3 A Theoretical Model

3.1 The Single-Country Model

The purpose of this chapter is to discuss the theoretical model that has guided my empirical work. The single-country model is discussed in this section, and then the model is expanded to two countries in Section 3.2. As noted in Section 2.1.6, the model is a simulation model in the sense that its properties are analyzed using simulation techniques. It should be repeated, however, that the model is not a simulation model of the kind that is used in applied general equilibrium analysis. The simulation results are only meant to be used to learn about the qualitative properties of the model; no significance is attached to the size of any of the effects. Knowledge of the qualitative properties of the model is used to guide the econometric specifications in Chapter 4. For ease of reference, the symbols for the variables in the model are listed in alphabetical order in Table 3-1.

3.1.1. Introduction

Nature of the Model

The model is an attempt to integrate three main ideas. The first is that macroeconomics should be based on better microeconomic foundations. In particular, macroeconomics should be consistent with the view that decisions are made by maximizing objective functions. The second idea is that macroeconomic theory should allow for the possibility of disequilibrium in some markets. The third, and perhaps somewhat less important, idea is that a model should account explicitly for balance-sheet and flow-of-funds constraints.

Relation to Previous Work

The implications of the first two ideas have generally been worked on together, beginning with the work of Patinkin (1956, chap. 13) and Clower (1965). Studies that have followed these two include Leijonhufvud (1968,

TABLE 3-1. The variables in the theoretical model in alphabetical order

Subscripts:	
b	bank
f	firm
g	government
h	household
i	machine of type i (i = 1, ..., M)
k	firm
p	private (sum of b, f, k, and h)
t	time period
m	number of periods a machine lasts
Symbols:	
A	end-of-period assets (+) or liabilities (-)
BO	bank borrowing from the monetary authority
BR	bank reserves
C	consumption
CF	cash flow
CG	capital gain or loss
D	dividends
d_1	personal income tax rate
d_2	profit tax rate
DEP	depreciation
e	exchange rate (spot)
F	forward exchange rate
g_1	reserve requirement rate
I	total investment in units of goods
IM	investment in units of the number of machines of the given type
K	actual number of machines of the given type on hand
KH	machine-hour requirements
KMIN	minimum number of machines of the given type required to produce the output
L	labor supplied or demanded
LA	aggregate constrained supply of labor to the firms
LAUN	aggregate unconstrained supply of labor to the firms
L^*	labor constraint
LL'	labor requirements
LMAX	maximum amount of labor to employ
LMIN	minimum amount of labor required to produce the output
LUN	aggregate unconstrained supply of labor to the firms and the government
M	money holdings
N	amount of time spent taking care of money holdings
P	price level
P'	price of investment goods
$\overline{P}$	average price
PS	value of stocks
Q	international reserve
R	interest rate
RD	discount rate
S	savings
T	taxes
T'	taxes due to fluctuations in the work force
TH	total number of hours in the period
TR	transfer payments
U	utility
UR	unemployment rate
V	stock of inventories, end of period
W	wage rate
$\overline{W}$	average wage rate
X	sales
XA	aggregate demand for goods
Y	total production for firms, taxable income for households
YY	production on machines of the given type
Π	profits

1973), Tucker (1968, 1971a, 1971b), Barro and Grossman (1971, 1976), and Grossman (1971, 1972a, 1972b). (Two related studies are Solow and Stiglitz 1968 and Korliras 1972, although the models developed in these papers are not constructed on a choice-theoretic basis and so are not concerned with the first idea.) This work has provided a more solid theoretical basis for the existence of the Keynesian consumption function and for the existence of unemployment; it has thus made the standard, textbook Keynesian theory somewhat less ad hoc. The existence of excess supply in the labor market is a justification for including income as an explanatory variable in the consumption function, and the existence of excess supply in the commodity market is a justification for the existence of unemployment.

The main problem with these disequilibrium studies is that they have not provided an explanation of why it is that prices and wages may not always clear markets. Prices and wages are either taken to be exogenous or are determined in an ad hoc manner. This is particularly restrictive in a disequilibrium context, since one of the key questions in this area is why there are market failures. Barro and Grossman are quite explicit in their book about this problem: "We provide no choice-theoretic analysis of the market-clearing process itself. In other words, we do not analyze the adjustment of wages and prices as part of the maximizing behavior of firms and households. Consequently, we do not really explain the failure of markets to clear, and our analyses of wage and price dynamics are based on ad hoc adjustment equations" (1976, p. 6).

This problem has persisted in the related work on fixed price equilibria (see Grandmont 1977 for a survey of this work). In a discussion of some of this work, for example, Malinvaud states: "A dynamic theory that would correctly describe the successive adjustments [of prices and wages] occurring in the real world is still more difficult to build than a long-run equilibrium theory under short-run rationing. At the present stage in the development of economic theory, one cannot expect to do more than provide a model of the first few steps of the dynamic adjustments initiated by demand pressures in the markets for goods and labour and by unwanted inventories, excess capacities or unemployment" (1977, pp. 101 – 102).

My model does provide a choice-theoretic explanation of market failures. The explanation is based on two postulates, both of which draw heavily on the studies in Phelps et al. (1970) and related work, which in turn have been influenced by Stigler's classic article (1961) on imperfect information and search. The first postulate is that firms have a certain amount of monopoly power in the short run in the sense that raising their prices above prices charged by other firms does not result in an immediate loss of all their customers, and lowering their prices below prices charged by other firms does not result in an immediate gain of everyone else's customers. There is, however, a tendency for high-price firms to lose customers over time and for low-price firms to gain customers. A similar statement holds for wages. This postulate can be justified on the basis of imperfect information about prices and wages on the part of customers and workers. The second postulate is that prices and wages are decision variables of firms, and firms choose these variables (along with others) in a profit-maximizing context.

If a firm's market share is a function of its price relative to the prices of other firms, then a firm's optimal price strategy is a function of this relationship. Models of this type have been developed by Phelps and Winter (1970) and Maccini (1972) for prices and by Phelps (1970) and Mortensen (1970) for wages. My model expands on this work by considering the price and wage decisions together (along with other decisions) and by assuming that firms expect that the future prices and wages of other firms are in part a function of their own past prices and wages.

It should be clear that disequilibrium can occur in models of this type. In the Phelps and Winter model, for example, disequilibrium occurs if the average price set by firms differs from the expected average price (1970, p. 335). In my model, as will be seen, disequilibrium also occurs because of expectation errors. The difference is that expectation errors in my model have much wider effects. In the Phelps and Winter model there is a straightforward way in which the system returns to equilibrium, whereas this is not true in my case. This is, I believe, an important difference between models of the Phelps and Winter type and more general models, something that will be stressed later. If the effects of expectation errors spill over into other markets, the effects of shocks and errors may be much more serious (larger and longer) than would seem to be implied by models of the Phelps and Winter type.

With respect to the previous literature, it is surprising that the studies in Phelps et al. (1970) and related work have had no impact on the work on fixed price equilibria, given the admittedly restrictive assumption of fixed prices or ad hoc price determination in the latter. In a 1980 study Malinvaud argues against the view that "price and wage changes are decided by firms as a rational reaction to the situation confronting them" (1980, p. 52). He argues that by following this approach "we may be fairly certain that we shall end up with a very partial representation of the real world; the representation will be so partial that the adequacy of the derived dynamic specification will be quite doubtful" (p. 53). This view seems so far to have prevailed in the fixed price equilibria literature. My view is obviously contrary to this: the linking of the Phelps et al. work to the disequilibrium models does seem to me to be an appealing way to close the disequilibrium models. At any rate, one should be able to test this in the long run by comparing models based on this idea with other models.

With regard to the third idea (the accounting for balance-sheet and flow-of-funds constraints), one of the main advantages of doing this is that it means that the government budget constraint is automatically accounted for. Christ (1968), among others, has emphasized this constraint. Accounting explicitly for balance-sheet constraints also means that it is easier to keep track of wealth effects.

I was also concerned with making the model general enough to include the main variables of interest in a macroeconomic context. The endogenous variables include sales, production, employment, investment, prices, wages, interest rates, and financial assets and liabilities. Previous disequilibrium models have not been this general.

A weakness of the model is that search has not been treated as a decision variable of any agent. As noted earlier, the existence of imperfect information and search can be used to justify the short-run monopoly power of firms with respect to prices and wages. It is thus a weakness of the model not to explain search and thus derive the degree of monopoly power of the firms. A much more complicated model would be needed to treat search as endogenous, and this has not been attempted.

Treatment of Expectations

Since the treatment of expectations is critical in any macro model, it will be useful to explain at the beginning how expectations have been handled. Individual agents in the model are assumed to form their expectations on the basis of a limited set of information. Agents do not know the complete model, and their expectations are in general different from the model's predictions. Expectations, in other words, are not rational. (The simulation model is deterministic, so "rational expectations" in this case means perfect foresight.) The nonrationality of expectations leads to expectation errors, which in turn lead to the system being in disequilibrium.

Another feature regarding expectations should be noted: expectations are assumed to be treated with certainty by the individual agents. In other words, agents ignore the fact that their expectations are uncertain when solving their optimization problems. The variables that are stochastic from the point of view of the individual agent are replaced with their expected values before the optimization problem is solved. Although this "certainty equivalent" treatment is only correct for linear models, it has been used here even though the models facing the individual agents are nonlinear. This is a common procedure in the optimal control literature (see, for example, Athans 1972), and it may provide a reasonable approximation in many cases. It does, however, rule out potentially important effects of uncertainty on decisions.

Treatment of Different Kinds of Financial Securities

The model treats different kinds of financial securities in a fairly simple way. The financial assets of households include demand deposits in banks, which will be called "money"; corporate stocks; and an all-other category, which will be called "bonds." The bonds are one-period securities. The expected one-period rate of return on bonds and stocks is assumed to be the same, and thus households are indifferent as to whether they hold bonds or stocks. Households have no financial liabilities. Firms have financial assets in the form of demand deposits and financial liabilities in the form of bonds. The government has financial assets in the form of bank borrowing; its liabilities consist of bonds and bank reserves. The liabilities of banks are demand deposits and borrowing from the government; their assets are bonds and bank reserves.

Comparison to the "Pitfalls" Approach of Brainard and Tobin

Because of the assumption that the expected one-period rate of return on bonds and stocks is the same, there are really only two securities in the model with respect to the maximization problem of households: bonds-stocks and money. This treatment ignores the main thrust of the "pitfalls" approach of Brainard and Tobin (1968). (Tobin's 1982 Nobel lecture provides a good review of this approach.) Brainard and Tobin stress the lack of perfect substitutability of different securities and develop a model for explaining the different rates of return on different securities.

There is little doubt that there is lack of perfect substitutability among many securities in the real world, and thus the pitfalls approach has considerable appeal. There are at present, however, some costs to adopting the approach, and it is an open question whether the potential gains are greater than these costs. The general strategy of the pitfalls approach has been to regard income account variables as exogenous for balance sheet behavior, and although this assumption can be relaxed, it is not trivial to do so given the basic strategy. It is also not easy within the approach to account for the effects of expected future short-term rates on current long-term rates and for the effects of expected future dividends on current stock prices. There is also a practical difficulty in trying to estimate pitfalls models. Different interest rates are highly collinear (because there is considerable substitutability among different securities, even though possibly not perfect substitutability), and it is difficult to get precise estimates of the effects of interest rate differences on security holdings. (See, for example, Smith and Brainard 1976, who attempt to get around this problem by the use of a Bayesian procedure.) It may be that the degree to which different securities are not perfect substitutes is too small to be capable of being picked up with the use of macro time series data.

It will be useful in understanding my model to consider another important difference between my approach and the pitfalls approach. This can best be explained by seeing how consumption is determined in the two approaches. As just mentioned, income account variables are generally taken to be exogenous by the pitfalls approach, but Tobin's 1982 Nobel lecture provides an example of the endogenous treatment of these variables within the context of the pitfalls approach.

Consider the following two specifications. The first is

(3.1) $C = f(Y, R, A_{-1}, \ldots) + \epsilon,$ [consumption]

(3.2) $Y = W \cdot L + R \cdot A_{-1},$ [income]

(3.3) $S = Y - C,$ [savings]

(3.4) $A = A_{-1} + S,$ [end-of-period assets]

where C is consumption, Y is income, S is savings, A is end-of-period assets, R is the interest rate, W is the wage rate, L is the number of hours worked, A_{-1} is beginning-of-period assets, and ϵ is an error term. The price level is assumed to be fixed and equal to 1. W, L, and R are taken to be exogenous. The second specification is

(3.5) $A = g(Y, R, \ldots) + \mu,$ [end of period assets]

(3.6) $Y = W \cdot L + R \cdot A_{-1},$ [income]

(3.7) $S = A - A_{-1},$ [savings]

(3.8) $C = Y - S,$ [consumption]

where the variables are as before and μ is an error term.

The first set of equations is consistent with my treatment. Consumption is determined by an estimated equation, (3.1). Income, savings, and end-of-period assets are determined by identities. In particular, end-of-period assets are "residually" determined by (3.4), given the consumption decision and Y. (In practice, as will be seen in Section 3.1.2, both consumption and labor supply are determined jointly in my model of household behavior, which means that income is not exogenous and does not belong on the RHS of 3.1. For the sake of the present argument, however, nothing is lost by taking labor supply to be exogenous. Also, the income definition in my model uses $R \cdot A$ instead of $R \cdot A_{-1}$ for the interest revenue term, but this difference is of no consequence for the present argument. If $R \cdot A$ were used in 3.2, then A would be determined, given C, by the solution of 3.2, 3.3, and 3.4 rather than by 3.4 alone.) The variables on the RHS of (3.1) are the exogenous variables (that is, exogenous to the household) that affect the consumption decision. In my model consumption decisions are derived from multiperiod utility maximization, and so the RHS variables are variables that affect the solution of the maximization problem, including expectations of future variables.

The second set of equations is consistent with Tobin's treatment. End-of-period assets are determined by an estimated equation, (3.5). Income, savings, and consumption are determined by identities. In particular, consumption is "residually" determined by (3.8), given the asset decision and Y. The variables on the RHS of (3.5) are variables that affect the asset decision.

From the point of view of a utility-maximizing model, Tobin's treatment is awkward. In the simple model with labor supply exogenous, one maximizes utility with respect to consumption. The natural decision variable to consider is consumption, not assets. Given that $C = Y - A + A_{-1}$, one can, of course, replace C with this expression in the utility function and maximize with respect to A (remember that Y is exogenous), but this is not the natural thing to do.

If the only problem with the Tobin approach were a certain awkwardness of interpretation, there would be no real issue involved in choosing between the above two specifications. In practice, however, quite different models are likely to result from the two approaches. In the first approach much time is spent searching for the estimated equation that best explains C, whereas in Tobin's approach the time is spent searching for the estimated equation that best explains A. For example, C_{-1} is a natural variable to use in the consumption equation to try to capture expectational and lagged adjustment effects, whereas A_{-1} is the natural variable to use in the asset equation. If different RHS variables are chosen for the two equations, it is likely that the behavior of consumption implied by Tobin's approach will be considerably different from the behavior implied by the first approach. If this is true, the awkwardness of Tobin's approach becomes a real issue, and it may argue against its use. (Note that if the same set of RHS variables is used for both equations, if this set includes Y and A_{-1}, and if the equations are linear, then the same equation is being estimated by both approaches. The argument here is that this is unlikely to be the case in practice.)

The main thrust of the pitfalls approach is, of course, to disaggregate A into many different kinds of securities, which means estimating an equation like (3.5) for many different securities. It is straightforward to disaggregate A following this approach, whereas it is not straightforward to do so following the first approach. On the other hand, it is straightforward to disaggregate consumption into different categories following the first approach, whereas it is not following the pitfalls approach. There is again likely to be a real issue here regarding which is the better approach in practice.

Although the example just given is for household behavior, similar considerations apply to models of firm behavior. From the point of view of a profit-maximization model, the pitfalls approach is awkward. In my profit-maximization model, for example, which is discussed in Section 3.1.3, it would be awkward to treat end-of-period assets (or liabilities) as a direct decision variable and thus in the empirical work to estimate an equation with this variable on the LHS. If this were done, it is likely that the estimated model of firm behavior would be quite different from the one that is in fact estimated.

These difficulties with the pitfalls approach may be overcome in future work and, in the spirit of the methodology of this book, it should be possible in the long run to compare pitfalls and non-pitfalls models. The foregoing discussion indicates that the two types of models are likely to have important quantitative differences, which should increase the chances of choosing between them.

It should finally be noted that an approach that is in between the two just discussed would specify that both the consumption and asset equations have errors, where the covariance matrix of the errors would be singular because of the adding up constraints. I do not find this approach particularly appealing, since the theoretical arguments against it are the same as those against the Tobin approach, but it is a possible area for future research.

3.1.2 Household Behavior

There are four types of agents in the model: households *(h)*, firms *(f)*, banks *(b)*, and the government *(g)*. The behavior of each type of agent will be discussed in turn, beginning with households in this section. The complete model is discussed in Section 3.1.5.

In order to simplify the notation, no special symbols have been used to denote expectations. This is unlikely to cause any confusion, since it will be made clear in the discussion which variables are expectation variables and which are decision variables. Note also that the use of the certainty equivalent assumption discussed earlier means that the household decision problem can be analyzed as a deterministic problem.

The Decision Problem

The model of household behavior is fairly straightforward. The utility of household h in period t is a function of consumption and leisure:

(3.9) $U_{ht} = f_9(C_{ht}, TH - L_{ht} - N_{ht}),$ [utility function]

where C_{ht} is consumption, L_{ht} is the amount of labor supplied, N_{ht} is the amount of time spent taking care of money holdings, and TH is the total number of hours in the period. The objective of the household is to maximize

(3.10) $OBJ_h = g_{10}(U_{h1}, U_{h2}, \ldots, U_{hN}),$ [objective function]

where period 1 is the current period and N is the remaining length of life of the household.

Since the expected one-period rate of return on bonds and stocks is the same, one can deal with only one security when analyzing the decision problem of a household. Let A_{ht} denote the security holdings of the household. Before-tax income (Y_{ht}) is

(3.11) $Y_{ht} = W_{ht}L_{ht} + R_t A_{ht},$ [before-tax income]

where W_{ht} is the wage rate and R_t is the one-period interest rate. This equation merely states that before-tax income is equal to wage plus nonwage income. The tax-transfer schedule is

(3.12) $T_{ht} = d_{1t}Y_{ht} - TR_t,$ [net taxes]

where d_{1t} is the (proportional) income tax rate, TR_t is the level of transfer payments to the household (TR_t can be interpreted as a minimum guaranteed level of income), and T_{ht} is the amount of net taxes paid. The level of savings (S_{ht}) is equal to income minus taxes minus consumption expenditures:

(3.13) $S_{ht} = Y_{ht} - T_{ht} - P_{ht}C_{ht},$ [savings]

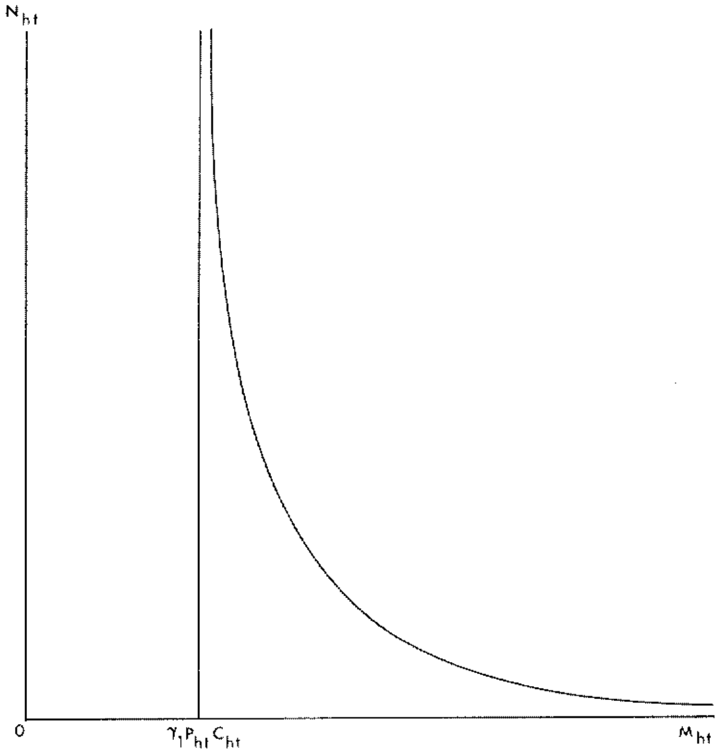

Figure 3-1 Relationship between N_{ht} and M_{ht}

where P_{ht} is the price of goods. The household budget constraint is

$$0 = S_{ht} - \Delta A_{ht} - \Delta M_{ht}, \tag{3.14}$$ [budget constraint]

where M_{ht} is the level of money holdings. The budget constraint states that any nonzero level of savings must result in a change in holdings of securities or money.

The relationship between the level of money holdings and the amount of time spent taking care of these holdings is depicted in Figure 3-1. For large values of M_{ht}, N_{ht} is small (few trips to the bank needed), whereas for values of M_{ht} that are small in the sense of being close to some proportion of expenditures, $\gamma_1 P_{ht} C_{ht}$, N_{ht} is large. This specification captures the idea that work is involved in keeping money balances small. The functional form that was used for the relationship in Figure 3-1 is

(3.15) $$N_{ht} = \frac{\gamma_2}{M_{ht} - \gamma_1 P_{ht} C_{ht}}.$$

[time spent taking care of money holdings]

Equations (3.9)–(3.15) hold for each period ($t = 1, \ldots, N$). The decision variables are C_{ht}, L_{ht}, and N_{ht} ($t = 1, \ldots, N$). The exogenous variables to the problem are W_{ht}, P_{ht}, R_t, d_{1t}, and TR_t ($t = 1, \ldots, N$). If future values of the exogenous variables are not known, expectations of these values must be made before the optimization problem is solved. In the solution of the complete model in Section 3.1.5 it is assumed that the household knows the values of the exogenous variables for period t, but not for periods $t + 1$ and beyond. There are two initial conditions: the initial stocks of securities and money, A_{h0} and M_{h0}. There is also assumed to be an exogenous terminal condition:

(3.16) $$A_{hN} + M_{hN} = \overline{AM},$$ [terminal condition]

where $\overline{AM}$ is exogenous. This means that bequests are exogenous.

There is a possible "disequilibrium" constraint on the household, which is that it may not be able to work as many hours as it would like:

(3.17) $$L_{ht} \leq L^*_{ht},$$ [labor constraint]

where L^*_{ht} is the maximum amount that the household can work in period t.

The decision problem of the household is to choose the paths of the decision variables to maximize (3.10), given the actual and expected values of the exogenous variables, the initial conditions, the terminal condition, and the possible labor constraint.

Simulation Results

The following values and functional forms were used for the simulation results. The functional form of the utility function was taken to be the constant elasticity of substitution (CES) form:

(3.9)′ $$U_{ht} = [\alpha_1 C_{ht}^{-\alpha_2} + (1 - \alpha_1)(TH - L_{ht} - N_{ht})^{-\alpha_2}]^{-1/\alpha_2},$$

where $\alpha_1 = .5$ and $\alpha_2 = -.5$. The elasticity of substitution is $1/(1 + \alpha_2)$, which in the present case is 2.0. The length of the decision period, N, was taken to be 3, and the objective function was taken to be

(3.10)′ $$OBJ_h = U_{h1} U_{h2}^{\lambda} U_{h3}^{\lambda^2},$$

where λ is the discount rate.

The exogenous variable values for period 1 were taken to be: $W_{h1} = 1.0$, $P_{h1} = 1.0$, $R_1 = .07$, $d_{11} = .2$, and $TR_1 = 0$. The household was assumed to know these values at the beginning of the period and to expect them to remain unchanged in periods 2 and 3. In other words, expectations were assumed to be static.

The values of the initial conditions were as follows: $A_{h0} = 1000.0$ and $M_{h0} = 100.0$. The value of the terminal condition was $\overline{AM} = 1100.0$. The remaining parameter values were chosen so as to lead to a flat optimal path of each decision variable; these were $\lambda = .944$, $TH = 1004.72366$, $\gamma_1 = .255905$, and $\gamma_2 = 1.0$. The value of λ is one minus the after-tax interest rate, where the after-tax interest rate is $.07 \times .8$.

The maximization problem of the household is to choose the three values of each of the decision variables, L_{ht}, C_{ht}, N_{ht} ($t = 1, 2, 3$), so as to maximize (3.10)′, subject to the terminal condition (3.16). This problem was solved by calculating the first-order equations analytically and then solving these equations using the Gauss-Seidel technique. The first-order equations were obtained as follows. The terminal condition allows one to write one of the nine decision variables as a function of the others, and this was done for C_{h3}. This expression was then substituted for C_{h3} in the objective function, leaving eight variables to be determined. The derivatives of the objective function with respect to the eight variables were taken, and the resulting eight first-order equations were used to solve for the eight unknowns. Some damping of the Gauss-Seidel technique was needed to solve the equations, but the time taken to solve them was trivial. A damping factor of .1 was generally used (although larger values also worked), and the time taken to solve a typical problem was about .75 seconds on the IBM 4341 at Yale. This procedure was chosen over the use of the DFP algorithm because it was undoubtedly much cheaper in terms of computer time and because the analytic work involved in obtaining the first-order equations was not very large.

The solution values are presented in the first column of Table 3-2. As noted in the table, the values are the same in each of the three periods. The choice of λ as one minus the after-tax interest rate means that the household has no incentive to save or dissave in any period, and thus the optimal value of savings each period is zero. Note that the variables just discussed, other than L_{ht}, C_{ht}, and N_{ht}, are "indirect" decision variables in the sense that they are residually determined given (1) the first three decision variable values, (2) the exogenous variable values, and (3) the parameter values.

The simulation experiments consisted of changing a particular variable from the value used for the base run, solving the household maximization

TABLE 3-2. Simulation results for household h

Variable	Base run values 1[a]	Experiment 1 $W_{ht}(+)$ 1 2 3	2 $P_{ht}(-)$ 1 2 3	3 $A_{h0}(+)$ 1 2 3	4[c] $R_t(+)$ 1 2 3	5 $d_{1t}(+)$ 1 2 3	6 $TR_t(-)$ 1 2 3	7 $L^*_{h1}(-)$ 1 2 3
L_{ht}	400.0	+ + +	+ + +	- - -	+ - -	- - -	+ + +	- + +
C_{ht}	376.0	+ + +	+ + +	+ + +	- + +	- - -	- - -	- - -
N_{ht}	26.0	- - -	0 0 0	0 0 0	+ + +	0 0 0	0 0 0	+ 0 0
M_{ht}	100.0	+ + +[b]	+ + +[b]	+ + +[b]	- - +	- - -[b]	- - -[b]	- - -[b]
S_{ht}	0.0	0 0 0	0 0 0	- - -	+ + -	- - +	0 0 0	- + +
A_{ht}	1000.0	- - -[b]	- - -[b]	+ + -[b]	- - -	+ + +[b]	+ + +[b]	- - +[b]
U_{ht}	483.48	+ + +	+ + +	+ + +	- + +	- - -	- - -	+ - -
OBJ_h	17.524	+	+	+	+	-	-	-

Notes: a. Values for periods 2 and 3 are the same as those for period 1.
b. Change in $M_{h3} + A_{h3} = 0$ since $M_{h3} + A_{h3} = 1100.0$.
c. For experiment 4 both A_{h0} and $\overline{AM}$ were lowered by 86.07.

- The +'s and -'s refer to changes from the values for the base run, not changes from period to period.
- The amounts of the changes were: .05 for W_{ht}, -.05 for P_{ht}, 50.0 for A_{h0}, .01 for R_t, .1 for d_{1t}, and -20.0 for TR_t. L^*_{h1} for the last experiment was 390.0.

problem using the new value, and observing the resulting changes in the optimal values. Seven experiments were performed: each of the five exogenous variables was changed, the initial condition A_{h0} was changed, and the labor constraint was made binding. The results are presented in Table 3-2. For the last experiment the labor constraint was binding, but for the others it was not. The five exogenous variables were changed for all three periods, which means that the household expected the changes to be permanent. In the last experiment the labor constraint was only made binding for the first period; the household was unconstrained in periods 2 and 3. The following paragraphs give a brief discussion of the results. Since only qualitative properties of the model are important, only pluses and minuses are presented in Table 3-2. This makes the results somewhat easier to discuss. When a quantitative result is needed in order to understand a property of the model, it is mentioned in the text. All the pluses and minuses are changes from the base run values, not changes from period to period.

Experiment 1: W_{ht} (+). The increase in the wage rate led the household to work and consume more. This is, of course, not an unambiguous result since there are both income and substitution effects operating. Given the particular parameter values chosen, the substitution effect dominates. The increase in the wage rate also led the household to spend less time taking care of money holdings. This is because an increase in the wage rate increases the opportunity cost of time spent both in leisure and in taking care of money holdings. Money holdings increased both because N_{ht} decreased and because consumption increased. Savings remained unchanged at zero each period. A_{ht} fell by the same amount that M_{ht} rose.

Experiment 2: P_{ht} *(−).* The signs of the results for the decrease in the price level are the same as those for the increase in the wage rate, with the exception of those for N_{ht}. Although N_{ht} did not change for this experiment, it fell for the wage rate increase. The change in price does not affect the opportunity cost of spending time taking care of money holdings, and so N_{ht} is not affected. Money holdings increased because consumption increased by a larger percentage than the price level decreased.

Experiment 3: A_{h0} *(+).* The increase in the initial value of assets led the household to work less and consume more. The terminal condition was not changed for this experiment, and so the household dissaved each period by enough to have the value of assets fall to the terminal condition value. The value of A_{ht} was lower in period 3 by the amount that M_{ht} was higher; M_{ht} was higher because consumption was higher.

Experiment 4: R_t *(+).* This experiment requires a little more explanation. Since part of the household's wealth is in the form of stocks, an increase in the interest rate implies a capital loss on stocks and thus a fall in wealth. In the base run for the complete model in Section 3.1.5, the value of stocks is equal to $48.2/R_t$, where 48.2 is the expected stream of after-tax cash flow. The interest rate for the present experiment was increased from .07 to .08, which implies a capital loss on stocks of 86.07. A_{h0} was thus lowered by this amount before the maximization problem was solved. The terminal value of wealth, $\overline{AM}$, was also lowered by this amount. Had the terminal value remained unchanged, the household would have had to save 86.07 over the three periods to make up for the loss. Instead, the household was merely assumed to lower its bequests by the amount of the loss.

The increase in the interest rate led the household to save in periods 1 and 2 and dissave in period 3. Work effort was higher in period 1 and lower in periods 2 and 3, consumption was lower in period 1 and higher in periods 2 and 3, and time spent taking care of money holdings was higher in all three periods. This last variable was higher because an increase in the interest rate increases the opportunity cost of holding money and thus increases the reward from keeping money holdings low. M_{ht} was lower in periods 1 and 2 and higher in period 3. It was higher in period 3 even though N_{ht} was higher because the positive effect from the increase in consumption dominated the negative effect from the increase in N_{ht}.

Experiment 5: d_{1t} (+). The increase in the tax rate led the household to work less and consume less. It worked less because the after-tax return to work was lower. It dissaved in periods 1 and 2 and saved in period 3. It dissaved in the first two periods because the after-tax interest rate was lower. The increase in the tax rate had no effect on N_{ht}. Although an increase in the tax rate lowers the after-tax return to work, which increases N_{ht}, it also lowers the after-tax interest rate, which decreases N_{ht}. These two effects exactly cancel each other out, and so a change in the tax rate has no effect on N_{ht}. Money holdings decreased for this experiment because consumption decreased.

Experiment 6: TR_t (−). The decrease in transfer payments led the household to work more and consume less. N_{ht} was not affected. Money holdings decreased because consumption decreased. Since a decrease in transfer payments is an increase in net taxes, experiments 5 and 6 show an important difference between raising net taxes by increasing the tax rate and raising net taxes by decreasing transfer payments. In both cases consumption is lower, but in the first case work effort is less, whereas in the second case work effort is greater.

Experiment 7: L^*_{h1} (−). Making the labor constraint binding forced the household to work less in period 1. It consumed less and dissaved in period 1. It also spent more time taking care of money holdings. It then worked more in periods 2 and 3 to make up in part for the forced cutback in period 1. It saved in periods 2 and 3 to make up for the dissaving in period 1. Consumption was less in all three periods.

Other Experiments. Experiments 1–6 were also performed with the signs of the changes reversed. The signs of the changes in the optimal values were opposite to those given above. The quantitative results were almost, but not quite, symmetric. For example, L_{ht} responded slightly more to a wage rate decrease than to a wage rate increase. Also, L_{ht} responded more to a change in the wage rate than to a change in the price level.

Summary of Household Behavior

The maximization problem of the household is fairly standard, and so its optimal behavior is not surprising. When the wage rate increases or the price level decreases, the household works more and consumes more. When the initial value of wealth increases, it works less and consumes more. When the interest rate increases, it saves at the beginning and dissaves at the end. It responds to an increase in the tax rate by working less and consuming less, and it responds to a decrease in transfer payments by working more and consuming less. A binding labor constraint forces the household to work less and leads it to consume less.

The only unusual feature about the maximization problem is the addition of N_{ht}, time spent taking care of money holdings, to the utility function. N_{ht} responds negatively to the wage rate and positively to the interest rate. In other words, the household spends more time keeping money balances low when the wage rate is low or the interest rate is high. The model thus provides an explanation of the interest sensitivity of the demand for money.

3.1.3 Firm Behavior

General Features

There are a number of features of the following model of firm behavior that distinguish it from others. One is the treatment of prices and wages. As discussed in Section 3.1.1, firms are assumed to have some monopoly power in the short run in their price and wage setting behavior, and they are assumed to set prices and wages in a profit-maximizing context. The number of decision variables of the firm is also larger than usual. In addition to prices and wages, the variables include production, investment, and employment.

The assumptions about technology and costs are also somewhat different. The underlying technology of a firm is assumed to be of a "putty-clay" type, where at any one time there are a number of different types of machines that can be purchased. The machines differ in price, in the number of workers that must be used with each machine per unit of time, and in the amount of output that can be produced per machine per unit of time. The worker-machine ratio is assumed to be fixed for each type of machine. With respect to costs, there are assumed to be costs involved in changing the size of the work force and the size of the capital stock. Because of these costs, it may be optimal for a firm to operate some of the time below capacity and "off" its production function. This means that some of the time the number of worker hours paid for may be greater than the number of hours that the workers are effectively working. Similarly, some of the time the number of machine hours available for use may be greater than the number of machine hours actually used. The difference between hours paid for by a firm and hours worked will be called "excess labor," and the difference between the number of machines on hand and the number of machines required to produce the output will be called "excess capital."

The model of firm behavior is somewhat tedious to present, since the optimization problem is complicated. In the following discussion, subscript f refers to firm f and subscript i refers to a machine of type i. The number of different types of machines is M, and i always runs from 1 through M. All coefficients are positive unless indicated otherwise.

As was the case for the model of household behavior, no special symbols have been used to denote expectations. It should be clear in the discussion which variables are decision variables and which are expectation variables. Again note that because of the certainty equivalence assumption, the maximization problem can be analyzed as a deterministic one.

The Technology

It will be useful to present the equations representing the technology first. The following two equations reflect the putty-clay nature of the technology:

(3.18) $$LL'_{fit} = \frac{YY_{fit}}{\lambda_i},$$ [labor required to produce YY_{fit}]

(3.19) $$KH_{fit} = \frac{YY_{fit}}{\mu_i}.$$ [machine hours required to produce YY_{fit}]

YY_{fit} is the amount of output produced on machines of type i in period t. Remember that i always runs from 1 through M. There is assumed to be no technical progress, so that λ_i and μ_i are not functions of time. The machines are assumed to wear out completely after m periods, but they are assumed not to be subject to physical depreciation before that time. λ_i and μ_i are thus not functions of the age of the machines.

The next equation defines the minimum number of machines of type i required to produce YY_{fit}:

(3.20) $$KMIN_{fit} = \frac{KH_{fit}}{\overline{H}}.$$

[minimum number of machines required to produce YY_{fit}]

It is assumed that $\overline{H}$, the maximum number of hours that each machine can be used each period, is constant across time. The actual number of machines of each type on hand in period t is

$$(3.21) \qquad K_{fit} = K_{fit-1} + IM_{fit} - IM_{fit-m}.$$

[actual number of machines of type i on hand]

Machines purchased in a period are assumed to be able to be used in the production process in that period. IM_{fit} is the number of machines of type i purchased in period t, and IM_{fit-m} is the number that wear out at the end of period $t-1$ and thus cannot be used in the production process in period t. The firm is subject to the restriction

$$(3.22) \qquad K_{fit} \geq KMIN_{fit},$$

[number of machines of type i on hand must be greater than or equal to the minimum number required]

which says that the actual number of machines of type i on hand must be greater than or equal to the minimum number required.

There is one good in the model, which can be used for either consumption or investment. In the following equation the number of machines purchased in period t is translated into the equivalent number of goods purchased:

$$(3.23) \qquad I_{ft} = \sum_{i=1}^{M} \theta_i IM_{fit}.$$

[number of goods purchased for investment]

θ_i is the number of goods it takes to create one machine of type i.

The total amount of output is

$$(3.24) \qquad Y_{ft} = \sum_{i=1}^{M} YY_{fit},$$

[total amount of output]

and the stock of inventories is

$$(3.25) \qquad V_{ft} = V_{ft-1} + Y_{ft} - X_{ft}.$$

[stock of inventories]

Equation (3.25) merely states that the stock of inventories is equal to last period's stock plus production minus sales. X_{ft} is the level of sales of the firm.

The next three equations define various adjustment costs facing the firm, with the costs taking the form of increased labor requirements:

$$(3.26) \qquad LL'_{fM+1t} = \beta_2 |V_{ft} - \beta_1 X_{ft}|,$$

[labor required to maintain deviations of inventories from β_1 times sales]

$$(3.27) \qquad LL'_{fM+2t} = \beta_3 (\Delta X_{ft})^2,$$

[labor required to handle fluctuations in sales]

$$(3.28) \qquad LL'_{fM+3t} = \beta_4 \left(\sum_{i=1}^{M} K_{fit} - \sum_{i=1}^{M} K_{fit-1} \right)^2.$$

[labor required to handle fluctuations in the capital stock]

Equation (3.26) reflects the assumption that there are costs in having inventories that are either greater than or less than a certain proportion of sales. Equations (3.27) and (3.28) reflect the assumptions that there are costs in having sales and the capital stock fluctuate. The minimum amount of labor required is

$$\text{(3.29)} \qquad LMIN_{ft} = \sum_{i=1}^{M+3} LL'_{fit}. \qquad \text{[minimum amount of labor required]}$$

The firm is subject to the restriction that labor paid for must be greater than or equal to labor requirements:

$$\text{(3.30)} \qquad L_{ft} \geq LMIN_{ft}. \qquad \text{[labor paid for must be greater than or equal to the minimum required]}$$

It is also assumed that there are adjustment costs in having the work force fluctuate. These costs take the form of increased taxes:

$$\text{(3.31)} \qquad T'_{ft} = \beta_5 |L_{ft} - L_{ft-1}|, \qquad \text{[taxes due to fluctuations in the work force]}$$

where T'_{ft} is the amount of taxes paid as a result of fluctuations in the work force.

The Financial Variables and Objective Function

The next set of equations pertains to the financial variables of the firm and to the firm's budget constraint. Depreciation is assumed to be straight line:

$$\text{(3.32)} \qquad DEP_{ft} = (1/m) \sum_{j=1}^{m} P'_{ft-j+1} I_{ft-j+1}. \qquad \text{[depreciation]}$$

The price of investment goods in (3.32) is denoted P' rather than P. The variable P is the price that the firm sets, and the firm is assumed not to buy its own goods for investment purposes. The variable P' is the price that it pays for these goods from other firms.

The value of before-tax profits on an accounting basis is

$$\text{(3.33)} \qquad \pi_{ft} = P_{ft} Y_{ft} - W_{ft} L_{ft} - DEP_{ft} + R_t A_{ft} + (P_{ft} - P_{ft-1}) V_{ft-1}. \qquad \text{[before-tax profits]}$$

If the firm is a debtor, the term $R_t A_{ft}$ is negative; it represents the interest costs of the firm. Negative values of A are liabilities, and so $-A_{ft}$ is the amount of borrowing of the firm. The last term in (3.33) is the gain or loss on the stock of inventories due to a price change. The level of taxes paid is

$$T_{ft} = d_{2t}\pi_{ft} + T'_{ft}, \tag{3.34}$$ [taxes paid]

where d_{2t} is the profit tax rate and T'_{ft} is the amount of taxes paid because of fluctuations in the work force. T'_{ft} is determined by (3.31).

The firm is assumed not to retain any earnings, and thus the level of dividends is merely the difference between before-tax profits and taxes:

$$D_{ft} = \pi_{ft} - T_{ft}. \tag{3.35}$$ [dividends paid]

The value of cash flow before taxes and dividends is

$$CF_{ft} = P_{ft}X_{ft} - W_{ft}L_{ft} - P'_{ft}I_{ft} + R_tA_{ft}, \tag{3.36}$$ [cash flow before taxes and dividends]

and the value of cash flow after taxes and dividends is

$$\begin{aligned} S_{ft} &= CF_{ft} - T_{ft} - D_{ft} \\ &= CF_{ft} - \pi_{ft} \\ &= P_{ft}(X_{ft} - Y_{ft}) - P'_{ft}I_{ft} + DEP_{ft} - (P_{ft} - P_{ft-1})V_{ft-1} \\ &= -P_{ft}V_{ft} + P_{ft-1}V_{ft-1} - P'_{ft}I_{ft} + DEP_{ft}. \end{aligned} \tag{3.37}$$ [savings: cash flow after taxes and dividends]

Cash flow after taxes and dividends is the savings of the firm. Since all after-tax profits are paid out in dividends, cash flow after taxes and dividends is merely cash flow minus profits, which is depreciation minus investment minus the change in the value of inventories. The budget constraint is

$$0 = S_{ft} - \Delta A_{ft} - \Delta M_{ft}. \tag{3.38}$$ [budget constraint]

M_{ft} is the level of money holdings of the firm. The budget constraint says that any nonzero value of savings must result in a change in A_f or M_f. The demand for money by the firm is simply assumed to be proportional to the value of sales:

$$M_{ft} = \gamma_3 P_{ft} X_{ft}. \tag{3.39}$$

Equations (3.18)–(3.39) hold for each period of the horizon ($t = 1, \ldots, T$). The objective of the firm is to maximize the present discounted value of after-tax cash flow, where the discount rates are the after-tax interest rates:

$$OBJ_f = \sum_{t=1}^{T} \frac{CF_{ft} - T_{ft}}{[1 + R_t(1 - d_{2t})]^t}. \tag{3.40}$$ [objective function]

$R_t(1 - d_{2t})$ is the after-tax interest rate for period t. The firm is assumed to be subject to the following two terminal conditions:

$$V_{fT} = \overline{V}, \qquad \text{[terminal condition for the stock of inventories]} \tag{3.41}$$

$$\sum_{i=1}^{M} K_{fiT} \geq \overline{K}. \qquad \text{[terminal condition for the capital stock]} \tag{3.42}$$

The first condition states that the stock of inventories at the end of the decision horizon is equal to a given number $\overline{V}$, and the second condition states that the number of machines held at the end of the horizon is greater than or equal to a given number $\overline{K}$. These conditions were imposed to avoid quirks that would otherwise occur in the optimal paths near the end of the horizon.

The decision problem of the firm is to choose paths of the decision variables to maximize (3.40), subject to the two terminal conditions and a number of initial conditions. The main decision variables are the firm's price, P_{ft}, its wage rate, W_{ft}, the number of each type of machine to buy, IM_{fit}, production, Y_{ft}, and the amount of labor to employ, L_{ft} $(t = 1, 2, \ldots, T)$. The main exogenous variables are the interest rate, R_t, the tax rate, d_{2t}, and the price of investment goods, P'_{ft} $(t = 1, 2, \ldots, T)$. The decision problem also requires that a number of expectations be formed, and these will now be discussed.

Determination of Expectations

The main expectations of a firm are those regarding other firms' prices and wages. For simplicity it will be assumed that there are just two firms, firm f and firm k. All expectations are firm f's. All values for the period prior to the first period of the decision horizon are known. Values for all other periods are either decision values or expectations.

The first equation pertains to firm f's expectation of firm k's price-setting behavior:

$$\frac{P_{kt}}{P_{kt-1}} = \left(\frac{P_{ft-1}}{P_{kt-1}}\right)^{\beta_6}\left(\frac{V_{kt-1}}{\beta_1 X_{kt-1}}\right)^{\beta_7}, \qquad \beta_7 < 0. \tag{3.43}$$

[expected price of firm k]

The first term in parentheses on the RHS of this equation reflects the assumption that firm f expects its price-setting behavior in period $t - 1$ to have an effect on firm k's price-setting behavior in period t. The second term represents the effect of market conditions on firm f's expectation of firm k's price. If firm k's stock of inventories at the end of period $t - 1$, V_{kt-1}, is greater than a certain proportion of sales, $\beta_1 X_{kt-1}$, firm f is assumed to expect that firm k will respond to this by lowering its price in period t in an effort to increase sales and draw down inventories.

The second term in (3.43) is assumed to pertain only to the first period of the horizon; (3.43) for periods $t+1$ and beyond includes just the first term:

$$(3.43)' \quad \frac{P_{kt+j}}{P_{kt+j-1}} = \left(\frac{P_{ft+j-1}}{P_{kt+j-1}}\right)^{\beta_6}, \qquad j = 1, \ldots, T.$$

[expected price of firm k for period $t+j$]

Equation (3.43)′ means that firm f expects that firm k is always adjusting its price toward firm f's price. If firm f's price is constant over time, then firm f expects that firm k's price will gradually approach this value. Firm f's expectation of the average price level is assumed to be the geometric average of its price and its expectation of firm k's price:

$$(3.44) \quad \overline{P}_t = (P_{ft} P_{kt})^{\frac{1}{2}}.$$

[expected average price]

The next equation determines firm f's expectation of the aggregate demand for goods, XA_t. This expectation is a function of the expected average price level:

$$(3.45) \quad XA_t = XA_{t-1}\left(\frac{\overline{P}_t}{\overline{P}_{t-1}}\right)^{\beta_8}, \qquad \beta_8 < 0.$$

[expected aggregate demand for goods]

Firm f's expectation of its market share of goods is

$$(3.46) \quad \frac{X_{ft}}{XA_t} = \frac{X_{ft-1}}{XA_{t-1}}\left(\frac{P_{ft}}{P_{kt}}\right)^{\beta_9}, \qquad \beta_9 < 0.$$

[expected market share of goods]

This equation reflects the assumption that a firm expects that its market share is a function of its price relative to the prices of other firms. The equation states that firm f's expected market share is equal to last period's share times a function of the ratio of its price to firm k's price.

This completes the equations regarding prices and demand. The next five equations pertain to wages and labor supply. The first determines firm f's expectation of firm k's wage rate:

$$(3.47) \quad \frac{W_{kt}}{W_{kt-1}} = \left(\frac{W_{ft-1}}{W_{kt-1}}\right)^{\beta_{10}}.$$

[expected wage rate of firm k]

This equation is similar to (3.43) for prices, but without the second term in

(3.43). Firm f's expectation of the average wage rate is

$$\overline{W}_t = (W_{ft} W_{kt})^{\frac{1}{2}}. \quad \text{[expected average wage rate]} \tag{3.48}$$

This equation is similar to (3.44) for prices.

Firm f's expectation of the aggregate unconstrained supply of labor is

$$LAUN_t = LAUN_{t-1}\left(\frac{\overline{W}_t}{\overline{W}_{t-1}}\right)^{\beta_{11}}\left(\frac{\overline{P}_t}{\overline{P}_{t-1}}\right)^{\beta_{12}}, \quad \beta_{12} < 0. \tag{3.49}$$

[expected aggregate unconstrained supply of labor]

$LAUN_t$ is the amount of labor that firm f expects will be supplied to the firm sector if the labor constraint is not binding on households. Equation (3.49) states that firm f expects that this amount is a positive function of the average wage rate and a negative function of the average price level. The next equation reflects the assumption that firm f expects households to be unconstrained in their labor supply decisions:

$$LA_t = LAUN_t, \quad \text{[expected aggregate constrained supply of labor]} \tag{3.50}$$

where LA_t denotes the actual amount of labor that firm f expects will be supplied. This assumption is discussed below. The final equation regarding wages and labor supply determines firm f's expectation of its market share of labor:

$$\frac{L_{ft}}{LA_t} = \frac{L_{ft-1}}{LA_{t-1}}\left(\frac{W_{ft}}{W_{kt}}\right)^{\beta_{13}}. \quad \text{[expected market share of labor]} \tag{3.51}$$

This equation is similar to (3.46) for goods. Firm f expects that its share is a function of its wage rate relative to firm k's wage rate.

This completes the expectational equations regarding prices, wages, demand, and labor supply. One last point in this regard concerns the firm's response to the possibility that it underestimates the supply of labor available to it at the wage rate that it sets. A firm is assumed to prepare for this possibility by announcing to households not only the wage rate that it will pay, but also the maximum amount of labor that it will employ, denoted $LMAX_{ft}$. This maximum is assumed to be set equal to the amount of labor that the firm expects to pay for, L_{ft}:

$$LMAX_{ft} = L_{ft}. \quad \text{[maximum amount of labor to employ]} \tag{3.52}$$

L_{ft} is determined by (3.51). By setting $LMAX_{ft}$ equal to L_{ft}, the firm is assured that it will never have to hire more labor than it expects to hire. This treatment is one exception to the general practice discussed in Section 3.1.1 of ignoring the effects of uncertainty on decisions. Note the similarity between (3.52) and (3.50). According to (3.52) the firm does not expect to turn any workers away, and according to (3.50) it does not expect any workers to be turned away in the aggregate.

Note that (3.49) implicitly assumes that firm f observes the lagged aggregate unconstrained supply of labor. If the labor constraint is binding on households, firms will be turning away workers, which should give firms some idea of the unconstrained supply. Firms are not, however, assumed to observe the lagged aggregate unconstrained demand for goods. If the labor constraint is binding on households, they will demand fewer goods than otherwise, and so the aggregate unconstrained demand for goods will be greater than the aggregate constrained demand. In this case there is no mechanism comparable to turning workers away for firms to observe the unconstrained demand, and thus it has been assumed that they do not observe it. In other words, firms have no way of knowing, say, how much (if any) of a drop in demand occurs because households are constrained in their labor supply. This assumption means that (3.45) is in terms of the actual (perhaps constrained) aggregate demand, not the unconstrained aggregate demand.

Characteristics of the Maximization Problem

The maximization problem of the firm is fairly complicated, and it may help to outline its main features. A key decision variable is the firm's price. The firm expects that it will gain customers by lowering its price relative to the expected prices of other firms. The main expected costs from doing this, in addition to the lower price it is charging per good, are the adjustment costs (3.26), (3.27), (3.28), and (3.31) involved in increasing sales, investment, and employment. The firm also expects that other firms will follow it if it lowers its price, so it does not expect to be able to capture an ever-increasing share of the market without further and further price reductions.

The firm expects that it will lose customers by raising its price relative to the expected prices of other firms. The main costs from doing this, aside from the lost customers, are the adjustment costs. On the plus side, the firm expects that other firms will follow it if it raises its price, so it does not expect to lose an ever-increasing share of the market without further and further price increases.

The firm expects that it will gain workers if it raises its wage rate relative to the expected wage rates of other firms and lose workers if it lowers its wage rate relative to the expected wage rates of other firms. The firm also expects that other firms will follow it if it raises (lowers) its wage rate, so it does not expect to capture (lose) an ever-increasing share of the market without further and further wage rate increases (decreases).

Because of the various adjustment costs, the firm, if it chooses to lower production, may choose in the current period not to lower its employment and capital stock to the minimum levels required. In other words, it may be optimal for the firm to hold either excess labor or excess capital or both during certain periods.

It may help in understanding the maximization problem to consider the algorithm that was used to solve it. The algorithm first searched over different price paths. For a given price path, the expected sales path can be computed using (3.43), (3.44), (3.45), and (3.46). For a given expected sales path, different output paths were tried. Two extreme output paths were tried: one in which the level of output remains as close as possible to the level of sales each period, and one in which the level of output remains as close as possible to the level of the previous period. In other words, for the first path output fluctuates roughly as sales do, and for the second path output fluctuates very little. The paths must satisfy the terminal condition (3.41) for inventories, and for each path production was adjusted to have this condition met. There is also a constraint that the stock of inventories cannot be negative in any period, and production was also adjusted if necessary to have this constraint met. The other output paths that were tried were weighted averages of the two extreme paths.

At the beginning of the first period there are a certain number of machines of each type on hand. If it is assumed, say, that only machines of type 1 are purchased, it is possible to compute for a given output path the number of machines that must be purchased to produce the output each period. This is done by first calculating the amount of output that can be produced with the current number of machines of all types on hand and then calculating the number of machines of type 1 required to produce the remaining output. These calculations are done using (3.19), (3.20), and (3.21). For a given output path, each of the M types of machines was tried, which means that it was first assumed that only type 1 machines are purchased, then only type 2 machines, and so on through type M machines.

For a given output path and a given type of machine, different investment paths were tried. Again, two extreme paths were tried: one in which the number of machines purchased equals the number required to produce the output and meet the terminal condition (3.42), and one in which the number of machines purchased remains as close as possible to the amount required to keep the number of machines unchanged from the previous period. The first path is one in which the capital stock fluctuates as much as the amount required, and the second path is one in which the capital stock fluctuates very little. The second path is subject to the constraint (3.22) that the number of machines must be sufficient to produce the output each period and to the terminal condition (3.42), and investment was adjusted if necessary to meet these conditions. Other paths were tried as weighted averages of the two extreme paths.

For each investment path different employment paths were tried. Given all the paths just mentioned, including the paths of the amount of output produced on each type of machine, it is possible to compute the amount of labor required to produce the total output. This is done using (3.18) and (3.26)–(3.29). Two extreme employment paths were tried: one in which the amount of labor equals the amount required, and one in which the amount of labor remains as close as possible to the amount of the previous period. The first path is one in which the amount of labor fluctuates as much as the amount required to produce the output, and the second path is one in which the amount of labor fluctuates very little. The second path is subject to the constraint (3.30) that the amount of labor must be sufficient to produce the output, and the amount of labor was adjusted if necessary to meet this. Other paths were tried as weighted averages of the two extreme paths.

Given the price path and the employment path, it is possible from (3.43)–(3.44) and (3.47)–(3.51) to compute the wage path that is necessary to have the employment path met. In other words, it is possible to compute the wage path that the firm expects is necessary to attract the amount of labor that it wants.

Given all these paths, it is possible to compute the objective function of the firm. This is done using (3.31)–(3.40). Since the algorithm consists of many layers of searching, the objective function is computed many times in the process of searching for the optimum. If, say, 5 output paths are tried for each sales path, if there are 3 types of machines, if 5 investment paths are tried for each output path and type of machine, and if 5 employment paths are tried for each investment path, then 375 objective function values ($5 \times 3 \times 5 \times 5$) are computed in the process of finding the optimum for the given sales path. If, say, 25 price paths (and thus 25 sales paths) are tried, the total number of objective function evaluations is 9,375 (25×375). Searching for the optimum price path was done by changing a price for a given period or a set of prices for a number of periods until the objective function stopped increasing and then trying another price or set of prices. The base price path that was used was the one in which the firm expects its market share of goods to remain unchanged. In other words, the base price path is one in which the firm is not trying to increase or decrease its market share.

Simulation Results

The length of the decision horizon, T, was taken to be 3 for the simulation results. The number of different types of machines, M, was taken to be 3, and the length of life of a machine, m, was taken to be 2.

The following values of the initial conditions were used.

Initial Conditions

$(t = 1)$

$A_{ft-1} = -100.0$	$M_{ft-1} = 25.0$
$I_{ft-1} = 27.0$	$P_{ft-1} = 1.0$
$IM_{f1t-1} = 0.0$	$P'_{ft-1} = 1.0$
$IM_{f1t-2} = 0.0$	$P_{kt-1} = 1.0$
$IM_{f2t-1} = 27.0$	$\bar{P}_{t-1} = 1.0$
$IM_{f2t-2} = 27.0$	$V_{ft-1} = 50.0$
$IM_{f3t-1} = 0.0$	$V_{kt-1} = 50.0$
$IM_{f3t-2} = 0.0$	$W_{ft-1} = 1.0$
$K_{f1t-1} = 0.0$	$W_{kt-1} = 1.0$
$K_{f2t-1} = 54.0$	$\bar{W}_{t-1} = 1.0$
$K_{f3t-1} = 0.0$	$X_{ft-1} = 263.0$
$L_{ft-1} = 185.0$	$X_{kt-1} = 263.0$
$LA_{t-1} = 370.0$	$XA_{t-1} = 526.0$
$LAUN_{t-1} = 370.0$	

Note that all machines on hand were assumed to be type 2 machines.

With respect to the exogenous variables, the interest rate for period 1, R_1, was taken to be .07, and the tax rate for period 1, d_{21}, was taken to be .5. The firm was assumed to know these values at the beginning of period 1 and to expect them to remain unchanged for periods 2 and 3. The firm was assumed to expect the price of investment goods for periods 1, 2, and 3 to be unchanged from its initial value given above of 1.0 (that is, $P'_{ft} = 1.0$, $t = 1, 2, 3$).

The two terminal-condition values were taken to be $\bar{K} = 54.0$ and $\bar{V} = 50.0$. The following parameter values were used.

Parameter Values

$\overline{H} = 1.0$	$\gamma_3 = 25.0/263.0 = .095057$
$\beta_1 = 50.0/263.0 = .190114$	$\lambda_2 = 263.0/185.0 = 1.421622$
$\beta_2 = .08$	$\lambda_1 = 1.006\lambda_2 = 1.422475$
$\beta_3 = .08$	$\lambda_3 = \lambda_2/1.006 = 1.413143$
$\beta_4 = .04$	$\mu_2 = 263.0/54.0 = 4.870370$
$\beta_5 = .04$	$\mu_1 = \mu_2$
$\beta_6 = .5$	$\mu_3 = \mu_2$
$\beta_7 = -.03$	$\theta_1 = 1.0315$
$\beta_8 = -1.0$	$\theta_2 = 1.0$
$\beta_9 = -5.0$	$\theta_3 = .97$
$\beta_{10} = .5$	
$\beta_{11} = 1.0$	
$\beta_{12} = -1.0$	
$\beta_{13} = 5.0$	

Note that all three types of machines have the same μ_i value. Type 1 machines are the most efficient with respect to labor requirements (that is, λ_1 is the largest) and cost the most (that is, θ_1 is the largest). Type 3 machines are the least efficient with respect to labor requirements and cost the least.

The algorithm discussed in the previous section was used to solve the maximization problem. In the search for the optimal price path, the smallest change in a price that was allowed was .001. For each price path, five output paths were tried (the two extreme paths and three weighted averages). For each output path and each type of machine, five investment paths were tried (the two extreme paths and three weighted averages). For each investment path, five employment paths were tried (the two extreme paths and three weighted averages). The weights were .5, .5; .1, .9; and .9, .1. It is clear that it would be necessary to try more paths in order to obtain the exact optimum, but for present purposes it is unlikely to matter that the exact optimum was not reached. Enough searching was done to make it likely that the computed optimum is close to the exact optimum, and for qualitative purposes this should be sufficient.

Each solution of the maximization problem took about 38 seconds on the IBM 4341 at Yale. Neither the DFP algorithm nor the procedure of obtaining first-order conditions analytically and solving them using Gauss-Seidel was tried, since the problem is really too complex for these methods. The problem has an inequality constraint, (3.42), which the methods cannot handle directly, but even if adjustments could be made for this, the problem is still too involved. It is not obvious that the DFP algorithm could have found the optimum given that it takes no advantage of the structure of the problem, and it seemed too risky to try. With respect to the other method, considerable work would have been required to obtain the first-order conditions, and this did not seem worth the effort.

The solution using the initial conditions and parameters just given was one in which the value of each decision variable was the same in all three periods. The values for selected variables are presented in the first column of Table 3-3. The ratio $L_{fit}/LMIN_{ft}$ in row 20 is a measure of the amount of excess labor held, where a value of 1.0 means no excess labor held. Likewise, the ratios $K_{fit}/KMIN_{fit}$, $i = 1, 2, 3$, in rows 21–23 are measures of the amount of excess capital held.

The simulation experiments consisted of changing initial conditions or exogenous variable values or parameter values, solving the maximization problem again, and observing the changes in the solution values from those for the base run. Results for nine experiments are presented in Table 3-3. The following paragraphs provide a discussion of these results.

Experiment 1: Increase in P_{k0}, the initial value of firm k's price. From (3.43), P_{k0} has a positive effect on firm f's expectation of firm k's price for period 1 and beyond (row 2). Firm f responded to the increase in P_{k0} by raising its own price (row 1). Had it raised its price by the same amount that it expected firm k's price to be raised, its expected market share would have remained constant (Eq. 3.46). In fact, its expected market share increased in all three periods (row 4). Although this is not shown in the table, firm f raised its price less in period 1 and slightly more in periods 2 and 3, the net result being an increase in market share for all three periods.

The expected aggregate demand for goods decreased because of the increase in prices (row 3; Eq. 3.45). Since firm f's expected market share rose and the expected aggregate demand for goods fell, firm f's expected sales could go either way. In fact, expected sales rose in period 1 and fell in periods 2 and 3 (row 5). Although this is not shown in the table, the sum of sales over the three periods rose. Production was smoothed relative to sales and was higher in all three periods (row 6). The stock of inventories was lower in periods 1 and 2 and equal to the terminal condition of 50.0 in period 3 (row 7).

The firm retained its investment in type 2 machines (rows 8–13). Investment was higher in periods 1 and 3 to meet the increased production (rows 12 and 14). Employment was also higher (row 15). Firm f's wage was higher to attract the extra employment (row 16). This in turn led firm f to expect that firm k's wage would be higher in periods 2 and 3 (row 17; Eq. 3.47). The expected aggregate supply of labor was lower because (although not shown) prices rose more than wages (row 18; Eqs. 3.49 and 3.50). Firm f's expected market share of labor rose because it had to attract the extra employment (row 19).

The firm planned to hold no excess labor or excess capital (rows 20–23). Profits and cash flow were higher because of the expansion and the higher prices relative to wages (rows 25 and 28). The level of savings was lower (row 30), primarily due to the fact that the increase in prices led to an increase in the value of inventories, which increases profits but not cash flow (Eqs. 3.33 and 3.36). Since the level of savings equals cash flow minus profits, it falls, other things being equal, when prices rise (Eq. 3.37). Money holdings rose because prices and sales rose (row 32; Eq. 3.39). The level of borrowing, which is $-A_{ft}$, rose because savings fell and money holdings rose (row 31; Eq. 3.38).

Although this is not shown in Table 3-3, roughly the opposite happened when P_{k0} was decreased rather than increased. Firm f did not lower its price as much as it expected firm k to do, and therefore it lost some market share. Its level of sales was lower in all three periods, as was its production. Investment and employment were lower; the wage rate was lower; profits and cash flow fell. The results were not exactly opposite in sign because the level of sales of firm f was lower in all three periods, whereas in Table 3-3 it is higher only in period 1. Moreover, the level of inventories, which is lower in periods 1 and 2 in Table 3-3, was also lower when P_{k0} was decreased. In both experiments firm f chose to produce less than it sold in period 1.

Experiment 2: Increase in W_{k0}, the initial value of firm k's wage. From (3.47), W_{k0} has a positive effect on firm f's expectation of firm k's wage in period 1 and beyond. The increase in W_{k0} thus led firm f to expect firm k's wage to be higher (row 17). Firm f responded to this by raising its wage (row 16). Although this is not shown in the table, firm f raised its wage less than it expected firm k to do. Its expected market share thus fell (row 19; Eq. 3.51). The expected aggregate supply was higher because of the higher wage rates (row 18; Eqs. 3.49 and 3.50). Profits and cash flow were lower because of the higher labor costs. The increase in W_{k0} had no effect on firm f's price, output, and investment decisions.

Although this is not shown in Table 3-3, the opposite signs were obtained when W_{k0} was decreased rather than increased.

Experiment 3: Increase in the λ_i's, the labor efficiency parameters. An increase in the λ_i's means that labor is now more efficient. With no other changes, this means that the firm is now holding excess labor. It responded to

TABLE 3-3. Simulation results for firm f

Row no.	Variable	Base run values 1[a]	Experiment 1: Increase in P_{k0} (1 2 3)	2: Increase in W_{k0} (1 2 3)	3: Increase in λ_i's (1 2 3)	4: Increase in μ_i's (1 2 3)	5: Interest rate increase to .20 (1 2 3)	6: Interest rate increase to .15 (1 2 3)	7: Tax rate increase (1 2 3)	8: Sales decrease (0 1 2 3)	9: Sales increase (0 1 2 3)
1	P_{ft}	1.0	+ + +	0 0 0	0 0 0	0 0 0	0 + +	0 + +	0 0 0	0 - - -	0 + + +
2	P_{kt}	1.0	+ + +	0 0 0	0 0 0	0 0 0	0 0 +	0 0 +	0 0 0	0 - - -	0 + + +
3	XA_t	526.0	- - -	0 0 0	0 0 0	0 0 0	0 - -	0 - -	0 0 0	- - - -	+ + + +
4	X_{ft}/XA_t	.5	+ + +	0 0 0	0 0 0	0 0 0	0 - -	0 - -	0 0 0	0 0 0 0	0 0 0 0
5	X_{ft}	263.0	+ - -	0 0 0	0 0 0	0 0 0	0 - -	0 - -	0 0 0	- - - -	+ + + +
6	Y_{ft}	263.0	+ + +	0 0 0	0 0 0	0 0 0	- - -	- - -	0 0 0	- - - -	- + + +
7	V_{ft}	50.0	- - 0	0 0 0	0 0 0	0 0 0	- - 0	- - 0	0 0 0	+ + + 0	- - - 0
8	Y_{f1t}	0.0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0 0	0 0 0 0
9	Y_{f2t}	263.0	+ + +	0 0 0	0 0 0	0 0 0	- - -	- - -	- - -	- - - -	- + + +
10	Y_{f3t}	0.0	0 0 0	0 0 0	0 0 0	0 0 0	+ + +	0 0 0	+ + +	0 0 0 0	0 0 0 0
11	IM_{f1t}	0.0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0 0	0 0 0 0
12	IM_{f2t}	27.0	+ 0 +	0 0 0	0 0 0	0 0 0	- - -	- 0 0	- - -	0 - 0 0	0 + 0 +
13	IM_{f3t}	0.0	0 0 0	0 0 0	0 0 0	0 0 0	+ + +	0 0 0	+ + +	0 0 0 0	0 0 0 0
14	I_{ft}	27.0	+ 0 +	0 0 0	0 0 0	- 0 0	- - -	- 0 0	- - -	0 - 0 0	0 + 0 +
15	L_{ft} ($= LMAX_{ft}$)	185.0	+ + +	0 0 0	- - -	+ 0 +	- + +	- - -	+ + +	0 - - -	0 + + +
16	W_{ft}	1.0	+ + +	+ + +	- - -	+ - +	- + +	- - -	+ + +	0 - - -	- + + +
17	W_{kt}	1.0	0 + +	+ + +	0 - -	0 + 0	0 - +	0 - -	0 + +	0 0 - -	0 0 + +
18	LA_t ($= LAUN_t$)	370.0	- - -	+ + +	- - -	+ 0 +	- - -	- - -	+ + +	0 + + +	0 - - -

19	L_{ft}/LA_t	.5	+ + +	- - -	- - -	+ 0 +	- + +	- - -	+ + +	0 - - -	0 + + +
20	$L_{ft}/LMIN_{ft}$	1.0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0 0	0 0 0 0
21	$K_{f1t}/KMIN_{f1t}$[b]	1.0	0 0 0	0 0 0	0 0 0	0 0 +	0 0 0	0 0 0	0 0 0	0 0 0 0	0 0 0 0
22	$K_{f2t}/KMIN_{f2t}$[b]	1.0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 +	0 0 0	+ 0 0 0	+ 0 0 0
23	$K_{f3t}/KMIN_{f3t}$[b]	1.0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 +	0 0 0	0 0 0	0 0 0 0	0 0 0 0
24	DEP_{ft}	27.0	+ + +	0 0 0	0 0 0	- - 0	- - -	- - 0	- - -	0 - - 0	0 + + +
25	Π_{ft}	44.0	+ + +	- - -	+ + +	+ + -	- - -	- - -	- - -	- - - -	- + + +
26	T_{ft}	22.0	+ + +	- - -	+ + +	+ + -	- - -	- - -	+ + +	- - - -	- + + +
27	D_{ft}	22.0	+ + +	- - -	+ + +	+ + -	- - -	- - -	- - -	- - - -	- + + +
28	CF_{ft}	44.0	+ + +	- - -	+ + +	+ + -	- - -	- - -	+ - -	- + - -	+ - + -
29	$CF_{ft} - T_{ft}$	22.0	+ + +	- - -	+ + +	+ - -	- - -	- - -	- - -	- + - +	+ - - -
30	S_{ft}	0.0	- - -	0 0 0	0 0 0	+ - 0	+ + -	+ - -	+ 0 0	- + + +	+ - - -
31	$-A_{ft}$	100.0	+ + +	0 0 0	0 0 0	- 0 0	- - -	- - -	- - -	+ + + -	- - - +
32	M_{ft}	25.0	+ + +	0 0 0	0 0 0	0 0 0	0 - -	0 - -	0 0 0	- - - -	+ + + +
33	OBJ_f	61.636	+	-	+	+	-	-	-	+	-

Notes: a. Values for periods 2 and 3 are the same as those for period 1.

b. If no capital of type i is held, then both K_{fit} and $KMIN_{fit}$ are 0. In this case 0/0 is defined to be 1.

- For experiment 1, P_{k0} was increased to 1.05.
- For experiment 2, W_{k0} was increased to 1.05.
- For experiment 3, each λ_i was increased by 2.0 percent.
- For experiment 4, each μ_i was increased by 2.0 percent.
- For experiment 7, the tax rate (d_{2t}) was increased to .75 (t = 1, 2, 3).
- For experiment 8, XA_0 was decreased to 525.0.
- For experiment 9, XA_0 was increased to 527.0.
- For both experiments 8 and 9, firm f expected XA_0 to be 526.0 when it made its decision for period 0.

this by lowering employment (row 15); its wage rate was lower because it needed to attract less labor (row 16). The firm chose to hold no excess labor (row 20), which means that all excess labor was eliminated in period 1. Profits and cash flow were higher because of the lower labor costs.

Experiment 4: Increase in the μ_i's, the capital efficiency parameters. An increase in the μ_i's means that the machines are now more efficient, which with no other changes means that the firm is holding excess capital. It responded to this by lowering investment enough in period 1 to eliminate all excess capital (rows 14 and 21). Although excess capital was not held in period 1, it was held in period 3 (row 21). The amount of capital held in period 3 was the amount required by the terminal condition (3.42), which was more than the amount required to produce the output. (The terminal condition was not changed for this experiment.)

Experiment 5: Interest rate increase to .20. In this case the firm switched to the cheaper, more labor-intensive type 3 machines (rows 8 – 13). It also raised its price in periods 2 and 3 and contracted. Investment was lower in all three periods (row 14). Employment was lower in period 1, but it was higher in periods 2 and 3 because of the increased labor requirements on the type 3 machines. The increase in the interest rate thus led to higher prices and lower investment and output.

Experiment 6: Interest rate increase to .15. In this case the interest rate increase was not large enough to lead the firm to switch to the type 3 machines. It was still optimal, however, for the firm to raise its prices in periods 2 and 3 and contract. Note that sales are unchanged in period 1, but that production is lower (rows 5 and 6), which means that the stock of inventories is lower (row 7). Since the interest rate contributes to the opportunity cost of holding inventories, an increase in the interest rate may lead the firm to hold fewer inventories, which is what happened here. The stock of inventories was unchanged in period 3 because of the terminal condition. Since the initial stock of inventories and the terminal condition are the same, any optimal plan of the firm must have the sum of production across the three periods equal the sum of sales. The way in which the firm can bring this about and still have the stock of inventories be less in periods 1 and 2 is to sell more in period 1 than in periods 2 and 3 and yet produce the same amount in all three periods. This is what the firm did in this experiment.

Although this is not shown in Table 3-3, the firm responded to an interest rate decrease (to .04) by switching to the type 1 machines and increasing investment. It did not, however, change its price and production plans, so there was no planned change in inventories. Employment was lower even though production was unchanged because of the use of the less labor-intensive machines.

Experiment 7: Tax rate increase. The increase in the profit tax rate led the firm to switch to the cheaper type 3 machines. Investment was lower because of this. Prices and production were unchanged. The main reason for the switch to the cheaper type 3 machines is the following. The objective of the firm is to maximize the present discounted value of after-tax cash flow. Two of the terms in the expression for after-tax cash flow are $-P'_{ft}I_{ft} + d_{2t}DEP_{ft}$, which means that investment lowers after-tax cash flow but depreciation raises it. The higher the tax rate d_{2t}, the more advantageous it is for the firm to have investment be low relative to depreciation. One way in which this can be done is to switch to the cheaper type 3 machines. This change lowers investment but does not require a lowering of production as long as more labor is hired. Depreciation does not fall as much as investment because it is a function of investment lagged one period as well as of current investment (Eq. 3.32). Although depreciation is lower in Table 3-3 (row 24), it is not as low as investment in period 1. (Note that from row 15 employment is higher, and that from row 16 the wage is higher, in order to attract the extra labor.) This negative effect of the tax rate on investment would, of course, not exist if investment expenditures could be written off completely in the current period. The effect is simply due to the firm's taking advantage of the effect of past investment expenditures on current depreciation.

Although this is not shown in Table 3-3, a decrease in the tax rate led the firm to switch to the type 1 machines, raise investment, and lower employment. The results were exactly opposite in sign to those for the increase in the tax rate.

Experiment 8: Unexpected decrease in sales. This experiment requires somewhat more explanation than the others. As will be discussed in Section 3.1.5, a firm solves its maximization problem at the beginning of the period before any transactions have taken place. Once transactions have taken place, many of the variables will be different from what the firm expected them to be. For experiment 8 the firm was first assumed to solve its maximization problem with no changes in any variables, so the decision values were those for the base run. The level of sales was then decreased. The effects of this change on the variables for the current period are presented in column 0 in Table 3-3 under experiment 8. The sales decrease took the form of a drop in aggregate demand (XA_t), and thus there is a negative sign in row 3. The firm's market share was assumed to remain unchanged, so its sales dropped (row 5). Because a change in sales increases labor requirements (Eq. 3.27) and because the firm was not planning to hold any excess labor, production had to be cut slightly from its planned level in order to meet the employment constraint (3.30). This is the reason for the minus sign in row 6. Production was cut less than sales fell, and therefore inventories rose (row 7). Because of the lower level of production, the firm ended up with slightly more capital than it needed to produce the output (row 22). In other words, meeting the labor constraint resulted in some excess capital being held. Profits and cash flow were lower because of the drop in production and sales. The drop in aggregate demand was also assumed to affect firm *k*, the other firm in the model. Firm *k* is assumed to be identical to firm *f*, and so the results are the same for firm *k*.

Any variable in column 0 that is not changed is a decision variable or an expectation variable that is not affected by the transactions of the period. The important decision variables for which this is true are the firm's price, investment, employment, and wage rate. Given the new set of initial conditions, the firm's maximization problem was solved again, where the horizon was still assumed to be three periods. The results are in columns 1, 2, and 3 in the table under experiment 8.

The firm responded to the sales decrease by lowering its price, production, investment, employment, and wage rate. Firm *f* expected firm *k* to lower its price because it knew that firm *k*'s stock of inventories exceeded $\beta_1 X_{k0}$. Firm *f* lowered its price by the same amount that it expected firm *k* to, thus leaving its market share unchanged (row 4). The lower prices have a positive effect on expected aggregate demand, but the lower initial level of aggregate demand has a negative effect (Eq. 3.45). The net effect was negative (row 3). Given the unchanged market share, the level of sales of firm *f* was lower (row 5). This then led to lower production, investment, employment, and the like.

Cash flow after taxes was larger for two of the three periods (row 29), and the objective function was larger (row 33). This is, however, somewhat misleading in that the firm is not better off because of the sales decrease. The firm suffered a loss of cash flow after taxes in period 0, and the objective function sign in row 33 pertains only to periods 1, 2, and 3. The firm started off at the beginning of period 1 with a higher level of inventories than was the case for the base run, and it gained cash flow by selling these off over the periods to reach the terminal condition of 50.0.

Experiment 9: Unexpected increase in sales. For this experiment sales were increased rather than decreased. The results are roughly the opposite to those for the sales decrease, but there is one important exception: production in period 0 was lower in both cases. This occurred because of the increased labor requirements due to the change in sales, which in both cases required cutting production in period 0.

Summary of Firm Behavior

The results of these experiments give a fairly good idea of the properties of the model of firm behavior. Some of the main effects are the following.

1. A change in the expected price (wage) of firm k leads firm f to change its own price (wage) in the same direction.
2. Excess labor on hand leads to a fall in employment, and excess capital on hand leads to a fall in investment.
3. An increase (decrease) in the interest rate leads to a substitution away from (toward) less labor-intensive machines and a decrease (increase) in investment expenditures. Changes in the interest rate also affect the opportunity cost of holding inventories, and thus the interest rate may affect the price and production decisions through this channel.
4. The firm responds to a decrease in aggregate demand by lowering its price and contracting. It responds to an increase in aggregate demand by raising its price and expanding.

It should be stressed that the results in Table 3-3 are for a particular set of parameter values. At least slightly different qualitative results are likely to be obtained for different sets. It seems unlikely, however, that the general properties of the model would be much affected by changes in the parameters. For the purpose of using the model to guide the specification of the econometric model, the results seem sufficient.

One point to note about the results is that for none of the experiments did the firm plan to hold excess labor. Similarly, the firm never planned to hold excess capital except in the last period. There are at least two reasons for this. One is that the cost-of-adjustment parameters regarding labor and capital, β_5 and β_4, are fairly small; the second is that it is relatively easy for the firm to smooth production, and with a smooth production path the employment and investment paths can be fairly smooth without deviating from the required amounts. Production can be smoothed not merely by using inventories as a buffer, but also by smoothing the expected sales path through changes in prices. In order for the results to show excess labor and excess capital being routinely held, the costs of smoothing production would have to rise relative to the costs of adjusting labor and capital. Again, however, for present purposes the results given above seem adequate.

3.1.4 Bank and Government Behavior

Bank Equations

Banks play a passive role in the model in the sense that no maximization problem is specified for them. Each bank, say bank b, receives money from households and firms in the form of demand deposits. Let $-M_{bt}$ denote the amount of demand deposits held in bank b, where M_{bt} is negative because demand deposits are a liability of a bank. Banks must hold a proportion g_{1t} of their demand deposits in the form of bank reserves:

(3.53) $$BR_{bt} = -g_{1t}M_{bt},$$ [bank reserves]

where BR_{bt} is the level of bank reserves and g_{1t} is the reserve requirement rate.

Bank borrowing from the monetary authority, BO_{bt}, is assumed to be a function of the difference between the discount rate, RD_t, and the interest rate, R_t:

(3.54) $$\frac{BO_{bt}}{BR_{bt}} = \gamma_4(RD_t - R_t), \qquad \gamma_4 < 0.$$ [bank borrowing]

No interest is assumed to be paid on demand deposits, and thus the level of before-tax profits of a bank is the difference between the interest revenue from its loans and the interest costs of its borrowing from the monetary authority:

(3.55) $$\pi_{bt} = R_tA_{bt} - RD_tBO_t,$$ [before-tax profits]

where A_{bt} is the amount of loans of the bank. The amount of taxes is

(3.56) $$T_{bt} = d_{2t}\pi_{bt},$$ [taxes paid]

where T_{bt} is the amount of taxes and d_{2t} is the profit tax rate. A bank is assumed to pay all of its after-tax profits in dividends:

(3.57) $$D_{bt} = \pi_{bt} - T_{bt},$$ [dividends paid]

where D_{bt} is the amount of dividends paid.

A bank's after-tax cash flow is merely its after-tax profits. Because it pays all of its after-tax profits in dividends, its level of savings is always zero, which means that a savings variable for a bank does not have to be specified. The bank's budget constraint is

(3.58) $$0 = \Delta A_{bt} + \Delta M_{bt} + \Delta BR_{bt} - \Delta BO_{bt}$$

or

(3.58)′ $$0 = A_{bt} + M_{bt} + BR_{bt} - BO_{bt}.$$ [budget constraint]

Government Equations

The government is defined here to be both the fiscal authority and the monetary authority. It collects taxes from households, firms, and banks, and it earns interest revenue on its loans to banks. If the government is a net debtor, which is assumed here, it pays interest on its borrowings. The other costs are wage costs and costs of goods purchased. The level of savings of the government, S_{gt}, is

$$(3.59) \qquad S_{gt} = \Sigma_h T_{ht} + \Sigma_f T_{ft} + \Sigma_b T_{bt} + RD_t \Sigma_b BO_{bt} + R_t A_{gt} - W_{gt} L_{gt} - P_{gt} C_{gt}. \qquad \text{[savings]}$$

The respective summations are over all the households, all the firms, and all the banks. A_{gt} is the value of net assets of the government (not counting $\Sigma_b BO_{bt}$), and it is negative if the government is a net debtor. The term $R_t A_{gt}$ is thus negative. L_{gt} is the amount of labor employed by the government, and W_{gt} is the wage rate paid by the government. C_{gt} is the amount of goods purchased, and P_{gt} is the price paid per good.

The budget constraint of the government is

$$(3.60) \qquad 0 = S_{gt} + \Sigma_b \, \Delta BR_{bt} - \Sigma_b \, \Delta BO_{bt} - \Delta A_{gt}. \qquad \text{[budget constraint]}$$

This equation states that any nonzero level of savings of the government must result in a change in nonborrowed reserves (that is, high-powered money) or government borrowing, $-A_{gt}$. For convenience, $-A_{gt}$ will be referred to as "the amount of government securities outstanding," even though there is no distinction in the model between government securities and any other type of securities.

Government behavior with respect to the tax-rate and expenditure variables is taken to be exogenous. In other words, fiscal policy is exogenous. The exogenous fiscal policy variables are d_{1t}, d_{2t}, TR_t, L_{gt}, and C_{gt}.

The three monetary policy variables are g_{1t}, RD_t, and A_{gt}. If all three of these variables are taken to be exogenous, the interest rate is implicitly determined in the model. Its value must be such as to have (3.60) satisfied, and in this loose sense it can be matched to (3.60). An alternative treatment is to assume that the government follows some reaction function with respect to its monetary policy. The reaction function that was assumed here is an interest rate reaction function:

$$(3.61) \qquad R_t = f(\; . \; . \; . \;), \qquad \text{[interest rate reaction function]}$$

where the arguments of the function are variables that affect the interest rate decision. Another possible reaction function is one in which the money supply, M_{bt}, is on the LHS, and another is one in which the variable nonborrowed reserves, $\Sigma_b BR_{bt} - \Sigma_b BO_{bt}$, is on the LHS. If a reaction function is postulated, one of the three monetary policy variables must be taken to be endogenous, where the most likely candidate is A_{gt}. If A_{gt} is taken to be endogenous, this means that open-market operations are used to meet the target LHS variable each period.

3.1.5 The Complete Model

There are two main questions to consider when putting together a model like the present one. One is how the agents are to be aggregated, and the other is the order in which the transactions take place. Aggregation will be discussed first.

One way in which the model could be put together would be to specify a number of different households, firms, and banks; have each one make its decisions; and then have them trade with each other. In order to do this one would have to specify mechanisms for deciding who trades with whom, and one would have to keep track of each individual trade. Questions of search behavior invariably arise in this context, as do distributional questions.

The other way is to ignore search and distributional issues. Even here, however, there are at least two ways in which these issues can be ignored: one is to postulate only one firm and treat it as a monopolist; the other is to postulate more than one firm but treat all firms as identical. This latter approach is the one that was taken. The advantage of postulating more than one firm is that models can be specified in which the behavior of an individual firm is influenced by its expectations of the behavior of other firms. Models like this, in which market share considerations can play a role, seem more reasonable in macroeconomics than do models of pure monopoly behavior.

An apparent disadvantage of postulating more than one firm and yet treating all firms as identical is that whenever a firm expects other firms to behave differently from the way it plans to behave, the firm is always wrong. Although firms always behave in the same way, they almost always expect that they will not. Firms never learn, in other words, that they are identical. Fortunately, this disadvantage is more apparent than real. If one is ignoring search and distributional questions anyway, there is no real difference (as far as ignoring the questions is concerned) whether one postulates one firm or many identical firms. Both postulates are of the same order of approximation, namely the complete ignoring of search and distributional questions, and if one feels that a richer model can be specified by postulating more than one firm, one might as well do so. The added richness will be gained without losing any more regarding search and distributional issues than is already lost in the monopoly model.

The aggregation that was used here consists of one household, two identical firms, and one bank. The household will be denoted h, the firms f and k, and the bank b. With respect to the order of transactions, information flows in one direction in the model: from the government, to the firms, to the household. Decisions are made at the beginning of the period before any transactions take place, and transactions occur throughout the rest of the period. A brief outline of the information flows will be given, and then the complete model will be set up. Note that the order of transactions is important in a model like the present one in which there can be disequilibrium. If transactions take place at nonmarket clearing prices, it is necessary to postulate who goes unsatisfied. In an equilibrium model in which no transactions take place until the market clearing prices are determined, the order of transactions does not matter.

A Brief Outline

Let t be the period under consideration. Before transactions take place, the following events occur. (1) The government determines the fiscal and monetary policy variables for period t. This includes the determination of the interest rate, which means that whatever variables are in the interest rate reaction function (3.61) are assumed to be known by the government at the beginning of period t. (2) Each firm receives information on the profit tax rate and the interest rate for period t from the government, forms expectations of these two variables for all relevant future periods, and solves its maximization problem. Determined by this solution are, among other things, its price, wage rate, and the maximum amount of labor to employ. (3) The household receives information for period t on the tax rate, the level of transfer payments, the interest rate, the wage rate, the price of goods, and the maximum amount that it will be able to work. It forms expectations of these variables for all relevant future periods and then solves its maximization problem. Determined by this solution are, among other things, its labor supply and consumption. (4) After the household makes its decision, transactions take place.

Note that the model is recursive in the sense that information flows in only one direction. The firms are not given an opportunity to change their decisions for the current period after the household has made its decisions; the firms only find out the decisions of the household after transactions have taken place. Note also that because the household makes its decisions after receiving information on the labor constraint, the system is guaranteed that the amount of labor supplied will not exceed the maximum allowed.

If the model is to be solved for more than one period, the whole procedure is repeated for period $t + 1$ after the transactions have taken place for period t. The decisions for period $t + 1$ are based on knowledge of the transactions for period t. Although values of the decision variables are computed for all periods of the horizon each time a maximization problem is solved, it is important to keep in mind that only the values for the current period are used in computing the transactions that take place. In each period new time paths are computed, based on the transactions that have taken place in the previous period, and thus the optimal values of the decision variables for periods other than the current period are of importance only insofar as they affect the optimal values for the current period.

The Model

When the complete model is put together a distinction must be made between the stock holdings and the bond holdings of the household. This distinction was unnecessary in the discussion of the household maximization problem because the expected rates of return on stocks and bonds are the same. The actual rates of return are not in general the same, and so this must be modeled.

The household owns all the stock in the model. Let PS_{t-1} denote the value of this stock at the end of period $t - 1$ or the beginning of period t. PS_{t-1} is assumed to be equal to the present discounted value of expected future after-tax cash flow of the firms and the bank, where the discount rates are the expected future one-period interest rates. Let ${}_{t-1}E_{t-1}$ denote the expected value of after-tax cash flow for period $t - 1$ that was made at the beginning of period $t - 1$, and let ${}_tE_t$ denote the expected value of after-tax cash flow for period t that is made at the beginning of period t. The variable ${}_tE_t$ is assumed to be a weighted average of ${}_{t-1}E_{t-1}$ and the actual value of after-tax cash flow in period $t - 1$:

$$(3.62) \qquad {}_tE_t = \lambda({}_{t-1}E_{t-1}) + (1 - \lambda)(CF_{ft-1} - T_{ft-1} + CF_{kt-1} - T_{kt-1} + D_{bt-1}), \qquad 0 < \lambda < 1.$$

[expected value of after-tax cash flow for period t]

The expected values of after-tax cash flow for periods $t + 1$ and beyond are all assumed to be equal to ${}_tE_t$. Similarly, the expected values of the interest rate for periods $t + 1$ and beyond are all assumed to be equal to the rate for period t, R_t. R_t is known at the beginning of period t. These expectational assumptions imply that

$$(3.63) \qquad PS_{t-1} = \frac{{}_tE_t}{R_t}. \qquad \text{[value of stocks at the beginning of period } t\text{]}$$

Let A'_{ht-1} denote the bond holdings of the household at the beginning of period t. Then the total value of stock and bond holdings at the beginning of period t, which was denoted A_{ht-1} in the discussion of the household maximization problem in Section 3.1.2, is $A'_{ht-1} + PS_{t-1}$. These variables will be used in the equations that follow.

There is a potential constraint on the output of the firms, which was briefly discussed in Section 3.1.3. Although the firms expect that they will be able to produce the amount of output that is computed from the maximization problem, this may not be the case. If the level of sales and the stock of inventories turn out to be different from what they were expected to be, labor requirements in (3.26) and (3.27) will be different from what they were expected to be. If the requirements are higher and if the firm was not planning to hold any excess labor, output will have to be cut from its planned value. Also, the firm may not get as much labor as it expected, and this will force it to cut output unless there is excess labor on hand to make up the difference. These adjustments are included in the model below.

The complete description of the model is as follows. The government determines

$$(\text{M1}) \qquad d_{1t}, d_{2t}, TR_t, L_{gt}, C_{gt}, R_t, g_{1t}, RD_t.$$

These decisions are exogenous except for the decision regarding R_t. R_t is determined by the reaction function (3.61). The value of stocks for the beginning of period t is determined by (3.63):

$$(\text{M2}) \qquad PS_{t-1}.$$

The value of the stock and bond holdings of the household at the end of period $t - 1$ or the beginning of period t is

$$(\text{M3}) \qquad A_{ht-1} = A'_{ht-1} + PS_{t-1},$$

where A'_{ht-1} is determined in period $t - 1$.

Given d_{2t} and R_t, firms f and k solve their maximization problems. Since the firms are identical, only the values for firm f need to be noted. The following variables, among others, are determined from this solution:

(M4) $P_{ft}, IM_{fit}, K_{fit}, I_{ft}, LMAX_{ft}, W_{ft}.$

All the different prices in the model are assumed to be equal to P_{ft}, and all the different wage rates are assumed to be equal to W_{ft}:

(M5) $P_{ht} = P_{gt} = P'_{ft} = P_{ft},$

(M6) $W_{ht} = W_{gt} = W_{ft}.$

The maximum amount that the household can work, L^*_{ht}, is

(M7) $L^*_{ht} = LMAX_{ft} + LMAX_{kt} + L_{gt}.$

Given d_{1t}, TR_t, R_t, A_{ht-1}, P_{ht}, W_{ht}, and L^*_{ht}, the household solves its maximization problem. Determined from this are

(M8) $L_{ht}, C_{ht}, N_{ht}, M_{ht}.$

The household can also be thought of as solving its maximization problem under the assumption of no labor constraint. Let LUN_{ht} denote the amount of labor that would be supplied if the constraint were not binding. Firms are assumed to observe this value after transactions have taken place, and therefore it is a variable of the model:

(M9) $LUN_{ht}.$

After the household makes its decisions, transactions take place. The rest of the model describes these transactions. The level of total sales is

(M10) $XA_t = C_{ht} + I_{ft} + I_{kt} + C_{gt}.$

Each firm receives half the sales:

(M11) $X_{ft} = X_{kt} = .5XA_t.$

The total amount of labor supplied to the firms is

(M12) $LA_t = L_{ht} - L_{gt}.$

This assumes that the government gets its labor first; what is left over goes to the firms. Each firm gets half the labor:

(M13) $L_{ft} = L_{kt} = .5LA_t.$

If the household were unconstrained, the amount of labor that would be supplied to the firms would be

(M14) $LAUN_t = LUN_{ht} - L_{gt}.$

Given X_{fi} and L_{ft}, it can now be seen whether firm f can produce the amount of output that it expected when it solved its maximization problem. If it cannot, output is cut back by the necessary amount. This is done in the most efficient way possible, which is by using the most labor-efficient machines first, the next most labor-efficient machines second, and so on. Y_{ft} will be used to denote the actual amount of output produced:

(M15) $\quad Y_{ft}$.

Given d_{2t}, R_t, P'_{ft}, I_{ft}, L_{ft}, X_{ft}, Y_{ft}, and the various lagged values, the following variables are determined by (3.25) and (3.31)–(3.39):

(M16) $\quad V_{ft}, T'_{ft}, DEP_{ft}, \pi_{ft}, T_{ft}, D_{ft}, CF_{ft}, S_{ft}, A_{ft}, M_{ft}$.

Because A_{ft} appears in (3.36) as well as in the budget constraint (3.38), the solution for some of these variables requires solving a small linear model.

The bank variables are determined next. The following equation determines M_{bt}:

(M17) $\quad M_{bt} = -M_{ht} - M_{ft} - M_{kt}$,

where the RHS variables are determined above. This equation merely states that the demand deposits of the household and firms are held in the bank. Given $d_{2t}, g_{1t}, R_t, RD_t, M_{bt}$, and various lagged values, the following variables are determined by (3.53)–(3.58):

(M18) $\quad BR_{bt}, BO_{bt}, \pi_{bt}, T_{bt}, D_{bt}, A_{bt}$.

In order to complete the variables for the household, the value of stocks at the end of period t must be known. This can be done if R_{t+1} is known, and so it is assumed that the government sets this rate at the end of period t but before the remaining variables for the household are determined:

(M19) $\quad R_{t+1}$.

Given that $CF_{ft}, T_{ft}, CF_{kt}, T_{kt}$, and D_{bt} have already been determined, ${}_{t+1}E_{t+1}$ can be computed from (3.62) with the time subscript moved ahead one period. ${}_{t+1}E_{t+1}$ is the expected value of after-tax cash flow for period $t+1$ made at the beginning of period $t+1$ (or the end of period t). Given R_{t+1} and ${}_{t+1}E_{t+1}$, PS_t can be computed from (3.63) with the time subscript moved ahead one period:

(M20) $\quad PS_t$.

The value of capital gains on stocks for period t, denoted CG_t, is

(M21) $\quad CG_t = PS_t - PS_{t-1}$.

Capital gains are assumed to be taxed like regular income. Given d_{1t}, TR_t, R_t, W_{ht}, P_{ht}, D_{ft}, D_{kt}, D_{bt}, M_{ht}, CG_t, M_{ht-1}, and A'_{ht-1}, the following four equations are used to solve for the four LHS variables:

(M22) $\quad Y_{ht} = W_{ht}L_{ht} + R_tA'_{ht} + D_{ft} + D_{kt} + D_{bt},$

(M23) $\quad T_{ht} = d_{1t}(Y_{ht} + CG_t) - TR_t,$

(M24) $\quad S_{ht} = Y_{ht} - T_{ht} - P_{ht}C_{ht},$

(M25) $\quad A'_{ht} = A'_{ht-1} + S_{ht} - \Delta M_{ht}.$

Equation (M22) is like (3.11), where nonwage income is now disaggregated into interest and dividend income. Equation (M23) is like (3.12), where capital gains are now included in the taxable income base. Equation (M24) is the same as (3.13). The budget constraint (M25) is like (3.14) except for the replacement of A' for A. Because A'_{ht} appears in both (M22) and (M25), the solution for the four LHS variables requires solving a linear model.

The last two variables to be determined are the government variables S_{gt} and A_{gt}. These are determined by (3.59) and (3.60):

(M26) $\quad S_{gt}, A_{gt}.$

There is one important redundant equation in the model, which states that the sum of bond holdings across all agents is zero:

(M27) $\quad 0 = A'_{ht} + A_{ft} + A_{kt} + A_{bt} + A_{gt}.$

This equation is redundant because the sum of savings across all agents is zero, and each agent's budget constraint has been used to solve for its bond holdings.

This completes the solution for period t. Given the solution values for this period, the model can be solved for period $t+1$. The initial conditions for period $t+1$ are the solution values for period t.

Simulation Results

Before the model is solved, the interest rate reaction function (3.61) must be specified. It is taken to be

$$(3.61)' \quad R_t = R_{t-1} - .1UR_t + \frac{P_{ft} - P_{ft-1}}{P_{ft-1}},$$

where UR_t is the unemployment rate. The unemployment rate is defined to be one minus the ratio of the constrained to the unconstrained supply of labor:

(3.64) $UR_t = 1 - \frac{L_{ht}}{LUN_{ht}}$. [unemployment rate]

Equation (3.61)′ is a "leaning against the wind" equation. The government raises the interest rate when unemployment falls and inflation rises, and it lowers the rate when unemployment rises and inflation falls. Given that the reaction function is used, A_{gt} it taken to be endogenous. The other two monetary policy variables, RD_t and g_{1t}, are exogenous.

The initial conditions and parameter values that were presented earlier for the household and firms were used for the results for the complete model. The other initial conditions and parameter values that are needed are the following.

$$
\begin{aligned}
&(t = 1)\\
A'_{ht-1} &= 311.42857\\
D_{bt-1} &= 4.2\\
BR_{bt-1} &= 30.0\\
BO_{bt-1} &= 0.0\\
A_{gt-1} &= -231.42857\\
{}_{t-1}E_{t-1} &= 48.2\\
CF_{ft-1} &= 44.0\\
CF_{kt-1} &= 44.0\\
T_{ft-1} &= 22.0\\
T_{kt-1} &= 22.0\\
R_{t-1} &= .07\\
UR_{t-1} &= 0.0\\
\gamma_4 &= -1.0\\
\lambda &= .9
\end{aligned}
$$

The reason for the choice of the above value for A'_{ht-1} is the following. From (M3) the value of wealth of the household at the beginning of period t, A_{ht-1}, is equal to $A'_{ht-1} + PS_{t-1}$, where from (3.63) $PS_{t-1} = {}_tE_t/R_t$. Given the above initial conditions, ${}_tE_t$ equals 48.2 and R_t equals .07, which implies a value of PS_{t-1} of 688.57143. This value plus the above value of 311.42857 for A'_{ht-1} equals 1,000, which is the value of A_{ht-1} used in Section 3.1.2 for the simulation results for the household.

With respect to the terminal value of wealth of the household, $\overline{AM}$ in (3.16), it was taken to be $311.42857 + PS_{t-1}$ for all of the experiments with the complete model, where PS_{t-1} is the value of stocks at the end of the previous period. If the model has been solved for at least one period, then the value of PS_{t-1} will in general differ from 48.2/.07, since in general both ${}_tE_t$ and R_t will be different. The terminal value of wealth thus differs from period to period depending on the value of stocks.

The government values that were used for the base run are as follows.

$(t = 1)$

$$
\begin{aligned}
d_{1t} &= .2 \\
d_{2t} &= .5 \\
TR_t &= 0.0 \\
L_{gt} &= 30.0 \\
C_{gt} &= 96.0 \\
g_{1t} &= .2 \\
RD_t &= .07
\end{aligned}
$$

The results of solving the model for the above values are presented in the first column of Table 3-4. A solution of the model for, say, period 1 requires running through steps (M1)-(M26). This entails the household and firms solving their maximization problems for periods 1-3, although only the decision values for period 1 ever get used. Once the model is solved for period 1, it can be solved for period 2. As the model is solved forward, it is assumed that the length of the decision horizon for the household and firms always remains at 3.

The cost of solving the complete model for one period is dominated by the cost of solving the maximization problem of the firm, since the other calculations are more or less trivial. The time taken on the IBM 4341 at Yale for the solution of the model for one period was about 39 seconds, of which about 38 seconds was used for the firm's maximization problem.

When the household solves its problem in, say, period 1, it must form expectations of W_{ht}, P_{ht}, R_t, d_{1t}, and TR_t for periods 2 and 3. In the analysis of household behavior in Section 3.1.2 it was assumed that the household expects these variables to remain unchanged in periods 2 and 3 from the observed period 1 values, and this assumption has been retained for the solution of the complete model. Regarding the labor constraint, it was assumed for experiment 7 in Table 3-2 that the household expected the constraint to be binding only for period 1, and this assumption has also been retained for the solution of the complete model. The labor constraint is thus binding on the household for at most the first period. In the analysis of firm behavior in Section 3.1.3 it was assumed that the firm expects the interest rate (R_t) and the tax rate (d_{2t}) to remain unchanged from the observed period 1 values, and the price of investment goods (P'_{ft}) to remain unchanged from the observed period 0 value. This assumption has been retained here.

When the model is solved period after period using the above initial conditions and parameter values and the above set of government values, the same solution value is obtained for each variable for each period. In other words, a "self-repeating" run is obtained. The values for selected variables from this run are presented in Table 3-4 in the column headed "Base run values." The self-repeating run is an equilibrium run in the sense that all the expectations are equal to the actual values. No errors are made anywhere in the model.

The experiments consisted of changing one of the government values and solving the model again. The value was changed for the current and all future periods. Most of the important properties of the model can be discovered by analyzing just two experiments: an increase in the interest rate and a decrease in government purchases of goods. For the interest rate experiment, the interest rate reaction function was dropped from the model and the interest rate was taken to be exogenous. This allows the interest rate to be taken to be a policy variable and changed exogenously. The results of the two experiments are presented in Table 3-4. Both the pluses and minuses and the actual numbers are presented for each experiment. Although the numbers have no empirical content, knowledge of them sometimes helps in understanding the results. The following paragraphs present a discussion of the results.

Experiment 1: An increase in the interest rate. The reader should remember that for this experiment there is no interest rate reaction function. The interest rate is exogenous, and the experiment consists of increasing it to .071 from its base period value of .070. Call the first period of the experiment period 1. The increase in the interest rate in period 1 causes the household to suffer a capital loss on its stocks at the beginning of the period (Eq. 3.63). Although this is not shown in the table, the value of stocks is $48.2/.071 = 678.87$, which compares to the base run value of $48.2/.07 = 688.57$.

The increase in the interest rate was not large enough to affect the firms' decisions for period 1 (rows 1, 6, 8 – 12). The household wanted to work more (row 19), but it was constrained from doing so because the firms did not want to hire any more labor. The household thus worked the same amount (row 13). It consumed less, spent more time taking care of money holdings, and planned to save more (rows 14, 15, 17). When transactions took place, sales were less (row 2) because of the drop in demand from the household. Production was slightly less (row 3) because the firms were forced to cut production from the planned values due to the increased labor requirements resulting from the change in sales. This cut was small, and sales dropped more than production. The level of inventories thus rose (row 4). The firms' profits

TABLE 3-4. Simulation results for the complete model

Row no.		Base run values 1[a]	Experiment $R_t(+)$ Signs 1 2 3 4 5	$R_t(+)$ Values 1	2	3	4	5	$C_{gt}(-)$ Signs 1 2 3 4 5	$C_{gt}(-)$ Values 1	2	3	4	5
Some key variables:														
1	P_{ft}	1.0	0 - - - -	1.0	.9999	.9998	.9997	.9996	0 - - - -	1.0	.9998	.9996	.9994	.9991
2	X_{ft}	263.0	- - - - -	262.82	262.78	262.83	262.86	262.93	- - - - -	262.75	262.61	262.54	262.38	262.21
3	Y_{ft}	263.0	- - - - -	262.97	262.79	262.75	262.82	262.83	- - - - -	262.96	262.70	262.56	262.47	262.29
4	V_{ft}	50.0	+ + + + -	50.16	50.17	50.09	50.05	49.96	+ + + + +	50.21	50.30	50.31	50.40	50.49
5	UR_t	0.0	+ + + + 0	.00052	.00068	.00056	.00018	0.0	0 + + + +	0.0	.00086	.00159	.00217	.00307
6	W_{ft}	1.0	0 - - - -	1.0	.9997	.9796	.9995	.9995	0 - - - -	1.0	.9998	.9995	.9991	.9985
7	R_{t+1}	.07	+ + + + +	.071	.071	.071	.071	.071	0 - - - -	.07	.0698	.0694	.0689	.0683
Firm f's decisions (other than P_{ft} and W_{ft}):														
8	X^e_{ft}	263.0	0 - - - -	263.0	262.85	262.81	262.85	262.87	0 - - - -	263.0	262.79	262.67	262.61	262.46
9	Y^e_{ft}	263.0	0 - - - -	263.0	262.80	262.75	262.82	262.85	0 - - - -	263.0	262.72	262.57	262.50	262.33
10	V^e_{ft}	50.0	0 + + + +	50.0	50.10	50.11	50.06	50.03	0 + + + +	50.0	50.14	50.20	50.21	50.27
11	I_{ft}	27.0	0 - - - -	27.0	26.96	26.99	26.97	27.00	0 - - - -	27.0	26.94	26.97	26.93	26.93
12	L^e_{ft} (= $LMAX_{ft}$)	185.0	0 - - - -	185.0	184.87	184.84	184.88	184.91	0 - - - -	185.0	184.82	184.73	184.67	184.56
Household's decisions:														
13	L_{ht}	400.0	0 - - - -	400.0	399.74	399.68	399.76	399.77	0 - - - -	400.0	399.64	399.44	399.35	399.12
14	C_{ht}	376.0	- - - - -	375.64	375.63	375.68	375.77	375.85	0 - - - -	376.0	375.84	375.65	375.41	375.04
15	N_{ht}	.2646	+ + + + +	.2665	.2665	.2666	.2665	.2665	0 + - - -	.2646	.2647	.2643	.2637	.2629
16	M_{ht}	100.0	- - - - -	99.88	99.87	99.87	99.88	99.90	0 - - - -	100.0	99.94	99.88	99.80	99.69
17	S^c_{ht}	0.0	+ + + + +	.66	.41	.32	.31	.26	0 - - - -	0.0	-.18	-.27	-.31	-.40
18	A^e_{ht}	1000.0	- - - - -	991.08	990.99	991.20	991.54	991.83	0 - + + +	1000.0	999.29	1000.70	1003.25	1006.30
19	LUN_{ht}	400.0	+ + - - -	400.21	400.01	399.90	399.83	399.77	0 - + + +	400.0	399.98	400.08	400.22	400.35

Transactions-determined variables (other than X_{ft}, Y_{ft}, V_{ft}, UR_t):														
20	L_{ft}	185.0	0 - - - -	185.0	184.87	184.84	184.88	184.88	0 - - - -	185.0	184.82	184.72	184.67	184.56
21	π_{ft}	44.0	- - - - -	43.86	43.85	43.85	43.88	43.88	- - - - -	43.95	43.88	43.88	43.90	43.93
22	D_{ft}	22.0	- - - - -	21.93	21.92	21.93	21.94	21.94	- - - - -	21.97	21.93	21.94	21.95	21.96
23	$CF_{ft} - T_{ft}$	22.0	- - - - +	21.78	21.93	21.99	21.99	22.02	- - - - -	21.76	21.89	21.92	21.89	21.89
24	S_{ft}	0.0	- + + + +	-.16	.02	.07	.05	.08	- - - - -	-.21	-.04	-.01	-.06	-.07
25	$-A_{ft}$	100.0	+ + + - -	100.14	100.12	100.05	100.00	99.92	+ + + + +	100.19	100.22	100.22	100.25	100.30
26	M_{ft}	25.0	- - - - -	24.98	24.98	24.98	24.98	24.98	- - - - -	24.98	24.96	24.95	24.93	24.90
27	$-M_{bt}$	150.0	- - - - -	149.85	149.82	149.82	149.84	149.87	- - - - -	149.95	149.86	149.77	149.65	149.49
28	BR_{bt}	30.0	- - - - -	29.97	29.96	29.96	29.97	29.97	- - - - -	29.99	29.97	29.95	29.93	29.90
29	BO_{bt}	0.0	+ + + + +	.030	.030	.030	.030	.030	0 0 - - -	0.0	0.0	-.007	-.019	-.032
30	D_{bt}	4.2	+ + + + +	4.256	4.255	4.255	4.255	4.256	- - - - -	4.199	4.196	4.179	4.153	4.121
31	A_{bt}	120.0	- - - - -	119.91	119.89	119.89	119.90	119.92	- - - - -	119.96	119.89	119.81	119.70	119.56
32	PS_t	688.57	- - - - -	678.32	678.26	678.38	678.48	678.67	- + + + +	687.89	690.03	693.64	697.95	703.97
33	CG_t	0.0	- - + + +	-.55	-.06	.12	.10	.19	- + + + +	-.68	2.14	3.61	4.31	6.02
34	S_{ht}	0.0	+ + + + +	.70	.36	.25	.25	.19	+ - - - -	.10	-.71	-1.10	-1.28	-1.72
35	A'_{ht}	311.43	+ + + + +	312.25	312.62	312.87	313.11	313.28	+ - - - -	311.53	310.88	309.84	308.63	307.02
36	A_{ht}	1000.0	- - - - -	990.58	990.89	991.26	991.59	991.95	- + + + +	999.42	1000.90	1003.48	1006.59	1011.00
37	S_{gt}	0.0	- - - - -	-.39	-.39	-.38	-.36	-.36	+ + + + +	.33	.80	1.13	1.39	1.87
38	$-A_{gt}$	231.43	+ + + + +	231.28	232.28	232.66	233.01	233.36	- - - - -	231.11	230.33	229.21	227.83	225.98

Notes: a. Values for all future periods the same as those for period 1.

- Superscript e denotes an expected value at the time the maximization problem was solved. The actual value that is determined when transactions takes place may differ from the expected value.
- For experiment 1, R_t was increased to .071 for all t.
- For experiment 2, C_{gt} was decreased to 95.0 for all t.

and cash flow were down because of the decrease in production and sales (rows 21, 23). The level of profits of the bank was higher because of the higher interest rate (row 30). The sum of after-tax cash flow of the firms and after-tax profits of the bank was lower, and this caused a fall in the value of stocks at the end of period 1 (rows 32 and 33). This capital loss, contrary to the capital loss at the beginning of the period, was caused by a fall in cash flow rather than a rise in the interest rate. The government ran a deficit in period 1 (row 37). There are a number of reasons for this. Firms' taxes were lower because of the fall in profits, and the household's taxes were lower because of the capital loss; the government's interest payments were higher because of the higher interest rate. The increase in the bank's taxes works the other way, but this increase was quite small, and thus the net effect on the government's budget was negative.

The response of a firm to a decrease in sales has been discussed in Section 3.1.3. The decrease in sales in period 1 led the firms in period 2 to lower prices, expected sales, planned production, investment, employment, and wage rates (rows 1, 6, 8–12). The household was again constrained in its labor supply, but this time because of the decrease in labor demand by the firms. (Unconstrained, the household wanted to work essentially the same amount as the base run value; see row 19.) The unemployment rate was higher in period 2 than in period 1 (row 5) because of the more severe labor constraint on the household. Sales were again lower in period 2 because of the lower consumption of the household.

The system continued at a lower level of sales and production throughout the five periods presented in the table. The main reason for this is the lower level of consumption of the household resulting from the higher interest rate. By period 5 the firms had reduced their inventories to essentially the base run value (row 4). The unemployment rate was back to zero by period 5. Given the particular parameter values used, the wage rate falls more than the price level each period (rows 1 and 6). This fall in the real wage leads the household to want to work less, and by period 5 its unconstrained supply of labor (row 19), while lower than the base run value, is no longer greater than the maximum amount allowed. The drop in the real wage is also the main reason that after-tax cash flow is higher than the base run value in period 5 (row 23) even though sales are lower. The government budget is in deficit throughout the period.

Experiment 2: A decrease in government purchases of goods. Part of this experiment has been discussed in Section 3.1.3 in the analysis of firm behavior. The decrease in goods purchases has no effect on anyone's decisions in period 1, but it does lead to lower sales, slightly lower production, and a higher level of inventories. The lower production is again due to the increased labor requirements resulting from the change in sales. Profits are lower, which causes a capital loss on stocks. Dividends are also lower. The reaction function does not change the interest rate at the end of period 1 (row 7) because the unemployment rate is zero and prices are unchanged.

In period 2 the firms responded to the sales decrease in the same manner as discussed in Section 3.1.3, namely by contracting. Although the price level and wage rate are the same to four digits in Table 3-4, the wage rate dropped slightly more. This led the household to lower very slightly its unconstrained supply of labor (row 19), but it was forced to supply even less because of the drop in the demand for labor from the firms (row 13). This is the main reason for the decrease in consumption in period 2. Sales were thus even lower in period 2 than they were in period 1 because of the consumption decrease. At the end of period 2 the reaction function lowered the interest rate (row 7) because of the positive level of unemployment and the fall in prices. This resulted in a capital gain at the end of period 2 (rows 32–33).

The system continued at the lower level of sales and production throughout the five periods in the table. The main factor that prevents the system from falling more than it does and that will eventually lead it to stop falling is the interest rate. As the unemployment rate rises and prices fall, the interest rate falls. A falling interest rate leads the household to consume more, both because of the fall in the interest rate itself (the intertemporal substitution effect) and the rise in wealth due to the capital gains on stocks. A fall in the interest rate may also lead the firms to switch to more expensive, less labor-intensive machines, which increases investment. Although this happened in the analysis of firm behavior in Table 3-3, the interest rate decreases were not large enough in Table 3-4 for this to take place in the current experiment. Although this is not presented in Table 3-4, the firms did switch to the more expensive machines in period 6 and thus increased their investment expenditures. It should be noted that one consequence that this switch has is to lower the demand for employment, which further constrains the household and leads it to lower consumption further. The substitution of more expensive machines is thus not in itself enough to stop the system from falling.

Other Contractionary Experiments

Given an understanding of the two experiments in Table 3-4, other contractionary experiments are easy to follow. If for any reason demand is lowered

—either government demand, firm demand, or household demand—a contractionary situation is likely to develop in which firms lower employment, the household lowers consumption because of the labor constraint, the firms lower employment more because of the further fall in sales, and so on.

Two of the experiments that were run involved an increase in the personal income tax rate, d_{1t}, and a decrease in the level of transfer payments, TR_t. Both led to decreased consumption by the household. The main difference between the two experiments is that the increase in d_{1t} leads, other things being equal, to a decrease in the unconstrained supply of labor, whereas the decrease in TR_t leads to an increase in the supply. The unemployment rate, which is a positive function of the unconstrained supply of labor, is thus higher in the transfer payment experiment than it is in the other.

An increase in the profit tax rate, d_{2t}, led to a fall in after-tax cash flow, dividends, and the price of stocks. The lower dividends and wealth of the household led it to consume less, which then started a contraction. This is the main channel through which an increase in the profit tax rate affects the economy, namely by first affecting the income and wealth of the household. As discussed in Section 3.1.3, an increase in d_{2t} may also lead the firm to switch to the less expensive machines, which lowers investment, but this is of rather minor importance.

An increase in the discount rate, RD_t, lowered the profits and dividends of the bank and thus the price of stocks. The lower dividends and wealth of the household led it to consume less. To the extent that bank profits are a small fraction of total profits in the economy, this effect on households is not likely to be a very large one in practice. A change in RD_t has no direct effect on the interest rate since it does not appear in the interest rate reaction function. An increase in RD_t does lead to a decrease in bank borrowing from the government, BO_t, which from (3.60) means that there are fewer government securities outstanding than otherwise (that is, $-A_{gt}$ is smaller). Remember that A_{gt} is the instrument by which the government achieves the target interest rate each period as dictated by the interest rate reaction function. Because of the interest rate reaction function, RD_t has little effect on R_t. The government merely offsets any changes in bank borrowing that result from changes in RD_t by changes in A_{gt}.

An increase in the reserve requirement rate, g_{1t}, also lowered bank profits and dividends, which then affected the household. Again, this effect is likely to be small in practice if bank profits are a small fraction of total profits. Bank reserves were higher because of the higher requirement rate, which from (3.60) means that there were fewer government securities outstanding than otherwise. g_{1t}, like RD_t, has little effect on R_t because the government merely offsets any changes in bank reserves that result from changes in g_{1t} by changes in A_{gt}.

Expansionary Experiments

Two "expansionary" experiments that were run involved a decrease in the interest rate and an increase in government purchases of goods. Expansionary experiments from a position of equilibrium are of somewhat less interest than contractionary ones in terms of learning about the properties of the model. When the system is in equilibrium, as it is in the base run, there are only two ways in which more output can be produced: one is for the household to work more, and the other is for the firms to switch to less labor-intensive machines. The household's work effort is a positive function of the real wage and the interest rate; it is a negative function of the initial value of wealth, the tax rate, and the level of transfer payments. The firms' switching to less labor-intensive machines is a positive function of the real wage and a negative function of the interest rate. The disequilibrium features of the model are thus not likely to be apparent for expansionary experiments, and the effects on output hinge on the labor supply response of the household and the investment response of the firms. The following is a brief discussion of the expansionary experiments.

When the interest rate was decreased, the household worked less in period 1. The real wage was unchanged because the interest rate decrease was not large enough to affect the firms' decisions in period 1. Given this and given the lower interest rate and the higher initial value of wealth from the interest rate decrease, the effect on household work effort was negative. Household consumption was higher in period 1, and thus sales were higher. Production was lower because of the increased labor requirements due to the change in sales and because of the decrease in labor supply. The stock of inventories was thus lower at the end of period 1. The lower work effort and higher consumption meant that the household dissaved in period 1.

The firms responded in period 2 to the higher sales, lower inventories, and lower labor supply by raising prices and wages. The price level was raised less than the wage rate, and this increase in the real wage led the household to increase its work effort in period 2 compared to the base run value. It continued to dissave in period 2. The real wage began to fall in period 3, but labor supply remained higher than its base run value. The main reason for this has to do with the saving behavior of the household. As noted, the lower interest rate led the household to dissave; this decreases wealth, which has a positive effect on labor supply in the next period. By period 3 the positive effect from the lower wealth outweighed the negative effects from the lower interest rate and the lower real wage.

The unemployment rate was zero for the first four periods, but in period 5 it was positive. Although labor supply and production were higher than they were in the base run, the household wanted to work slightly more than the labor constraint allowed, and so the unemployment rate was positive.

For the experiment in which government purchases of goods were increased, labor supply was the same in period 1, higher in period 2, and lower in periods 3 and beyond. It was unchanged in period 1 because the increase in goods purchases has no effect on the decisions in period 1. It was higher in period 2 primarily because the real wage was higher, and it was lower in periods 3 and beyond primarily because the real wage was lower.

The unemployment rate was zero throughout the five periods of the experiment; production was lower because of the lower labor supply; and prices and wages were higher because of the increase in sales and decrease in inventories. The interest rate was higher beginning in period 3 because of the increase in prices. Capital losses on stocks began occurring at the end of period 2 because of the higher interest rate.

3.1.6 Summary and Further Discussion

1. One of the main properties of the model is that disequilibrium can occur because of expectation errors. Once the system is in disequilibrium in the sense that expected values differ from actual values, it will remain so. In particular, a multiplier reaction can take place in which the firms constrain the household in its labor supply; the household responds by lowering consumption and thus sales of the firms; the firms respond by lowering production and their demand for labor, which further constrains the household; the household responds by lowering consumption even more; and so on.

2. Contrary to a model like the one of Phelps and Winter that was discussed in Section 3.1.1, the present model does not return to equilibrium in a straightforward way once it is shocked. In fact, the model never returns to equilibrium. No agent knows or ever learns the complete model, and thus decisions are always being made on the basis of expectations that turn out not to be correct. There is no convergence of expectations to the true values. This feature of the model does not depend on the expectations being formed in simple ways; it would be true even if agents formed their expectations on the

basis of predictions from sophisticated models as long as the models were not the true model and did not converge to the true model.

This feature of less than perfect expectations seems sensible in the present context. In order for agents to form correct expectations, they would have to know the maximization problems of all other agents. They also would have to know the exact way that transactions take place once the decisions have been solved for. In a model like the present one it seems unreasonable to assume that agents have this much information. (This is contrary to simple models of the Phelps and Winter type, where the assumption does not necessarily seem implausible.) It also seems unreasonable to assume that agents all learn the correct model over time. At the least, if they did finally learn it, the length of time needed to do so seems so long as to be for all practical purposes infinity.

The imposition of long-run constraints on models was discussed in Section 2.1.5, where it was noted that these constraints can play a critical role in the development of a model. It can now be seen why I believe that long-run constraints may be playing too much of a role in recent work. In order for a model like the present one to return to equilibrium once it is shocked, one has to make what seem to be unreasonable assumptions about the ability of agents to learn the complete model. Unless these assumptions are made, no long-run equilibrium constraints can be imposed on the model.

3. No price and wage rigidities have been postulated in the model. If this were done, it would provide another explanation of the existence of disequilibrium aside from expectations errors. One reason this was not done is to show that disequilibrium phenomena can easily arise without such rigidities.

4. The interest rate is the key variable that prevents the system from contracting indefinitely. As unemployment increases or prices fall, the interest rate is lowered by the interest rate reaction function. A fall in the interest rate results in a capital gain on stocks. Both the lower interest rate and the higher wealth have a positive effect on the consumption of the household. The lower interest rate may also lead the firms to switch to more expensive, less labor-intensive machines, which increases investment expenditures.

5. The fact that the interest rate has such important effects in the model means that monetary policy is quite important. With the interest rate reaction function included in the model, monetary policy is endogenous, and therefore monetary policy experiments cannot be run. One can, however, drop the reaction function and take the interest rate as exogenous. Monetary policy experiments can then be run by changing the interest rate, and, as just noted, this will have important effects on the system.

With the reaction function dropped, it is possible to take all three monetary instruments—the amount of government securities outstanding $(-A_{gt})$, the reserve requirement rate (g_{1t}), and the discount rate (RD_t)—as exogenous. In this case R_t is endogenous and is implicitly determined. Monetary policy experiments can then be run by changing one or more of these variables. The primary way that these changes would affect the system is through their effect on the interest rate.

6. The unemployment rate is a positive function of the supply of labor, which in turn is a function of variables such as the real wage, the interest rate, the income tax rate, and the level of transfer payments. The effects of a policy change on the unemployment rate thus depend in part on the labor supply response to the policy change. For example, increasing the income tax rate lowers labor supply, whereas decreasing the level of transfer payments raises it. Given the many factors that affect labor supply, there is clearly no stable relationship in the model between the unemployment rate and real output and between the unemployment rate and the rate of inflation. There is, in other words, no stable Okun's law and no stable Phillips curve in the model.

7. An interesting question about the long-run properties of the model is whether it is possible to concoct a self-repeating run in which there exists unemployment. It can be seen from (3.50) that this is not possible. The firm expects the unconstrained and constrained aggregate supplies of labor to be the same. If this is not true for, say, period t, which the firm knows at the beginning of period $t + 1$, the firm will not make the same decisions in period $t + 1$ as it did in period t.

The key assumption that allows there to be no self-repeating run with unemployment is that the firms observe the unconstrained as well as the constrained aggregate supplies of labor. Assume instead that the firms do not observe the unconstrained supply, and consider a self-repeating run with no unemployment. Now change the utility function of the household in such a way that it desires to work more and consume more, but keep the same levels of money holdings and wealth. Assume also that when constrained by the old self-repeating value of labor supply, the household chooses the same labor supply, consumption, and money holdings as it did before (and thus the same value of wealth as before). If the firms do not know the unconstrained supply of labor, there is no way for the information on the change in the utility function to be communicated to them. They only observe the actual demand for goods and supply of labor, which are the same as before. The firms thus make the same decisions as before, the household is subject to the same labor constraint as before (and so makes the same decisions as before), and so on. A self-repeating run will thus exist, but now in a situation where there is unemployment. Although this result is artificial, it does help to illustrate a feature of the model regarding information flows.

3.1.7 Comparison of the Model to the IS-LM Model and to a Class of Rational Expectations Models

The IS-LM Model

It may help in understanding the present model to compare it to two well-known models. The first is the IS-LM model, which has undoubtedly been the most popular model of the last three decades. A standard version of the IS-LM model consists of the following ten equations: (1) a consumption function in income and assets (the level of assets is exogenous), (2) an investment function in the rate of interest and income, (3) an income identity, where income is consumption plus investment plus government spending, (4) a real money demand function in the rate of interest and income, (5) a money supply function in the rate of interest (or the money supply taken to be exogenous), (6) an equilibrium condition equating money supply to money demand, (7) a production function in labor and the capital stock (the capital stock is exogenous), (8) a demand for labor equation equating the marginal product of labor to the real wage rate, (9) a labor supply function in either the money wage (the "Keynesian" version) or the real wage (the "classical" version), (10) an equilibrium condition equating the supply of labor to the demand for labor. These ten equations determine the following ten unknowns: consumption, investment, income, demand for money, supply of money, demand for labor, supply of labor, the price level, the wage rate, and the interest rate.

One of the main differences between my model and the IS-LM model is the treatment of consumption and labor supply. In my model the consumption and labor supply decisions are jointly determined. Both are a function of the same variables: the wage rate, the price level, the interest rate, the tax rate, the level of transfer payments, and wealth. In the IS-LM model, on the other hand, the decisions are not integrated. Labor supply is a function of the money wage or the real wage, and consumption is a function of income and assets. From a microeconomic point of view these decisions are not consistent. The only justification for using income as an explanatory variable in the consumption function is if the households are always constrained in their labor supply decisions. This is, however, inconsistent with the labor supply equation, where it is implicitly assumed that the households are not constrained.

Another important difference is the treatment of investment and employment. In my model the investment and employment decisions are jointly determined. Both decisions are a function of the various factors that affect the solutions of the firms' maximization problems. These decisions are not integrated in the IS-LM model. Investment is a function of the interest rate and income, and the demand for employment is a function of the real wage rate and the shape of the production function.

A third difference is that the IS-LM model is a static equilibrium one, whereas my model is dynamic and allows for the possibility of disequilibrium. Because of its static nature, there are no wealth, inventory, or capital-stock effects in the IS-LM model. These effects play an important role in my model. Wealth effects are easy to handle in the model because of the accounting for the flow-of-funds and balance-sheet constraints. This also means that there is no confusion regarding the government budget constraint: the constraint is automatically accounted for, so that any savings or dissavings of the government must result in a change in at least one of its assets or liabilities. This constraint is not part of the IS-LM model, and it has caused considerable discussion (see, for example, Christ 1968).

The equilibrium nature of the IS-LM model means that there is no unemployment. In the Keynesian version of the model it is possible to increase output by increasing government spending, but this comes about not by lessening some disequilibrium constraint but by inducing the households to work more by increasing the money wage. As discussed in the previous sections, disequilibrium effects can be quite important in my model. Unemployment can exist, and multiplier reactions can take place over time.

One of the key variables that affect consumption in my model is the interest rate. This comes about because of the multiperiod nature of the utility maximization problem, where intertemporal substitution effects are allowed. There are no such effects in the IS-LM model because it is static, and thus the interest rate does not affect consumption.

A Class of Rational Expectations Models

A class of rational expectations (RE) models has recently been developed that has become quite popular. This class includes the models in Lucas (1973), Sargent (1973, 1976), Sargent and Wallace (1976), and Barro (1976). Although the models in these five studies are not identical, they are similar enough to be able to be grouped together for purposes of the present comparison.

Three characteristics of the RE models are (1) the assumption that expectations are rational, (2) the assumption that information is imperfect regarding the current state of the economy, and (3) the postulation of an aggregate supply equation in which aggregate supply is a function of exogenous terms plus the difference between the actual and the expected price level. The models have the important property that government actions affect real output only if they are unanticipated. Because information is imperfect, unanticipated government actions can affect the difference between the actual and the expected price level, and so they can affect, for at least one period, aggregate supply. Anticipated government actions, on the other hand, do not affect this difference (because, since expectations are rational, all the information regarding anticipated government actions has already been incorporated into the actual and expected price levels), and so they cannot affect aggregate supply.

A key difference between the RE models and my model is that expectations are not rational in my model. The implications of the nonrationality of expectations have already been discussed and will not be repeated here. There is, however, another important difference between the models, which is that the RE models are not choice-theoretic. While agents are assumed to be rational in the sense that they know the model and use all the available information in the system in forming their expectations, they are at the same time irrational in the sense that their decisions are not derived from the assumption of maximizing behavior.

To the extent that the aggregate supply equation in the RE models has any microeconomic justification, it is based on the Lucas and Rapping (LR) model (1969). In this model a household is assumed to maximize a two-period utility function in consumption and leisure subject to a two-period budget constraint. Current labor supply is a function of the current wage rate and price level, the discounted future wage rate and price level, and the initial value of assets. The discount rate is the nominal interest rate. The signs of the derivatives of this function are ambiguous for the usual reasons. If it is assumed, as Lucas and Rapping do, that current and future consumption and future leisure are substitutes for current leisure and that income and asset effects are small, then current labor supply is a positive function of the current wage rate and a negative function of the current and future price level and the future wage rate. This model is used to justify, in at least a loose sense, the assumption in the RE models that the difference between the actual and the expected price level has a positive effect on aggregate supply. An actual price level higher than expected is analogous to an increase in the current wage rate relative to the current and future price level and the future wage rate.

Although the LR model is used in part as a justification for the aggregate supply equation in the RE models, there are some important features of the LR model that are not incorporated into the supply equation. One variable that is omitted is the interest rate. As just discussed, the interest rate has an effect on the current supply of labor in the LR model, and thus it should be included in the supply equation in the RE models. The interest rate clearly belongs in an equation whose justification is based in part on an appeal to intertemporal substitution effects. The RE models, with the exception of Barro's (1976), also exclude from the supply equation any asset variables, even though the initial value of assets has an effect on the current supply of labor in the LR model. Another omission of both the LR and RE models is the exclusion of personal tax rates from the analysis. It is well known that personal tax rates have an effect on the labor supply of a utility-maximizing household.

It is also true that many of the other equations of the RE models are not based on the assumption of maximizing behavior. Sargent and Wallace, for example, note that their model is ad hoc, where "by *ad hoc* we mean that the model is not derived from a consistent set of assumptions about individuals' and firms' objective functions and the information available to them" (1976, p. 241).

The RE models can thus be criticized on theoretical grounds in that it seems odd to postulate rationality with respect to the formation of expectations (in particular that agents are sophisticated enough to know the complete model) but not with respect to overall behavior.

Regarding policy effects, it seems likely that in models in which there are both rational expectations and maximizing agents, anticipated government actions will affect the economy. To the extent that the government affects, directly or indirectly, variables that influence the solutions of the households' utility maximization problems, real output will be affected. It would be an unusual model that insulated the households' decision problems from everything that the government affects. The policy property of the RE models that anticipated government actions do not affect real output is thus not likely to be true in a model in which there are rational expectations and maximizing agents.

3.2 The Two-Country Model

3.2.1 Introduction

The way in which I approached the construction of a two-country model was to consider how one would link my single-country model to another model exactly like it. Because the flow-of-funds and balance-sheet constraints are met in the single-country model, they are also met in the two-country model, which distinguishes it in an important way from previous models. Stock and flow effects are completely integrated in the model. There is, for example, no natural distinction between stock-market and flow-market determination of the exchange rate, a distinction that has played an important role in the literature on the monetary approach to the balance of payments. (See, for example, Frenkel and Rodriguez 1975; Frenkel and Johnson 1976; Dornbusch 1976; Kouri 1976; and the survey by Myhrman 1976.) The exchange rate is merely one endogenous variable out of many, and in no rigorous sense can it be said to be *the* variable that clears a particular market. In other words, there is no need for a stock-flow distinction in the model. (Other studies in which the stock-flow distinction is important include Allen 1973; Black 1973; Branson 1974; and Girton and Henderson 1976.)

In the following sections capital letters denote variables for country 1, lowercase letters denote variables for country 2, and an asterisk (*) on a variable denotes the other country's purchase or holding of the variable. The exchange rate, denoted e_t, is the price of country 2's currency in terms of country 1's currency. There is assumed to be an international reserve, denoted Q_t for country 1's holdings and q_t for country 2's holdings, which is denominated in the currency of country 1. The total amount of this reserve is assumed to be constant across time. There is assumed to be one good per country.

3.2.2 Trade Linkages

A way of introducing trade in the model is to add c^*_{ht} to the utility function (3.9) of the household:

$$(3.9)'' \qquad U_{ht} = f_{9''}(C_{ht}, TH - L_{ht} - N_{ht}, c^*_{ht}),$$

where c^*_{ht} is household h's consumption of the foreign good. The term $-e_t p_{ht} c^*_{ht}$ is then added to the savings equation, (3.13):

$$(3.13)'' \quad S_{ht} = Y_{ht} - T_{ht} - P_{ht}C_{ht} - e_t p_{ht} c^*_{ht},$$

where p_{ht} is the price of the foreign good. This adds one decision variable, c^*_{ht}, and two exogenous variables, e_t and p_{ht}, to the maximization problem of the household. The demand for the home good will be, among other things, a function of the two prices and the exchange rate, and similarly for the demand for the foreign good.

3.2.3 Price Linkages

In addition to the obvious trade linkages between countries, there may be price linkages. In particular, prices of domestic goods may be influenced by the prices of foreign goods. One way of introducing this into the model is to modify the equation determining firm f's expected aggregate demand for (domestic) goods, (3.45). Since a household's demand for domestic goods is a function of the price of domestic goods and the price of foreign goods, it is reasonable to assume that a firm expects that the aggregate demand for domestic goods is a function of the average price of domestic goods and the average price of foreign goods:

$$(3.45)'' \quad XA_t = XA_{t-1}\left(\frac{\bar{P}_t}{\bar{P}_{t-1}}\right)^{\beta_8}\left(\frac{e_t\bar{p}_t}{e_{t-1}\bar{p}_{t-1}}\right)^{\beta_{14}}, \qquad \beta_8 < 0, \qquad \beta_{14} > 0,$$

where $\bar{p}_t$ is the average price of foreign goods.

Replacing (3.45) with (3.45)″ adds two exogenous variables to the maximization problem of the firm: the exchange rate and the average price of foreign goods. If the product of these two, which is the average price of foreign goods in domestic currency, increases, the firm expects, other things being equal, that the demand for domestic goods will increase. An increase in the domestic currency price of foreign goods is thus like a demand increase, and the firm responds to a demand increase by raising its price. Higher import prices thus lead to higher domestic prices through this channel.

3.2.4 Introduction of a Foreign Security

Although it is easy to introduce a foreign good into the model, it is not as easy to introduce a foreign security; the model is not set up to handle different securities in a convenient way. One way of introducing a foreign security is the following. Assume that only banks hold foreign securities, and let a^*_{bt} denote the amount of the security held by bank b. Foreign securities, like domestic securities, are assumed to be one-period bonds. Bank b's demand for foreign securities is assumed to be a function, among other things, of each country's interest rate:

(3.65) $a^*_{bt} = f_{65}(R_t, r_t, \ . \ . \ . \ ),$

where r_t is country 2's interest rate. This assumption is ad hoc in that the equation is not derived from the solution of a maximization problem for bank b, but for present purposes it is sufficient for illustrating the main features of the model. In the empirical work this assumption is not used because perfect substitutability between foreign and domestic securities is assumed.

The introduction of a^*_{bt} to the model requires that (3.55) determining bank profits be modified:

(3.55)″ $\pi_{bt} = R_t A_{bt} - RD_t BO_t + r_t e_t a^*_{bt},$

where the last term is the interest revenue in domestic currency on the foreign security holdings. The bank's budget constraint (3.58) is also modified:

(3.58)″ $0 = \Delta A_{bt} + \Delta M_{bt} + \Delta BR_{bt} - \Delta BO_{bt} + e_t \Delta a^*_{bt}.$

Finally, (M27) is modified to reflect foreign holdings of domestic securities:

(M27)″ $0 = A'_{ht} + A_{ft} + A_{kt} + A_{bt} + A_{gt} + A^*_{bt}.$

3.2.5 Determination of the Exchange Rate

The basic feature of the two-country model with respect to the determination of the exchange rate can be most easily seen by aggregating the household, firms, and bank into one sector, called the "private sector." Let S_{pt} denote the level of savings of the private sector, which is the sum of the savings of the household and firms. (As discussed in Section 3.1.4, the savings of the bank is always zero.) Let A_{pt} denote the sum of A'_{ht}, A_{ft}, A_{kt}, and A_{bt}. Also, change the b subscript on BR_{bt}, BO_{bt}, and a^*_{bt} to p to keep the notation consistent. The same aggregation hold for country 2, with capital and lowercase letters reversed.

Although the level of savings of an agent is determined by a definition in the model, it will be convenient to represent the determination of savings in the following way:

(T1) $S_{pt} = f_{T1}(. \ . \ .),$

(T2) $S_{gt} = f_{T2}(. \ . \ .),$

(T3) $s_{pt} = f_{T3}(. \ . \ .),$

(T4) $s_{gt} = f_{T4}(. \ . \ .).$

Equation (T1) represents the determination of the savings of the private sector of country 1. Almost every variable in the model, including the variables that pertain to country 2, has at least an indirect effect on savings, and thus the argument list of the function in (T1) is long. This is also true of (T2), which represents the determination of the savings of the government of country 1. Equations (T3) and (T4) are similar equations for country 2.

The next thing to be done is to aggregate the budget constraints of the individual agents into a budget constraint of the private sector. This cancels out the securities that are only held within the private sector, which in the present case are money holdings. Adding the budget constraints (3.14) for the household, (3.38) for each firm, and (3.58)″ for the bank yields:

$$\text{(T5)} \qquad 0 = S_{pt} - \Delta BR_{pt} + \Delta BO_{pt} - \Delta A_{pt} - e_t \Delta a^*_{pt}.$$

The government budget constraint (3.60) in the present notation is

$$\text{(T6)} \qquad 0 = S_{gt} + \Delta BR_{pt} - \Delta BO_{pt} - \Delta A_{gt} - \Delta Q_t,$$

where the term ΔQ_t, which is the change in holdings of the international reserve of the government, is added to the equation. Similar equations hold for country 2:

$$\text{(T7)} \qquad 0 = s_{pt} - \Delta br_{pt} + \Delta bo_{pt} - \Delta a_{pt} - \frac{1}{e_t}\Delta A^*_{pt},$$

$$\text{(T8)} \qquad 0 = s_{gt} + \Delta br_{pt} - \Delta bo_{pt} - \Delta a_{gt} - \frac{1}{e_t}\Delta q_t.$$

The level of bank reserves, BR_{bt}, is determined by (3.53). It is equal to $-g_{1t}M_{bt}$, where g_{1t} is the reserve requirement rate and $-M_{bt}$ is the level of demand deposits. M_{bt} drops out of the model in the aggregation to the private sector, and so an equation like (3.53) cannot be written down for BR_{pt}. BR_{pt} is, of course, still determined in the model, and for the purpose of the equations here its determination can be represented in the same manner as in (T1)-(T4) for the savings variables:

$$\text{(T9)} \qquad BR_{pt} = f_{T9}(\ldots).$$

This equation stands for the determination of BR_{pt}, where nearly every variable in the model is in the argument list. Bank borrowing from the monetary authority is determined by (3.54), which in the present notation is

(T10) $\frac{BO_{pt}}{BR_{pt}} = \gamma_4(RD_t - R_t), \qquad \gamma_4 < 0.$

Similar equations hold for country 2:

(T11) $br_{pt} = f_{T11}(\ldots),$

(T12) $\frac{bo_{pt}}{br_{pt}} = \gamma_4(rd_t - r_t).$

Equation (3.65), the equation determining the domestic demand for the foreign security, in the present notation is

(T13) $a^*_{pt} = f_{T13}(R_t, r_t, \ldots).$

A similar equation holds for country 2's demand for country 1's security:

(T14) $A^*_{pt} = f_{T14}(R_t, r_t, \ldots).$

The following three definitions close the model:

(T15) $0 = A_{pt} + A_{gt} + A^*_{pt},$

(T16) $0 = a_{pt} + a_{gt} + a^*_{pt},$

(T17) $0 = \Delta Q_t + \Delta q_t.$

Equation (T15) states that the sum of the holdings of country 1's bond across holders is zero. (Remember that liabilities are negative values.) Equation (T16) is the similar equation for country 2. Equation (T17) states that there is no change in total world reserves.

The savings variables satisfy the property that $S_{pt} + S_{gt} + e_t s_{pt} + e_t s_{gt} = 0$, and therefore one of the equations (T1)–(T8) and (T15)–(T17) is redundant. It will be useful to drop (T17); this leaves 16 independent equations. There are 19 variables in the model: S_{pt}, S_{gt}, s_{pt}, s_{gt}, A_{pt}, a^*_{pt}, a_{pt}, A^*_{pt}, BR_{pt}, br_{pt}, BO_{pt}, bo_{pt}, Q_t, q_t, e_t, R_t, r_t, A_{gt}, and a_{gt}. In the case of fixed exchange rates e_t is exogenous and Q_t is endogenous, and in the case of flexible exchange rates e_t is endogenous and Q_t is exogenous. Given that one of these two variables is taken to be exogenous, the model can be closed by taking A_{gt} and a_{gt} to be the exogenous monetary policy variables.

It should be clear from this representation that e_t is not determined solely in stock markets or in flow markets; it is simultaneously determined along with the other endogenous variables. This, as discussed earlier, is an important difference between this model and previous models.

3.2.6 Properties of the Model

I have not obtained any simulation results for the two-country model. Given the results for the single-country model and given (as will be seen in Section 4.2.2) the special case of the two-country model that had to be used to guide the econometric specifications, simulation results for the two-country model seemed unnecessary. The main features to be remembered about the model are the following.

1. Adding a foreign good to the utility function of the household means that the demand for the foreign good will be a function of the same variables that affect the household's consumption decision in the single-country model plus two new variables: the price of the foreign good and the exchange rate.

2. Adding the price of the foreign good to the equation determining the firm's expected aggregate demand for the domestic good means that the price of the foreign good and the exchange rate will affect the domestic price level.

3. Any model of exchange rate determination that is used for the empirical work should be consistent with (T1)–(T17). In particular, no distinction should be necessary between stock-market and flow-market determination of the exchange rate.