Comparing Information in Forecasts from Econometric Models

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The information contained in one model's forecast compared to that in another can be assessed from a regression of actual values on predicted values from the two models. We do this for forecasts of real GNP growth rates for different pairs of models. The models include a structural model (the Fair (1976) model), various versions of the vector autoregressive (VAR) model, and various versions of a model we call the "autoregressive components" (AC) model. Our procedure requires that forecasts make no use of future information, and we have been careful to try to insure this, including using the version of the Fair model that existed in 1976, the beginning of our test period. (JEL 132)

Many econometric models are used to forecast economic activity. These models differ in structure and in the data used, and so their forecasts are not perfectly correlated with each other. How should we interpret the differences in forecasts? Does each model have a strength of its own, so that each forecast represents useful information unique to it, or does one model dominate in the sense of incorporating all the information in the other models plus some?

Structural econometric models often make use of large information sets in forecasting a given variable. The information set used in a large-scale macroeconometric model is typically so large that the number of predetermined variables exceeds the number of observations available for estimating the model. Estimation can proceed effectively only because of the large number of a priori restrictions imposed on the model, restrictions that do not work out to be simple exclusion restrictions on the reduced form equation for the variable forecasted.

Vector autoregressive (VAR) models are typically much smaller than structural models and in this sense use less information. The above question with respect to VAR models versus structural models is thus whether the information not contained in VAR models (but contained in structural models) is useful for forecasting purposes. In other words, are the a priori restrictions of large-scale models useful in producing derived reduced forms that depend on so much information, or is most of the information extraneous?

One cannot answer this question by doing conventional tests of the restrictions in a structural model. These restrictions might be wrong in important ways and yet the model contain useful information. Even ignoring this point, however, one cannot perform such tests with most large-scale models because, as noted above, there are not enough observations to estimate unrestricted reduced forms.

We will examine the question whether one model's forecast of real GNP carries different information from another's by regressing the actual change in real GNP on the forecasted changes from the two models. This procedure, which is discussed in the next section, is related to the literature on encompassing tests[1] and the literature on the opti-

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[1]See, for example, Russell Davidson and James G. MacKinnon (1981), David F. Hendry and Jean-Francois Richard (1982), Yock Y. Chong and David F. Hendry

mal combination of forecasts.[2] From our point of view, this procedure has two advantages over the standard procedure of computing root mean squared errors (RMSEs) to compare alternative forecasts. First, if the RMSEs are close for two forecasts, little can be concluded about the relative merits of the two. With our procedure, as will be seen, one can sometimes discriminate more. Second, even if one RMSE is much smaller than the other, it may still be that the forecast with the higher RMSE contains information not in the other forecast. There is no way to test for this using the RMSE framework.

It should be stressed that our procedure does not allow us to discover whether all the variables in a model contribute useful information for forecasting. If, say, our regression results reveal that a large model contains all the information in smaller models plus some, it may be that the good results for the large model are due to a small subset of it. We can only say that the large model contains all the information in the smaller models that it has been tested against, not that it contains no extraneous variables.

We compare the forecasts from the structural model in Ray C. Fair (1976) with those of various atheoretical models. The atheoretical models include various versions of the VAR model and various versions of a model we call the "autoregressive components" (AC) model. The AC model divides GNP into components and estimates an autoregressive equation for each component. Like the Fair model, the AC model uses a fairly large amount of information on the components of GNP and yet it has a simple structure. Like parts of the Fair model, it uses lagged values of the components in the determination of the current values. The various versions of the AC model differ in their degree of disaggregation of the components, which allows us to examine how disaggregation affects the forecasts.

The AC model is of use to compare to the VAR model as well as to a structural model like the Fair model. It will be of interest to see within the class of atheoretical models how it compares to the VAR model. If, for example, it is found to contain information not in the VAR model, this indicates that the VAR model is missing useful information in the components. The AC model can be looked upon as an alternative to the VAR model when simple atheoretical models are desired.

Our procedure requires that forecasts be based only on information available prior to the forecast period. We call forecasts that meet this requirement "*quasi ex ante*" forecasts. To guard against future information creeping into the specification of the model and thus the forecasts, we chose the version of the Fair model that existed in 1976 and based all the tests on the period since 1976. We also converted the model to a model with no exogenous variables by adding an autoregressive equation for each exogenous variable in the model. Finally, we estimated the model (including the exogenous-variable equations) through period $t-1$ for each new beginning period t of the forecast. Using these "rolling" estimation forecasts is important because in doing so we are producing the actual forecasts that one could make with the model as time progresses.

We followed the same rolling estimation procedure for the VAR and AC models. These models have no exogenous variables, and so no adjustment was needed here. The Robert B. Litterman prior (1979) that we use for one of the versions of the VAR model was published near the beginning of the test period. The AC model is new, but it has such a simple structure that it is unlikely to be accused of having had future information used for its specification.

The quasi *ex ante* forecasts that we generate may have different properties from forecasts made with a model estimated with future data. If the model is misspecified (for example, parameters change through time), then the rolling estimation forecasts (where estimated parameters vary through time) may

(1986), and Grayham Mizon and Jean-Francois Richard (1986). See also Charles R. Nelson (1972) and J. Phillip Cooper and Charles R. Nelson (1975) for an early use of encompassing-like tests.

[2]See, for example, Clive W. J. Granger and Paul Newbold (1986).

carry rather different information from forecasts estimated over the entire sample.[3] Also, some models may use up more degrees of freedom in estimation than others, and with varied estimation procedures it is often very difficult to take formal account of the number of degrees of freedom used up. In the extreme case where there were so many parameters in a model that the degrees of freedom were completely used up when it was estimated (an obviously over parameterized model), it would be the case that the forecast value equals the actual value and there would be a spurious perfect correspondence between the variable forecasted and the forecast. One can guard against this degrees of freedom problem by requiring that no forecasts be within-sample forecasts.[4]

I. The Procedure

Let ${}_{t-s}\hat{Y}_{1t}$ denote a forecast of Y_t (in our application, log real gross national product at time t) made from model 1 using information available at time $t-s$ and using the model's estimation procedure and forecasting method each period. Let ${}_{t-s}\hat{Y}_{2t}$ denote the same thing for model 2. (In the notation above, these two forecasts are "*quasi ex ante*" forecasts.) The parameter s is the length ahead of the forecast, $s>0$. Note that the estimation procedure used to estimate a model and the model's forecasting method are considered by us as part of the model; we take no account of these procedures here.

[3]Even if the model is not misspecified, estimated parameters will change through time due to sampling error. If our purpose were to evaluate the forecasting ability of the *true* model (i.e., the model with the true coefficients), we would face a generated regressor problem. However, we are interested here in the performance of the model *and* its associated estimation procedure. If one were interested in adjusting for generated regressors, the correction discussed in Kevin M. Murphy and Robert H. Topel (1985) could not be directly applied here because the covariance matrix of the coefficient estimates used to generate the forecasts changes through time because of the use of the rolling regressions. Murphy and Topel require a single covariance matrix.

[4]Nelson (1972) and Cooper and Nelson (1975) do not require the forecasts to be based only on information through the previous period.

We consider the following regression equation:

$$(1)\quad Y_t - Y_{t-s} = \alpha + \beta\left({}_{t-s}\hat{Y}_{1t} - Y_{t-s}\right) + \gamma\left({}_{t-s}\hat{Y}_{2t} - Y_{t-s}\right) + u_t.$$

If neither model 1 nor model 2 contains any information useful for s-period-ahead forecasting of Y_t, then the estimates of β and γ should both be zero. In this case the estimate of the constant term α would be the average s-period-change in Y. If both models contain independent information[5] for s-period-ahead forecasting, then β and γ should both be nonzero. If both models contain information, but the information in, say, model 2 is completely contained in model 1 and model 1 contains further relevant information as well, then β but not γ should be nonzero. (If both models contain the same information, then the forecasts are perfectly correlated, and β and γ are not separately identified.)

The procedure we are proposing in this paper is to estimate equation (1) for different models' forecasts and test the hypothesis H_1 that $\beta=0$ and the hypothesis H_2 that $\gamma=0$. H_1 is the hypothesis that model 1's forecasts contain no information relevant to forecasting s periods ahead not in the constant term and in model 2, and H_2 is the hypothesis that model 2's forecasts contain no information not in the constant term and in model 1.

As we noted above, our procedure bears some relation to encompassing tests, but our setup and interests are somewhat different. For example, it does not make sense in our case to constrain β and γ to sum to one, as is usually the case for encompassing tests. If in our case both models' forecasts are just noise, the estimates of both β and γ should be zero. Also, say that the true process generating Y_t is Y_t equal to $X_t + Z_t$, where X_t and Z_t are independently distributed. Say that model 1 specifies that Y_t is a function of

[5]If both models contain "independent information" in our terminology, their forecasts will not be perfectly correlated. This can arise either because the models use different data or because they use the same data but impose different restrictions on the reduced form.

X_t only and that model 2 specifies that Y_t is a function of Z_t only. Both forecasts should thus have coefficients of one in equation (1), and so in this case β and γ would sum to two. It also does not make sense in our setup to constrain the constant term α to be zero. If, for example, both models' forecasts were noise and we estimated equation (1) without a constant term, then the estimates of β and γ would not generally be zero when the mean of the dependent variable is nonzero.

It is also not sensible in our case to assume that u_t is identically distributed. It seems quite likely that u_t is heteroskedastic. If, for example, $\alpha = 0$, $\beta = 1$, and $\gamma = 0$, u_t is simply the forecast error from model 1, and in general forecast errors are heteroskedastic. Also, we will be considering four-period-ahead forecasts in addition to one-period-ahead forecasts, and this introduces a third-order moving average process to the error term in equation (1).[6] We correct for both heteroskedasticity and the moving average process in the estimation of the standard errors of the coefficient estimates. We use the procedure given by Lars Peter Hansen (1982), Robert E. Cumby, John Huizinga, and Maurice Obstfeld (1983), and Halbert White and Ian Domowitz (1984) for the estimate of the asymptotic covariance matrix of the estimate $\hat{\theta}$ of the parameter vector $\theta \equiv (\alpha\ \beta\ \gamma)'$ in (1). Define X as the $T \times 3$ matrix of variables, whose row t is $X_t = [1, ({}_{t-s}\hat{Y}_{1t} - Y_{t-s}), ({}_{t-s}\hat{Y}_{2t} - Y_{t-s})]$, and let $\hat{u}_t = Y_t - Y_{t-s} - X_t\delta$. We estimate the covariance matrix of $\hat{\theta}$, $V(\hat{\theta})$, as:

$$(2) \qquad V(\hat{\theta}) = (X'X)^{-1}S(X'X)^{-1},$$

where

$$(3) \qquad S = \Omega_0 + \sum_{j=1}^{s-1} (\Omega_j + \Omega_j'),$$

$$(4) \qquad \Omega_j = \sum_{t=j+1}^{T} (u_t u_{t-j}) \hat{X}_t' \hat{X}_{t-j},$$

[6]The error term in equation (1) could, of course, be serially correlated even for the one-period-ahead forecasts. Such serial correlation does not appear to be a problem with any of the models we study here, however, and we have assumed it to be zero.

where $\hat{\theta}$ is the ordinary least squares estimate of θ and s is the forecast horizon. When s equals 1 the second term on the right hand side of (3) is zero, and the covariance matrix is simply Halbert White's (1980) correction for heteroskedasticity.

Note that as an alternative to equation (1) we could have regressed the *level* (rather than the change) of GNP on the forecasted *levels* and a constant. GNP has a very strong low frequency component. It may be an integrated process, and any sensible forecast of GNP will be cointegrated with GNP itself. The sum of β and γ will thus be constrained in effect to one, and in the levels regression we would be estimating in effect one less parameter. If GNP is an integrated process, running the levels regression with an additional independent variable Y_{t-1} (thereby estimating β and γ without constraining their sum to one) is essentially equivalent to our differenced regression (1).

It should finally be noted that there are cases in which an optimal forecast does not tend to be singled out as best in regressions of the form (1), even with many observations. Say the truth is $Y_t - Y_{t-1} = aX_{t-1} + e_t$. Say that model 1 does rolling regressions of $Y_t - Y_{t-1}$ on X_{t-1} and uses these regressions to forecast. Say that model 2 always takes the forecast to be bX_{t-1}, where b is some number other than a, so that model 2 remains forever an incorrect model. In equation (1) regressions the two forecasts tend to be increasingly collinear as time goes on; essentially they are collinear after the first part of the sample. Thus, the estimates of β and γ tend to be erratic. Adding a large number of observations does not cause the regressions to single out the first model; it only has the effect of enforcing that $\hat{\beta} + (\hat{\gamma}b)/a = 1$.

II. Quasi *Ex Ante* Forecasts

As noted above, we want forecasts from models that are based only on information through the period prior to the beginning of the forecast period (through period $t-s$ for a forecast for period t). There are four ways in which future information can creep into a current forecast. The first is if actual values of the exogenous variables for periods after $t-s$ are used in the forecast. The second is if the coefficients of the model have been estimated over a sample period that includes observations beyond $t-s$. The third is if information beyond $t-s$ has been used in the specification of the model even though for purposes of the tests the model is only estimated through period $t-s$. The fourth is if information beyond period $t-s$ has been used in the revisions of the data for periods $t-s$ and back, such as revised seasonal factors and revised benchmark figures.

The VAR, AC, and AR models discussed below have no exogenous variables, and so there are no exogenous-variable problems for these models. The way we have handled the problem for the Fair model is to add autoregressive equations for the exogenous variables to the model. For each exogenous variable in the model an eighth-order autoregressive equation (with a constant term and time trend included) has been postulated. When these equations are added to the model, the model effectively has no exogenous variables in it. This method of dealing with exogenous variables in structural models was advocated by Cooper and Nelson (1975) and Stephen K. McNees (1981). McNees, however, noted that the method handicaps the model: "It is easy to think of exogenous variables (policy variables) whose future values can be anticipated or controlled with complete certainty even if the historical values can be represented by covariance stationary processes; to do so introduces superfluous errors into the model solution" (McNees, 1981, p. 404).

For the coefficient-estimate problem, we use rolling estimations for all the models. For the forecast for period t, we estimate the model through period $t-s$; for the forecast for period $t+1$, we estimate the model through period $t-s+1$; and so on. By "model" in this case we mean the model inclusive of any exogenous-variable equations. The beginning observation is held fixed for all the regressions; the sample expands by one observation each time a time period elapses.

The third problem—the possibility of using information beyond period $t-s$ in the specification of the model—is more difficult to handle. Models are typically changed through time, and model builders seldom go back to or are interested in "old" versions. We have, however, attempted to account for this problem in this paper regarding the Fair model. We consider the version of the Fair model that existed as of the second quarter of 1976.

We have done nothing about the data-revision problem in this paper. The data that have been used are the latest revised data. It would be extremely difficult to try to purge these data of the possible use of future information, and we have not tried. Note that it is not enough simply to use data that existed at any point in time (say period $t-s$) because data on the s-period-ahead value (period t) are needed to estimate equation (1). We would have to try to construct data for period t that are consistent with the old data for period $t-s$.

III. The Models

A. *The Fair Model (FAIR)*

The first version of the Fair model was presented in Fair (1976) along with the estimation method and method of forecasting with the model. This version was based on data through 1975 I. One important addition that was made to the model from this version was the inclusion of an interest rate reaction function in the model. This work is described in Ray C. Fair (1978), which is based on data through 1976 II. The version of the model in Fair (1976) consists of 26 structural stochastic equations, and with the addition of the interest rate reaction function, there are 27 stochastic equations. There are 106 exogenous variables, and for each of these variables an eighth-order autoregressive equation with a constant and time trend was added to the model. This gave a model of 133 stochastic equations, and this is the version that was used.

For the rolling estimations, the first estimation period ended in 1976 II, which is the first quarter in which the model could definitely be said to exist. This allowed the model to be estimated 40 times (through 1986 I).

Because the version of the Fair model used here existed as of 1976 II, because it has in effect no exogenous variables, and because it is estimated via rolling estimation, the forecasts from it can be said to be forecasts that are truly based only on information through the period prior to the first period of the forecast (except for the data revision problem).[7] This may be the first time that a large model this old has been tested.

B. *The VAR Models (VAR4, VAR4P1, VAR4P2, VAR4P3, VAR1, and VAR2)*

We consider six VAR models in this paper. The first, VAR4, estimated by ordinary least squares, is the same as the model used in Christopher A. Sims (1980) except that we have added the three-month Treasury bill rate to the model. There are seven variables in the model: real GNP, the GNP deflator, the unemployment rate, the nominal wage rate, the price of imports, the money supply, and the bill rate. All but the unemployment rate and the bill rate are in logs. Each equation consists of each variable lagged one through four times, a constant, and a time trend, for a total of 30 coefficients to estimate.

The next three VAR models—VAR4P1, VAR4P2, and VAR4P3—have Bayesian priors imposed on the coefficients of VAR4. We impose the Litterman prior that the variables follow univariate random walks. The standard deviations of the prior take the form

$$(5)\quad S(i,j,k) = \gamma g(k) f(i,j)(s_j/s_i),$$

where i indexes the left-hand side variable, j indexes the right-hand side variables, and k indexes the lag. s_i is the standard error of the unrestricted equation for variable i. VAR4P1 imposes parameter values that imply fairly loose priors. They are: 1) $f(i,j)=1$ for all i and j, 2) $g(k)=1$ for all k, and 3) $\gamma=0.2$. VAR4P2 imposes parameter values that imply much tighter priors: 1) $f(i,j)=1$ for $i=j$, $f(i,j)=0.5$ for $i\neq j$, 2) $g(k)=k^{-1}$, and 3) $\gamma=0.1$. VAR4P3 is the same as VAR4P2 except that $f(i,j)=0.2$ for $i\neq j$, which implies even tighter priors than for VAR4P2. The parameter values for VAR4P2 are those imposed by Litterman (1979, p. 49).

The fifth VAR model, VAR2, uses only the first two lags of each variable, for a total of 16 coefficients in each equation. The sixth model, VAR1, uses only each variable lagged once, for a total of 9 coefficients. No priors were imposed on VAR2 and VAR1; they were estimated by ordinary least squares.

Each VAR model was estimated 40 times using the same sample periods as were used for the Fair model. Each model was then used to make 40 forecasts of real GNP.

C. *The AC Models (AC-6, AC-13, AC-17, AC-48, AC-E6, AC-E13, AC-E17, and AC-E48)*

Time-series models like VAR models typically ignore the components of GNP. For example, the VAR models used in this paper contain no components. The current model used by Christopher A. Sims (serial) for forecasting includes only the component nonresidential fixed investment. Including many components in a VAR model rapidly uses up degrees of freedom, and this is undoubtedly one of the main reasons the components are seldom used. A possible alternative to the VAR approach, but one that also does not use much economic theory, is to

[7]This statement needs to be qualified slightly. Although the structural stochastic equations used for the Fair model are exactly as in Fair (1976) and (1978)—same left-hand side and right-hand side variables and same functional forms—the data revisions in the National Income Accounts since 1976 have required slight modifications to some of the identities in the model. Also, the identities in Fair (1976) for the government sector are for the total government sector, whereas in the version used here there are separate identities for the federal government sector and the state and local government sector. This disaggregation of the government sector does not affect anything except that it means that there are more exogenous variables (and thus more exogenous-variable equations) in the version used here than there were in Fair (1976). No structural stochastic equations need to be modified because of this disaggregation because the government variables do not appear as explanatory variables in these equations. The government variables appear only in the identities.

model each of the components of real GNP by a simple autoregressive equation (but not real GNP itself) and then determine real GNP as the sum of the components, (i.e., by the GNP identity).

The AC model may be regarded as an extreme caricature of large-scale macroeconometric models. Like the latter, lagged values of components are used to predict components, and the predicted components are added up to predict GNP. The AC model, however, treats all components in a simple and symmetrical way and carries the number of lags further than is often the case with large-scale models. Note also that the AC model could be adapted to forecast variables other than GNP. For any variable that is determined by an identity, autoregressive equations could be used to forecast the right-hand side variables and then the identity used to forecast the variable itself. Also, the AC model for real GNP could be added to equations explaining other variables to create a more complete macroeconometric model.

We have considered eight AC models in this paper, all estimated by ordinary least squares. The models are first distinguished by whether they include 6, 13, 17, or 48 components. Increasing the disaggregation of the components allows one to examine how much additional useful information for forecasting purposes is contained in the more disaggregated components. Each equation for a component contains the first eight lagged values of the component, a constant, and a time trend. None of the AC equations is in log form. For the AC-6, AC-13, AC-17, and AC-48 models, these are all the variables included in the equations. For the AC-E6, AC-E13, AC-E17, and AC-E48 models (E for "extended"), the first four lagged values of real GNP are added to each equation. This allows for an impact of aggregate economic activity on each component and uses up only four degrees of freedom per equation. The components used for each model are listed in the Appendix. The 17 component models (AC-17 and AC-E17) use the same components as does the Fair model. The same sample periods and procedures were used for the AC models as were used for the Fair and VAR models except the for the 6, 13, and 48 component models the beginning quarter for the estimation periods was 1961 I rather than 1954 I.[8]

The AC models are of interest in two respects. First, if the Fair model turns out to dominate the VAR models (which it does), it is of interest to know if this is due simply to the fact that the Fair model is dealing with the lagged components of GNP. If it is as simple as this, then the AC models might do even better, and this can be tested. Second, the AC models are to some extent competitors of the VAR models within the class of atheoretical models, at least regarding the predictions of GNP. Both models are based on very little economic theory. It is thus of interest to see if one type of model dominates the other.

D. *The Autoregressive Models (AR4 and AR8)*

AR4 and AR8 are simple benchmark models, estimated by ordinary least squares. For AR4 real GNP was regressed on its first four lagged values, a constant, and a time trend. For AR8 real GNP was regressed on its first eight lagged values, a constant, and a time trend. The same sample periods were used here as were used for the Fair and VAR models.

IV. The Results

The results comparing the Fair model to the other models are presented in Tables 1 and 2. The sample period used for the one-quarter-ahead results is 1976 III–1986 II, for a total of 40 observations. The sample period for the four-quarter-ahead results is 1972

[8]This choice was dictated by the available data. Fortunately, the results do not appear to be sensitive to the use of the later beginning quarter. For the 17 component model the estimation periods could begin in 1954 I, and so two versions of AC-17 and AC-E17 were estimated, one for the periods beginning in 1954 I and one for the periods beginning in 1961 I. The two versions gave very similar results.

TABLE 1—BIAS AND RMSE FOR EACH MODEL'S FORECAST

	One-Quarter-Ahead Forecasts Sample Period = 1976 III–1986 II				Four-Quarter-Ahead Forecasts Sample Period = 1977 II–1986 II			
	const	SE	DW	RMSE	const	SE	DW	RMSE
FAIR	0.0034 (2.43)	0.00883	2.04	0.00945	0.0087 (3.60)	0.0146	1.04	0.0170
VAR1	−0.0014 (0.78)	0.01179	1.47	0.01188	−0.0127 (2.93)	0.0263	0.56	0.0292
VAR2	−0.0021 (1.23)	0.01090	1.36	0.01111	−0.0157 (3.33)	0.0285	0.40	0.0326
VAR4	0.0012 (0.68)	0.01145	1.22	0.01152	−0.0011 (0.24)	0.0291	0.53	0.0291
VAR4P1	0.0004 (0.24)	0.01105	1.41	0.01106	−0.0028 (0.59)	0.0291	0.43	0.0292
VAR4P2	−0.0017 (1.07)	0.01004	1.68	0.01018	−0.0105 (2.90)	0.0220	0.52	0.0244
VAR4P3	−0.0010 (0.66)	0.00976	1.75	0.00981	−0.0063 (1.65)	0.0231	0.43	0.0239
AC-6	0.0016 (0.93)	0.01070	1.64	0.01081	0.0052 (1.26)	0.0251	0.42	0.0256
AC-13	0.0007 (0.39)	0.01071	1.49	0.01073	0.0007 (0.18)	0.0259	0.38	0.0259
AC-17	0.0018 (1.06)	0.01080	1.56	0.01095	0.0031 (0.72)	0.0264	0.38	0.0266
AC-48	0.0008 (0.46)	0.01141	1.18	0.01144	0.0019 (0.41)	0.0281	0.31	0.0282
AC-E6	0.0020 (1.23)	0.01047	1.96	0.01066	0.0068 (1.69)	0.0245	0.54	0.0254
AC-E13	0.0005 (0.34)	0.00957	1.93	0.00958	0.0001 (0.02)	0.0222	0.52	0.0222
AC-E17	0.0017 (1.18)	0.00936	2.25	0.00952	0.0035 (0.95)	0.0228	0.62	0.0231
AC-E48	0.0009 (0.60)	0.00993	1.84	0.00997	0.0037 (0.84)	0.0266	0.50	0.0268
AR4	0.0026 (1.60)	0.01023	2.02	0.01055	0.0134 (2.94)	0.0277	0.48	0.0308
AR8	0.0028 (1.63)	0.01067	1.98	0.01102	0.0147 (3.13)	0.0286	0.52	0.0321

Notes: const is the estimate of the constant term in a regression of the forecast error on the constant term. The forecast error is ${}_{t-1}\hat{Y}_{1t} - Y_t$ for the one-quarter-ahead results and ${}_{t-4}\hat{Y}_{1t} - Y_t$ for the four-quarter-ahead results. Y is the log of real GNP.
t-statistics in absolute value are in parentheses.
SE is the estimated standard error of the regression not adjusting for degrees of freedom.
RMSE is the root mean squared error of the forecast.

II–1986 II, for a total of 37 observations. Remember that each observation for a model's forecast is based on a different set of coefficient estimates of the model—the rolling estimation. Remember also that for the Fair model all exogenous variable values are generated from the autoregressive equations; no actual values are used. Finally, remember that in Table 2 the estimated standard errors of the coefficient estimates are corrected for heteroskedasticity and (for the four-quarter-ahead results) for the moving average process of the error term.[9]

[9]In two cases for the four-quarter-ahead results the matrix $V(\hat{\theta})$ was singular or nearly singular. In these two cases we assumed a second-order MA process for the error term instead of a third order, which solved the problem. Had this been a more widespread problem, we would have used one of the estimators in Donald W. K. Andrews (1987), but this seemed unnecessary given only two failures. The two failures are FAIR versus VAR4 and FAIR versus VAR4P1.

TABLE 2—FAIR MODEL VERSUS THE OTHERS: ESTIMATES OF EQUATION (1)

	One-Quarter-Ahead Forecasts Dependent Variable is $Y_t - Y_{t-1}$ Sample Period = 1976 III–1986 II					Four-Quarters-Ahead Forecasts Dependent Variable is $Y_t - Y_{t-4}$ Sample Period = 1977 II–1986 II				
Other Model	const	FAIR $_{t-1}\hat{Y}_{1t} - Y_{t-1}$	OTHER $_{t-1}\hat{Y}_{2t} - Y_{t-1}$	SE	DW	const	FAIR $_{t-4}\hat{Y}_{1t} - Y_{t-4}$	OTHER $_{t-4}\hat{Y}_{2t} - Y_{t-4}$	SE	DW
VAR1	−0.0034	1.05	−0.10	0.00915	2.02	−0.0135	1.06	0.20	0.0134	1.66
	(1.20)	(4.51)	(0.38)			(2.28)	(6.78)	(3.09)		
VAR2	−0.0029	0.89	0.14	0.00912	2.02	−0.0135	1.08	0.18	0.0135	1.59
	(1.00)	(3.47)	(0.69)			(2.08)	(6.30)	(2.66)		
VAR4	−0.0031	0.86	0.14	0.00908	1.93	−0.0164	1.07	0.20	0.0130	1.62
	(1.06)	(3.53)	(0.89)			(2.72)	(6.53)	(3.31)		
VAR4P1	−0.0033	0.91	0.11	0.00914	2.02	−0.0159	1.08	0.19	0.0132	1.64
	(1.07)	(3.83)	(0.70)			(2.55)	(6.32)	(2.79)		
VAR4P2	−0.0031	0.89	0.17	0.00913	2.06	−0.0145	1.04	0.29	0.0133	1.61
	(1.05)	(3.56)	(0.64)			(2.47)	(6.20)	(2.76)		
VAR4P3	−0.0041	0.87	0.37	0.00910	2.08	−0.0209	1.07	0.49	0.0135	1.57
	(1.15)	(3.73)	(0.89)			(3.69)	(6.70)	(2.24)		
AC-6	−0.0041	0.94	0.16	0.00913	2.10	−0.0201	1.13	0.22	0.0140	1.50
	(1.11)	(3.98)	(0.67)			(3.09)	(6.89)	(1.43)		
AC-13	−0.0034	0.99	0.02	0.00918	2.04	−0.0193	1.18	0.16	0.0144	1.45
	(0.92)	(3.93)	(0.05)			(2.39)	(7.34)	(0.58)		
AC-17	−0.0031	1.00	−0.04	0.00917	2.03	−0.0164	1.22	−0.00	0.0145	1.42
	(0.84)	(4.07)	(0.14)			(1.86)	(8.11)	(0.00)		
AC-48	−0.0039	0.97	0.10	0.00915	2.04	−0.0191	1.19	0.14	0.0143	1.48
	(1.15)	(4.00)	(0.50)			(2.85)	(7.63)	(0.85)		
C-E6	−0.0042	0.91	0.20	0.00910	2.14	−0.0195	1.15	0.17	0.0142	1.51
	(1.23)	(3.69)	(0.80)			(2.80)	(7.05)	(1.15)		
AC-E13	−0.0043	0.81	0.40	0.00899	2.16	−0.0171	1.17	0.10	0.0145	1.41
	(1.35)	(2.81)	(1.05)			(2.47)	(6.30)	(0.41)		
AC-E17	−0.0045	0.76	0.42	0.00894	2.31	−0.0168	1.20	0.04	0.0145	1.42
	(1.41)	(2.79)	(1.50)			(2.50)	(6.19)	(0.16)		
AC-E48	−0.0047	0.86	0.36	0.00894	2.21	−0.0168	1.21	0.03	0.0145	1.43
	(1.42)	(3.64)	(1.65)			(2.62)	(7.16)	(0.19)		
AR4	−0.0084	0.96	0.59	0.00885	2.47	−0.0168	1.22	0.01	0.0145	1.42
	(2.01)	(4.51)	(1.87)			(1.12)	(9.24)	(0.03)		
AR8	−0.0067	0.98	0.38	0.00901	2.32	−0.0144	1.22	−0.05	0.0145	1.41
	(1.72)	(4.48)	(1.33)			(1.70)	(9.54)	(0.27)		

Notes: Y = log of real GNP.
t-statistics in absolute value are in parentheses.
See text for discussion of estimation methods.

The bias and RMSE for each forecast are presented in Table 1.[10] The bias is estimated by regressing the forecast error (predicted change minus actual change) on a constant. If the constant term is zero in this regression then the standard error (SE) of the regression is the same as the RMSE; otherwise the RMSE is larger. The errors are roughly in percentage points (0.01 is a 1 percent error) because real GNP is in logs. For the one-quarter-ahead results the Fair model has the largest bias, but even this bias is only 0.34 percent. It also has the smallest RMSE, although a number of RMSEs are quite close. The second best model in terms of RMSE is AC-E17. The best VAR model is VAR4P3. For the four-quarter-ahead results a number of the estimated biases are significant. The Fair model has by far the smallest RMSE. The best VAR model is again VAR4P3, and the best AC model is now AC-E13. AC-E13 and AC-E17 are better than VAR4P3.

[10] For reference purposes, the predicted values and errors for three models—Fair, VAR4P3, and AC-E17—are presented in Table A in the appendix.

Consider now the results in Table 2. The coefficient estimate for the Fair model forecast is always significant at the 5 percent level for both the one-quarter-ahead and four-quarter-ahead results. None of the coefficient estimates for the other models' forecasts is significant at this level for the one-quarter-ahead results. The regression gives a fairly large weight to the AR4 and AC-E17 forecasts, although these are not quite statistically significant. For the four-quarter-ahead results the only significant estimates (aside from those for the Fair model) are for the VAR models. The results across the different VAR models are fairly close, with perhaps VAR4 performing the best.

The results in Table 2 are thus rather striking. They provide strong support for the hypothesis that the Fair model carries useful information not in the other models. The significance of the VAR forecasts for the four-quarter-ahead results indicates that some information is in the VAR forecasts that the Fair model is not using for the four-quarter-ahead forecasts, but this is the only significantly negative aspect of the results for the Fair model. It perhaps indicates some dynamic misspecification for the Fair model.

Comparing Tables 1 and 2 shows one of the advantages of our procedure over the RMSE procedure. For the one-quarter-ahead results in Table 1 the RMSEs are all fairly close, and even though the RMSE is smallest for the Fair model, it is only slightly smaller. One might conclude from this table that the models are all about the same. The results in Table 2, on the other hand, show that the Fair model dominates the others by a fairly large amount. Conversely, the four-quarter-ahead results in Table 1 show the Fair model dominating the others by a large amount, but the results in Table 2 show that the VAR forecasts contain information not in the Fair forecasts. One would not have known this from Table 1.

Table 3 goes on to compare the VAR models with the AC and AR models. It is hard from Table 2 to pick out which AC and VAR model performs the best because the models are so dominated by the Fair model, but Table 3 provides more ability to discriminate. Regarding the AC models, the best results in terms of significant coefficient estimates are obtained for the 13 and 17 component versions and for the extended (AC-E) versions. In other words, going from 6 to 13 or 17 components does help, but going beyond this does not seem to add further useful information, and adding the lagged values of real GNP to the equations seems to add useful information.

The best performing AC model in Table 3 is probably AC-E17, and so consider the comparisons of AC-E17 with the VAR models. AC-E17 performs better than any VAR model for the one-quarter-ahead results. No VAR model coefficient estimate is significant for these comparisons, although some are close to being significant. For the four-quarter-ahead results the VAR models perform about as well as does AC-E17. The best fit is for AC-E17 versus VAR4P2, where the t-statistic for VAR4P2 is 3.73 and the t-statistic for AC-E17 is 3.41. In other words, both AC-E17 and the VAR models appear to contain independent information useful for forecasting four quarters ahead. Comparing across the VAR models, VAR4P2 and VAR4P3 are probably the best, although the results are quite close across the models. The results in Table 3 also show that the VAR models dominate the AR models. Clearly the VAR models contain information not in the AR models, but not vice versa.

The results in Table 3 thus indicate that AC models like AC-E17 contain useful forecasting information not contained in even the best VAR model. In other words, there appears to be useful forecasting information in the components of GNP that is not captured in the VAR models, and so within the class of fairly atheoretical models, AC models appear to be useful alternatives to the VAR models. This conclusion is strengthened by the fact that for the one-quarter-ahead forecasts the VAR models appear to contain little information not already in AC-E17.

Regarding the AC models, note from Table 2 that the AC forecasts are not statistically significant at the 5 percent level when compared with the Fair forecasts. Although the AC-E17 forecast gets a weight of 0.42 for the

TABLE 3—VAR MODELS VERSUS AC AND AR MODELS: ESTIMATES OF EQUATION (1)
C = CONSTANT, V = VAR MODEL, A = AC OR AR MODEL
ONE-QUARTER-AHEAD FORECASTS DEPENDENT VARIABLE IS $Y_t - Y_{t-1}$ SAMPLE PERIOD = 1976 III–1986 II

	VAR1			VAR2			VAR4			VAR4P1			VAR4P2			VAR4P3		
	C	*V*	*A*	*C*	*V*	*A*	*C*	*V*	*A*	*C*	*V*	*A*	*C*	*V*	*A*	*C*	*V*	*A*
AC-6	0.0029	0.14	0.37	0.0034	0.39	0.17	0.0019	0.36	0.23	0.0022	0.36	0.23	0.0025	0.56	0.16	−0.0004	0.94	0.21
	(0.98)	(0.59)	(1.94)	(1.20)	(2.18)	(0.74)	(0.62)	(2.05)	(0.97)	(0.70)	(2.21)	(0.99)	(0.83)	(2.24)	(0.71)	(0.10)	(2.39)	(0.93)
		[0.01057]			[0.01014]			[0.01003]			[0.01020]			[0.01011]			[0.01008]	
AC-13	0.0035	0.21	0.28	0.0039	0.43	0.10	0.0023	0.38	0.18	0.0023	0.40	0.21	0.0028	0.60	0.11	−0.0004	1.01	0.19
	(0.95)	(0.94)	(0.74)	(1.12)	(2.39)	(0.25)	(0.66)	(2.13)	(0.44)	(0.64)	(2.37)	(0.51)	(0.78)	(2.38)	(0.29)	(0.10)	(2.52)	(0.49)
		[0.01068]			[0.01018]			[0.01008]			[0.01024]			[0.01013]			[0.01011]	
AC-17	0.0026	0.25	0.32	0.0027	0.43	0.23	0.0014	0.39	0.25	0.0011	0.42	0.31	0.0016	0.61	0.24	−0.0018	1.05	0.30
	(0.67)	(1.05)	(1.05)	(0.74)	(2.49)	(0.75)	(0.39)	(2.31)	(0.82)	(0.29)	(2.60)	(1.01)	(0.41)	(2.50)	(0.80)	(0.38)	(2.66)	(0.99)
		[0.01065]			[0.01013]			[0.01005]			[0.01019]			[0.01008]			[0.01005]	
AC-48	0.0039	0.23	0.20	0.0040	0.43	0.10	0.0026	0.39	0.13	0.0027	0.40	0.14	0.0025	0.60	0.16	−0.0008	1.04	0.22
	(1.48)	(1.00)	(1.09)	(1.69)	(2.38)	(0.50)	(1.00)	(2.21)	(0.69)	(1.06)	(2.43)	(0.73)	(0.92)	(2.48)	(0.81)	(0.22)	(2.68)	(1.14)
		[0.01067]			[0.01017]			[0.01007]			[0.01024]			[0.01009]			[0.01004]	
AC-E6	0.0019	0.14	0.48	0.0022	0.36	0.32	0.0012	0.33	0.33	0.0014	0.32	0.34	0.0014	0.50	0.31	−0.0011	0.87	0.33
	(0.57)	(0.55)	(1.89)	(0.69)	(1.91)	(1.31)	(0.39)	(1.81)	(1.27)	(0.45)	(1.81)	(1.29)	(0.44)	(1.96)	(1.27)	(0.27)	(2.19)	(1.39)
		[0.01037]			[0.01000]			[0.00992]			[0.01009]			[0.00997]			[0.00995]	
AC-E13	0.0000	0.13	0.82	0.0003	0.31	0.69	−0.0007	0.30	0.70	−0.0005	0.28	0.71	−0.0003	0.42	0.67	−0.0023	0.73	0.68
	(0.03)	(0.58)	(2.54)	(0.09)	(1.88)	(2.12)	(0.21)	(1.87)	(1.99)	(0.14)	(1.86)	(2.06)	(0.07)	(1.85)	(2.03)	(0.57)	(2.05)	(2.10)
		[0.00988]			[0.00958]			[0.00946]			[0.00963]			[0.00958]			[0.00957]	
AC-E17	−0.0008	0.09	0.83	−0.0005	0.27	0.70	−0.0015	0.28	0.71	−0.0011	0.24	0.72	−0.0009	0.36	0.96	−0.0026	0.62	0.69
	(0.22)	(0.40)	(3.26)	(0.15)	(1.64)	(2.89)	(0.47)	(1.82)	(2.66)	(0.35)	(1.67)	(2.73)	(0.27)	(1.54)	(2.74)	(0.67)	(1.74)	(2.87)
		[0.00967]			[0.00944]			[0.00930]			[0.00949]			[0.00946]			[0.00945]	
AC-E48	0.0011	0.14	0.64	0.0013	0.32	0.51	0.0003	0.31	0.51	0.0006	0.29	0.52	0.0004	0.47	0.51	−0.0019	0.82	0.52
	(0.35)	(0.63)	(3.29)	(0.50)	(1.85)	(2.63)	(0.11)	(1.82)	(2.36)	(0.21)	(1.83)	(2.51)	(0.15)	(2.05)	(2.75)	(0.50)	(2.27)	(2.88)
		[0.01009]			[0.00978]			[0.00967]			[0.00985]			[0.00971]			[0.00969]	
AR4	−0.0003	0.21	0.63	0.0004	0.39	0.48	−0.0021	0.38	0.62	−0.0005	0.36	0.49	0.0004	0.54	0.39	−0.0014	0.92	0.31
	(0.07)	(0.88)	(1.65)	(0.11)	(2.17)	(1.31)	(0.52)	(2.44)	(1.69)	(0.11)	(2.17)	(1.26)	(0.10)	(2.03)	(1.00)	(0.32)	(2.03)	(0.76)
		[0.01044]			[0.01001]			[0.00978]			[0.01010]			[0.01003]			[0.01008]	
AR8	0.0021	0.23	0.36	0.0025	0.42	0.24	0.0003	0.39	0.35	0.0017	0.40	0.23	0.0023	0.59	0.15	0.0002	1.05	0.06
	(0.50)	(0.94)	(0.96)	(0.63)	(2.31)	(0.67)	(0.08)	(2.36)	(0.97)	(0.43)	(2.28)	(0.60)	(0.53)	(3.10)	(0.55)	(0.06)	(2.29)	(0.15)
		[0.01063]			[0.01013]			[0.00999]			[0.01023]			[0.01012]			[0.01014]	
AC-6	0.0124	0.45	0.25	0.0127	0.37	0.31	0.0073	0.36	0.32	0.0082	0.36	0.30	0.0095	0.69	0.19	−0.0052	1.03	0.34
	(1.27)	(1.88)	(0.59)	(1.28)	(1.61)	(0.74)	(0.82)	(1.91)	(0.79)	(0.91)	(1.71)	(0.73)	(1.05)	(1.87)	(0.43)	(0.49)	(1.57)	(0.86)
		[0.0230]			[0.0236]			[0.0236]			[0.0237]			[0.0222]			[0.0230]	
AC-13	0.0142	0.51	0.19	0.0164	0.44	0.19	0.0100	0.43	0.21	0.0107	0.43	0.21	0.0110	0.75	0.13	−0.0059	1.17	0.32
	(1.04)	(2.34)	(0.32)	(1.13)	(2.00)	(0.30)	(0.79)	(2.32)	(0.33)	(0.82)	(2.15)	(0.32)	(0.86)	(2.27)	(0.21)	(0.50)	(1.90)	(0.55)
		[0.0232]			[0.0240]			[0.0240]			[0.0240]			[0.0223]			[0.0234]	
AC-17	0.0049	0.50	0.49	0.0079	0.44	0.47	0.0020	0.42	0.46	0.0024	0.43	0.47	0.0016	0.72	0.45	−0.0155	1.22	0.59
	(0.42)	(3.66)	(1.18)	(0.64)	(2.93)	(1.04)	(0.17)	(3.63)	(1.03)	(0.20)	(3.38)	(1.04)	(0.14)	(3.35)	(1.06)	(1.22)	(2.87)	(1.39)
		[0.0225]			[0.0235]			[0.0235]			[0.0235]			[0.0218]			[0.0226]	
AC-48	0.0199	0.56	−0.04	0.0234	0.52	−0.09	0.0163	0.51	−0.10	0.0170	0.51	−0.09	0.0155	0.82	−0.07	−0.0013	1.28	0.07
	(1.77)	(2.48)	(0.09)	(1.93)	(2.18)	(0.18)	(1.55)	(2.56)	(0.20)	(1.59)	(2.31)	(0.18)	(1.49)	(2.40)	(0.16)	(0.11)	(1.97)	(0.15)
		[0.0233]			[0.0219]			[0.0241]			[0.0241]			[0.0224]			[0.0237]	
AC-E6	0.0087	0.42	0.36	0.0092	0.35	0.40	0.0044	0.34	0.40	0.0051	0.34	0.40	0.0064	0.64	0.30	−0.0075	0.99	0.42
	(0.78)	(2.53)	(1.19)	(0.79)	(2.14)	(1.40)	(0.39)	(2.49)	(1.37)	(0.45)	(2.27)	(1.33)	(0.59)	(2.38)	(0.93)	(0.61)	(1.96)	(1.41)
		[0.0226]			[0.0232]			[0.0233]			[0.0233]			[0.0219]			[0.0226]	
AC-E13	0.0003	0.39	0.78	0.0016	0.33	0.81	−0.0027	0.31	0.80	−0.0025	0.32	0.81	−0.0020	0.57	0.73	−0.0145	0.96	0.82
	(0.03)	(4.02)	(3.17)	(0.13)	(2.69)	(2.92)	(0.22)	(3.35)	(2.77)	(0.21)	(3.25)	(2.86)	(0.19)	(3.80)	(3.19)	(1.14)	(3.12)	(3.70)
		[0.0199]			[0.0207]			[0.0209]			[0.0207]			[0.0195]			[0.0199]	
AC-E17	−0.0015	0.42	0.74	−0.0003	0.36	0.76	−0.0049	0.34	0.76	−0.0044	0.34	0.76	−0.0036	0.60	0.69	−0.0171	1.01	0.77
	(0.14)	(3.97)	(3.52)	(0.03)	(2.81)	(3.15)	(0.42)	(3.43)	(2.95)	(0.39)	(3.31)	(3.00)	(0.37)	(3.73)	(3.41)	(1.47)	(3.20)	(3.81)
		[0.0201]			[0.0209]			[0.0210]			[0.0210]			[0.0197]			[0.0202]	
AC-E48	0.0111	0.47	0.29	0.0131	0.40	0.30	0.0081	0.39	0.28	0.0083	0.39	0.29	0.0078	0.69	0.25	−0.0064	1.12	0.34
	(1.24)	(3.35)	(1.20)	(1.40)	(2.66)	(1.19)	(0.87)	(3.05)	(1.06)	(0.91)	(2.90)	(1.08)	(0.87)	(2.99)	(0.98)	(0.55)	(2.48)	(1.34)
		[0.0226]			[0.0234]			[0.0236]			[0.0235]			[0.0219]			[0.0227]	
AR4	0.0075	0.54	0.29	0.0069	0.48	0.36	−0.0017	0.46	0.41	0.0018	0.47	0.34	0.0106	0.78	0.09	−0.0044	1.31	0.11
	(0.36)	(3.94)	(0.58)	(0.34)	(3.32)	(0.68)	(0.08)	(4.15)	(0.74)	(0.08)	(3.77)	(0.59)	(0.48)	(3.70)	(0.16)	(0.19)	(2.71)	(0.16)
		[0.0232]			[0.0240]			[0.0240]			[0.0240]			[0.0224]			[0.0237]	
AR8	0.0212	0.55	−0.06	0.0162	0.49	0.12	0.0075	0.47	0.17	0.0124	0.47	0.07	0.0218	0.80	−0.20	0.0086	1.36	−0.23
	(1.13)	(4.08)	(0.14)	(0.95)	(3.38)	(0.32)	(0.41)	(4.13)	(0.42)	(0.65)	(3.78)	(0.16)	(1.23)	(3.97)	(0.49)	(0.46)	(2.89)	(0.47)
		[0.0233]			[0.0241]			[0.0241]			[0.0241]			[0.0223]			[0.0236]	

Notes: Y = log of real GNP.
t-statistics in absolute value are in parentheses.
Estimated standard errors of the regressions are in brackets.
See text for discussion of estimation methods.
$V = {}_{t-1}\hat{Y}_{1t} - Y_{t-1}$ for one-quarter-ahead results for VAR models.
$V = {}_{t-y}\hat{Y}_{1t} - Y_{t-4}$ for four-quarter-ahead results for VAR models.
$A = {}_{t-1}\hat{Y}_{2t} - Y_{t-1}$ for one-quarter-ahead results for AC and AR models.
$A = {}_{t-4}\hat{Y}_{2t} - Y_{t-4}$ for four-quarter-ahead results for AC and AR models.

one-quarter-ahead results in Table 2, it is not statistically significant, and for the four-quarter-ahead results the weight on the AC-E17 forecast is much smaller. These results may be interpreted as indicating that the Fair model captures most of the information in the components of GNP.

V. Conclusion

The procedure used in this paper for examining models appears to be useful in comparing the different models. Using this procedure we have learned that the Fair model does very well relative to the other models. The Fair model cannot be dismissed as being based on the same information used in the other forecasts. The fact that the forecasts from the Fair model are significant shows that they are not collinear with the other forecasts and that the differences between the Fair model and the other models are meaningful.

We have also learned that information about components matters. The information about components of the kind incorporated in the AC models does help improve forecasts when compared with the VAR and AR models, but it is at best of modest benefit in improving the Fair model forecasts. In this sense the useful information in the Fair model not in the VAR or AR models includes information about components of GNP. We have also learned something about how to combine forecasts. While it appears that the VAR and AC forecasts do not contain a lot of information not in the Fair model forecasts for one-quarter-ahead forecasting, it may be that a combination of the forecasts from the Fair and VAR models is useful for four-quarter-ahead forecasting. While the AC model was dominated by the Fair model, it clearly contains information not in the VAR model. The VAR models seem to be losing useful information by ignoring the components of GNP and the GNP identity.

We should conclude with a warning about the interpretation of our results. The fact that one model does well or poorly for one sample period (in our case 1976 III–1986 II) does not necessarily mean that it will do well or poorly in future sample periods. The results could change if the structure of the economy is changing, which is, of course, true of any econometric result. In our case, however, the results could also change if the magnitudes of the forecast errors of the different models are changing at different rates through time. The errors could, for example, be changing at different rates because the data are providing different rates of improvement of the models' parameters.

APPENDIX
Components of AC Models

AC-6:
1. Personal consumption expenditures
2. Gross private fixed investment
3. Change in business inventories
4. Government purchases of goods and services
5. Exports
6. Imports

AC-13:
1. Personal consumption expenditures, durable goods
2. Personal consumption expenditures, nondurable goods
3. Personal consumption expenditures, services
4. Gross private fixed investment, nonresidential
5. Gross private fixed investment, residential
6. Change in business inventories, nonfarm
7. Change in business inventories, farm
8. Government purchases of goods, federal
9. Government purchases of goods, state and local
10. Government purchases of services, federal
11. Government purchases of services, state and local
12. Exports
13. Imports

AC-17:
1. Personal consumption expenditures, durable goods
2. Personal consumption expenditures, nondurable goods
3. Personal consumption expenditures, services
4. Gross private fixed investment, nonresidential, firm sector
5. Gross private fixed investment, nonresidential, financial sector
6. Gross private fixed investment, nonresidential, household sector
7. Gross private fixed investment, residential, firm sector
8. Gross private fixed investment, residential, financial sector
9. Gross private fixed investment, residential, household sector
10. Change in business inventories, firm sector
11. Change in business inventories, household sector
12. Government purchases of goods, federal
13. Government purchases of goods, state and local
14. Government purchases of services, federal
15. Government purchases of services, state and local
16. Exports

17. Imports

Note: See Ray C. Fair (1984) for the definitions of firm, financial, and household sectors. This breakdown is from the Flow of Funds Accounts.

AC-48:

Personal consumption expenditures, durable goods:
1. Motor vehicles and parts
2. Furniture and household equipment
3. Other

Personal consumption expenditures, nondurable goods:
4. Food
5. Clothing and shoes
6. Gasoline and oil
7. Fuel oil and coal
8. Other

Personal consumption expenditures, services:
9. Housing
10. Household operation, electricity and gas
11. Household operation, other
12. Transportation
13. Medical care
14. Other

Gross private fixed investment:
15. Nonresidential structures
16. Nonresidential producers' durable equipment
17. Residential

Change in business inventories:
18. Farm
19. Nonfarm, manufacturing, durable goods
20. Nonfarm, manufacturing, nondurable goods
21. Nonfarm, merchant wholesalers, durable goods
22. Nonfarm, merchant wholesalers, nondurable goods
23. Nonfarm, nonmerchant wholesalers, durable goods
24. Nonfarm, nonmerchant wholesalers, nondurable goods
25. Nonfarm, retail trade, durable goods
26. Nonfarm, retail trade, nondurable goods
27. Nonfarm, other, durable goods
28. Nonfarm, other, nondurable goods

Government purchases of goods and services, federal:
29. Durable goods
30. Nondurable goods
31. Services, compensation of employees, national defense, military
32. Services, compensation of employees, national defense, civilian
33. Services, compensation of employees, nondefense
34. Services, other services
35. Structures

Government purchases of goods and services, state and local:
36. Durable goods
37. Nondurable goods
38. Services, compensation of employees
39. Services, other services
40. Structures

Exports of goods and services:
41. Merchandise, durable goods
42. Merchandise, nondurable goods
43. Services, factor income
44. Services, other

Imports of goods and services:
45. Merchandise, durable goods
46. Merchandise, nondurable goods
47. Services, factor income
48. Services, other

TABLE A—PREDICTED VALUES AND ERRORS FOR THREE MODELS
ONE-QUARTER-AHEAD FORECASTS

Quarter	Actual Change	FAIR Forecast Change	FAIR Error	VAR4P3 Forecast Change	VAR4P3 Error	AC-E17 Forecast Change	AC-E17 Error
1976.3	0.0042	0.0115	0.0073	0.0050	0.0008	0.0147	0.0105
4	0.0099	0.0104	0.0004	0.0043	−0.0056	0.0106	0.0007
1977.1	0.0136	0.0157	0.0021	0.0051	−0.0085	0.0221	0.0085
2	0.0159	0.0136	−0.0023	0.0060	−0.0099	0.0150	−0.0010
3	0.0200	0.0122	−0.0078	0.0061	−0.0139	0.0152	−0.0048
4	−0.0027	0.0116	0.0143	0.0066	0.0093	0.0124	0.0151
1978.1	0.0088	0.0105	0.0016	0.0038	−0.0051	0.0092	0.0004
2	0.0310	0.0103	−0.0207	0.0033	−0.0277	0.0019	−0.0291
3	0.0087	0.0112	0.0025	0.0054	−0.0033	0.0142	0.0056
4	0.0123	0.0066	−0.0057	0.0042	−0.0081	0.0110	−0.0013
1979.1	0.0000	0.0049	0.0048	0.0032	0.0031	0.0043	0.0042
2	−0.0010	0.0102	0.0112	0.0010	0.0020	0.0062	0.0073
3	0.0091	0.0087	−0.0003	0.0001	−0.0090	0.0017	−0.0073
4	−0.0019	0.0076	0.0095	0.0013	0.0032	0.0074	0.0093
1980.1	0.0099	0.0009	−0.0090	0.0002	−0.0097	0.0059	−0.0040
2	−0.0238	0.0055	0.0293	0.0005	0.0243	0.0054	0.0291
3	0.0006	0.0133	0.0127	−0.0005	−0.0010	0.0008	0.0002
4	0.0127	0.0188	0.0062	0.0017	−0.0109	0.0051	−0.0076

TABLE A—CONTINUED

		FAIR		VAR4P3		AC-E17	
Quarter	Actual Change	Forecast Change	Error	Forecast Change	Error	Forecast Change	Error
1981.1	0.0191	0.0059	−0.0132	0.0020	−0.0171	0.0101	−0.0090
2	−0.0033	0.0021	0.0054	0.0036	0.0069	0.0032	0.0065
3	0.0045	0.0030	−0.0014	0.0023	−0.0022	0.0084	0.0039
4	−0.0140	−0.0031	0.0109	0.0022	0.0163	−0.0005	0.0135
1982.1	−0.0153	−0.0017	0.0136	0.0026	0.0179	0.0024	0.0177
2	0.0030	0.0052	0.0022	0.0021	−0.0009	−0.0008	−0.0037
3	−0.0080	0.0044	0.0124	0.0048	0.0128	0.0049	0.0129
4	0.0016	0.0034	0.0019	0.0060	0.0044	0.0018	0.0002
1983.1	0.0085	0.0229	0.0143	0.0081	−0.0004	0.0200	0.0115
2	0.0223	0.0198	−0.0025	0.0099	−0.0124	0.0158	−0.0065
3	0.0147	0.0190	0.0044	0.0125	−0.0021	0.0136	−0.0011
4	0.0177	0.0125	−0.0051	0.0119	−0.0058	0.0191	0.0014
1984.1	0.0233	0.0154	−0.0079	0.0112	−0.0121	0.0146	−0.0087
2	0.0123	0.0078	−0.0044	0.0113	−0.0010	0.0078	−0.0044
3	0.0057	0.0072	0.0015	0.0100	0.0042	0.0073	0.0016
4	0.0037	0.0070	0.0033	0.0079	0.0043	0.0076	0.0039
1985.1	0.0076	0.0128	0.0053	0.0079	0.0003	0.0032	−0.0044
2	0.0058	0.0131	0.0073	0.0084	0.0026	0.0072	0.0014
3	0.0101	0.0125	0.0024	0.0093	−0.0008	0.0044	−0.0056
4	0.0052	0.0145	0.0094	0.0107	0.0055	0.0094	0.0042
1986.1	0.0092	0.0179	0.0087	0.0103	0.0011	0.0074	−0.0018
2	0.0027	0.0139	0.0112	0.0107	0.0080	0.0031	0.0004

Four-Quarter-Ahead Forecasts

		FAIR		VAR4P3		AC-E17	
Quarter	Actual Change	Forecast Change	Error	Forecast Change	Error	Forecast Change	Error
1977.2	0.0436	0.0559	0.0122	0.0163	−0.0274	0.0524	0.0088
3	0.0595	0.0569	−0.0025	0.0161	−0.0434	0.0548	−0.0047
4	0.0468	0.0693	0.0224	0.0189	−0.0279	0.0695	0.0226
1978.1	0.0421	0.0539	0.0118	0.0205	−0.0216	0.0482	0.0061
2	0.0571	0.0492	−0.0079	0.0192	−0.0379	0.0429	−0.0142
3	0.0458	0.0467	0.0010	0.0196	−0.0261	0.0321	−0.0136
4	0.0608	0.0437	−0.0171	0.0131	−0.0477	0.0218	−0.0389
1979.1	0.0520	0.0402	−0.0117	0.0122	−0.0398	0.0234	−0.0286
2	0.0200	0.0364	0.0165	0.0156	−0.0043	0.0293	0.0094
3	0.0204	0.0375	0.0171	0.0125	−0.0078	0.0222	0.0018
4	0.0062	0.0267	0.0205	0.0091	0.0029	0.0181	0.0120
1980.1	0.0161	0.0337	0.0177	0.0042	−0.0118	0.0145	−0.0016
2	−0.0067	0.0245	0.0311	0.0028	0.0095	0.0096	0.0163
3	−0.0152	0.0213	0.0365	0.0066	0.0218	0.0208	0.0359
4	−0.0006	0.0158	0.0164	0.0017	0.0023	0.0276	0.0282
1981.1	0.0086	0.0151	0.0066	0.0015	−0.0071	0.0322	0.0237
2	0.0290	0.0315	0.0025	0.0084	−0.0206	0.0058	−0.0233
3	0.0329	0.0494	0.0165	0.0142	−0.0187	0.0263	−0.0066
4	0.0062	0.0159	0.0097	0.0075	0.0013	0.0335	0.0273
1982.1	−0.0282	0.0113	0.0395	0.0105	0.0387	0.0275	0.0557
2	−0.0219	0.0081	0.0300	0.0092	0.0311	0.0241	0.0460
3	−0.0344	−0.0154	0.0190	0.0102	0.0446	0.0104	0.0448
4	−0.0188	0.0040	0.0227	0.0177	0.0364	0.0223	0.0411
1983.1	0.0050	0.0155	0.0105	0.0169	0.0119	0.0270	0.0220
2	0.0244	0.0164	−0.0080	0.0249	0.0006	0.0317	0.0073
3	0.0471	0.0227	−0.0244	0.0296	−0.0174	0.0273	−0.0197
4	0.0632	0.0788	0.0156	0.0368	−0.0264	0.0563	−0.0069
1984.1	0.0779	0.0680	−0.0099	0.0394	−0.0385	0.0516	−0.0264
2	0.0679	0.0621	−0.0058	0.0447	−0.0231	0.0581	−0.0098
3	0.0590	0.0486	−0.0104	0.0407	−0.0182	0.0583	−0.0006
4	0.0450	0.0501	0.0052	0.0374	−0.0076	0.0385	−0.0064
1985.1	0.0292	0.0309	0.0017	0.0364	0.0071	0.0205	−0.0087
2	0.0228	0.0230	0.0002	0.0338	0.0110	0.0191	−0.0037
3	0.0271	0.0246	−0.0025	0.0298	0.0026	0.0106	−0.0165
4	0.0286	0.0416	0.0130	0.0327	0.0041	0.0105	−0.0181
1986.1	0.0303	0.0407	0.0104	0.0348	0.0045	0.0147	−0.0156
2	0.0272	0.0415	0.0143	0.0382	0.0110	0.0134	−0.0137

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