for the Nixon-Ford Administrations

Quarter	Actual Values Y	Actual Values 100 $\%\Delta PF$	Actual Values 100 UR	Actual Values $RBILL$	Actual Values Target $RBILL$	Actual Values Y^*	Optimal Values for $\gamma = 1.0$ ΔXG	Optimal Values for $\gamma = 1.0$ ΔVBG	Optimal Values for $\gamma = 1.0$ Y	Optimal Values for $\gamma = 1.0$ 100 $\%\Delta PF$	Optimal Values for $\gamma = 1.0$ 100 UR	Optimal Values for $\gamma = 0.1$ ΔXG	Optimal Values for $\gamma = 0.1$ ΔVBG	Optimal Values for $\gamma = 0.1$ Y	Optimal Values for $\gamma = 0.1$ 100 $\%\Delta PF$	Optimal Values for $\gamma = 0.1$ 100 UR
1969I	162.5	4.1	3.4	6.1	5.5	161.5	−1.2	−1.0	161.5	3.8	3.5	−1.0	−0.9	161.6	3.8	3.5
II	163.2	4.4	3.5	6.2	5.6	162.9	−0.4	−1.2	162.8	4.2	3.6	−1.2	−1.7	162.0	4.1	3.7
III	163.8	4.7	3.6	7.0	5.7	164.4	−0.0	−1.6	164.2	4.3	3.7	−1.1	−2.7	162.9	4.1	3.9
IV	162.9	4.4	3.6	7.3	5.8	165.9	1.9	−0.9	165.5	4.4	3.3	0.5	−2.6	163.8	4.1	3.6
1970I	161.9	4.5	4.2	7.3	5.9	167.4	3.3	0.3	167.0	4.9	3.3	1.8	−2.1	165.0	4.5	3.8
II	161.9	4.5	4.8	6.8	6.0	168.9	3.6	1.1	168.5	5.0	3.4	1.8	−2.0	166.1	4.7	3.9
III	163.2	3.2	5.2	6.4	6.2	170.5	2.6	0.6	169.8	3.6	3.7	0.3	−3.5	166.6	3.4	4.4
IV	161.2	7.2	5.8	5.4	6.3	172.0	5.6	2.6	171.1	7.9	4.2	3.2	−2.5	167.4	7.7	4.9
1971I	165.6	3.6	6.0	3.9	6.3	173.6	2.1	1.7	173.0	4.8	4.5	0.2	−3.7	169.4	4.7	5.2
II	166.8	4.1	5.9	4.2	6.3	175.1	4.3	2.9	174.8	4.8	4.8	2.7	−2.9	171.6	4.7	5.4
III	167.8	2.6	6.0	5.1	6.3	176.7	4.5	3.7	176.4	3.0	4.9	3.1	−2.2	173.4	3.0	5.4
IV	170.6	0.4	6.0	4.2	6.3	178.3	3.4	4.6	177.9	1.4	5.1	1.9	−1.6	174.9	1.4	5.5
1972I	173.6	5.3	5.8	3.4	6.3	179.9	3.5	6.5	179.4	6.7	5.3	2.1	−0.1	176.6	6.6	5.7
II	177.5	1.7	5.7	3.7	6.3	181.6	2.7	8.1	181.2	2.6	5.5	1.7	1.4	178.7	2.5	5.8
III	180.2	2.2	5.6	4.2	6.3	183.2	3.0	10.7	182.9	2.9	5.6	1.9	3.8	180.3	2.9	5.9
IV	184.0	3.0	5.3	4.9	6.3	184.9	1.7	12.6	184.5	3.4	5.6	0.4	5.3	181.7	3.4	6.0
1973I	188.7	3.3	5.0	5.6	6.3	186.5	−0.4	13.2	186.2	3.5	5.7	−2.1	5.4	182.6	3.4	6.1
II	189.6	5.7	4.9	6.6	6.3	188.2	1.3	15.9	187.8	5.5	5.7	−0.8	7.4	183.4	5.4	6.2
III	190.5	5.4	4.8	8.4	6.3	189.9	0.9	17.1	189.0	4.7	5.5	−2.3	7.4	183.0	4.5	6.1
IV	191.7	9.5	4.7	7.5	6.3	191.7	−0.3	18.1	189.6	9.3	5.3	−5.4	6.0	180.7	9.0	6.4
1974I	187.8	14.5	5.1	7.6	6.3	193.4	3.8	22.7	190.2	14.1	5.4	−2.6	7.8	178.4	13.8	6.8
II	186.9	15.5	5.1	8.3	6.3	195.1	3.6	25.2	191.3	15.0	4.8	−3.1	8.4	177.7	14.5	6.5
III	185.9	13.0	5.5	8.3	6.3	196.9	5.1	28.5	193.2	12.7	4.6	−0.6	11.3	179.9	12.1	6.4
IV	181.0	13.3	6.6	7.3	6.3	198.7	10.6	37.6	195.9	13.4	4.9	6.8	21.2	185.3	13.0	6.3
1975I	174.7	11.0	8.3	5.9	6.3	200.5	15.1	47.9	199.1	11.6	5.5	13.8	34.7	192.1	11.3	6.4

than the actual average rate. The optimal output series is smoother than the actual output series, which, because of the nonlinearities in the model, allows more output to be produced on average with the same average rate of inflation.

For $\gamma = 1.0$ in Table 10–1, the optimum corresponds to more output, but also to a higher average rate of inflation. Comparing the two sets of optimal results in Table 10–1, it can be seen that the optimum for $\gamma = 1.0$ corresponds to 43.3 billion dollars more in output being produced over the 30 quarters and to a higher average rate of inflation of 0.11 percent per year. The difference between the optimal average unemployment rates over the 30 quarters is 0.33 percentage points.

The summary results for Table 10–2 show that both optima correspond to more output and more inflation than actually existed. Comparing the two sets of optimal results, it can be seen that the optimum for $\gamma = 1.0$ corresponds to 58.7 billion dollars more in output over the 32 quarters, to a higher average rate of inflation of 0.09 percent per year, and to a lower average unemployment rate of 0.29 percentage points. It is interesting to note that the average unemployment rate for both optima are higher than the actual rate, even though both optima correspond to more output being produced. There are two main reasons for this. The first is that the bill rates that were targeted for the two runs are generally larger than the actual bill rates. Interest rates have a positive effect on the work effort of the household sector; in particular the mortgage rate has a positive effect on the labor force participation of all persons 16 and over except men 25–54. The higher interest rates for the optimal runs thus cause the labor force to be larger than otherwise, which in turn causes the unemployment rate to be larger than otherwise.

The other main reason for the higher unemployment rates for the optimal runs is that the optima correspond to higher real wages. When the economy expands in the model, the money wage rate, WF, rises faster initially than does the price level. The real wage thus increases initially, which has a positive effect on the labor force and thus on the unemployment rate. It was mentioned in section 1.1 and in Chapter Nine that there are many factors that have an effect on the unemployment rate, and the results in Table 10–2 provide a good example of how the unemployment rate can be higher in one run than in another even though real output is also higher.

The summary results for Table 10–3 show that the optimal values for $\gamma = 0.1$ are close to the actual values. The optimal value of output over the 25 quarters is only 1.6 billion dollars higher than the actual value. It is again the case that the optimal average unemployment rate is larger than the actual rate even though the optimal value of output is greater than the actual value. Comparing the two sets of optimal results, the optimum for $\gamma = 1.0$ corresponds to 117.7 billion dollars more in output over the 25 quarters, to a higher average rate of inflation of 0.18 percent per year, and to a lower average unemployment rate of 0.65 percentage points.

An important feature of the results in the three tables is that for $\gamma = 1.0$ the optimal output series correspond closely to the target series. In Table 10–1, for example, the difference between the sum of Y^* and the sum of the optimal output values over the 30 quarters is only 4.7 billion dollars. In Tables 10–2 and 10–3 the respective differences are 7.8 and 24.9 billion dollars. Since the starting values used for XG corresponded to the output targets being achieved exactly, this closeness may be due merely to a failure on the part of the algorithm to find the true optima. This, however, did not appear to be the case from some experimentation that was carried out to see if the true optima had been attained.

What these and other results show is that the model has the property that output can be increased to some reasonable target value (from a lower value) without having too serious an affect on the rate of inflation. It is not, however, generally possible to decrease the rate of inflation to, say, zero percent (from a higher rate) without having serious effects on the level of output. Consequently, when a loss function like (10.3) is minimized, with equal weights attached to the output and inflation targets, the optimum tends to correspond more closely to the output target being achieved than it does to the inflation target being achieved. Even when the weight on the output target is only one-tenth of the weight on the inflation target, it is still the case that the inflation target of zero percent is not close to being achieved.

It is possible to use the results in Tables 10–1, 10–2, and 10–3 to examine the question of the "trade-off" between, say, the rate of inflation and the level of output. One must be very careful in doing this, however, because of the many diverse factors that affect both variables. It was argued in section 9.4 that there is no reason to expect there to be a stable relationship between the rate of inflation and the level of output, and this holds true whether the values of the policy variables are historic or optimal values. The trade-off that one observes in tables like 10–1, 10–2, and 10–3 for one control period and one set of problems may not hold true for other control periods and other sets of problems.

Comparing the two sets of optimal results in Table 10–1 shows that a yearly gain of output of 5.8 billion dollars (43.3 ÷ 7.5 years) is achieved at a cost of an extra 0.11 percent inflation per year. In Table 10–2 the yearly gain is 7.3 billion dollars (58.7 ÷ 8 years) at a cost of 0.09 percent inflation per year, and in Table 10.3 the yearly gain is 18.8 billion dollars (117.7 ÷ 6.25 years) at a cost of 0.18 percent inflation per year. These figures show, as already mentioned, that the trade-off in the three tables is such that it is costly in terms of lost output to lower the rate of inflation, or, the other way around, that it is not costly in terms of extra inflation to increase the level of output.

It should be stressed again, however, that these figures should not necessarily be extrapolated to other periods. Because of the nonlinearities in the model, the figures in particular should not be extrapolated to situations in which the two sets of optimal results that are compared correspond to much larger differences in the state of the economy than the differences in the current three tables.

It is finally of interest to note that the optimal values for $\gamma = 0.1$ in the three tables correspond more closely to the actual values than do the optimal values for $\gamma = 1.0$. One possible conclusion from this fact is that the people who were responsible for controlling the economy weighted inflation more heavily than output in their loss functions. This would be true, however, only if the people believed that the trade-offs between inflation and output were similar to those in the present model and they had targets for inflation and output that were similar to the targets in the loss function (10.3).