# TESTING THE NAIRU MODEL FOR THE UNITED STATES

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*Abstract*—This paper tests, using U.S. data, the dynamics implied by the NAIRU view of the relationship between inflation and the unemployment rate. The results are somewhat sensitive to the measure of inflation used, but they generally reject the dynamics. An alternative way of thinking about the relationship between inflation and the unemployment rate is suggested.

## I. The NAIRU Model

The main purpose of this paper is to test the dynamics implied by the NAIRU view of the relationship between inflation and the unemployment rate. This view is that there is a value of the unemployment rate (the NAIRU) below which the price level forever accelerates and above which the price level forever decelerates. The simplest version of the NAIRU equation is

$$\pi_t - \pi_{t-1} = \beta(u_t - u^*) + \gamma s_t + \epsilon_t, \qquad \beta < 0, \quad \gamma > 0, \tag{1}$$

where $t$ is the time period, $\pi_t$ is the rate of inflation, $u_t$ is the unemployment rate, $s_t$ is a cost shock variable, $\epsilon_t$ is an error term, and $u^*$ is the NAIRU. If $u_t$ equals $u^*$ for all $t$, the rate of inflation will not change over time aside from the short-run effects of $s_t$ and $\epsilon_t$ (assuming $s_t$ and $\epsilon_t$ have zero means). Otherwise, the rate of inflation will increase over time (the price level will accelerate) if $u_t$ is less than $u^*$ for all $t$ and will decrease over time (the price level will decelerate) if $u_t$ is greater than $u^*$ for all $t$.

A more general version of the NAIRU specification is

$$\pi_t = \alpha + \sum_{i=1}^{n} \delta_i \pi_{t-i} + \sum_{i=0}^{m} \beta_i u_{t-i} + \sum_{i=0}^{q} \gamma_i s_{t-1} + \epsilon_t, \qquad \sum_{i=1}^{n} \delta_i = 1. \tag{2}$$

For this specification the NAIRU is $-\alpha/\Sigma_{i=0}^{m}\beta_i$. If the unemployment rate is always equal to this value, the inflation rate will be constant in the long run aside from the short-run effects of $s_t$ and $\epsilon_t$.

A key restriction in equation (2) is that the $\delta_i$ coefficients sum to 1 (or in equation (1) that the coefficient of $\pi_{t-1}$ is 1). This restriction is used in much of the recent literature. See, for example, the equations in Akerlof et al. (1996, p. 38), Fuhrer (1995, p. 46), Gordon (1997, p. 14), Layard et al. (1991, p. 379), and Staiger et al. (1997, p. 35). The specification has even entered the macro textbook literature; see, for example, Mankiw (1994, p. 305). Also, there seems to be considerable support for the NAIRU view in the policy literature. For example, Krugman (1996, p. 37) in an article in the *New York Times Magazine* writes "the theory of the Nairu has been highly successful in tracking inflation over the last 20 years. Alan Blinder, the departing vice chairman of the Fed, has described this as the 'clean little secret of macroeconomics'."

The results in this paper, on the other hand, suggest that equations like (2) are in general *not* good approximations of the actual dynamics of the inflation process. The basic test that is performed is the following. Let $p_t$ be the log of the price level for period $t$, and let $\pi_t$ be measured as $p_t - p_{t-1}$. Using this notation, equation (1) and (2) can be written in terms of $p$ rather than $\pi$. Equation (1), for example, becomes

$$p_t = 2p_{t-1} - p_{t-2} + \beta(u_t - u^*) + \gamma s_t + \epsilon_t. \tag{3}$$

In other words, equation (1) can be written in terms of the current and past two price levels,[1] with restrictions on the coefficients of the past two price levels. Similarly, if in equation (2) $n$ is, say, 4, the equation can be written in terms of the current and past five price levels, with two restrictions on the coefficients of the five past price levels. (Denoting the coefficients on the past five price levels as $a_1$ through $a_5$, the two restrictions are $a_4 = 5 - 4a_1 - 3a_2 - 2a_3$ and $a_5 = -4 + 3a_1 + 2a_2 + a_3$.) The main test in this paper is of these two restrictions. The restrictions are easy to test by simply adding $p_{t-1}$ and $p_{t-2}$ to the NAIRU equation and testing whether they are jointly significant.

An equivalent test is to add $\pi_{t-1}$ (that is, $p_{t-1} - p_{t-2}$) and $p_t - 1$ to equation (2). Adding $\pi_{t-1}$ breaks the restriction that the $\delta_i$ coefficients sum to 1, and adding both $\pi_{t-1}$ and $p_{t-1}$ breaks the summation restriction and the restriction that each price level is subtracted from the previous price level before entering the equation. This latter restriction can be thought of as a first-derivative restriction, and the summation restriction can be thought of as a second-derivative restriction.

Under what theory would $p_{t-1}$ and $p_{t-2}$ (or $\pi_{t-1}$ and $p_{t-1}$) be added to equation (2)? One such theory has been used to guide the specification of the price and wage equations in my macroeconometric model. This theory was first presented in Fair (1974), and more recent discussions are in Fair (1984, chapter 3; 1994, chapter 2). It is briefly outlined in the appendix, along with another, simpler theory. There may, however, be other theories than the two in the appendix that lead to $p_{t-1}$ and $p_{t-2}$ being added to the NAIRU equation. The main aim of this paper is to test the dynamics of the

Received for publication June 9, 1997. Revision accepted for publication February 18, 1999.

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I would like to thank James Stock for making available a program to compute the DF-GLS statistic and Skander Van den Heuvel for research assistance. All the data used in this paper can be downloaded from http://fairmodel.econ.yale.edu.

[1] "Price level" will be used describe $p$ even though $p$ is actually the log of the price level.

NAIRU specification, and not to argue strongly in favor of one theory over another.

## II. The Data and Unit Root Tests

Many estimates of equations like (2) use the GDP deflator as the measure of the price level. Other popular measures are the consumer price index (CPI) and the personal consumption deflator (PCD). Gordon (1997), for example, uses all three. If, however, the aim is to measure prices set by U.S. firms, none of these measures seems very good. The GDP deflator includes prices of government output and indirect business taxes, for example, which are clearly not decision variables of firms. The CPI and PCD are to some extent even worse, because they include import prices in addition to indirect business taxes.

The main price variable used in this paper is a business nonfarm price deflator, denoted *PNF*. Let *YY* be nominal business nonfarm output (NIPA table 1.7, line 3), let *IBT* be total indirect business taxes (NIPA table 3.1, line 4), and let *Y* be business nonfarm output in 1992 dollars (NIPA table 1.8, line 3). Then *PNF* is defined to be $(YY - IBT)/Y$. *PNF* is net of indirect business taxes, farm output, government output, and imports.

The civilian unemployment rate is used for the unemployment rate. The import price deflator—the ratio of nominal imports (NIPA table 1.1, line 17) to imports in 1992 dollars (NIPA table 1.2, line 17)—is used as the cost shock variable.

In what follows $p$ will denote the log of *PNF*; $u$ will denote the unemployment rate; and $pm$ will denote the log of the import price deflator. The data are quarterly and were collected for the 1952:1–1998:1 period.

Using a variety of tests, $p$, $u$, and $pm$ were all tested for unit roots, and the null hypothesis of a unit root was not rejected for any of them. In addition, the null hypothesis of a unit root was not rejected for $\Delta p$. For an example, consider $u$ and the augmented Dickey-Fuller (ADF) test using a constant, time trend, and four lags. The equation estimated for the test is

$$\Delta u_t = \alpha_1 + \alpha_2 t + \alpha_3 \Delta u_{t-1} + \alpha_4 \Delta u_{t-2} + \alpha_5 \Delta u_{t-3} + \alpha_6 \Delta u_{t-4} + \alpha_7 u_{t-1} + \xi_t. \qquad (4)$$

The null hypothesis of a unit root is the hypothesis that $\alpha_7 = 0$. This equation was estimated for the 1955:3–1998:1 period (the same period used for the tests in the next section), and the estimate of $\alpha_7$ was $-0.0416$ with a $t$-statistic of $-2.41$. The 5% and 1% critical values from MacKinnon (1991) are $-3.37$ and $-4.21$, respectively, and so the null hypothesis is not rejected.

To check whether the MacKinnon critical values are accurate, the following experiment was done

1) Equation (4) without $u_{t-1}$ included was estimated. This equation was taken as the "base" equation, and the error term was assumed to be normally distributed with mean zero and variance equal to the estimated variance.
2) Using the normality assumption and the estimated variance, a value of the error term was drawn for each quarter of the estimation period. These error terms were added to the base equation, and the equation was solved dynamically to generate new data for $u$. Given the new data for $u$, equation (4) was estimated, and the $t$-statistic for the estimate of $\alpha_7$ was recorded.
3) The procedure in step 2 was done 1,000 times, producing 1,000 $t$-statistics.
4) The $t$-statistics were sorted by size. Five percent of the $t$-statistics were below $-3.37$ and one percent were below $-4.21$. These two values compare closely to the MacKinnon values of $-3.44$ and $-4.01$, and the same conclusion is reached using these values as was reached using the MacKinnon values, namely that the unit root hypothesis is not rejected.

TABLE 1.—UNIT ROOT TESTS

$\Delta y_t = \alpha_1 + \alpha_2 t + \alpha_3 \Delta y_{t-1} + \alpha_4 \Delta y_{t-2} + \alpha_5 \Delta y_{t-3} + \alpha_6 \Delta y_{t-4} + \alpha_7 y_{t-1} + \xi_t$

| | | | Computed Critical | | Power Using Critical | |
|---|---|---|---|---|---|---|
| $y$ | $\hat{\alpha}_7$ | $t$-Stat. | 5% | 1% | 5% | 1% |
| $u$ | −.0416 | −2.41 | −3.37 | −4.21 | .306 | .051 |
| $p$ | −.0049 | −1.94 | −3.53 | −4.11 | .142 | .039 |
| $\Delta p$ | −.1045 | −2.34 | −3.42 | −4.13 | .250 | .048 |
| $pm$ | −.0066 | −1.19 | −3.41 | −4.13 | .086 | .016 |
| $\Delta pm$ | −.2787 | −4.13 | −3.41 | −4.03 | .962 | .755 |

Estimation period: 1955:3–1998:1.

A similar experiment was then performed to estimate the power of the test. In this case equation (4) *with* $u_{t-1}$ *included* was estimated and taken to be the base equation. (The truth is now that $u$ does not have a unit root.) Again, the error term was assumed to be normally distributed with mean zero and variance equal to the estimated variance. For each trial, a value of the error term was drawn for each quarter, and these error terms were added to the base equation. The equation was solved dynamically to generate new data for $u$. Given the new data for $u$, equation (4) was estimated, and the $t$-statistic for the estimate of $\alpha_7$ was recorded. This was done 1,000 times, and the 1,000 $t$-statistics were sorted by size. Of these t-statistics, 30.6% were below $-3.44$, the above-estimated 5% critical value, and 5.1% were below $-4.01$, the above-estimated 1% critical value. The hypothesis that $\alpha_7$ equals zero was thus rejected only 30.6 (or 5.1%) of the time, depending on the critical value used, even though it is in fact false. The power of the test is thus quite low.

This same overall procedure was followed for $p$, $\Delta p$, $pm$, and $\Delta pm$. The results are shown in table 1, including the results for $u$. Table 1 shows that, except for $\Delta pm$, the hypothesis of a unit root for the variable is not rejected, and the power of the test is low. All the computed critical values are close to the MacKinnon values.

The five variables in table 1 were also tested for a unit root using the DF-GLS test in Elliott et al. (1996). The DF-GLS values for $u, p, \Delta p, pm,$ and $\Delta pm$ were $-2.51, -1.67, -2.17, -1.30,$ and $-3.98$, respectively. The 5% and 1% critical values are $-2.96$ and $-3.49$, respectively.[2] The hypothesis of a unit root is thus not rejected except for $\Delta pm$, the same conclusion that was reached using the ADF test. No attempt was made to estimate the power of this test, although the results from Elliott et al. suggest that the power is likely to be higher than that for the ADF test.

It is not clear what to make of the unit root results, given the lower power of at least the ADF test. Since it may be that some of the variables have unit roots, it is probably unwise to rely on standard asymptotic distributions in hypothesis testing using these variables. Fortunately, it is straightforward to estimate "exact" distributions for the tests in the next section, and this has been done. The procedure for doing this is similar to what has just been done for the unit root tests.

## III. The Test Results

The specification of equation (2) that was used for the tests is

$$\pi_t = \alpha_0 + \sum_{i=1}^{12} \delta_i \pi_{t-i} + \sum_{i=1}^{3} \beta_i u_{t-i} + \sum_{i=1}^{3} \gamma_i s_{t-i} + \epsilon_t, \qquad \sum_{i=1}^{12} \delta_i = 1. \tag{5}$$

where $s_t$ is postulated to be $pm_t - \tau_0 - \tau_1 t$, the deviation of $pm$ from a trend line. A fairly general specification was chosen to lessen the chances of the results being due to a particular choice of lags. The lag length on the past inflation rates was taken to be twelve, and three lags each of the unemployment rate and cost shock variable were used, with each of the two variables taken to begin in period $t - 1$.

Equation (5) was estimated in the following form:

$$\Delta\pi_t = \lambda_0 + \lambda_1 t + \sum_{i=1}^{11} \theta_i \Delta\pi_{t-i} + \sum_{i=1}^{3} \beta_i u_{t-i} + \sum_{i=1}^{3} \gamma_i pm_{t-i} + \epsilon_t, \tag{6}$$

where $\lambda_0 = \alpha_0 + (\gamma_1 + \gamma_2 + \gamma_3)\tau_0 + (\gamma_1 + 2\gamma_2 + 3\gamma_3)\tau_1$ and $\lambda_1 = (\gamma_1 + \gamma_2 + \gamma_3)\tau_1$. $\alpha_0$ and $\tau_0$ are not identified in equation (6), but, for purposes of the tests, this does not matter. If, however, one wanted to compute the NAIRU (that is, $-\alpha_0/\Sigma_{i=1}^{3}\beta_i$), one would need a separate estimate of $\tau_0$ in order to estimate $\alpha_0$.[3]

For reference, it will be useful to write equation (6) with $\pi_{t-1}$ and $p_{t-1}$ added:

$$\Delta\pi_t = \lambda_0 + \lambda_1 t + \sum_{i=1}^{11} \theta_i \Delta\pi_{t-i} + \sum_{i=1}^{3} \beta_i u_{t-i} + \sum_{i=1}^{3} \gamma_i pm_{t-i} + \phi_1 \pi_{t-1} + \phi_2 p_{t-1} + \epsilon_t. \tag{7}$$

TABLE 2.—ESTIMATES OF EQUATION (6) AND (7)

| | Equation (6) | | Equation (7) | |
|---|---|---|---|---|
| Variable | Estimate | $t$-Stat. | Estimate | $t$-Stat. |
| cnst | .0052 | 1.34 | −.0176 | −2.69 |
| $t$ | −.000002 | −0.12 | .000155 | 3.94 |
| $u_{t-1}$ | −.274 | −3.04 | −.248 | −2.96 |
| $u_{t-2}$ | .262 | 1.67 | .277 | 1.91 |
| $u_{t-3}$ | −.079 | −0.85 | −.162 | −1.87 |
| $pm_{t-1}$ | .061 | 4.14 | .072 | 5.11 |
| $pm_{t-2}$ | −.043 | −1.56 | −.042 | −1.68 |
| $pm_{t-3}$ | −.018 | −1.14 | −.004 | −0.23 |
| $\Delta\pi_{t-1}$ | −.752 | −9.85 | −.280 | −2.51 |
| $\Delta\pi_{t-2}$ | −.554 | −5.97 | −.204 | −1.90 |
| $\Delta\pi_{t-3}$ | −.326 | −3.30 | −.072 | −0.71 |
| $\Delta\pi_{t-4}$ | −.282 | −2.99 | −.084 | −0.88 |
| $\Delta\pi_{t-5}$ | −.237 | −2.66 | −.086 | −0.97 |
| $\Delta\pi_{t-6}$ | −.297 | −3.60 | −.174 | −2.13 |
| $\Delta\pi_{t-7}$ | −.208 | −2.56 | −.108 | −1.35 |
| $\Delta\pi_{t-8}$ | −.213 | −2.70 | −.115 | −1.49 |
| $\Delta\pi_{t-9}$ | −.175 | −2.23 | −.091 | −1.18 |
| $\Delta\pi_{t-10}$ | −.226 | −2.98 | −.156 | −2.14 |
| $\Delta\pi_{t-11}$ | −.079 | −1.17 | −.041 | −0.65 |
| $\pi_{t-1}$ | | | −.593 | −5.45 |
| $p_{t-1}$ | | | −.041 | −4.82 |
| SE | 0.00296 | | 0.00273 | |
| $\chi^2$ | | | 26.28 | |

Estimation period: 1955:3–1998:1.
Note: When $p_{t-1}$ and $p_{t-2}$ are added in place of $\pi_{t-1}$ and $p_{t-1}$, the respective coefficient estimates are −0.634 and 0.593 with $t$-statistics of −5.47 and 5.45. All else is the same.

### A. $\chi^2$ Tests Using PNF

The estimation period for the tests was 1955:3–1998:1. The results of estimating equation (6) and (7) are presented in table 2. With $\pi_{t-1}$ and $p_{t-1}$ added, the standard error of the equation falls from 0.00296 to 0.00273. The $t$-statistics for the two variables are $-5.45$ and $-4.82$, respectively, and the $\chi^2$ value for the hypothesis that the coefficients of both variables are zero is 26.28.[4]

[2] Interpolated for 171 observations from table 1, part C, in Elliott et al. (1996).

[3] The present specification assumes that the NAIRU is constant, although, if the NAIRU had a trend, this would be absorbed in the estimate of the coefficient of the time trend in equation (6) and would change the interpretation of $\lambda_1$. In recent work, Gordon (1997) has argued that the NAIRU may be time varying, and in future work it may be interesting to consider this case as well as other cases in which the NAIRU is postulated to change over time.

[4] Note that there is a large change in the estimate of the coefficient of the time trend when $\pi_{t-1}$ and $p_{t-1}$ are added. This is to be expected from the discussion in the appendix. To some extent, the time trend is serving a similar role in equation (7) as the constant term is in equation (6).

The 5% critical $\chi^2$ value for two degrees of freedom is 5.99 and the 1% critical value is 9.21. If the $\chi^2$ distribution is a good approximation to the actual distribution, the two variables are highly significant and thus the NAIRU dynamics strongly rejected. If, however, equation (6) is in fact the way the price data are generated, the $\chi^2$ distribution may not be a good approximation for the test.[5] To check this, the "exact" distribution was computed using a procedure similar to that in the previous section.

The procedure in the present case is as follows. First, estimate equation (6) and record the coefficient estimates and the estimated variance of the error term. Call this the "base" equation. Assume that the error term is normally distributed with mean zero and variance equal to the estimated variance. Then:

1. Draw a value of the error term for each quarter. Add these error terms to the base equation and solve it dynamically to generate new data for $p$. Given the new data for $p$ and the data for $u$ and $pm$ (which have not changed), compute the $\chi^2$ value as in table 2. Record this value.
2. Perform step 1 1,000 times to produce 1,000 $\chi^2$ values. The distribution of these values is the "exact" distribution.
3. Sort the $\chi^2$ values by size, and choose the value above which 5% of the values lie and the value above which 1% of the values lie. These are the 5% and 1% critical values, respectively.

These calculations were done, and the 5% critical value was 18.10 and the 1% critical value was 24.91. These values are considerably larger than the critical values from the actual $\chi^2$ distribution (which is as expected if equation (6) is the actual data-generating process), but they are still smaller than the computed value of 26.28. The two price variables are thus significant at the 99%-confidence level even using the alternative values.

The above procedure treats $u$ and $pm$ as exogenous, and it may be that the estimated critical values are sensitive to this treatment. To check for this, the following two equations were postulated for $u$ and $pm$:

$$pm_t = a_1 + a_2 t + a_3 pm_{t-1} + a_4 pm_{t-2} + a_5 pm_{t-3} + a_6 pm_{t-4} + \nu_t, \tag{8}$$

and

$$u_t = b_1 + b_2 t + b_3 u_{t-1} + b_4 u_{t-2} + b_5 u_{t-3} + b_6 u_{t-4} + b_7 pm_{t-1} + b_8 pm_{t-2} + b_9 pm_{t-3} + b_{10} pm_{t-4} + \eta_t. \tag{9}$$

[5] If the $\chi^2$ distribution is not a good approximation, then the $t$-distribution will not be either, and so the $t$-statistics in table 2 will not be reliable. The following analysis focuses on correcting the $\chi^2$ critical values, and no use of the $t$-statistics is made.

Table 3.—Results for Equation (7) $\chi^2$ Test that $\phi_1 = 0$ and $\phi_2 = 0$

| | 1 PNF n = 12 | 2 PNF n = 24 | 3 GDPD n = 12 | 4 CPI n = 12 | 5 CPIC n = 12 |
|---|---|---|---|---|---|
| $\pi_{t-1}$ | −0.593 | −0.615 | −0.317 | −0.340 | −0.388 |
| | (−5.45) | (−4.97) | (−4.23) | (−3.37) | (−4.28) |
| $p_{t-1}$ | −0.041 | −0.045 | −0.023 | −0.022 | −0.016 |
| | (−4.82) | (−4.80) | (−3.54) | (−2.89) | (−2.66) |
| $\chi^2$ | 26.28 | 21.94 | 16.06 | 10.74 | 16.32 |
| $\chi^2_5$ | 15.01 | 12.41 | 12.48 | 12.13 | 14.05 |
| $\chi^2_1$ | 19.19 | 17.30 | 17.55 | 18.23 | 18.86 |

PNF = business nonfarm price deflator.
GDPD = GDP deflator.
CPI = consumer price index.
CPIC = core CPI.
$t$-statistics are in parentheses.
$\chi^2_5$ is the computed 5% critical value.
$\chi^2_1$ is the computed 1% critical value.
All the critical values were computed using 1,000 trials and equation (8) and (9) for $pm$ and $u$.

These two equations, along with equation (6), were taken to be the "model," and they were estimated along with equation (6) to get the "base" model. The error terms $\epsilon_t$, $\nu_t$, and $\eta_t$ were then assumed to be multivariate normal with mean zero and covariance matrix equal to the estimated covariance matrix (obtained from the estimated error terms). Each trial then consisted of draws of the three error terms for each quarter and a dynamic simulation of the model to generate new data for $p$, $pm$, and $u$, from which the $\chi^2$ value was computed. The computed critical values were not very sensitive to this treatment of $pm$ and $u$, and they actually fell slightly. The 5% value was 15.01 compared to 18.10 above, and the 1% value was 19.19 compared to 24.91 above. These results are summarized in column 1 in table 3.

To examine the sensitivity of the results to the use of twelve lags in equation (5), the test was run using 24 lags rather than twelve. For this test, the estimation period began in 1958:3 rather than 1955:3. As shown in column 2 in table 3, the $\chi^2$ value was 21.94 with computed 5% and 1% critical values of 12.41 and 17.30. (All the critical values reported in this rest of this section were computed using 1,000 trials and equation (8) and (9) for $pm$ and $u$.) The results are thus not sensitive to the use of more lags.

### B. $\chi^2$ Tests Using Other Price Measures

The results are somewhat sensitive to the use of other price measures. When the GDP deflator was used (with twelve lags), column 3 in table 3 shows that the $\chi^2$ value was 16.06 with computed 5% and 1% critical values of 12.48 and 17.55. In this case, the two price variables are significant at the 95% confidence level but not the 99% level. When the overall CPI was used, column 4 in table 3 shows that the $\chi^2$ value was 10.74 with computed 5% and 1% critical values of 12.13 and 18.23. In this case, the two price variables are not significant at even the 95% level. Finally, when the CPI excluding food and energy (the "core" CPI) was used, column 5 in table 3 shows that the $\chi^2$ value was 16.32 with computed 5% and 1% critical values of 14.05 and 18.86. In this case, as in the case using the GDP deflator, the two price variables are significant at the 95% but not the 99% level.

The choice of the price measure is thus somewhat important for purposes of the test. As argued in section II, the business nonfarm price deflator has the advantage over the GDP deflator of not including prices of government output and indirect business taxes. It has the advantage over the CPI of not including import prices and indirect business taxes. It thus seems that much less weight should be put on the results using the GDP deflator and the CPI, but, even for these measures, the two price variables are significant at the 95% level except for the overall CPI.[6]

### C. Recursive RMSE Tests

An alternative way to examine equation (6) and (7) is to consider how well they predict outside sample. To do this, the following root mean squared error (RMSE) test was performed. Each equation was first estimated for the period ending in 1969:4 (all estimation periods begin in 1955:3), and a dynamic eight-quarter-ahead prediction was made beginning in 1970:1. The equation was then estimated through 1970:1, and a dynamic eight-quarter-ahead prediction was made beginning in 1970:2. This process was repeated through the estimation period ending in 1997:4. Since observations were available through 1998:1, this procedure generated 113 one-quarter-ahead predictions, 112 two-quarter-ahead predictions, through 106 eight-quarter-ahead predictions, where all the predictions are outside sample. RMSEs were computed using these predictions and the actual values.

The actual values of $u$ and $pm$ were used for all these predictions. The aim here is not to generate predictions that could have in principle been made in real time, but to see how good the dynamic predictions from each equation are conditional on the actual values of $u$ and $pm$.

The RMSEs are presented in table 4 for the one-, four-, and eight-quarter-ahead predictions for $p$ and $\Delta p$. (Ignore for now the third set of results.) The one-quarter-ahead prediction accuracy is about the same for the two equations (0.42 versus 0.40 percentage points), but by four-quarters-ahead equation (7) is noticeably more accurate than equation (6). For the eight-quarter-ahead predictions, the RMSEs are 4.13 versus 2.94 for $p$ and 0.84 versus 0.57 for $\Delta p$, which are sizable differences.

It is thus the case that the addition of $\pi_{t-1}$ and $p_{t-1}$ has considerably increased the accuracy of the predictions, and so these variables are not only statistically significant but also economically important in a predictive sense.

TABLE 4.—RECURSIVE RMSE RESULTS

| | $p$ | | | $\Delta p$ | |
|---|---|---|---|---|---|
| | Quarters Ahead | | | | |
| | 1 | 4 | 8 | 4 | 8 |
| Eq. (6) | 0.42 | 1.58 | 4.13 | 0.59 | 0.84 |
| Eq. (7) | 0.40 | 1.38 | 2.94 | 0.50 | 0.57 |
| Eq. (10), (11) | 0.36 | 1.13 | 2.16 | 0.41 | 0.42 |

Prediction period: 1970:1–1998:1.
Errors are in percentage points.

## IV. A Simple Structural Model

It turns out that a simple structural model of price and wage determination leads to considerably lower RMSEs than even those for equation (7). The model is

$$p_t = \beta_0 + \beta_1 p_{t-1} + \beta_2 w_t + \beta_3 pm_{t-1} + \beta_4 u_{t-1} + \beta_5 t + \epsilon_t \tag{10}$$

$$w_t = \gamma_0 + \gamma_1 w_{t-1} + \gamma_2 p_t + \gamma_3 p_{t-1} + \gamma_4 u_{t-1} + \gamma_5 t + u_t, \tag{11}$$

where the new variable is $w$, the log of the nominal wage rate.[7] These two equations are identified in that $w_{t-1}$ is excluded from equation (10) and $pm_{t-1}$ is excluded from equation (11). The motivation for these equations is presented in the appendix.[8]

Equation (10) and (11) were estimated by two-stage least squares (2SLS) using as first-stage regressors the constant, $t$, $p_{t-1}$, $w_{t-1}$, $u_{t-1}$, and $pm_{t-1}$. Equation (10) was estimated as is, and the estimates of $\beta_1$ and $\beta_2$ were used to impose a coefficient constraint on equation (11) before it was estimated. This constraint is to ensure that the implied real wage equation from (10) and (11) does not have unreasonable

[6] This general rejection of the NAIRU dynamics may help explain two results in the literature. Staiger et al. (1997), using a standard NAIRU specification, estimate variances of NAIRU estimates and find them to be very large. This is not surprising if the NAIRU specification is misspecified, as the present results suggest. Similarly, Eisner (1997) finds the results of estimating NAIRU equations to be sensitive to various assumptions, particularly assumptions about whether the behavior of inflation is symmetric for unemployment rates above and below the assumed NAIRU. Again, this sensitivity is not surprising if the basic equations used are misspecified. It is clear from Eisner's paper that, although he is working with NAIRU equations, he does not like the concept of the NAIRU. The present results suggest that his doubts are well founded.

[7] The nominal wage rate used is variable *WF* in Fair (1994). It is a total compensation measure, including fringe benefits. It is available from the website mentioned in the introductory footnote.

[8] Equation (10) and (11) are simpler than the price and wage equations in my U.S. macroeconometric model. See Fair (1994) or the website for these equations. The main characteristics of equation (10) and (11) that carry over to the equations in the model are that the cost shock variable appears only in the price equation and that the current and lagged price levels appear in the wage equation, but only the current wage appears in the price equation. In addition, as will be seen in table 5, the demand-pressure variable (in the present case, the unemployment rate) has much more explanatory power in the price equation than in the wage equation, and the estimate of the coefficient of the wage variable in the price equation is fairly small. According to these equations, when there are cost shocks or changes in demand pressure, price changes primarily lead wage changes. For purposes of this paper, equation (10) and (11) have been made no more complicated than necessary to make the main points. In this spirit, the price measure used in this paper (PNF) is not the exact price measure used in the model. Again, PNF was made no more complicated than necessary: It can be easily computed from three variables in the NIPA tables.

long-run properties. The implied real wage equation is

$$
\begin{aligned}
w_t - p_t = \frac{1}{1-\beta_2\gamma_2}\{&(1-\beta_2)\gamma_1 w_{t-1} \\
&+ [(1-\beta_2)\gamma_3 - (1-\gamma_2)\beta_1]p_{t-1} \\
&- (1-\gamma_2)\beta_0 + (1-\beta_2)\gamma_0 \\
&+ [-(1-\gamma_2)\beta_3 + (1-\beta_2)\gamma_4]u_{t-1} \\
&- (1-\gamma_2)\beta_4 pm_{t-1} \\
&+ [-(1-\gamma_2)\beta_5 + (1-\beta_2)\gamma_5]t \\
&- (1-\gamma_2)\epsilon_t + (1-\beta_2)\mu_t\}.
\end{aligned} \tag{12}
$$

Unless the coefficient of $w_{t-1}$ equals the negative of the coefficient of $p_{t-1}$, equation (12) implies that, in the long run, the real wage depends on the level of $p$, which is not sensible. Consequently, the restriction that the two coefficients are equal in absolute value and of opposite signs is imposed in the estimation. The restriction on the structural coefficients is

$$\gamma_3 = \frac{\beta_1}{1-\beta_2}(1-\gamma_2) - \gamma_1. \tag{13}$$

Equation (11) was estimated by 2SLS using this restriction, where the values used for $\beta_1$ and $\beta_2$ were the estimated values from equation (10).

The coefficient estimates are presented in table 5. The estimation period began in 1954:1 instead of 1955:3 because fewer initial observations were needed for the lags. The unemployment rate has a large absolute $t$-value in the price equation, but a very small one in the wage equation. The coefficient estimate of $w_t$ in the price equation is fairly small, which means that the price equation is not a "wage push" equation. Instead, cost shocks and demand-pressure changes affect the price equation, and the wage equation is primarily a "price push" equation.

The same RMSE experiment was performed using equation (10) and (11) as was performed in the previous section. The equations were estimated (by 2SLS) 113 times, and 113 one-quarter-ahead predictions were generated, 112 two-quarter-ahead predictions, and so on. The results are presented in table 4. They show that the structural model is considerably more accurate in terms of outside sample prediction accuracy than even equation (7). For the eight-quarter-ahead prediction, for example, the RMSE for $p$ is 2.16 versus 2.96 and 4.15, and the RMSE for $\Delta p$ is 0.42 versus 0.57 and 0.85.

In the early 1980s, there began a movement away from the estimation of structural price and wage equations to the estimation of reduced-form price equations like equation (6).[9] The current results call into question this practice in that considerable predictive accuracy seems to be lost when this is done.

[9] See, for example, Gordon (1980) and Gordon and King (1982).

TABLE 5.—ESTIMATES OF EQUATION (10) AND (11)

$p_t = \beta_0 + \beta_1 p_{t-1} + \beta_2 w_t + \beta_3 pm_{t-1} + \beta_4 u_{t-1} + \beta_5 t + \epsilon_t$
$w_t = \gamma_0 + \gamma_1 w_{t-1} + \gamma_2 p_t + \gamma_3 p_{t-1} + \gamma_4 u_{t-1} + \gamma_5 t + \mu_t$

| | Estimate | $t$-Stat. | | Estimate | $t$-Stat. |
|---|---|---|---|---|---|
| $\beta_0$ | 0.0778 | 1.65 | $\gamma_0$ | −0.0709 | −1.60 |
| $\beta_1$ | 0.9225 | 284.47 | $\gamma_1$ | 0.9887 | 109.53 |
| $\beta_2$ | 0.0200 | 2.51 | $\gamma_2$ | 0.7513 | 8.86 |
| $\beta_3$ | 0.0403 | 13.61 | $\gamma_4$ | −0.0104 | −0.28 |
| $\beta_4$ | −0.1795 | −8.51 | $\gamma_5$ | 0.000181 | 2.61 |
| $\beta_5$ | 0.000088 | 1.01 | $\gamma_3$ | −0.7546 | [a] |
| SE | 0.00294 | | SE | 0.00817 | |

[a] Coefficient constrained.
Estimation period: 1954:1–1998:1.
Estimation method: 2SLS.
First-stage regressors: constant, $t$, $p_{t-1}$, $w_{t-1}$, $u_{t-1}$, $pm_{t-1}$.

## V. Properties

Coming back to equation (6) versus (7), it has so far been shown that the two added variables in equation (7) are statistically significant and that equation (7) predicts better outside sample. The last question considered here is how the properties of the two equations compare. If, say, the unemployment rate were permanently lowered by one percentage point, what would the two equations say the price consequences of this are?

To answer this, the following experiment was performed for each equation. A dynamic simulation was run beginning in 1998:2 using the actual values of all the variables from 1998:1 back. The values of $u$ from 1998:2 on were taken to be the actual value for 1998:1. $pm$ was assumed to grow at a 2% annual rate from 1998:2 on. Call this simulation the "base" simulation. A second dynamic simulation was then run in which the only change was that the unemployment rate was decreased permanently by one percentage point from 1998:2 on. The difference between the predicted value of $p$ from this simulation and that from the base simulation for a given quarter is the estimated effect of the change in $u$ on $p$.[10]

For comparison purposes, two other sets of results were obtained. For the first set, equation (7) was estimated with only $\pi_{t-1}$ added, which means that the summation (second-derivative) restriction is broken but not the first-derivative restriction. For this estimated equation, the $\delta_i$ coefficients summed to 0.846.[11] The above experiment was then performed for this equation.

[10] Because the equations are linear, it actually does not matter what values are used for $pm$ as long as the same values are used for both simulations. Similarly, it does not matter what values are used for $u$ as long as each value for the second simulation is one percentage point higher than the corresponding value for the base simulation.

[11] When only $\pi_{t-1}$ is added, the $\chi^2$ value is 5.21 with computed (as in table 3) 5% and 1% critical values of 9.18 and 14.11, respectively. $\pi_{t-1}$ is thus not significant at even the 5% level, even though the sum seems substantially less than 1. When $p_{t-1}$ is added to the equation with $\pi_{t-1}$ already added, the $\chi^2$ value is 20.40 with computed 5% and 1% critical values of 13.56 and 18.21, respectively. $p_{t-1}$ is thus highly significant when added to the equation with $\pi_{t-1}$ already added. Recursive RMSE results as in table 4 were also obtained for the equation with only $\pi_{t-1}$ added. The five RMSEs corresponding to those in table 4 are 0.42, 1.49, 3.20, 0.53, and 0.61. These are in-between those for equation (6) and equation (7).

TABLE 6.—EFFECTS OF A ONE PERCENTAGE POINT FALL IN $U$

| | Equation (6) | | Equation (7) | | Equation (7) $\phi_2 = 0$ | | Eq. (10), (11) | |
|---|---|---|---|---|---|---|---|---|
| Quar. | $p^{new} \div p^{base}$ | $\Delta p^{new} - \Delta p^{base}$ | $p^{new} \div p^{base}$ | $\Delta p^{new} - \Delta p^{base}$ | $p^{new} \div p^{base}$ | $\Delta p^{new} - \Delta p^{base}$ | $p^{new} \div p^{base}$ | $\Delta p^{new} - \Delta p^{base}$ |
| 1 | 1.0000 | 0.00 | 1.0000 | 0.00 | 1.0000 | 0.00 | 1.0000 | 0.00 |
| 2 | 1.0027 | 1.10 | 1.0025 | 0.99 | 1.0025 | 0.99 | 1.0018 | 0.73 |
| 3 | 1.0036 | 0.32 | 1.0024 | −0.03 | 1.0030 | 0.20 | 1.0035 | 0.68 |
| 4 | 1.0052 | 0.66 | 1.0038 | 0.57 | 1.0043 | 0.51 | 1.0052 | 0.64 |
| 5 | 1.0073 | 0.84 | 1.0055 | 0.67 | 1.0059 | 0.65 | 1.0067 | 0.60 |
| 6 | 1.0094 | 0.83 | 1.0069 | 0.56 | 1.0074 | 0.61 | 1.0081 | 0.56 |
| 7 | 1.0118 | 0.95 | 1.0085 | 0.62 | 1.0092 | 0.70 | 1.0094 | 0.53 |
| 8 | 1.0142 | 0.93 | 1.0098 | 0.51 | 1.0109 | 0.66 | 1.0107 | 0.50 |
| 9 | 1.0170 | 1.12 | 1.0113 | 0.62 | 1.0129 | 0.81 | 1.0118 | 0.46 |
| 10 | 1.0198 | 1.10 | 1.0125 | 0.48 | 1.0149 | 0.77 | 1.0129 | 0.43 |
| 11 | 1.0229 | 1.20 | 1.0138 | 0.50 | 1.0170 | 0.84 | 1.0140 | 0.41 |
| 12 | 1.0259 | 1.20 | 1.0148 | 0.41 | 1.0191 | 0.80 | 1.0149 | 0.38 |
| 40 | 1.2238 | 3.65 | 1.0320 | 0.06 | 1.1174 | 1.59 | 1.0265 | 0.05 |
| ∞ | ∞ | ∞ | 1.0330 | 0.00 | ∞ | 1.95 | 1.0248 | 0.00 |

$P$ = price level, $p = 400 \log P$.

For the other comparison set, the above experiment was performed for equation (10) and (11), in which the value used for $w$ for 1998:1 (which is needed for experiment) was taken to be the actual value.

Before looking at the results, it should be stressed that this experiment is not meant to be realistic. For example, it is unlikely that the Fed would allow a permanent fall in $u$ to take place as $p$ rose. This experiment is simply meant to help illustrate how the equations differ in a particular dimension.

The results for the four experiments are presented in table 6. Consider the very long-run properties first. For equation (6), the new price level grows without bounds relative to the base price level, and the new inflation rate grows without bounds relative to the base inflation rate. For equation (7) with only $\pi_{t-1}$ added, the new price level grows without bounds relative to the base, but the inflation rate does not. It is 1.95 percentage points higher in the long run. For equation (7) with both $\pi_{t-1}$ and $p_{t-1}$ added, the new price level is higher by 3.20% in the limit, and the new inflation rate is back to the base. Similarly, for equation (10) and (11), the new price level is higher by 2.48% in the limit, and the new inflation rate is back to the base.

The long-run properties are thus vastly different, as is, of course, obvious from the specifications. What is interesting, however, is that the effects are fairly close for the first few quarters. One would be hard pressed to choose among the equations on the basis of which short-run implications seem more "reasonable." Instead, one needs tests of the kind performed in this paper.

## VI. An Alternative View

The $\chi^2$ tests in section III generally reject the dynamics implied by the NAIRU specification, and the RMSE results in table 4 show that the NAIRU specification has poor predictive power relative to the equation with $\pi_{t-1}$ and $p_{t-1}$ included and relative to a simple structural price and wage model. It would be easy in future work for others to test NAIRU specifications in the manner done in this paper. The main test is simply to add the log of the price levels lagged one and two periods to the equation and to examine their significance.

If the NAIRU specification is rejected, this changes the way one thinks about the relationship between inflation and unemployment. One should not think that there is some unemployment rate below which the price level forever accelerates and above which it forever decelerates. On the other hand, neither equation (7) nor equation (10) and (11) seems to be a sensible alternative. Both these specifications imply that a lowering of the unemployment rate has only a modest long-run effect on the price level, regardless of how low the initial value of the unemployment rate is. For example, the results in table 6 for these two specifications are independent of the initial value of the unemployment rate.

A weakness of all four of the specifications examined in table 6 (in my view) is the linearity assumption regarding the effects of $u$ on $p$. It seems likely that there is a strongly nonlinear relationship between the price level and the unemployment rate at low levels of the unemployment rate. One possible specification, for example, would be to replace $u$ in, say, equation (10) with $1/(u - 0.02)$. In this case, as $u$ approaches 0.02, the estimated effects on $p$ become larger and larger. I have experimented with a variety of functional forms like this in estimating price equations like (10) to see if the data can pick up a nonlinear relationship. Unfortunately, there are so few observations of very low unemployment rates that the data do not appear capable of discriminating among functional forms. A variety of functional forms, including the linear form, lead to very similar results. This does not mean, however, that the true functional form is linear, only that the data are insufficient for estimating the true functional form.

The alternative view put forth here about the relationship between inflation and the unemployment rate can thus be summarized by the following two points. First, the NAIRU dynamics, namely the first- and second-derivative restrictions, are not accurate. Second, the relationship between the price level and the unemployment rate is nonlinear at low values of the unemployment rate. The data generally support the first point and have little to say about the second point.

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## APPENDIX

As noted in section I, the theory that has guided the specification of the price and wage equations in my macroeconometric model was first presented in Fair (1974), with more recent discussions in Fair (1984, chapter 3; 1994, chapter 2). The following is a brief outline of this theory. Firms are assumed to solve multiperiod profit-maximization problems in which prices, wages, investment, employment, and output are decision variables. The maximization problem requires that a firm form expectations of various variables before the problem is solved. A firm's market share is a function of its price relative to the prices of other firms. A firm expects that it will gain (lose) customers if it lowers (raises) its price relative to the expected prices of other firms. Similarly, a firm expects that it will gain (lose) workers if it raises (lowers) its wage rate relative to the wage rates of other firms. The properties of the theoretical model are examined via simulation runs. Two of the properties that are relevant for present purposes are

1. A change in the expected prices (wages) of other firms leads the given firm to change its own price (wage) in the same direction.
2. A firm responds to a decrease in demand by lowering its price and contracting output, and vice versa for an increase in demand.

In this setup, the natural decision variable is the price **level.** The objective of a firm is to choose its price-level path (along with the paths of the other decision variables) that maximizes the multiperiod objective function.

Another, simpler, model in which the price level is the natural decision variable is a duopoly game with asymmetric information discussed in Tirole (1988, section 9.1.1). The duopolists (firms 1 and 2) sell differentiated products, and firm 2 has incomplete information about firm 1's cost. The demand curves are symmetric and linear:

$$D_i(p_i, p_j) = a - bp_i + dp_j, \qquad 0 < d < b. \tag{14}$$

Both firms have constant marginal costs, $c_1$ and $c_2$, respectively, where $c_2$ is common knowledge, but only firm 1 knows $c_1$. Tirole shows that firm 2's profit-maximizing price is

$$p_2 = (a + dp_1^e + bc_2)/2b, \tag{15}$$

where $p_1^e$ is firm 2's expectation of firm 1's price. $p_1^e$ depends, among other things, on firm 2's expectation of firm 1's marginal cost. Equation (15) says that firm 2's price is a function of the demand parameter $a$, firms 2's marginal cost $c_2$, and firm 2's expectation of firm 1's price $p_1^e$.

The transition from theory to econometric specifications in macroeconomics is usually crude, and this is true in the present context in moving from the above theory to the specification of equation (10) and (11) in the text. The lagged price level in equation (10) can be thought of as picking up expectational effects, which are in both theoretical models mentioned above, represented, for example, by $p_1^e$ in equation (15) in the duopoly model. The wage variable and the import price variable can be thought of as picking up cost effects, which are also in both models, represented by $c_2$ in equation (15). Finally, the unemployment rate picks up demand effects, which are in both models, represented by $a$ in equation (15). The time trend in equation (10) is meant to pick up any trend effects on the price level not captured by the other variables. Adding the time trend to an equation like (10) is similar to adding the constant term to an equation specified in terms of changes rather than levels.

In the wage equation, equation (11), the wage rate is a function of the lagged wage rate, the current and lagged price levels, the unemployment rate, and the time trend. Given that the unemployment rate is not significant in the equation, the wage equation is one in which the wage rate simply adjusts to the price level over time.

Equation (10) and (11) can be used to justify adding $p_{t-1}$ and $p_{t-2}$ to the NAIRU specification. From equation (10) and (11), the final form of the price equation can be derived by lagging equation (10) one period, multiplying through by $\gamma_1$, subtracting this expression from equation (10), and then using equation (11) to substitute out the wage rate. The final form of the price equation is:[12]

$$\begin{aligned} p_t = \frac{1}{1-\beta_2\gamma_2}&[(\beta_0 + \beta_2\gamma_0 - \beta_0\gamma_1 + \beta_5\gamma_1) \\ &+ (\beta_1 + \beta_2\gamma_3 + \gamma_1)p_{t-1} - \beta_1\gamma_1 p_{t-2} \\ &+ \beta_3 pm_{t-1} - \beta_3\gamma_1 pm_{t-2} + (\beta_4 + \beta_2\gamma_4)u_{t-1} \\ &- \beta_4\gamma_1 u_{t-2} + (\beta_5 - \beta_5\gamma_1 + \beta_2\gamma_5)t \\ &+ (\epsilon_t - \gamma_1\epsilon_{t-1} + \beta_2 u_t)]. \end{aligned} \tag{16}$$

$p_{t-1}$ and $p_{t-2}$ appear as separate explanatory variables in this equation. The NAIRU equation (6) differs from equation (16) in having more lagged price levels, but with the first- and second-derivative restrictions imposed on the price levels. If equation (6) is correctly specified, $p_{t-1}$ and $p_{t-2}$ do not appear separately in the price equation, whereas if equation (10) and (11) are correctly specified, the variables do. A test is thus to add $p_{t-1}$ and $p_{t-2}$ to equation (6) and see if they are jointly significant.

[12] I am indebted to Phil Howrey for suggesting the use of the final form price equation in the present context.