Model Reliability

edited by

David A. Belsley
and
Edwin Kuh

The MIT Press
Cambridge, Massachusetts
London, England

This book was set in Times New Roman by Asco Trade Typesetting Ltd., Hong Kong, and printed and bound by Halliday Lithograph in the United States of America

Library of Congress Cataloging-in-Publication Data

Main entry under title:

Model reliability.

Includes bibliographies and index.
1. Econometric models—Addresses, essays, lectures. I. Belsley, David A.
II. Kuh, Edwin.
HB141.M58 1986 330'.028 85-18455
ISBN 0-262-02224-9

Contents

List of Contributors

Lewis S. Alexander
Yale University
New Haven, Conn.

David A. Belsley
Boston College and
Massachusetts Institute of Technology
Cambridge, Mass

Jean-Marie Dufour
University of Montreal
Montreal, Quebec, Canada

Robert F. Engle
University of California at San Diego
San Diego, Calif.

Ray C. Fair
Yale University
New Haven, Conn.

C. W. J. Granger
University of California at San Diego
San Diego, Calif.

Mark N. Greene
Federal Reserve System
Washington, D.C.

E. Philip Howrey
University of Michigan
Ann Arbor, Mich.

Peter Hollinger
Massachusetts Institute of Technology
Cambridge, Mass.

Saul H. Hymans
University of Michigan
Ann Arbor, Mich.

Edwin Kuh
Massachusetts Institute of Technology
Cambridge, Mass.

Edward E. Leamer
University of California at Los Angeles
Los Angeles, Calif.

John Neese
Massachusetts Institute of Technology
Cambridge, Mass.

R. P. Robins
University of California at San Diego
San Diego, Calif.

Stephen Swartz
Federal Reserve System
Washington, D.C.

Roy E. Welsch
Massachusetts Institute of Technology
Cambridge, Mass

Introduction

In the latter part of the seventies, several of us recognized a growing need for a generally improved understanding of notions about model reliability. The result was a five-year interuniversity research project that allowed us to exchange views on different approaches and also to work together on topics of common interest, often using a common computer environment, TROLL. This volume skims some of the cream from the top of these efforts, giving the reader an excellent feel for the directions that the thoughts and ideas of this group have taken.

The seventies witnessed greater criticism of and skepticism about econometric models. And, although much of this was justified, at least some slings and darts partook of maiming the messenger who brings the bad news (economic maladies like stagflation are beyond the power of econometricians to create or expunge). We sought to be more constructive in our criticism. We shared a common belief that the mechanical application of standard econometric methods was not enough and that the solution was not merely more bells and whistles. There was a clear need to evaluate models more intensively from new perspectives, ones extending beyond the confines of Neyman-Pearson hypothesis testing. This volume contains some of these ideas, and although the unifying theme of model reliability is the objective, the approaches toward it are highly diverse. We worked individually, meeting once or twice a year. These meetings fostered continuity in the evolution of our several view points. Sometimes several of our members would cooperate on the same topic or use the same analytical tools. The principal area of common interest was time series.

Robert Engle and Clive Granger at the University of California, San Diego, and Douglas Martin, University of Washington, Seattle, worked with members of the Center for Computational Research in Economics and Management Science at MIT. So did James Durbin of the London School of Economics, who visited the Center for three summers to work with us on related time-series problems. Alexander Samarov, Walter Vandaele, and Stephen Peters at the center worked in collaboration with this group on team research projects. Meetings over a four-year period provided one important means for continuity to the evolution of our individual thinking. Another means of cooperation existed through the TROLL econometric modeling system. This system was accessible over national telecommunication networks to those who were interested in common methods and was employed mainly in the several time-series topics.

Other participants included G. S. Maddala, University of Florida, Gainesville; Gregory Chow, Princeton University; Edward Leamer, University of California, Los Angeles; Saul Hymans, Philip Howrey, and Mark Greene, University of Michigan; Ray Fair, Yale University, and Jean-Marie Dufour, University of Montreal. John Neese, Peter Hollinger, David Belsley, Roy Welsch, and Edwin Kuh were actively engaged throughout on model reliability issues.

This volume includes pieces evenly divided into four, possibly overlapping, facets of model reliability: specification testing, outside information, diagnostics, and system analysis. In chapter 1 Engle, Granger, and Robins show that the common practice of forecasting on the basis of conditional means while ignoring conditional variances can be misleading both as to the form of the conditional mean and the nature of causality. In chapter 2 Dufour makes use of recursive techniques to analyze the stability over time of various models of the demand for money during the German hyperinflation. The techniques highlight structural instability and pinpoint the time of occurrence.

In chapter 3 Leamer illuminates the effect different priors can have on inferences about the course of U.S. inflation when applied to the same data set. His hope of aiding the formation of a consensus is frustrated, but the techniques displayed are a model for other research. In chapter 4 Greene, Howrey, and Hymans show how to include recently occurring monthly data to improve the forecast reliability of quarterly models.

In chapter 5 Belsley examines the proper form for data that are to be subjected to conditioning analysis. The "improved" conditioning often obtained through centering or first-differencing is shown typically to be illusory. In chapter 6 Swartz and Welsch make use of bounded-influence techniques to investigate the quality of the data-model fit in an energy model. These robust techniques notably improve certain forecasts.

In chapter 7 Fair and Alexander demonstrate means for comparing the equation-by-equation accuracy of two large-system macromodels. In this case the two models are Fair's own and the Michigan Quarterly Econometric Model (MQEM). In chapter 8 Kuh, Neese, and Hollinger apply their methods for cutting through the complexity of large multi-equation, dynamic models to help understand the main forces at work in producing inflation in the MQEM.

Although there is much diversity among the topics of these papers, they are connected by one important thread: a search for means beyond routine econometric practice to produce more reliable econometric models.

Econometric practice unavoidably incorporates, either explicitly or implicitly, two different sources of prior information: that brought by the investigator on the nature of the process being modeled and that inculcated by econometric practice as to what is conventionally acceptable. The former source arises from the concepts and questions of interest to a particular research topic. The latter source is frequently accepted without much question and is all too often highly restrictive. Thus, having formulated an econometric model on the basis of specific knowledge, it is often routine practice or routine software availability that determines, and limits, the estimator to be used, the hypotheses to be tested, the questions to be asked, and the ways for proceeding when problems arise.

Each study in this volume widens and deepens the set of techniques available to econometric investigation and extends our ability to assess many assumptions that have been traditionally less open to investigation. The combined result represents a dynamic view of econometric practice that we feel increases our ability to understand model reliability and to produce econometric results more relevant to their intended purpose.

CHAPTER 6

A Comparison of the Michigan and Fair Models

RAY C. FAIR and LEWIS S. ALEXANDER

This chapter compares the predictive accuracy of the Michigan and Fair models using the method developed in Fair (1980). These models are compared to each other and to an eighth-order autoregressive model. The method accounts for the four main sources of uncertainty of a forecast: uncertainty due to (1) the error terms, (2) the coefficient estimates, (3) the exogenous variables, and (4) the possible misspecification of the model. Because it accounts for these four sources, it can be used to make comparisons across models. In other words, it puts each model on an equal footing for purposes of comparison. The method has been used to compare the Fair model to autoregressive models, vector autoregressive models, Sargent's classical macroeconomic model, and a small linear model, but this is the first time it has been used to compare two relatively large structural models.

Ideally, model builders should not be the ones comparing their models to others. Although one may try to be objective, there is always the suspicion that one has stacked the cards in favor of her or his model. This chapter is not intended to be the final word on the relative merits of the Michigan and Fair models. Its primary aim is to demonstrate the application of the comparison method to large models.

As will be seen, the application of the method to the Michigan model reveals two potential shortcomings of the method. First, the results for the Michigan model are highly sensitive to plausible alternative assumptions about exogenous variable uncertainty. This makes comparison difficult because there is no obvious criterion for choosing between the competing assumptions. Second, the Michigan model relies fairly heavily on the use of dummy variables, and the part of the method that accounts for exogenous-variable uncertainty cannot handle dummy variables. It must be assumed that the dummy variables are known with certainty. The method may thus bias the results in favor of models that are heavily tied to dummy variables. It is uncertain how large this bias might be.

THE COMPARISON METHOD

The method was first proposed in Fair (1980), and the latest discussion of it is in Chapter 8 in Fair (1984). The following is a brief outline of the method.

Assume that the model has m stochastic equations, p unrestricted coefficients to

Table A5.6. Estimation Tests: Aggregate M_2 Logarithm versus Semilogarithmic Specification (Interval: 1965.1–1987.4)

		Coefficients (Standard errors)					
Functional form	Time frame	Transactions	Treasury bill	Own rate	Lagged deposits	$\bar{R}^2$	SE
Standard equation	Specification						
Log	Short run	0.088	−0.046	0.038	0.924	0.998	0.008
		(0.04)	(0.006)	(0.008)	(0.039)		
	Long run	1.16	−0.605	0.500	—		
Semilog	Short run	0.105	−0.620	0.524	0.901	0.998	0.008
		(0.05)	(0.09)	(0.11)	(0.04)		
	Long run	1.06	−6.25	5.30	—		
Inverted equation	Specification						
Log	Short run	0.294	−0.119	0.112	0.745	0.915	0.104
	Long run	1.15	−0.467	0.440	—	0.995	0.008
Semilog	Short run	0.403	−0.0176	0.016	0.631	0.922	0.762
	Long run	1.092	0.048	0.043	—	0.995	0.008

estimate, and T observations for the estimation. The model can be nonlinear, simultaneous, and dynamic. Let S denote the covariance matrix of the error terms, and let V denote the covariance matrix of the coefficient estimates. S is $m \times m$ and V is $p \times p$. An estimate of S, say $\hat{S}$, is $(1/T)UU'$, where U is an $m \times T$ matrix of estimated errors. The estimate of V, say $\hat{V}$, depends on the estimation technique used. Let $\hat{\alpha}$ denote a p-component vector of the coefficient estimates, and let u_t denote an m-component vector of the error terms for period t.

Uncertainty from the error terms and coefficient estimates can be estimated in a straightforward way by means of stochastic simulation. Given assumptions about the distributions of the error terms and coefficient estimates, one can draw values of both error terms and coefficients. For each set of values the model can be solved for the period of interest. Given, say, J trials, the estimated forecast mean and estimated variance of the forecast error for each endogenous variable for each period can be computed. Let $\tilde{y}_{itk}$ denote the estimated mean of the k-period-ahead forecast of variable i, where t is the first period of the forecast, and let $\tilde{\sigma}^2_{itk}$ denote the estimated variance of the forecast error. $\tilde{y}_{itk}$ is simply the average of the J predicted values from the J trials, and $\tilde{\sigma}^2_{itk}$ is the sum of squared deviations of the predicted values from the estimated mean divided by J.

It is usually assumed that the distributions of the error terms and coefficient estimates are normal, although the stochastic-simulation procedure does not require the normality assumption. The normality assumption has been used for the results in this chapter. Let u_t^* be a particular draw of the error terms for period t, and let α^* be a particular draw of the coefficients. The distribution of u_t^* is assumed to be $N(0,\hat{S})$, and the distribution of α^* is assumed to be $N(\hat{\alpha},\hat{V})$.

There are two polar assumptions that can be made about the uncertainty of the exogenous variables. One is that there is no uncertainty. The other is that the exogenous-variable forecasts are in some way as uncertain as the endogenous-variable forecasts. Under this second assumption one could, for example, estimate an autoregressive equation for each exogenous variable and add these equations to the model. This expanded model, which would have no exogenous variables, could then be used for the stochastic-simulation estimates of the variances. The assumption used in this chapter is in between the two polar assumptions. An eighth-order autoregressive equation was estimated for each exogenous variable (with a constant term and time trend included in the equation), and the estimated standard error from this regression was used as the estimate of the degree of uncertainty attached to forecasting the exogenous variable for each period. This procedure ignores the uncertainty of the coefficient estimates in the autoregressive equations, which is one of the reasons it is not as extreme as the second polar assumption. The procedure also assumes that the exogenous-variable errors are uncorrelated with each other and with the structural errors.

This assumption is implemented as follows. Let $\hat{s}_i$ denote the estimated standard error from the autoregressive equation for exogenous variable i. Let v_{it} be a normally distributed random variable with mean zero and variance $\hat{s}_i^2$:$v_{it} \sim N(0, \hat{s}_i^2)$ for all t. Let $\hat{x}_{it}$ be the "base" value of exogenous variable i for period t. The base values can be either actual values if the period in question is within the period for which data exist or guessed values otherwise. If the values are guessed, they need *not* be the predictions from the autoregressive equations. The autoregressive equations are used merely to get the values for $\hat{s}_i$.

Let x^*_{it} be the value of variable i for period t used for a particular trial. Given the above setup, one can assume that the v_{it} errors pertain to forecasting either the level of the variable or the change in the variable. If the level assumption is used, the value of x^*_{it} for a given trial is $\hat{x}_{it} + v_{it}$, where v_{it} is drawn from the above distribution. If the change assumption is used, the values are as follows. Let the beginning period be 1 and assume that the overall prediction period is of length K. The values of x^*_{it} $(t = 1, \ldots, K)$ for a given trial are

$$\begin{aligned} x^*_{i1} &= \hat{x}_{i1} + v_{i1} \\ x^*_{i2} &= \hat{x}_{i2} + v_{i1} + v_{i2} \\ &\vdots \\ x^*_{iK} &= \hat{x}_{iK} + v_{i1} + v_{i2} + \ldots + v_{iK} \end{aligned} \tag{1}$$

where each v_{it} $(t = 1, \ldots, K)$ is drawn from the $N(0, \hat{s}_i^2)$ distribution. Because of the assumption that the errors pertain to changes, the error term v_{i1} is carried along from period 1 on. Similarly, v_{i2} is carried along from period 2 on, and so on. Given the way that many exogenous variables are forecast, by extrapolating past trends or taking variables to be unchanged from their last observed values, it may be that any error in forecasting the level of a variable in, say, the first period will persist throughout the prediction period. If this is true, the change assumption is likely to result in a better approximation of exogenous-variable uncertainty.

The stochastic-simulation estimate of the forecast-error variance that is based on draws of the error terms, coefficients, *and* exogenous-variable errors will be denoted $\tilde{\tilde{\sigma}}^2_{itk}$. It differs from $\tilde{\sigma}^2_{itk}$ in that it takes into account exogenous-variable uncertainty.

Estimating the uncertainty from the possible misspecification of the model is the most difficult and costly part of the method. It requires successive reestimation and stochastic simulation of the model. It is based on a comparison of estimated variances computed by means of stochastic simulation with estimated variances computed from outside-sample (i.e., outside the estimation period) forecast errors. Assuming no stochastic-simulation error, the expected value of the difference between the two estimated variances for a given variable and period is zero for a correctly specified model. The expected value is not in general zero for a misspecified model, and this fact is used to try to account for misspecification.

Without going into details, the basic procedure is to estimate the model over a number of different estimation periods and for each set of estimates to compute the difference between the two estimated variances for each variable and lead time of the forecast. The average of these differences for each variable and lead time provides an estimate of the expected value. Let $\bar{d}_{ik}$ denote this average for variable i and lead time k. The stochastic simulations for this work are with respect to draws of error terms and coefficients only, not also draws of exogenous-variable errors. Given $\bar{d}_{ik}$, the final step is to add it to $\tilde{\tilde{\sigma}}^2_{itk}$. This sum, which will be denoted $\hat{\sigma}^2_{itk}$, is the final estimated variance; it takes into account all four sources of uncertainty. Another way of looking at $\bar{d}_{ik}$ is that it is the part of the forecast-error variance not accounted for by the stochastic-simulation estimate. Some of the specifics of the above procedure will become apparent in the discussion of the computations under Calculations of the Results.

SOME FEATURES OF THE MODELS

Table 6.1 provides an outline of the models. The Michigan model has 61 stochastic equations and 50 identities. The Fair model has 30 stochastic equations and 98 identities. The following is a brief discussion of some of the differences between the two models.

Even though the Michigan model has more stochastic equations than does the Fair model, it has to some extent more reduced-form like equations. For example, the level of corporate profits is determined by a stochastic equation in the Michigan model, whereas it is determined by an identity (revenue minus costs) in the Fair model. The identity in the Michigan model that would normally determine corporate profits instead determines the statistical discrepancy, which is endogenous. (The statistical discrepancy is exogenous in the Fair model.) Treating the statistical discrepancy as endogenous is a way of allowing a reduced-form like equation for corporate profits to be estimated and used in the model.

The Michigan model is also more reduced form in its determination of the unemployment rate. In the Fair model there are three stochastic equations explaining the labor force (equations for prime age men, prime age women, and all others), a stochastic equation explaining the number of people holding two jobs, and a stochastic equation explaining the demand for jobs by the firm sector. The unemployment rate is determined by an identity. It is equal to one minus the ratio of total employment to the total labor force. Total employment is equal to the total number of jobs minus the number of people holding two jobs. In the Michigan model the unemployment rate is determined by a stochastic equation. It is a function of a dummy variable (DFPR in Table 6.2), a time trend, and one minus the employment rate of adult men. The em-

Table 6.1. The Models

Michigan
- 61 stochastic equations
- 50 identities
- 96 exogenous variables, of which 39 are dummy variables
- Basic estimation period in Belton, Hymans, and Lown (1981): 1954.1–1979.4
- Estimation technique: ordinary least squares, sometimes accounting for first-order serial correlation of the error terms

Fair
- 30 stochastic equations
- 98 identities
- 106 exogenous variables, of which 11 are dummy variables
- Basic estimation period in Fair (1984): 1954.1–1982.3
- Estimation technique: two stage least squares, sometimes accounting for first-order serial correlation of the errors terms

Autoregressive
- One eighth-order autoregressive equation (with a constant term and time trend included) per relevant variable
- No exogenous variables other than the time trend
- Basic estimation period: same as for the Michigan model
- Estimation technique: ordinary least squares

Table 6.2. Dummy Variables in the Michigan and Fair Models

Name	Description	Equation	t-Statistic[d]
Michigan			
DAPACTM	Dummy variable to reflect Canadian auto pact	C16	2.08
DASTRIKE	Dummy variable for auto strikes	C1, C11	6.46, 2.62
DASTRIKE$_{-1}$		C1	−4.23
DAUTO	Dummy variable to reflect 1975 auto rebates and reaction to higher auto prices in 1974; equals 0.90 in 1974.2 and 1974.3, 0.95 in 1975.1 and 1975.2, equals 1.0 otherwise	C1	−5.04[a]
DEX65	Dummy variable for the change in Federal excise tax law, equal to 1 from 1954.1–1964.1, 0 otherwise	D8	3.30[a]
DFPR[b]	Dummy variable to reflect shift in relation between RUM and RUG values (RUM = unemployment rate, males 20 and over; RUG = global unemployment rate)	B3	2.21[a] −5.14[a]
DFROFF	Dummy variable for removal of price controls; equals 0.25 in 1974.2–1975.1, 0 otherwise	A2	4.71
DFRZ1	Dummy variables to reflect price freeze and Phase II effects on prices and compensation	A1	3.02
DFRZ2		A2	−1.83[a]
DFRZ3		A2	−1.83[a]
	DFRZ1 equals −1.0 in 1971.4 DFRZ2 equals 0.5 in 1971.3, 1.0 in 1971.4 DFRZ3 equals 1.0 in 1972.2–1972.4 equals 0 otherwise		
DGPAY	Dummy variable to reflect government pay increases	A11	4.10[a]
DJGPM	Dummy variable to reflect increased consumer awareness of gas mileage in the cost of running a new car, equal to 0 from 1954.1–1974.4, 1 otherwise	C1 C3	−5.04[a] 3.56[a]
DM72DOCK	Dummy variable for dock strikes	C16	6.63
DM72DOCK$_{-1}$		C16	−1.77
DM72SS	Dummy variable to reflect steel strike in import equation; equal to 0.5 in 1959.2, 1.0 in 1959.3, 0 otherwise	C16	1.73
DM72SS$_{-1}$		C16	−0.61
DPGAS	Dummy variable for availability of PGAS series, equal to 1 from 1954.1 to 1957.1, 0 otherwise (PGAS = price index for gasoline, motor oil, coolant, and other products)	A6	−1.97
DPROP13	Dummy variable for the effect of Proposition 13 on state and local indirect business taxes; equals 1 in 1978.3; 0 otherwise	D9	−13.36
DRAM	Dummy variable for the effect on MRAM of changes in the structure of reserve requirements on demand and time deposits (part of dependent variable of equation E4) (MRAM = reserve adjustment magnitude)		N.A.[c]

DSEAS1	Dummy variable equal to 1 in the first quarter, −1 in the fourth quarter, 0 otherwise	E2–E8, E10	Many coefficients
DSEAS2	Dummy variable equal to 1 in the second quarter, −1 in the fourth quarter, 0 otherwise	E2–E8, E10	Many coefficients
DSEAS3	Dummy variable equal to 1 in the third quarter, −1 in the fourth quarter, 0 otherwise	E2–E8, E10	Many coefficients
DSPRD	Dummy variable for anomaly in spread between RCP and RTB; equals 1 in 1974.2 and 1974.3, 0 otherwise (RCP = 4–6 month commercial paper rate; RTB = 90-day treasury bill rate)	E10	10.87
DTEX[b]	Dummy variable to reflect direct price effects of changes in excise tax laws in 1965	A3	1.51[a]
		A5	1.30[a]
DTIB[b]	Dummy variable to reflect changes in indirect business taxes	D8	16.01
DTP[b]	Dummy variable to reflect changes in personal taxes	An identity	N.A.
DTPR[b]	Dummy variable for personal tax rate	An identity	N.A.
DTSI	Dummy variable that assumes values equal to the revenue effect of changes in social insurance tax law	A1	5.33[a]
		D1	−3.18
DUBEXT	Dummy variable for the extension of unemployment benefits beyond 26 weeks	D5	3.77
DUM74	Dummy variable in IPD072 equation; equals 0 in 1954.1–1973.4, 1 otherwise (IPD072 = producers' durable equipment investment except in agriculture and production)	C11	2.90[a]
DUM75	Dummy variable in GDEBTP equation; equals 0 in 1954.1–1974.4, 1 otherwise (GDEBTP = gross public debt of the U.S. treasury held by private investors)	E5	5.09
DVDOWN	Dummy variable to reflect effects of winddown of Vietnam War on employment; equals 1 in 1970.1–1972.2, 0 otherwise	B2	−1.52
DVNUP	Dummy variable to reflect effects of Vietnam War build-up on employment; equals 1 in 1965.3–1966.4, 0 otherwise	B2	−0.68
D5467	Dummy variable for change in trend growth of productivity; equals 1 in 1954.1–1967.4, 0 otherwise	A2	10.99[a]
		B1	3.82
D5864	Dummy variable in JCAP equation; equals 1 in 1958.1–1964.4, 0 otherwise (JCAP = index of available capacity in manufacturing)	F3	−6.72
D66	Dummy variable in M1BPLUS equation; equals 0 in 1954.1–1965.4, 1 otherwise (M1PLUS = M1B plus total savings at all depository institutions)	E11	−2.97
D674	Dummy variable for state income tax law changes; equals 0 in 1954.1–1967.3, 1 otherwise	D14	2.13
D6873	Dummy variable for change in trend growth of productivity; equals 1 in 1968.1–1973.4, 0 otherwise	A2	2.50[a]
		B1	3.22

Table 6.2. *(Continued)*

Name	Description	Equation	*t*-Statistic[d]
D7074	Dummy variable in JCAP equation; equals 1 in 1970.1–1974.2, 0 otherwise (JCAP = index of available capacity in manufacturing)	F3	−6.31
D711	Dummy variable for state personal income tax law changes; equals 0 in 1954.1–1970.4, 1 otherwise	D14	0.91
D763	Dummy variable in IRC72 equation; equals 1 in 1976.3, 0 otherwise	C13	−2.93
D79	Dummy variable for change in trend growth of productivity; equals 0 in 1954.1–1978.4, 1 otherwise	B1	−0.31
Fair			
D593	1 in 1959.3; 0 otherwise	11, 13	1.86, 2.70
D594	1 in 1959.4; 0 otherwise	11, 13	0.64, 0.50
D601	1 in 1960.1; 0 otherwise	11	1.89
D651	1 in 1965.1; 0 otherwise	27	2.18
D652	1 in 1965.2; 0 otherwise	27	1.17
D691	1 in 1969.1; 0 otherwise	27	3.65
D692	1 in 1969.2; 0 otherwise	27	5.42
D714	1 in 1971.4; 0 otherwise	27	2.64
D721	1 in 1972.1; 0 otherwise	27	4.10
DD793	1 from 1979.3 on; 0 otherwise	30	4.20[a]
DD811	1 from 1981.1 on; 0 otherwise	21	6.29

[a] *t*-statistics are for explanatory variables that are functions of the relevant dummy variable and other variables.

[b] Autoregressive equation estimated from this variable for the estimation of exogenous-variable uncertainty under Calculations of the Results.

[c] N.A., not applicable.

[d] The *t*-statistics for the Michigan model are as computed for the results in this chapter. They may differ slightly from the values in Belton, Hymans, and Lown (1981).

ployment rate of adult men is determined by a stochastic equation. It is a function, among other things, of real GNP.

The Michigan model has more disaggregation with respect to the expenditure variables. The differences pertain to consumer durable expenditures and nonresidential fixed investment. In the Michigan model durable expenditures are disaggregated into four components: new autos, motor vehicles and parts less new autos, furniture and household equipment, and all other. There is one stochastic equation for each of these components. In the Fair model there is one stochastic equation explaining total durable expenditures. Nonresidential fixed investment is disaggregated into four components in the Michigan model: structures, producers' durable equipment in production, producers' durable equipment in agriculture, and producers' durable equipment except in agriculture and production. There is one stochastic equation for each of these components. In the Fair model there is one stochastic equation explaining total nonresidential fixed investment. There is also a separate equation in the Michigan model explaining the number of new car sales, which is used as an explanatory variable in the automobile expenditure equation. Considerable work has gone into the Michigan model in explaining automobile expenditures.

As noted in the introductory section, there is a fairly heavy use of dummy variables in the Michigan model. Also, many of the dummy variables appear to be subjective. The dummy variables in the Michigan model are listed in Table 6.2. This table also includes the number of the equation that each variable appears in and the associated *t*-statistic of its coefficient estimate. The description of the variables is taken from Belton, Hymans, and Lown (1981). Two of the more subjective variables are DJGPM, which is a dummy variable to reflect increased consumer awareness of gas mileage in the cost of running a new car, and DAUTO, which is a dummy variable to reflect auto rebates and reaction to higher auto prices. Of the 345 estimated coefficients in the Michigan model, 43 are coefficients of nonseasonal dummy variables or variables that are a function of nonseasonal dummy variables.

Dummy variables play a less important role in the Fair model. The dummy variables in the Fair model are also listed in Table 6.2. There are 11 dummy variables, 6 of which account for the effects of dock strikes in the import equation (equation 27). The other 5 dummy variables appear in 4 other stochastic equations. Of the 169 estimated coefficients in the Fair model, 13 are coefficients of dummy variables or variables that are a function of dummy variables.

The fairly heavy use of dummy variables in the Michigan model poses a problem for the comparison method. With a few exceptions, it is not sensible to estimate autoregressive equations for the dummy variables, and so they have to be taken as fixed for purposes of the stochastic-simulation draws of the exogenous-variable errors.[1] The method may thus underestimate the uncertainty from the exogenous variables for the Michigan model.

Even where autoregressive equations are estimated for dummy variables, it is not clear that the use of these equations is appropriate. Consider, for example, dummy variable DFPR, which plays an important role in the stochastic equation explaining the unemployment rate. It begins to take on positive values in 1965.1. It is 0 before 1965.1; it is 1 in 1965.1 and increases by 1 each quarter until 1970.4; it is flat until 1976.1; it increases by 1 from 1976.1 to 1979.4; and it is flat thereafter. The autoregressive equation for this variable was estimated only over the nonzero observations. The estimated standard error was 0.173. This estimated error is quite low, and so it means that very little uncertainty is assumed for the variable. It is almost like taking the variable to be fixed.

The DFPR variable links the employment rate of adult men to the overall unemployment rate. The former is easier to explain than the latter because the labor force of adult men fluctuates less than does the labor force of other groups. Thus, the Michigan model links a relatively easy-to-explain variable to a relatively hard-to-explain variable by the use of a time trend and the DFPR dummy variable. If the comparison method has underestimated the uncertainty of the DFPR variable, then the uncertainty of the unemployment rate forecasts will be underestimated.

Another example of the dummy variable problem concerns the key price equation in the Michigan model, Eq. (A2), which determines PPNF, the private nonfarm deflator. There are two dummy variables in the equation that pertain to the price freeze, and there is a productivity trend variable that is a function of three other dummy variables. One of the latter three variables takes on a value of 1 between 1954.1 and 1967.4 and 0 otherwise; one takes on a value of 1 between 1968.1 and 1973.4 and 0 otherwise; and one takes on a value of 1 between 1979.1 and 1979.4 and 0 otherwise. The specification of this equation may mean that a fairly large part of the fluctuations in the price deflator is explained by the dummy variables, and if this is true, the method will underestimate the uncertainty from the price equation.

The Michigan model has also used what seem to be questionable explanatory variables in some of the equations. For example, the discount rate is used as an explanatory variable in the bill rate equation. It is by far the most significant variable in the equation. On a quarterly basis the two variables are highly correlated, but this is because the discount rate generally follows the bill rate with a lag of a few weeks. The discount rate is not generally the policy instrument used by the Fed to influence short-term rates.[2] It is simply a passive instrument. Another example of this type is the use of the minimum wage in the wage rate equation. It seems more likely that the aggregate wage rate affects the minimum wage rate rather than vice versa. Both the discount rate and the minimum wage are exogenous in the model.

CALCULATIONS OF THE RESULTS

Many steps were involved in obtaining the final results, and it is easiest to discuss the computation of the results in the order in which they were done. The results for the Michigan model will be discussed first.

Duplication of the Basic Estimates

Data for the Michigan model were taken from the TROLL version of the model that was current at the beginning of 1983.[3] The specification of this version of the model is in Belton, Hymans, and Lown (1981) (BHL). The first step was to duplicate the basic sets of estimates, which we were able to do. For none of the 61 equations were the differences between our estimates and the BHL estimates large enough to call into question our duplication of the results.

Initial Stochastic Simulation Results

Given the basic coefficient estimates, the V and S covariance matrices were estimated. The number of unconstrained coefficients in the model is 345, and so V is 345×345. V was estimated as a block diagonal matrix, with the blocks being the estimated covariance matrices of the coefficient estimates of the individual equations.[4] The estimation of S required more thought (S is 61×61 since there are 61 stochastic equations). The problem was that estimation periods differ across equations. With three exceptions the periods ended in 1979.4, but they generally began with different quarters. The beginning quarters for the longest and shortest estimation periods were 1954.1 and 1963.2, respectively. There are two plausible ways to estimate S. One is to estimate the full S over the period that all the equations have in common, which is 1963.2–1979.4. The other is to take S to be a diagonal matrix and to estimate each diagonal element using the same estimation period that is used to estimate the corresponding equation. In this case the diagonal elements of S would be based on different estimation periods.

To see how sensitive the results are to alternative estimates of S, three stochastic

Table 6.3. Initial Stochastic Simulation Results for the Michigan Model[a–d]

	Estimated Standard Errors of Forecasts							
	1978				1979			
	1	2	3	4	1	2	3	4
Real GNP								
Full S—small	0.39	0.55	0.72	0.84	0.94	1.04	1.17	1.21
Diagonal S—small	0.43	0.56	0.74	0.87	1.01	1.12	1.23	1.30
Diagonal S—large	0.39	0.62	0.76	0.89	0.98	1.08	1.13	1.21
Private nonfarm deflator								
Full S—small	0.30	0.43	0.57	0.65	0.78	0.90	0.96	1.07
Diagonal S—small	0.29	0.41	0.53	0.65	0.75	0.85	0.96	1.03
Diagonal S—large	0.27	0.40	0.51	0.59	0.70	0.79	0.90	0.99
Unemployment rate								
Full S—small	0.19	0.30	0.38	0.44	0.51	0.55	0.61	0.67
Diagonal S—small	0.22	0.30	0.38	0.45	0.50	0.57	0.66	0.72
Diagonal S—large	0.23	0.35	0.44	0.54	0.58	0.65	0.72	0.76
Bill rate								
Full S—small	0.38	0.46	0.60	0.72	0.71	0.69	0.77	0.96
Diagonal S—small	0.34	0.46	0.62	0.73	0.69	0.63	0.72	0.94
Diagonal S—large	0.41	0.48	0.58	0.70	0.69	0.71	0.80	0.91
Money supply								
Full S—small	0.73	1.16	1.49	1.79	2.07	2.32	2.54	2.66
Diagonal S—small	0.75	1.32	1.78	2.12	2.47	2.77	2.98	3.20
Diagonal S—large	0.76	1.28	1.74	1.99	2.29	2.57	2.77	2.89

[a]Stochastic simulation is with respect to error terms only.

[b]250 trials for each set of results.

[c]Full S—small = full S estimated for 1963.2–1979.4 period. Diagonal S—small = S taken to be diagonal. Estimation period for diagonal elements is 1963.2–1979.4. Diagonal S—large = S taken to be diagonal. Estimation period for each diagonal element is the same as the period used to estimate the corresponding equation.

[d]All errors are in percentage points. Errors for real GNP, the GNP deflator, and the money supply are percentages of the forecast means.

simulations were performed. These results are presented in Table 6.3 for selected variables. The first simulation used the full S estimated for the common period; the second used the diagonal S estimated for the common period; and the third used the diagonal S estimated using the different estimation periods. The period of the simulation is 1978.1–1979.4. The number of trials for each stochastic simulation was 250. These simulations were with respect to draws from the error terms only, since this is all that is of interest with respect to the S matrix. As can be seen, the results are not very sensitive to the alternative S matrices. For the rest of the results in this chapter S has been estimated as a diagonal matrix with the estimation period for each diagonal element being the same as the period used to estimate the corresponding equation.

Although the estimation periods for the Michigan equations ended in 1979.4, the data base contained data through 1982.1. Some of the observations for 1982.1 did not seem sensible, but the data through 1981.4 seemed good. The Michigan model was reestimated through 1981.4. Specifically, new coefficient estimates were obtained along with new estimates of V and S. To see how sensitive the stochastic-simulation results are to the different estimation periods, two stochastic simulations were performed using the two sets of estimates. The simulation period for both simulations was 1978.1–1979.4; both simulations were based on 250 trials and both simulations were based on draws of error terms and coefficients. The results for selected variables are presented in Table 6.4. These results are also fairly close, which means that it does not make much difference which set is taken to be the basic set of estimates of the model. We decided to stay with the first set of estimates (i.e., the estimates through

Table 6.4. More Initial Stochastic Simulation Results for the Michigan Model[a–e]

	Estimated Standard Errors of Forecasts							
	1978				1979			
	1	2	3	4	1	2	3	4
Real GNP								
Basic	0.46	0.61	0.78	0.91	0.99	1.11	1.23	1.39
Extended	0.55	0.68	0.82	0.95	1.11	1.27	1.43	1.58
Private nonfarm								
Basic	0.28	0.40	0.53	0.66	0.74	0.88	1.01	1.17
Extended	0.32	0.46	0.56	0.67	0.79	0.92	1.04	1.20
Unemployment rate								
Basic	0.25	0.36	0.45	0.54	0.61	0.66	0.74	0.84
Extended	0.24	0.35	0.43	0.49	0.58	0.65	0.72	0.81
Bill rate								
Basic	0.41	0.57	0.73	0.96	1.03	1.06	1.26	1.49
Extended	0.49	0.61	0.75	1.08	1.11	1.03	1.21	1.51
Money supply								
Basic	0.88	1.51	2.10	2.67	3.18	3.42	3.87	4.58
Extended	0.94	1.65	2.15	2.63	3.11	3.53	3.89	4.32

[a]Stochastic simulation is with respect to error terms and coefficient estimates.

[b]250 trials for each set of results.

[c]S matrix is taken to be diagonal.

[d]Basic = estimation periods end in 1979.4. Extended = estimation periods end in 1981.4.

[e]All errors are in percentage points. Errors for real GNP, the GNP deflator, and the money supply are percentages of the forecast means.

1979.4), since this is the set presented in Belton, Hymans, and Lown (1981). We did, however, use the data through 1981.4 for the successive reestimation and stochastic simulation of the model that is discussed below.

Uncertainty with Respect to the Error Terms and Coefficient Estimates

Table 6.5 contains the main results of this chapter. The values in the *a* rows are stochastic-simulation estimates of the forecast standard errors based on draws of the error terms only. The values in the *b* rows are based on draws of both error terms and coefficients. The results are based on 250 trials for each of the two stochastic simulations.[5] The coefficient estimates and the estimates of S and V that were used for these simulations are based on the estimation periods that ended in 1979.4. The simulation period is 1978.1–1979.4. In terms of the notation previously given, the b-row values are values of $\tilde{\sigma}_{itk}$.[6]

Treatment of Exogenous-Variable Uncertainty

Eighth-order autoregressive equations were estimated for 48 exogenous variables in the model. The variables and estimation periods are listed in Table A.1 in the Appendix. Of the 39 dummy variables listed in Table 6.2, 5 had equations estimated for them. These are indicated by footnote *b* in Table 6.2. Two stochastic simulations were performed with respect to exogenous-variable uncertainty. The first was based on the assumption that the errors for the exogenous variables pertain to changes in the variables, and the second was based on the assumption that the errors pertain to the levels of the variables. These two assumptions are discussed under The Comparison Method. Both simulations were based on draws for the error terms, coefficients, and exogenous-variable errors, and both were based on 250 trials.[7] The results are presented in the c-rows in Table 6.5. The results in the left half of the table are for the change assumption, and the results in the right half are for the level assumption. In terms of the previously given notation, the c-row values are values of $\tilde{\tilde{\sigma}}_{itk}$.

Uncertainty from the Possible Misspecification of the Model

For the misspecification results the Michigan model was estimated and stochastically simulated 27 times. For the first set, the estimation periods ended in 1974.4 and the simulation period began two quarters later in 1975.2. For the second set, the estimation periods ended in 1975.1 and the simulation period began in 1975.3. For the final set, the estimation periods ended in 1981.2 and the simulation period began in 1981.4. The beginning quarters for the estimation periods remained unchanged from those for the basic period. The length of the first 20 simulation periods was eight quarters. Since the data ended in 1981.4, the length of the twenty-first simulation period, which began in 1980.2, was only seven quarters. Similarly, the length of the twenty-second period was six, and so on through the length of the twenty-seventh period, which was only one quarter. For each of the 27 sets of estimates, new estimates of V and S were obtained. Each of the 27 stochastic simulations was based on 50 trials.[8]

Table 6.5. Estimated Standard Errors of Forecasts for 1978.1–1979.4 for the Three Models[a–d]

	Change Assumption for Exogenous-Variable Uncertainty								Level Assumption for Exogenous-Variable Uncertainty							
	1978				1979				1978				1979			
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Real GNP																
Michigan																
a	0.39	0.62	0.76	0.89	0.98	1.08	1.13	1.21								
b	0.46	0.61	0.78	0.91	0.99	1.11	1.23	1.39								
c	0.51	0.86	1.17	1.61	2.13	2.73	3.34	3.94	0.48	0.74	0.92	1.14	1.30	1.51	1.72	1.84
d	0.83	1.24	1.60	1.86	2.24	2.87	3.91	5.01	0.81	1.16	1.43	1.48	1.47	1.75	2.66	3.61
Fair																
a	0.49	0.66	0.81	0.98	1.10	1.14	1.22	1.32								
b	0.51	0.69	0.89	1.03	1.09	1.22	1.30	1.35								
c	0.61	0.83	1.08	1.34	1.54	1.73	1.88	2.01	0.61	0.78	0.94	1.14	1.31	1.41	1.50	1.66
d	0.91	1.29	1.70	2.47	2.42	2.71	3.00	3.39	0.91	1.25	1.61	1.94	2.28	2.52	2.78	3.20
AR8																
a	0.65	1.01	1.27	1.56	1.67	1.72	1.75	1.76								
b	0.72	1.13	1.50	1.78	1.97	2.11	2.26	2.34								
d	1.14	1.73	2.16	2.08	1.96	2.08	2.47	2.74								
Private nonfarm deflator																
Michigan																
a	0.27	0.40	0.51	0.59	0.70	0.79	0.90	0.99								
b	0.28	0.40	0.53	0.66	0.74	0.88	1.01	1.17								
c	0.30	0.46	0.60	0.74	0.87	1.04	1.16	1.31	0.29	0.44	0.55	0.64	0.79	0.92	1.06	1.17
d	0.36	0.46	0.53	0.71	0.82	0.96	1.09	1.25	0.35	0.44	0.47	0.60	0.74	0.83	0.98	1.11
Fair																
a	0.38	0.55	0.68	0.77	0.84	0.91	0.92	0.98								
b	0.41	0.57	0.70	0.84	0.91	0.99	1.09	1.21								
c	0.44	0.59	0.68	0.80	0.93	1.03	1.18	1.26	0.41	0.63	0.74	0.83	0.93	1.01	1.08	1.16
d	0.68	1.14	1.54	2.03	2.45	2.81	3.20	3.51	0.66	1.16	1.57	2.04	2.45	2.81	3.16	3.47

AR8																
a	0.30	0.48	0.70	0.91	1.11	1.27	1.39	1.50								
b	0.34	0.53	0.77	1.05	1.30	1.55	1.78	1.99								
d	0.70	1.18	1.72	2.57	3.24	3.75	3.98	3.74								
Nominal GNP																
Michigan																
a	0.40	0.64	0.79	0.94	1.00	1.15	1.20	1.20								
b	0.46	0.68	0.89	1.01	1.07	1.24	1.40	1.52								
c	0.58	0.93	1.22	1.62	2.15	2.81	3.89	3.82	0.53	0.81	1.01	1.24	1.38	1.65	1.84	1.90
d	1.08	1.56	1.84	1.92	2.40	3.13	4.53	5.08	1.05	1.49	1.71	1.61	1.75	2.15	2.97	3.85
Fair																
a	0.59	0.85	1.07	1.23	1.36	1.47	1.57	1.62								
b	0.61	0.89	1.12	1.34	1.47	1.70	1.97	2.05								
c	0.79	1.04	1.31	1.56	1.82	2.12	2.50	2.54	0.76	1.06	1.24	1.46	1.71	1.93	2.11	2.21
d	1.05	1.39	1.80	2.06	2.34	2.55	2.77	2.80	1.03	1.40	1.75	1.98	2.25	2.39	2.43	2.50
AR8																
a	0.45	0.69	0.82	0.92	0.95	1.02	1.15	1.21								
b	0.50	0.79	1.01	1.18	.1.27	1.37	1.49	1.66								
d	1.25	1.87	2.35	2.42	2.58	3.03	3.66	4.10								
Unemployment rate																
Michigan																
a	0.23	0.35	0.44	0.54	0.58	0.65	0.72	0.76								
b	0.25	0.36	0.45	0.54	0.61	0.66	0.74	0.84								
c	0.25	0.41	0.59	0.76	0.94	1.18	1.46	1.75	0.24	0.37	0.49	0.61	0.69	0.78	0.86	0.96
d	0.34	0.56	0.77	0.92	1.00	1.11	1.40	1.87	0.34	0.53	0.70	0.80	0.78	0.66	0.76	1.17
Fair																
a	0.24	0.38	0.48	0.54	0.61	0.65	0.65	0.68								
b	0.26	0.42	0.52	0.61	0.70	0.77	0.78	0.80								
c	0.27	0.43	0.57	0.67	0.73	0.83	0.89	0.92	0.27	0.43	0.52	0.62	0.72	0.79	0.82	0.91
d	0.39	0.62	0.89	1.08	1.18	1.29	1.36	1.46	0.39	0.62	0.86	1.05	1.17	1.27	1.32	1.46
AR8																
a	0.29	0.58	0.81	0.97	1.06	1.11	1.17	1.23								
b	0.29	0.56	0.83	1.04	1.19	1.30	1.37	1.41								
d	0.31	0.37	0.39	0.29	i	i	i	i								

(continued)

Table 6.5. *(Continued)*

	Change Assumption for Exogenous-Variable Uncertainty								Level Assumption for Exogenous-Variable Uncertainty							
	1978				1979				1978				1979			
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Bill rate																
Michigan																
a	0.41	0.48	0.58	0.70	0.69	0.71	0.80	0.91								
b	0.41	0.57	0.73	0.96	1.03	1.06	1.26	1.49								
c	0.61	0.79	0.97	1.28	1.40	1.44	1.98	3.57	0.65	0.80	0.95	1.21	1.22	1.32	1.51	2.18
d	0.78	1.05	1.20	1.52	1.68	1.67	1.93	3.39	0.81	1.05	1.19	1.46	1.53	1.56	1.44	4.87
Fair																
a	0.71	1.00	1.07	1.13	1.17	1.21	1.17	1.19								
b	0.73	0.94	1.04	1.03	1.15	1.25	1.31	1.45								
c	0.72	0.96	1.09	1.16	1.17	1.34	1.49	1.60	0.71	0.99	1.08	1.17	1.29	1.28	1.50	1.37
d	1.37	2.13	2.40	2.54	2.67	2.87	3.08	3.29	1.36	2.15	2.40	2.55	2.72	2.85	3.09	3.18
AR8																
a	0.52	0.82	0.92	0.97	1.00	1.08	1.17	1.23								
b	0.54	0.86	1.00	1.13	1.22	1.35	1.39	1.40								
d	1.52	2.51	2.72	3.08	3.39	3.65	3.89	4.09								
Money supply																
Michigan																
a	0.76	1.28	1.74	1.99	2.29	2.57	2.77	2.89								
b	0.83	1.51	2.10	2.67	3.11	3.42	3.87	4.58								
c	0.89	1.52	2.18	2.88	3.39	3.95	4.62	6.09	0.85	1.39	1.90	2.35	2.78	3.31	3.93	5.00
d	1.60	2.14	2.81	3.81	4.56	5.45	6.54	8.15	1.58	2.05	2.59	3.43	4.12	5.01	6.07	7.37
Fair																
a	0.98	1.35	1.49	1.66	1.82	2.00	2.03	1.98								
b	0.95	1.37	1.57	1.77	2.11	2.32	2.38	2.54								
c	1.07	1.53	1.84	2.03	2.49	2.69	3.12	3.45	1.03	1.47	1.75	1.93	2.13	2.24	2.37	2.44
d	1.49	1.90	1.98	2.06	2.22	2.08	2.17	1.56	1.46	1.91	2.08	2.27	2.32	2.32	2.36	1.78

AR8																
a	0.57	1.11	1.55	1.95	2.43	2.91	3.42	3.92								
b	0.57	1.17	1.68	2.33	3.08	3.89	4.83	5.77								
d	2.10	3.50	4.26	5.27	5.91	7.05	8.85	10.39								
Consumer expenditures, services																
Michigan																
a	0.28	0.39	0.47	0.54	0.59	0.65	0.70	0.74								
b	0.30	0.41	0.52	0.62	0.67	0.74	0.80	0.85								
c	0.28	0.45	0.59	0.81	1.03	1.21	1.46	1.72	0.31	0.44	0.54	0.60	0.69	0.77	0.85	0.91
d	0.47	0.76	0.97	1.27	1.57	1.89	2.29	2.67	0.49	0.75	0.91	1.14	1.38	1.64	1.96	2.23
Fair																
a	0.30	0.40	0.53	0.61	0.72	0.81	0.89	1.00								
b	0.28	0.41	0.55	0.67	0.81	0.93	0.99	1.10								
c	0.28	0.43	0.60	0.76	0.91	1.06	1.25	1.37	0.29	0.44	0.56	0.71	0.86	0.99	1.11	1.22
d	0.35	0.56	0.90	1.24	1.72	1.75	2.08	2.36	0.36	0.57	0.87	1.21	1.43	1.71	2.00	2.27
AR8																
a	0.28	0.40	0.49	0.60	0.69	0.72	0.78	0.81								
b	0.30	0.44	0.54	0.64	0.76	0.84	0.95	1.05								
d	0.51	0.81	1.05	1.31	1.61	1.90	2.27	2.56								
Consumer expenditures, nondurables																
Michigan																
a	0.52	0.70	0.85	0.98	1.14	1.18	1.27	1.38								
b	0.52	0.70	0.88	1.02	1.14	1.25	1.39	1.45								
c	0.69	1.16	1.66	2.43	3.34	4.29	5.44	6.68	0.67	0.92	1.23	1.49	1.74	1.95	2.22	2.41
d	0.98	1.58	2.10	2.83	3.78	4.77	5.99	7.30	0.96	1.41	1.77	2.08	2.48	2.86	3.34	3.80
Fair																
a	0.58	0.73	0.82	0.99	0.99	1.07	1.07	1.16								
b	0.70	0.78	0.96	1.07	1.11	1.17	1.23	1.33								
c	0.68	0.93	1.08	1.30	1.45	1.60	1.76	1.95	0.68	0.84	0.96	1.18	1.28	1.27	1.36	1.43
d	0.82	1.07	1.13	1.42	1.56	1.59	1.62	1.73	0.82	0.99	1.02	1.31	1.40	1.26	1.18	1.11
AR8																
a	0.57	0.89	1.07	1.26	1.37	1.43	1.45	1.48								
b	0.61	0.97	1.16	1.33	1.51	1.62	1.71	1.81								
d	0.88	1.35	1.37	1.12	1.28	1.50	1.73	1.88								

(continued)

Table 6.5. (*Continued*)

	Change Assumption for Exogenous-Variable Uncertainty								Level Assumption for Exogenous-Variable Uncertainty							
	1978				1979				1978				1979			
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Consumer expenditures, durables																
Michigan																
a	1.15	1.53	1.82	2.20	2.36	2.73	2.98	3.18								
b	1.31	1.93	2.32	2.77	3.33	3.85	4.30	4.85								
c	1.43	2.41	3.71	5.23	7.11	9.09	10.97	12.93	1.19	1.88	2.63	3.16	3.84	4.29	4.87	5.49
d	3.57	4.22	5.70	6.35	8.17	10.43	12.78	14.98	3.48	3.94	5.06	4.80	5.57	6.67	8.16	9.35
Fair																
a	2.17	2.44	2.79	3.26	3.72	3.69	4.09	4.25								
b	2.11	2.55	3.16	3.52	3.94	4.15	4.40	4.61								
c	2.10	2.69	3.00	3.78	4.55	5.03	5.48	5.88	2.36	2.67	3.11	3.79	4.34	4.54	4.82	5.10
d	3.09	3.68	4.99	6.55	8.27	9.41	10.71	12.21	3.27	3.66	5.06	6.56	8.15	9.16	10.39	11.86
AR8																
a	1.91	2.60	3.15	3.36	3.60	3.92	4.18	4.19								
b	2.13	2.82	3.39	3.83	3.80	4.45	4.69	5.12								
d	4.32	5.54	6.39	6.64	6.93	7.97	9.09	9.92								
Housing investment																
Michigan																
a	2.09	3.07	3.68	4.69	5.54	6.41	7.16	7.40								
b	2.24	3.22	3.98	4.86	5.93	7.20	8.61	10.37								
c	2.40	3.51	4.62	5.61	7.00	8.87	10.52	12.36	2.29	3.62	4.52	5.65	6.94	8.52	9.85	11.26
d	5.43	8.65	9.71	10.32	12.69	16.12	19.73	22.52	5.38	8.70	9.66	10.34	12.65	15.93	19.39	21.93
Fair																
a	2.71	4.71	6.36	7.25	8.19	8.99	9.93	10.38								
b	2.80	4.83	6.83	8.46	9.58	10.77	11.88	13.35								
c	2.71	4.54	6.08	7.69	9.05	10.40	11.55	12.34	2.82	4.61	6.13	7.21	8.54	9.99	11.49	12.56
d	4.73	7.09	6.89	6.77	8.92	11.23	12.83	14.82	4.79	7.13	6.93	6.22	8.40	10.85	12.78	14.66

AR8																
a	2.61	4.22	5.92	7.06	7.66	8.02	8.21	8.23								
b	2.78	4.68	6.20	7.42	8.53	8.96	9.34	9.42								
d	6.43	11.25	12.24	10.89	9.58	10.35	12.46	12.77								
Nonresidential fixed investment																
Michigan																
a	1.13	1.47	1.71	1.94	2.20	2.51	2.79	3.09								
b	1.08	1.57	1.85	2.16	2.56	2.88	3.22	3.40								
c	1.16	1.60	2.01	2.51	3.06	3.70	4.42	5.46	1.02	1.44	1.84	2.33	2.65	3.05	3.50	4.09
d	2.56	4.29	5.65	5.90	5.14	6.69	13.80	29.49	2.50	4.23	5.59	5.82	4.91	6.35	13.53	29.27
Fair																
a	1.72	2.25	2.60	2.92	3.24	3.32	3.49	3.66								
b	1.81	2.56	2.97	3.05	3.48	3.77	4.00	4.09								
c	1.75	2.37	2.89	3.41	3.83	4.21	4.68	5.04	1.95	2.49	2.82	3.23	3.55	3.94	4.18	4.41
d	2.65	3.09	3.94	4.46	5.48	6.36	7.18	8.09	2.79	3.18	3.89	4.32	5.29	6.19	6.86	7.71
AR8																
a	1.24	2.13	2.75	3.29	3.80	4.21	4.30	4.46								
b	1.42	2.05	2.78	3.68	4.27	4.92	5.15	5.45								
d	2.26	2.35	3.41	3.35	2.69	2.03	i	i								
Inventory investment																
Michigan																
a	3.86	4.20	4.38	5.18	5.06	4.81	5.17	5.33								
b	4.59	4.75	4.53	5.10	4.76	5.11	5.45	5.57								
c	4.39	4.91	5.16	5.49	5.75	6.95	7.73	8.39	4.31	4.70	4.79	5.05	5.02	5.65	5.89	6.12
d	6.14	7.04	7.76	8.12	7.82	7.71	8.48	9.65	6.08	6.89	7.52	7.84	7.30	6.56	6.85	7.75
Fair																
a	4.66	5.22	5.10	5.52	6.12	5.54	5.38	5.77								
b	4.50	5.05	5.52	5.60	4.99	5.93	5.90	6.23								
c	4.64	5.34	5.68	6.01	5.81	5.95	6.13	6.22	4.57	5.28	5.68	5.91	5.53	5.44	5.40	6.23
d	6.42	8.11	9.13	9.35	9.66	9.67	10.02	10.49	6.38	8.07	9.13	9.29	9.49	9.37	9.59	10.49
AR8																
a	5.22	5.45	5.81	5.72	5.69	6.41	6.02	6.38								
b	5.33	6.18	6.45	6.75	6.64	6.67	6.75	7.32								
d	6.67	7.66	7.92	7.51	7.78	8.12	8.18	8.61								

(continued)

Table 6.5. (*Continued*)

	Change Assumption for Exogenous-Variable Uncertainty								Level Assumption for Exogenous-Variable Uncertainty							
	1978				1979				1978				1979			
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Imports																
Michigan																
a	2.47	3.42	3.68	3.80	3.80	3.65	3.64	3.67								
b	2.65	3.49	3.64	3.90	4.42	4.47	4.63	4.78								
c	2.68	3.34	3.67	3.90	4.60	5.21	5.69	6.66	2.73	3.63	3.75	4.10	4.55	4.81	4.82	5.12
d	3.68	4.91	5.66	6.29	7.10	8.12	9.70	11.65	3.72	5.11	5.71	6.41	7.07	7.87	9.22	10.84
Fair																
a	1.90	2.46	2.60	2.67	2.72	2.71	2.61	2.97								
b	2.22	2.44	2.66	2.76	2.81	3.28	3.56	3.70								
c	1.96	2.42	2.66	2.89	3.03	3.37	3.44	3.65	2.11	2.51	2.55	2.85	2.96	3.20	3.41	3.48
d	3.86	5.73	7.54	8.98	10.30	11.14	12.53	13.08	3.93	5.77	7.50	8.97	10.28	11.09	12.52	13.04
AR8																
a	2.63	3.53	3.98	4.64	4.71	4.87	5.14	5.22								
b	2.79	3.61	4.33	4.91	5.43	5.58	5.77	5.96								
d	5.04	7.54	9.69	11.70	13.11	13.71	14.25	15.26								
Wage rate																
Michigan																
a	0.31	0.42	0.59	0.66	0.78	0.87	0.97	1.10								
b	0.35	0.47	0.59	0.72	0.86	1.00	1.15	1.32								
c	0.35	0.46	0.58	0.68	0.80	0.91	1.06	1.20	0.32	0.44	0.57	0.70	0.86	0.99	1.13	1.25
d	0.28	0.42	0.58	0.71	0.77	0.90	1.05	1.26	0.25	0.40	0.57	0.73	0.83	0.98	1.12	1.30
Fair																
a	0.56	0.86	1.00	1.13	1.20	1.30	1.32	1.38								
b	0.59	0.84	1.01	1.15	1.35	1.48	1.70	1.85								
c	0.60	0.82	1.04	1.19	1.39	1.59	1.83	1.98	0.59	0.85	0.99	1.26	1.49	1.67	1.80	1.97
d	0.38	0.46	0.65	0.78	1.04	1.22	1.59	1.90	0.37	0.51	0.57	0.88	1.17	1.32	1.56	1.89
AR8																
a	0.21	0.33	0.43	0.53	0.60	0.63	0.68	0.73								
b	0.26	0.44	0.58	0.73	0.86	0.95	1.04	1.14								
d	0.43	0.75	1.04	1.35	1.75	2.11	2.51	2.96								

Profits																
Michigan																
a	3.80	4.99	5.90	6.54	7.63	8.87	9.87	10.12								
b	3.97	5.24	6.07	7.04	7.97	10.29	12.02	13.16								
c	4.26	5.74	7.45	8.87	11.85	15.77	18.83	21.89	3.90	5.41	6.37	7.25	8.83	10.84	12.05	13.14
d	5.97	6.59	8.50	8.16	9.60	12.64	19.07	29.08	5.31	6.31	7.57	6.36	5.45	5.35	12.43	23.21
Fair																
a	4.98	6.26	7.61	8.75	8.90	10.07	10.91	11.80								
b	5.02	6.55	8.35	9.26	10.27	11.55	13.46	15.11								
c	6.86	8.49	10.27	11.61	12.89	15.52	17.62	18.04	6.34	7.57	8.65	9.61	10.97	11.03	12.69	13.93
d	8.40	9.86	11.29	11.94	13.26	15.21	17.75	17.77	7.98	9.08	9.84	10.00	11.40	11.02	12.87	13.58
AR8																
a	3.22	4.93	6.39	7.10	7.37	7.59	7.93	8.60								
b	3.72	5.51	7.26	8.58	9.74	10.64	11.77	13.41								
d	9.56	16.32	23.29	27.89	30.69	33.82	37.47	40.43								

[a] a = uncertainty due to error terms; b = uncertainty due to error terms and coefficient estimates; c = uncertainty due to error terms, coefficient estimates, and exogenous-variable forecasts; d = uncertainty due to error terms, coefficient estimates, exogenous-variable forecasts, and the possible misspecification of the model; i = the total estimated variance was negative.

[b] 250 trials for each stochastic simulation.

[c] Errors are in percentage points except for inventory investment, where the errors are in billions of 1972 dollars at an annual rate. Errors for all variables except the unemployment rate, the bill rate, and inventory investment are percents of the forecast means.

[d] The exact variables tabled for each model are the following. See Belton, Hymans, and Lown (1981) for the Michigan notation, and see Fair (1984) for the Fair notation. The variables for the autoregressive model are the same as those for the Michigan model.

	Michigan	Fair
Real GNP	GNP72	GNPR
Private nonfarm deflator	PPNF	P
Nominal GNP	GNP	GNP
Unemployment rate	RUG	UR
Bill rate	RTB	RS
Money supply	M1BPLUS	M1
Consumer expenditures, services	CS72	CS
Consumer expenditures, nondurables	CN72	CN
Consumer expenditures, durables	C72–CS72–CN72	CD
Housing investment	IRC72	IH
Nonresidential fixed investment	IBF72	IK
Inventory investment	IINV72	IV
Imports	M72	IM
Wage rate	JCMH	W
Profits	YCP	Π_f

Table 6.6. Root Mean Squared Errors of Outside-Sample Forecasts for 1975.2–1981.4 for the Three Models[a–f]

	Number of Quarters Ahead							
	1	2	3	4	5	6	7	8
Real GNP								
Michigan	0.80	1.11	1.37	1.36	1.34	1.58	2.53	3.62
Fair	0.83	1.24	1.66	2.02	2.38	2.68	2.99	3.40
AR8	1.14	1.75	2.20	2.20	2.23	2.38	2.73	3.03
Private nonfarm deflator								
Michigan	0.36	0.46	0.51	0.68	0.82	0.95	1.11	1.30
Fair	0.69	1.18	1.64	2.17	2.62	3.03	3.47	3.87
AR8	0.72	1.26	1.92	2.93	3.80	4.57	5.09	5.24
Nominal GNP								
Michigan	1.06	1.46	1.66	1.49	1.63	1.97	2.84	3.90
Fair	0.96	1.38	1.86	2.22	2.54	2.79	3.01	3.28
AR8	1.28	1.90	2.38	2.49	2.70	3.18	3.82	4.29
Unemployment rate								
Michigan	0.34	0.54	0.70	0.77	0.77	0.66	0.78	1.24
Fair	0.41	0.66	0.93	1.14	1.27	1.38	1.48	1.62
AR8	0.34	0.54	0.70	0.81	0.88	0.93	1.01	1.07
Bill rate								
Michigan	0.78	1.04	1.15	1.29	1.42	1.45	1.42	1.42
Fair	1.28	2.05	2.30	2.44	2.58	2.74	2.90	3.08
AR8	1.57	2.58	2.75	3.09	3.42	3.65	3.87	4.04
Money supply								
Michigan	1.60	2.20	2.78	3.64	4.37	5.15	6.10	6.95
Fair	1.43	1.86	2.09	2.45	2.63	2.85	3.10	3.02
AR8	2.22	3.66	4.61	5.83	6.76	8.23	10.25	11.85
Consumer expenditures, services								
Michigan	0.46	0.72	0.90	1.11	1.33	1.60	1.92	2.19
Fair	0.40	0.64	0.98	1.31	1.56	1.87	2.21	2.54
AR8	0.50	0.80	1.05	1.33	1.63	1.92	2.29	2.58
Consumer expenditures, nondurables								
Michigan	0.89	1.33	1.61	1.83	2.17	2.51	2.96	3.42
Fair	0.82	1.08	1.21	1.43	1.58	1.59	1.57	1.65
AR8	0.87	1.34	1.40	1.27	1.46	1.64	1.86	2.01
Consumer expenditures, durables								
Michigan	3.58	4.04	5.10	4.89	5.60	6.79	8.31	9.69
Fair	3.32	4.14	5.67	7.04	8.52	9.67	10.97	12.47
AR8	4.53	5.84	6.81	7.22	7.78	8.55	9.74	10.41
Housing investment								
Michigan	5.65	9.05	10.30	11.18	13.48	16.61	20.41	23.62
Fair	5.39	8.57	9.69	10.65	12.41	14.33	15.70	17.12
AR8	6.88	11.91	13.26	12.42	11.35	12.10	14.03	14.43
Nonresidential fixed investment								
Michigan	2.66	4.58	6.09	6.39	5.63	7.28	14.99	32.03
Fair	2.52	3.08	3.99	4.56	5.66	6.64	7.54	8.51
AR8	2.32	2.90	4.23	4.52	4.56	4.61	4.59	4.61
Inventory investment								
Michigan	6.14	6.72	7.41	7.67	7.05	6.25	6.44	7.57
Fair	6.20	7.64	8.44	8.60	9.04	9.24	9.44	9.92
AR8	6.96	7.95	8.51	8.33	8.79	9.07	9.08	8.98

	Number of Quarters Ahead							
	1	2	3	4	5	6	7	8
Imports								
Michigan	3.83	5.05	5.75	6.33	6.86	7.58	8.99	10.63
Fair	3.89	6.02	8.15	9.96	11.76	13.27	15.46	17.28
AR8	5.33	8.06	10.33	12.55	14.10	15.26	16.43	18.42
Wage rate								
Michigan	0.29	0.48	0.67	0.83	0.94	1.11	1.27	1.49
Fair	0.57	0.89	1.24	1.58	1.92	2.24	2.59	3.00
AR8	0.44	0.78	1.10	1.44	1.88	2.27	2.71	3.21
Profits								
Michigan	5.84	7.31	9.08	9.04	8.99	9.36	16.24	27.80
Fair	7.21	8.79	9.98	10.66	11.80	12.73	13.96	14.57
AR8	9.59	16.56	23.86	28.99	32.15	35.15	38.84	41.71

[a]The results are based on 27 sets of coefficient estimates for each model.

[b]Each prediction period began two quarters after the end of the estimation period.

[c]The predicted values used were the mean values from the 27 stochastic simulations to get the $\bar{d}_{ik}$ values for each model.

[d]There are 27 observations for the one-quarter-ahead forecasts, 26 for the two-quarter-ahead forecasts, and so on.

[e]See note *c* to Table 6.5 for the units of the errors.

[f]See note *d* to Table 6.5 for the notation for the variables.

These results produced for the one-quarter-ahead forecast for each endogenous variable 27 values of the difference between the estimated forecast-error variance based on outside-sample errors (i.e., the squared forecast errors) and the estimated forecast-error variance based on stochastic simulation. The average of these 27 values was taken for each variable. In terms of the previous notation, this average is $\bar{d}_{i1}$, where the i refers to variable i, and the 1 refers to the one-quarter-ahead forecast. The total variance for the one-quarter-ahead forecast of variable i is $\tilde{\sigma}^2_{it1} + \bar{d}_{i1}$, which in terms of the notation is $\hat{\sigma}^2_{it1}$. For the results in Table 6.5, t is 1978.1, and the d-row value for 1978.1 for each variable is the square root of $\hat{\sigma}^2_{it1}$. The calculations for the two-quarter-ahead forecasts are the same except that there are only 26 values of the difference between the two estimated variances for each variable. Similarly, there are only 25 values for the three-quarter ahead forecast, and so on.

The d-row values in Table 6.5 take into account the four main sources of uncertainty, and they are the values to be compared across models. This will be done in the Discussion. Two sets of d-row values are presented in Table 6.5 for each variable. The first is for the change assumption regarding the exogenous variables, and the second is for the level assumption. The $\bar{d}_{ik}$ values are the same for both sets of results, but the c-row values (i.e., the values of $\tilde{\sigma}_{itk}$) are not.

Outside-Sample Root Mean Squared Errors

For the misspecification calculations one has for each variable 27 one-quarter-ahead outside-sample forecast errors, 26 two-quarter-ahead outside-sample forecast errors, and so on. From these individual errors, one can calculate root mean squared errors. The results of doing this are presented in Table 6.6. The RMSEs in Table 6.6 and the d-row values in Table 6.5 differ in two major respects. First, the d-row values take into account exogenous variable uncertainty, which the RMSEs do not. The outside-sample errors that are used for the RMSE results are all based on actual values of the exogenous variables. Second, the d-row values are for a particular quarter—1978.1 for the one-quarter-ahead forecast, 1978.2 for the two-quarter-ahead forecast, and so on. The RMSEs are averages across all the quarters—27 quarters for the one-quarter-ahead forecast, 26 quarters for the two-quarter forecast, and so on. The RMSEs do not take account of the fact that forecast-error variances vary across time. If the variances did not vary across time and if there were no exogenous variable uncertainty, the d-row values and the RMSEs would be the same except for stochastic-simulation error.

Although the d-row values are better than the RMSEs for comparison purposes, the RMSE results in Table 6.6 provide a rough check on the results in Table 6.5. If a particular d-row value differs substantially from the corresponding RMSE, it is of some interest to determine why this is.

Results for the Fair Model

The results for the Fair model in Table 6.5 are taken from the results in Fair (1984). For the results in Fair (1984) the d-row values were based on 51 sets of estimates of the model. For the present results only the relevant 27 sets of these estimates were used. The values in the a- and b-rows in Table 6.5 for the Fair model are exactly those in Table 8-2 in Fair (1984), although in the present case results for more variables are tabled. The values in the c-rows in the right half of Table 6.5 differ slightly from the c-row values in Table 8-2 of Fair (1984) because a different sequence of random draws was used for the present results. The differences are thus due to stochastic-simulation error. The values in the c-rows in the left half of Table 6.5 are new. The change assumption with respect to the exogenous-variable errors was not used for the work in Fair (1984). Remember that the $\bar{d}_{ik}$ values that are used for the Fair model in Table 6.5 are different from those used in Table 8-2 of Fair (1984) because they are based on 27 rather than on 51 sets of estimates.

Results for the Autoregressive Model (AR8)

The Michigan data base was used for the autoregressive model. The estimation periods are the same as those for the Michigan model.[9] The model consists of a set of eighth-order autoregressive equations with a constant term and time trend. The equations are completely separate from each other. The same steps were followed for the autoregressive model as were followed for the Michigan model except that 100 rather than 50 trials were used for each of the 27 sets of stochastic simulations. The results for the autoregressive model are also presented in Tables 6.5 and 6.6. There are no c-row values for this model because there are no exogenous variables except for the time trend.

A Digression about Stochastic-Simulation Error

Some evidence about the size of stochastic-simulation error is available from the present results. First, there are two sets of c-row values in Table 6.5, and the one-quarter-ahead values for each set should be the same for each variable aside from stochastic-simulation error. (The change versus level difference does not affect the one-quarter-ahead results.) Different random draws were used for the two sets. As can be seen in Table 6.5, the simulation errors are fairly small. Some of the larger errors for Michigan are 1.43 vs 1.19 for durable expenditures, 2.40 vs 2.29 for housing investment, and 4.26 vs 3.90 for profits. Some of the larger errors for Fair are 2.10 vs 2.36 for durable expenditures, 1.75 vs 1.95 for nonresidential fixed investment, 1.96 vs 2.11 for imports, and 6.86 vs 6.34 for profits.

Second, the values in the c-rows in the right half of Table 6.5 for the Fair model should be the same as the c-row values in Table 8-2 in Fair (1984) aside from simulation error. Both sets of results are based on 250 trials, but the random-variable draws were different. The comparisons for the eight-quarter-ahead results are 1.60 vs 1.66 for real GNP, 1.13 vs 1.15 for the GNP deflator, 0.82 vs 0.91 for the unemployment rate, 1.40 vs 1.37 for the bill rate, 2.28 vs 2.44 for the money supply, 1.94 vs 1.97 for the wage rate, and 15.00 vs 13.93 for profits.

Although simulation error is certainly not close to zero for the present results, it seems small enough so as not to affect the basic conclusions that are drawn from the results.

A Digression about Computer Work

The Fair–Parke program (1984) was used for all the computations in this chapter. Once a model is set up in the program, all the estimation and stochastic simulation that are needed for the results in Table 6.5 can be done with a few commands. The program provides an easy way to debug the setting up of the model, and once this debugging has been done, few other errors are likely to arise.

The computer work was done on an IBM 4341 at Yale. The computer time needed for the estimation of the Michigan model was trivial because the estimation technique is simply ordinary least squares. With respect to solution times, the time needed to solve the model for one quarter was about 0.9 second, although this time could be considerably lowered. The Fair–Parke program has an option for efficient coding of the subroutines that are needed to set up the model in the program. This option was not used for the Michigan model. It was used for the Fair model, and the solution time for the Fair model was about 0.2 second per quarter. It is likely that the Michigan time could be lowered to about this value with efficient coding. The total time for an eight-quarter stochastic simulation using 250 trials at 0.9 second per quarter is $250 \times 8 \times 0.9 = 1800$ seconds, or about 30 minutes. Each of the a-, b-, and c-row calculations for Table 6.5 thus took about 30 minutes for the Michigan model, since there is little to the calculations other than solving the model over and over. With efficient coding this time could be reduced to about 7 minutes, which is about the time taken for the Fair model calculations.

DISCUSSION OF THE RESULTS

Sensitivity to Exogenous-Variable Assumptions

The Michigan results in Table 6.5 are in general much more sensitive to the two assumptions about exogenous-variable uncertainty than are the Fair results. The Michigan c-row values for the change assumption, which are in the left half of the table, are in many cases much larger than the corresponding values for the level assumption, which are in the right half of the table. This is unfortunate from the point of view of the method because it makes comparisons more difficult. As previously discussed, the change assumption may be a better approximation, and we have concentrated on the change-assumption results in the following discussion. This is the worst case for the Michigan model. Michigan does best for the RMSE results in Table 6.6, which are based on the assumption of no exogenous variable uncertainty. The results in Table 6.5 for the level assumption are in between the RMSE results in Table 6.6 and the results in Table 6.5 for the change assumption.

It should be noted that the sensitivity of the Michigan results to the exogenous-variable assumptions is not due to the fact that the model is heavily tied to dummy variables. All but four of the dummy variables have been taken to be fixed for the calculations. The sensitivity instead indicates that the Michigan model is more heavily tied to nondummy exogenous variables than is the Fair model. This is probably because variables such as the discount rate and the minimum wage rate have been taken to be exogenous.

Michigan versus Fair

The top half of Table 6.7 contains for each variable and quarter the ratio of the Michigan d-row value in Table 6.5 to the corresponding Fair d-row value. (In what follows M denotes the Michigan model and F denotes the Fair model.) The following is a discussion of the results in Table 6.7.

1. In general, the shorter the lead time of the forecast, the better M is relative to F. For real GNP, for example, M is better than F for the first five quarters and worse than F for the remaining three.
2. The best variable for M is the private nonfarm deflator, where M is about three times more accurate than F. M is also more accurate than F for the wage rate, although not by as much as for the price deflator.
3. M is considerably better than F for the bill rate except for the eight-quarter-ahead forecast, where F is slightly better. F is considerably better than M for the money supply (the money supply variable is M_1).
4. With respect to the components of GNP, F is better than M for the three consumption variables, housing investment, and nonresidential fixed investment. M is better than F for inventory investment and imports. F is thus in general better than M with respect to the components of GNP. There is, however, more error cancellation for M than for F with respect to the predictions of real GNP. As noted above, M is actually better than F for the first five quarters for real

GNP. For nominal GNP F is better than M for all but the four-quarter-ahead forecast.

5. For the unemployment rate M is better than F for the first six quarters and worse for the remaining two. The same is true for profits.

Michigan and Fair versus Autoregressive

The bottom half of Table 6.7 presents the M versus autoregressive and F versus autoregressive ratios. (In what follows AR8 denotes the autoregressive model.) The d-row values in Table 6.5 for AR8 are not sensible for the unemployment rate and nonresidential fixed investment. It sometimes turns out in the successive reestimation and stochastic simulation of the model that the stochastic simulation estimates of the variances are on average much larger than the estimates based on outside-sample errors. This results in large negative values of $\bar{d}_{ik}$, and these values when added to the square of the c-row values (or b-row values in the case of AR8) can yield negative values of the total variance, which is not sensible. What this means is that the sample is not large enough to produce sensible results. This problem occurred for the unemployment rate and nonresidential fixed investment for AR8, and so these two variables have been omitted from the bottom half of Table 6.7.

The results in Table 6.7 in general show that M and F are better than AR8. The main exceptions are as follows. M is worse than AR8 for real GNP for the last four quarters, for nominal GNP for the last three quarters, for nondurable consumption for all quarters, and for durable consumption and housing investment for the last four quarters. F is worse than AR8 for real GNP for the last five quarters, for durable consumption for the last four quarters, and for inventory investment for all but the first quarter.

General Remarks

If the current results are taken at face value, they are obviously mixed. M and F are generally better than AR8, but there is no obvious winner between M and F. M is much better than F for the price deflator and the wage rate. M is also much better than F for the bill rate except for the last quarter. F is much better than M for nondurable consumption, housing investment, nonresidential fixed investment, and the money supply. For the other variables the results are closer.

When all is said and done, however, one may not want to take the current results at face value. There are at least three reasons for this. First, the results are sensitive to the assumptions about exogenous-variable uncertainty. M is more sensitive than F to the exogenous-variable assumptions. If the change assumption has overestimated exogenous-variable uncertainty, then the results are biased in favor of F. If, on the other hand, the change assumption has underestimated uncertainty, which may be true for variables like DFPR (see the discussion under Some Features of the Models), then the results are biased in favor of M.

Second, the heavy use of dummy variables in the Michigan model may have biased the results in favor of M. As previously noted, there are a number of dummy variables in the Michigan price equation, and at least part of the good showing by M for the

Table 6.7. Ratios of d-Row Values from Table 6.5

	Michigan/Fair							
	1978				1979			
	1	2	3	4	1	2	3	4
Real GNP	0.91	0.96	0.94	0.75	0.93	1.06	1.30	1.48
Private nonfarm deflator	0.53	0.40	0.34	0.35	0.33	0.34	0.34	0.36
Nominal GNP	1.03	1.12	1.02	0.93	1.03	1.23	1.64	1.81
Unemployment rate	0.87	0.90	0.87	0.85	0.85	0.86	1.03	1.28
Bill rate	0.57	0.49	0.50	0.60	0.63	0.58	0.63	1.03
Money supply	1.07	1.13	1.42	1.85	2.05	2.62	3.01	5.22
Consumer expenditures, services	1.34	1.36	1.08	1.02	0.91	1.08	1.10	1.13
Consumer expenditures, nondurables	1.20	1.48	1.86	1.99	2.42	3.00	3.70	4.22
Consumer expenditures, durables	1.16	1.15	1.14	0.97	0.99	1.11	1.19	1.23
Housing investment	1.15	1.22	1.41	1.52	1.18	1.10	1.54	1.52
Nonresidential fixed investment	0.97	1.39	1.43	1.32	0.94	1.05	1.92	3.65
Inventory investment	0.96	0.87	0.85	0.87	0.81	0.80	0.85	0.92
Imports	0.95	0.86	0.75	0.70	0.69	0.73	0.77	0.89
Wage rate	0.74	0.91	0.89	0.91	0.74	0.74	0.66	0.66
Profits	0.71	0.67	0.75	0.68	0.72	0.83	1.07	1.64

	Michigan/Autoregressive								Fair/Autoregressive							
	1978				1979				1978				1979			
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Real GNP	0.73	0.72	0.74	0.89	1.14	1.39	1.58	1.83	0.80	0.75	0.79	1.19	1.23	1.30	1.21	1.24
Private nonfarm deflator	0.51	0.39	0.31	0.28	0.25	0.26	0.27	0.33	0.97	0.97	0.90	0.79	0.76	0.75	0.80	0.94
Nominal GNP	0.86	0.83	0.78	0.79	0.93	1.03	1.24	1.24	0.84	0.74	0.77	0.85	0.91	0.84	0.76	0.68
Bill rate	0.51	0.42	0.44	0.49	0.50	0.46	0.50	0.83	0.90	0.85	0.88	0.82	0.79	0.79	0.79	0.80
Money supply	0.76	0.61	0.66	0.72	0.77	0.77	0.74	0.78	0.71	0.54	0.46	0.39	0.38	0.30	0.25	0.15
Consumer expenditures, services	0.92	0.94	0.92	0.97	0.98	0.99	1.01	1.04	0.69	0.69	0.86	0.95	1.06	0.92	0.92	0.92
Consumer expenditures, nondurables	1.11	1.17	1.53	2.53	2.95	3.18	3.46	3.88	0.93	0.79	0.82	1.27	1.22	1.06	0.94	0.92
Consumer expenditures, durables	0.83	0.76	0.89	0.96	1.18	1.31	1.41	1.51	0.72	0.66	0.78	0.99	1.19	1.18	1.18	1.23
Housing investment	0.84	0.77	0.79	0.95	1.32	1.56	1.58	1.76	0.74	0.63	0.56	0.62	0.93	1.09	1.03	1.16
Inventory investment	0.92	0.92	0.98	1.08	1.01	0.95	1.04	1.12	0.96	1.06	1.15	1.25	1.24	1.19	1.22	1.21
Imports	0.73	0.65	0.58	0.54	0.54	0.59	0.68	0.76	0.77	0.76	0.78	0.77	0.79	0.81	0.88	0.86
Wage rate	0.65	0.56	0.56	0.53	0.44	0.43	0.42	0.43	0.88	0.61	0.63	0.58	0.59	0.58	0.63	0.64
Profits	0.62	0.40	0.36	0.29	0.31	0.37	0.51	0.72	0.88	0.60	0.48	0.43	0.43	0.45	0.47	0.44

price deflator may be due to this. The same problem may also exist for the unemployment rate, whose equation is heavily tied to the use of a dummy variable.

Third, the misspecification estimates are based on only 27 observations, which is a fairly small sample. More observations are clearly needed before any strong conclusions can be drawn.

NOTES

1. This problem pertains, of course, only to the dummy variables that change values over the simulation periods. The first simulation period in the work below began in 1975.2. Some of the dummy variables in Table 6.2 pertain only to the period before this.
2. Note that if our argument here is correct, many of the policy implications of the Michigan model are suspect. If the discount rate is treated as exogenous for purposes of policy experiments, the interest rate responsiveness to the policy change is likely to be underestimated.
3. We are indebted to Edwin Kuh and Steve Schwartz for providing us with a tape of the data. We are also indebted to Joan Crary for answering a number of questions about the model. These individuals are not accountable for the results in this chapter. We assume responsibility for all errors.
4. The 345 coefficients include serial correlation coefficients. These coefficients were treated as structural coefficients, and so the covariance matrix of the coefficient estimates includes them.
5. It sometimes happens that a particular draw fails to result in a solution of the model. In this case the trial is discarded. There were no failures for the a-row simulation. There was one failure for the b-row simulation, and so the number of trials for this simulation was 249 rather than 250.
6. As indicated in note *c* to Table 6.5, most of the errors are in units of percentage of the forecast mean. See the discussion in Chapter 8 in Fair (1984) for the exact way in which the percentage errors are computed.
7. There were no failures of the model to solve for the c-row calculations.
8. Of the $27 \times 50 = 1350$ trials, 5 failed to result in a solution of the model.
9. Five of the variables for which autoregressive equations were estimated are determined by identities in the Michigan model—real GNP, the GNP deflator, nominal GNP, consumer durable expenditures, and nonresidential fixed investment. The estimation period used for these variables is 1956.1–1979.4.

REFERENCES

Belton, Terrence, Saul H. Hymans, and Cara Lown (1981). "The Dynamics of the Michigan Quarterly Econometric Model of the U.S. Economy." Discussion Paper R-108.81, Department of Economics, University of Michigan, December.

Fair, Ray C. (1980). "Estimating the Expected Predictive Accuracy of Econometric Models." *International Economic Review* 21, 355–378.

Fair, Ray C. (1984). *Specification, Estimation, and Analysis of Macroeconometric Models.* Cambridge: Harvard University Press.

Table A6.1. Exogenous Variables of the Michigan Model for Which Autoregressive Equations Were Estimated[a]

Variable	Estimation Period	Variable	Estimation Period
AUTOSIZE	1956.1–1979.4	PFP	1956.1–1979.4
BTRP	1956.1–1979.4	PGAS	1959.1–1979.4
DFPR	1967.1–1979.4	PIINV	1956.1–1979.4
DTEX	1956.1–1979.4	PM	1956.1–1979.4
DTIB	1956.1–1979.4	PX	1956.1–1979.4
DTP	1956.1–1979.4	RDIS	1956.1–1979.4
DTPR	1956.2–1979.4	RRDEM	1956.1–1979.4
EGOV	1956.1–1979.4	SDR	1972.1–1979.4
GAID	1956.1–1979.4	SLCSF	1956.1–1979.4
GFD	1956.1–1979.4	TCFR	1956.1–1979.4
GFO	1956.1–1979.4	TCO	1956.1–1979.4
GOLD	1956.1–1979.4	TDEPRAG	1956.1–1979.4
GSL	1956.1–1979.4	TDEPRNC	1956.1–1979.4
GTRF	1956.1–1979.4	TDEPRO	1956.1–1979.4
GTROF	1956.1–1979.4	TDEPRQ	1956.1–1979.4
GTRSL	1956.1–1979.4	TITCR	1956.1–1979.4
IVA	1956.1–1979.4	TSIFR	1956.1–1979.4
JGPM	1956.1–1979.4	TSISL	1956.1–1979.4
JICS	1956.1–1979.4	WCEIL	1956.1–1979.4
KCAC	1956.1–1979.4	WUSMIN	1956.1–1979.4
KCCA	1956.1–1979.4	X72	1956.1–1979.4
MBASE	1956.1–1979.4	YGWS	1956.1–1979.4
PAUTO	1956.1–1979.4	YPINT	1956.1–1979.4
PCRUDE	1956.1–1979.4	YPRENT	1956.1–1979.4

[a]See Belton, Hymans, and Lown (1981) for a description of the variables. See Fair (1984 p. 285) for a discussion of the exogenous variables of the Fair model for which autoregressive equations were estimated.