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ORIGINAL ARTICLE

What do price equations say about future inflation?

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Abstract
This paper uses an econometric approach to examine the inflation consequences of the American Rescue Plan Act of 2021. Price equations are estimated and used to forecast future inflation. The main results are: (1) The data suggest that price equations should be specified in level form rather than in first or second difference form. (2) There is some slight evidence of nonlinear demand effects on prices. (3) There is no evidence that demand effects have gotten smaller over time. 4) The stimulus from the act combined with large wealth effects from past household saving, rising stock prices, and rising housing prices is large and is forecast to drive the unemployment rate down to below 3.5 percent by the middle of 2022. 5) Given this stimulus, the inflation rate is forecast to rise to slightly under 5 percent by the middle of 2022 and then comes down slowly. 6) There is considerable uncertainty in the point forecasts, especially two years out. The probability that inflation will be larger than 6 percent next year is estimated to be 31.6 percent. 7) If the Fed were behaving as historically estimated, it would raise the interest rate to about 3 percent by the end of 2021 and 3.5 percent by the end of 2022 according to the forecast. This would lower inflation, although slowly. By the middle of 2022 inflation would be about 1 percentage point lower. The unemployment rate would be 0.5 percentage points higher.

Keywords Price equations · Inflation · Fed policy

1 Introduction

The passage of the American Rescue Plan Act in March 2021 has led to much debate about its future inflation consequences. Larry Summers (2021), among others, has argued that the inflation consequences could be severe. The Biden administration and the Fed have argued there is likely to be a blip in inflation in 2021 but nothing long-lasting. Most of this discussion is based on casual empiricism rather than econometric estimates. This paper takes an econometric approach and examines what estimated price equations imply about future inflation. As of this writing data are available for the first quarter of 2021, so the forecast period begins with the second quarter of 2021. It ends in the fourth quarter of 2023.

The price and wage equations in my U.S. macroeconometric model (the US model) are used as a base, but a number of price equations are examined. In previous work[1] I have argued that the data do not support the dynamics of the expectations-augmented Phillips curve, and this issue is examined further in this paper. The dynamics of price equations are crucial for examining long-run inflation consequences from a short-run blip. For example, are the Administration and the Fed right in their view that there are no long run consequences? It will be seen that the data support the specification of price equations in level form rather than in first difference or second difference form. The NAIRU specification does not appear to be supported by the data.

Another issue regarding the specification of price equations is which demand variable to use. A common choice is the unemployment rate, perhaps subtracted for a time varying "natural" rate. A problem with this choice is the linearity assumption. As the economy moves into a regime of low unemployment rates, one might expect a nonlinear response. One possibility is to use the reciprocal of the unemployment rate, which is tried here. An output gap measure and its reciprocal are also tried. Estimating nonlinear effects is difficult because there are few periods of very low unemployment rates; the Fed usually intervenes. Unfortunately,

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[1] Fair (2000) and updated in Fair (2018, Section 3.1.3).

as will be seen, inflation forecasts are sensitive to reciprocals versus levels.

Given a particular estimated price equation with, say, the unemployment rate as an explanatory variable, one needs a forecast of future unemployment rates to make an inflation forecast. The US model is used for this purpose. More will be said about this below.

The US model is described in detail in a document on my website, *Macroeconometric Modeling: 2018*, which will be abbreviated "MM" (Fair 2018). Most of my past macro research, including the empirical results, is in MM. It includes chapters on methodology, econometric techniques, numerical procedures, theory, empirical specifications, testing, and results. The results in my previous macro papers have been updated through 2017 data, which provides a way of examining the sensitivity of the original results to the use of additional data. It is too much to explain the model in one paper, and I will rely on MM as the reference. Think of MM as the appendix to this paper. In what follows the relevant sections in MM will be put in brackets. The forecast used in this paper is also on the website. The paper Fair (2020b) summarizes the main properties of the model.

2 Single price equations

2.1 Price equations

Consider first a price equation not embedded in a wage-price sector. The expectations augmented Phillips curve is:

$$\pi_t = \pi^e_{t+1} + \beta\left(u_t - u^*\right) + \gamma s_t + \epsilon_t \quad , \quad \beta < 0, \quad \gamma > 0, \tag{1}$$

where t is the time period, π_t is the rate of inflation, π^e_{t+1} is the expected rate of inflation for period $t+1$, u_t is the unemployment rate, s_t is a cost shock variable, ϵ_t is an error term, and u^* is the NAIRU.[2]

A key question is how π^e_{t+1} is determined. A common assumption is that

$$\pi^e_{t+1} = \sum_{i=1}^{n} \delta_i \pi_{t-i} \quad , \quad \sum_{i=1}^{n} \delta_i = 1. \tag{2}$$

This says that agents look only at past inflation in forming their expectations of future inflation. An alternative is to embed equation (1) in a complete model and assume rational expectations. One could solve (1) forward and use the model's future predictions of u and s to solve for π^e_{t+1}. This is not done here. Inflation expectations are assumed to depend only on past inflation. An early paper supporting this is Fuhrer (1997). Coibion *et al.* (2020) review the recent literature on how inflation expectations are formed. Household and firm expectations tend to differ considerably from market expectations and those of professional forecasters. There is evidence that the strongest predictor of household's and firms' inflation forecasts are what they believe inflation has been in the recent past, which are not always accurate beliefs. There is also little evidence that firms know much about monetary policy targets. Further survey evidence regarding firms is in Candia *et al.* (2021), which support these conclusions. It seems clear that firms' inflation expectations are not rational, nor even very sophisticated. The assumption used here, that inflation expectations depend only on past inflation, may be the best that one can do. The story consistent with this assumption is that as actual inflation increases (from some shock) firms begin to perceive this, perhaps with a lag, which affects their inflation expectations and pricing decisions.

Combining (1) and (2) yields:

$$\pi_t = \sum_{i=1}^{n} \delta_i \pi_{t-i} + \beta\left(u_t - u^*\right) + \gamma s_t + \epsilon_t \quad , \quad \sum_{i=1}^{n} \delta_i = 1. \tag{3}$$

One restriction in equation (3) is that the δ_i coefficients sum to one. A second restriction is that each price level is subtracted from the previous price level before entering the equation. These two restrictions are straightforward to test. Let p_t be the log of the price level for period t, and let π_t be measured as $p_t - p_{t-1}$. Using this notation, equation (3) can be written in terms of p rather than π. If, for example, $n = 1$, equation (3) becomes

$$p_t = 2p_{t-1} - p_{t-2} + \beta(u_t - u^*) + \gamma s_t + \epsilon_t. \tag{4}$$

In other words, equation (3) can be written in terms of the current and past two price levels,[3] with restrictions on the coefficients of the past two price levels. Similarly, if, say, $n = 4$, equation (3) can be written in terms of the current and past five price levels, with two restrictions on the coefficients of the five past price levels. (Denoting the coefficients on the past five price levels as a_1 through a_5, the two restrictions are $a_4 = 5 - 4a_1 - 3a_2 - 2a_3$ and $a_5 = -4 + 3a_1 + 2a_2 + a_3$.) The restrictions are easy to test by simply adding p_{t-1} and p_{t-2} to equation (3) and testing whether they are jointly significant.

An equivalent test is to add π_{t-1} (i.e., $p_{t-1} - p_{t-2}$) and p_{t-1} to equation (3). Adding π_{t-1} breaks the restriction that the δ_i coefficients sum to one, and adding both π_{t-1} and p_{t-1} breaks the summation restriction and the restriction that each price

[2] Some specifications take u^* to be time varying. This is not done here. It's hard to avoid subjectivity in the choice of how the natural rate varies over time.

[3] "Price level" will be used to describe p even though p is actually the log of the price level.

Table 1 Equation Estimates Dependent Variable is $\Delta\pi_t$

	(1)		(2)		(3)	
	Equation (5)		Equation (5)		Equation (5)	
			π_{t-1} added		π_{t-1} added and p_{t-1} added	
Variable	Estimate	t-stat.	Estimate	t-stat.	Estimate	t-stat.
cnst	0.0017	0.60	0.0071	2.20	-0.0346	-6.00
t	0.0000004	0.05	-0.0000173	-1.63	0.0001821	7.09
UR_t	-0.0319	-1.56	-0.0367	-1.83	-0.1279	-6.12
$\log PIM_t$	-0.0002	-0.16	0.0017	1.40	0.0342	8.47
$\pi_{t-2} - \pi_{t-1}$	0.272	4.16	0.233	3.57	0.085	1.40
$\pi_{t-3} - \pi_{t-1}$	0.209	3.20	0.169	2.59	0.080	1.35
$\pi_{t-4} - \pi_{t-1}$	0.125	2.00	0.076	1.21	0.033	0.59
π_{t-1}			-0.171	-3.22	-0.663	-8.78
p_{t-1}					-0.053	-8.34
SE	0.00435		0.00428		0.00380	
χ^2					82.81	

- $p_t = \log PF_t$, $\pi_t = \log(PF_t/PF_{t-1}$, UR_t = unemployment rate, $\log PIM$ = log of the price of imports
- Estimation method: ordinary least squares
- Estimation period: 1954.1–2019:4
- Five percent χ^2 critical value = 5.99; one percent χ^2 critical value = 9.21

level is subtracted from the previous price level before entering the equation. This latter restriction can be thought of as a first derivative restriction, and the summation restriction can be thought of as a second derivative restriction.

2.2 Data

A widely cited price deflator in the media is the price deflator for personal consumption expenditures (*PCE*). This is the price deflator targeted by the Fed. If, however, one is interested in explaining the pricing behavior of agents in the U.S. economy, *PCE* is not appropriate because it includes import prices (as well as excluding export prices). The same is true of the consumer price index (*CPI*). Import prices reflect decisions of foreign agents and the behavior of exchange rates, which are not decision variables of domestic agents. The price deflator used in the following analysis is the price deflator of the U.S. firm sector, variable *PF* in the US model, which reflects private, domestic, decisions.

The measure of demand used in this section is the unemployment rate, denoted *UR*. Data on *UR* are from the BLS household survey. These data are re-benchmarked each year and are not revised back, which can cause spikes in some of the variables. This problem is not always addressed in the literature, especially in DSGE modeling—Fair (2020a). I have adjusted for this in the US model using backward interpolation—(MM, Table A.5). The cost shock variable used in the analysis is taken to be the import price deflator (*PIM*).

The estimation period is 1954.1–2019.4, ending in the last quarter before the pandemic. For the estimation of equation (3) n was taken to be 4, $p_t = \log PF_t$. $\pi_t = p_t - p_{t-1}$, and $u_t = UR_t$. s_t is postulated to be $\log PIM_t - \tau_0 - \tau_1 t$, the deviation of $\log PIM$ from a trend line. Given these variables and the restriction on the δ_i coefficients, the equation estimated is:

$$\Delta\pi_t = \lambda_0 + \lambda_1 t + \sum_{i=2}^{4} \delta_i(\pi_{t-i} - \pi_{t-1}) + \beta UR_t + \gamma \log PIM_t + \epsilon_t, \quad (5)$$

where $\lambda_0 = -\beta u^* - \gamma\tau_0$ and $\lambda_1 = \gamma\tau_1$. u^* is not identified in equation (5), but for purposes of the tests this does not matter. If, however, one wanted to compute the NAIRU (i.e., u^*), one would need a separate estimate of τ_0 in order to estimate u^*.[4]

2.3 Estimates

The results of estimating equation (5) are presented in column (1) in Table 1. In column (2) π_{t-1} is added, and in column (3) both π_{t-1} and p_{t-1} are added. Comparing columns

[4] Note that if u^* follows a linear time trend, this will be picked up by the inclusion of t in the equation.

Table 2 Effects of a One Percentage Point Fall in *UR*

Quar.	Equation (5)		Equation (5) π_{t-1} added		Equation (5) π_{t-1} and p_{t-1} added	
	$P^{new} - P^{base}$	$\pi^{new} - \pi^{base}$	$P^{new} - P^{base}$	$\pi^{new} - \pi^{base}$	$P^{new} - P^{base}$	$\pi^{new} - \pi^{base}$
1	0.0003	0.13	0.0004	0.15	0.0014	0.51
2	0.0008	0.18	0.0009	0.20	0.0028	0.56
3	0.0014	0.23	0.0016	0.25	0.0044	0.58
4	0.0023	0.29	0.0024	0.31	0.0060	0.59
5	0.0033	0.36	0.0034	0.36	0.0076	0.59
6	0.0045	0.42	0.0045	0.40	0.0091	0.56
7	0.0058	0.48	0.0058	0.44	0.0106	0.53
8	0.0074	0.54	0.0071	0.48	0.0120	0.49
12	0.0157	0.79	0.0134	0.60	0.0164	0.35
40	0.1905	2.52	0.0761	0.83	0.0260	0.03
80	1.0446	4.99	0.1789	0.86	0.0267	0.00
120	3.4704	7.45	0.2868	0.86	0.0267	0.00

• P = price level (PF), $\pi = \log PF - \log PF_{-1}$

(1) and (2), Table 1 shows that when π_{t-1} is added, it is significant with a t-statistic of -3.22. When both π_{t-1} and p_{t-1} are added in column (3), both are significant with t-statistics of -8.78 and -8.34 respectively. The χ^2 value for the hypothesis that the coefficients of both variables are zero is 82.81.[5]

The results thus strongly reject equation (5) and equation (5) with π_{t-1} added. Only the lagged inflation variables are significant, and there are very large changes in the coefficient estimates when π_{t-1} and p_{t-1} are added. In particular, the coefficient estimates of the unemployment rate are much smaller in absolute value without the two variables added.

2.4 Dynamics

The three equations in Table 1 have quite different dynamics, and it will be useful to examine the differences. The question considered is the following: if the unemployment rate were permanently lowered by one percentage point, what would the price level and inflation consequences be? To answer this question, the following experiment was performed for each equation. A dynamic simulation was run beginning in 2021.2 using the actual values of all the variables from 2021.1 back. The values of *UR* and *PIM* from 2021.2 forward were taken to be the actual values for 2021.1. Call this simulation the "base" simulation. A second dynamic simulation was then run where the only change was that the unemployment rate was decreased permanently by one percentage point from 2021.2 on. The difference between the predicted value of π from this simulation and that from the base simulation for a given quarter is the estimated effect of the change in *UR* on π. Similarly for p.[6]

The results for the three equations are presented in Table 2. It should be stressed that these experiments are not meant to be realistic. For example, there is no Fed reaction to the rise in inflation. The experiments are simply meant to help illustrate how the equations differ in a particular dimension.

Consider the very long run properties in Table 2 first. For equation (5), the new price level grows without bounds relative to the base price level and the new inflation rate grows without bounds relative to the base inflation rate. For equation (5) with π_{t-1} added, the new price level grows without bounds relative to the base, but the inflation rate does not. It is 0.86 percentage points higher in the long run. For equation (5) with both π_{t-1} and p_{t-1} added, the new price level is higher by 2.67 percent in the limit and the new inflation rate is back to the base.

The long run properties are thus vastly different, as is, of course, obvious from the specifications. What is interesting, however, is that the effects on inflation are close after, say, 8

[5] Note that there is a large change in the estimate of the coefficient of the time trend when π_{t-1} and p_{t-1} are added. The time trend is serving a similar role in this equation as the constant term is in equation (5).

[6] Because the equations are linear, it does not matter what values are used for *PIM* as long as the same values are used for both simulations. Similarly, it does not matter what values are used for *UR* as long as each value for the second simulation is one percentage point higher than the corresponding value for the base simulation. Also, unless *UR* is exactly at the NAIRU, the base simulation for equation (5) will either have an accelerating or decelerating inflation and price path. The computed differences in this case are differences from the accelerating or decelerating path. For equation (5) with π_{t-1} added, the base simulation will have an accelerating or decelerating price path. For this reason results are presented in Table 2 only out 120 quarters.

quarters. The inflation differences, new minus base, are 0.54, 0.48, and 0.49, respectively. It is hard to distinguish among the equations based only on their short run properties.

3 Price and wage equations

The results above support the specification of the price equation in level form, and this form is used for the price and wage equations in the US model. Three new variables are added to the analysis: *WF*, a wage rate of the firm sector, *D5G*, the employer social security tax rate, and *LAM*, a measure of potential labor productivity. The wage rate that measures the cost to the firm sector is $WF \cdot (1 + D5G)$, the wage rate inclusive of employer social security taxes. *LAM* is constructed from peak-to-peak interpolation of the log of actual labor productivity, output divided by worker hours, for the 1952.1–2021.1 period. Its growth rate reflects the growth rate of potential productivity.[7]

Let $p = \log PF$, $wa = \log[WF \cdot (1 + D5G)] - \log LAM$, $s = \log PIM$, and d denote the demand variable. Then the price equation is

$$p_t = \beta_1 p_{t-1} + \beta_2 wa_t + \beta_0 + \beta_3 t + \beta_4 d_t + \beta_5 s_t + \epsilon_t. \tag{6}$$

This equation is equation (5) with π_{t-1} and p_{t-1} added, with the wage rate added, and with only one lag of the price level.[8]

In the wage rate equation the wage rate runs off the price level. Let $w = \log WF - \log LAM$. Then the wage rate equation is

$$w_t = \gamma_1 w_{t-1} + \gamma_2 p_t + \gamma_3 p_{t-1} + \gamma_0 + v_t. \tag{7}$$

This equation says that wages respond to prices, but are not directly affected by demand. Demand and cost shocks affect the price level, which then affects the wage rate. The price equation is identified because the wage rate equation includes the lagged wage rate, which the price equation does not. The wage rate equation is identified because the price equation includes d_t and s_t, which the wage rate equation does not.

A constraint is imposed on the coefficients in the wage rate equation to ensure that the determination of the real wage implied by the two equations is sensible. The relevant parts of the two equations regarding the constraint are

$$p_t = \beta_1 p_{t-1} + \beta_2 w_t + \dots, \tag{8}$$

$$w_t = \gamma_1 w_{t-1} + \gamma_2 p_t + \gamma_3 p_{t-1} + \dots. \tag{9}$$

The implied real wage equation from these two equations should not have $w_t - p_t$ as a function of either w_t or p_t separately, since one does not expect the real wage to grow simply because the levels of w_t and p_t are growing. The desired form of the real wage equation is thus

$$w_t - p_t = \delta_1 (w_{t-1} - p_{t-1}) + \dots, \tag{10}$$

which says that the real wage is a function of its own lagged value plus other terms. The real wage in equation (10) is not a function of the level of w_t or p_t separately. The constraint on the coefficients in equations (8) and (9) that imposes this restriction is:

$$\gamma_3 = [\beta_1 / (1 - \beta_2)] (1 - \gamma_2) - \gamma_1. \tag{11}$$

This constraint is imposed in the estimation by first estimating the price equation to get estimates of β_1 and β_2 and then using these estimates to impose the constraint on γ_3 in the wage rate equation.

The time trend, t, in the price equation is meant to pick up any trend effects on the price level not captured by the other variables. Adding the time trend to an equation like this (in level form) is similar to adding the constant term to an equation specified in terms of changes rather than levels. The time trend will also pick up any trend mistakes made in constructing *LAM*. It also accounts for the trend in *PIM*.

The demand variable used in the previous section is the unemployment rate, *UR*. Three other variables are tried here: $1/UR$, GAP, and $1/(GAP + .07)$, where *GAP* is an estimate of the output gap. The .07 is added to *GAP* in the reciprocal to ensure that the denominator does not go negative.[9] The form of the demand variable is an important question for forecasting 2021 and beyond since the economy may be pushed to capacity, which is the reason for the use of the reciprocals.

Table 2 includes four estimates of equation (6), for the four demand variables. Each is highly significant. The estimated standard errors are, respectively, 0.003769, 0.003711, 0.003846, and 0.003927. $1/UR_t$ has the lowest standard error and $1/(GAP_t + .07)$ has the highest, but they are all close. The estimates of the other coefficients are not sensitive to

[7] The peaks are 1955.2, 1963.3, 1966.1, 1973.1, 1992.4, and 2010.4, where the first line is extended back to 1952.1 and where from 2011.1 on the annual growth rate was taken to be 1.50 percent. The annual growth rates between the six peaks are 3.40, 2.73, 2.54, 1.56, and 2.01, respectively.

[8] In equation (5) s equaled $\log PIM - \tau_0 - \tau_1 t$. Here s is just $\log PIM$ since equation (6) already includes a constant term and time trend.

[9] The output gap in the US model is defined as $(YS - Y)/YS$, where Y is the actual output of the firm sector and *YS* is a measure of potential output. *YS* is computed from peak to peak interpolations of $\log Y$ over the 1952.1–2021.1 period. The peaks are 1953.2, 1966.1, 1973.2, 1999.4, 2006.4, and 2019.1, where tre the first line is extended back to 1952.1 and the last line is extended forward to 2021.1. The annual growth rates between the six peaks are 4.09, 3.67, 3.24, 2.65, and 1.83, respectively.

Table 3 Equation (6) Estimates Dependent Variable is $\log PF_t$

Variable	d=UR		d=1/UR		d=GAP		d=1/(GAP+.07)	
	Estimate	t-stat.	Estimate	t-stat.	Estimate	t-stat.	Estimate	t-stat.
$\log PF_{t-1}$	0.882	88.93	0.877	88.35	0.913	92.11	0.915	90.16
wa_t	0.0471	4.67	0.0550	5.47	0.0191	1.83	0.0188	1.76
$cnst$	-0.0181	-2.23	-0.0320	-4.05	-0.0361	-4.40	-0.0507	-5.84
t	0.000243	11.98	0.000230	11.54	0.000220	10.64	0.000217	10.29
$\log PIM_t$	0.0495	21.96	0.0496	22.19	0.0448	21.45	0.0440	20.78
d_t	-0.176	9.30	0.000624	9.53	-0.111	-9.12	0.001123	8.51
SE	0.003769		0.003711		0.003846		0.003927	

- $wa_t = \log[WF_t(1 + D5G_t)] - \log LAM_t$
- Estimation method: two stage least squares
- Estimation period: 1954.1–2019:4

the demand variable used except for the coefficient estimate of wa_t when GAP is used. Although not shown in the table, when when both $1/UR_t$ and UR_t are included together in the equation, the coefficient estimate for $1/UR_t$ is 0.000364 with a t-statistic of 1.96 and the coefficient estimate for UR_t is -0.079 with a t-statisitc of -1.49. The estimated standard error is 0.003717. $1/UR_t$ is thus slightly better.

An estimate of the wage rate equation (7) is presented in Table 4. The constraint for this estimate is based on the coefficient estimates of the price equation with $1/UR$ as the demand variable, the second equation in Table 3. As noted above, this equation simply reflects the assumption that wages follow prices.

The equations in Tables 3 and 4 are estimated by two stage least squares (2SLS). The main first stage regressors, aside from the one-quarter lagged values of the explanatory variables in the equation, are one-quarter lagged values of the log of real per capita government purchases of goods and services, the log of real per capita government transfer payments other than unemployment benefits, and the log of real per capita exports. No current quarter values are used as first stage regressors. The complete list of first stage regressors is in MM, Table A.9.

A popular question in current work is whether the Phillips curve has become flatter. Focusing on the second equation in Table 3, the price equation in the US model with $1/UR$ as the demand variable, the question is whether the coefficient of $1/UR$ has become smaller over time. The coefficient estimate is in fact relatively stable. The estimation period begins in 1954.1. When the equation is estimated through 1971.1, 69 observations, the coefficient estimate is 0.000755, which compares to 0.000624 in Table 3. When the end point is extended one quarter at a time, the largest estimate is 0.000762 in 1972.3 and the smallest estimate is 0.000549 in 2008.2. All the coefficient estimates are significant. This is a small range for this kind of work.

What does not work, however, is to do a rolling regression of, say, 20 years (80 quarters). Here the variation in the coefficient estimates is large. The problem with this procedure, in my view, is that the sample size is too small. As one rolls out of the mid-1980's, the inflation experience in the late 1960's, 1970's, and mid-1980's is lost, and one enters a much smoother period regarding inflation. Using 80 quarters, the last sample period is 2000.1–2019.4, which is clearly not typical of the historical experience of inflation. It should not be surprising that price equations estimated for this period are considerably different from ones estimated earlier or for a longer period. Not using information through the 1980's is problematic.

4 Demand assumptions

There are seven price equations to consider: the three in Table 1 and the four in Table 3. Five require future values of UR and two require future values of GAP. The forecast period is 2021.2–2023.4, 11 quarters. The US model is used for the forecasts. A key issue for the forecasts is how to account for the American Rescue Plan Act (ARPA) passed in March 2021. The Congressional Budget Office (CBO) and the Joint Committee on Taxation (JCT) have estimated

Table 4 Equation (7) Estimates Dependent Variable is $\log WF_t - \log LAM_t$

Variable	Estimate	t-stat.
$\log WF_{t-1} - \log LAM_{t-1}$	0.943	52.04
$\log PF_t$	0.926	34.05
$cnst$	-0.0371	-3.19
$\log PF_{t-1}$	0.928	
SE	0.007824	

- Coefficient for $\log PF_{t-1}$ constrained.
- Estimation method: two stage least squares.
- Estimation period: 1954.1–2019:4

the budget outlays from this act: $1,088 billion in FY2021, $476 billion in FY2022, $115 billion in FY2023, and then relatively small amounts after that. Some of this was spent in 2021.1. From the national income and product accounts (NIPA) released April 29, 2021, federal transfer payments to persons (*TR*) was larger in 2021.1 versus 2020.1, the last "normal" quarter before the pandemic, by $686 billion at a quarterly rate. Grants-in-aid to state and local (S&L) governments (*GIA*) was larger by $39 billion, and subsidies (*SUB*) was larger by $82 billion. This total, $807 billion, is assumed to be due to the ARPA. This leaves $281 billion left for 2021.2 and 2021.3 using the CBO and JCT estimate of $1,088 billion for FY2021. I have allocated this 60/40 in the two quarters, so $169 billion in 2021.2 and $112 billion in 2021.3. For the next four quarters I have allocated the $476 evenly, so $119 billion each. For the next four quarters I have allocated the $115 billion evenly, so $29 billion each.

These values are in nominal terms. To convert them to real terms, I took the value of the GDP deflator in 2021.1, let it grow at an annual rate of 3 percent, and used these values to deflate the nominal values. The 11 values over the 11 quarters in billions of dollars are 145, 96,101, 100, 99, 99, 24, 24, 24, and 23. (The actual $807 billion nominal value in 2021.1 is $699 billion in real terms.) Although some of this additional spending will take the form of increased real *GIA* and increased *SUB*, for purposes of the forecast all has been put in real *TR*. *SUB* was taken to be $20 billion in each of the 11 quarters, roughly its value in 2020.1. Real *GIA* was taken to grow at an annual rate of 3 percent from 2021.2 on using as a base value its value in 2020.1. In addition, S&L government transfer payments to persons was taken to grow at an annual rate of 3 percent using as a base value its value in 2020.1.[10] Real *TR* was taken to grow at an annual rate of 3 percent using as a base value its value in 2020.1 and then the additions discussed above were added to these values. *TR* is part of disposable income, which in the model affects household expenditures, both consumption and housing investment.

Since some of the additional spending from ARPA will go to subsidies and *GIA*, the implicit assumption used here is that the multiplier effects from these two variables are the same as the multiplier effects from *TR*. The real output multipliers from increasing real *TR* by 1 for the 11 quarters are respectively: 0.11, 0.25, 0.36, 0.45, 0.51, 0.55, 0.58, 0.60, 0.62, 0.63, and 0.64. The initial effects are thus small, rising to a multiplier of about half after 4 quarters. As is obvious from the large increases in the personal saving rate after the pandemic stimulus payments, households initially save much of the increased transfer payments.

Table 5 Forecasts for 2021.2–2023.4

	$\%\Delta Y$	GAP	UR
2021.1[a]	7.4	0.022	6.2
2021.2	12.8	-0.001	5.5
2021.3	9.6	-0.017	4.7
2021.4	6.3	-0.025	4.0
2022.1	4.2	-0.028	3.6
2022.2	3.4	-0.029	3.4
2022.3	3.3	-0.029	3.2
2022.4	2.6	-0.027	3.2
2023.1	2.5	-0.027	3.2
2023.2	2.7	-0.027	3.3
2023.3	2.9	-0.027	3.3
2023.4	2.9	-0.027	3.3

- [a]Actual
- $\%\Delta Y$ = percentage change in real output, annual rate

Some of the other assumptions for the forecast are as follows (all growth rates are at annual rates): tax rates unchanged from their 2021.1 values, real exports growing at 3 percent, the price of imports growing at 3 percent, *YS* growing at 3 percent, *LAM* growing at 1.5 percent, and the relative price of housing growing at 5 percent. In addition, the estimated Fed rule is dropped and the short term interest rate in the model (*RS*, the three-month Treasury bill rate) is assumed to be unchanged from its 2021.1 value, which is 5 basis points. The forecast and all the assumptions are on my website.

Nothing was done about possible tax and spending changes from the Biden administration's proposed infrastructure plans. The current forecast is obviously a conditional forecast, conditional on nothing new done after the ARPA. As least for the first year or two "nothing new" is likely not a bad approximation since it will take time for the legislation to be passed, if it is, and for the beginning of the increased spending and taxes.

The second price equation in Table 3, the one using $1/UR$, was used for the forecast. It makes little difference to the forecasts of the unemployment rate and output which price equation is used. The results for output, the gap, and the unemployment rate are presented in Table 5. The predicted output growth rate is 12.8 percent for 2021.2. (All growth rates are at annual rates.) This large rate is in part due to household wealth, which is large from past transfer payments saved and from past large increases in stock and housing prices. This has a large effect on household expenditures, including housing investment. The high growth rate is also due in part to a large predicted inventory correction in 2021.2 (inventory investment was negative and large in

[10] The values of S&L transfer payments were higher during the pandemic as S&L governments passed on some of the increased *GIA* to persons. Since only normal growth is assumed for real *GIA* for the forecast, only normal growth was assumed for S&L transfer payments to persons.

Table 6 Inflation Forecasts for 2021.2–2023.4 Using Various Price Equations

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
2021.1[a]	3.7	3.7	3.7	3.7	3.7	3.7	3.7
2021.2	2.7	2.5	2.3	2.1	1.6	2.7	3.2
2021.3	3.4	2.7	2.8	2.7	2.5	3.5	5.2
2021.4	3.4	2.6	3.1	3.2	3.5	3.8	6.6
2022.1	3.6	2.6	3.4	3.5	4.2	3.9	6.9
2022.2	3.7	2.5	3.5	3.7	4.6	3.9	6.8
2022.3	3.9	2.5	3.7	3.7	4.8	3.9	6.6
2022.4	4.0	2.5	3.8	3.7	4.8	3.8	6.0
2023.1	4.2	2.5	3.8	3.6	4.6	3.7	5.3
2023.2	4.4	2.5	2.8	3.5	4.3	3.6	4.9
2023.3	4.5	2.4	3.7	3.5	4.1	3.5	4.6
2023.4	4.7	2.4	3.7	3.5	4.0	3.5	4.3

- [a]Actual
- Inflation is the percentage change in *PF* at an annual rate.
- Price equations are as follows:
- (1): Table 1 (1) *UR*
- (2): Table 1 (2) *UR*
- (3): Table 1 (3) *UR*
- (4): Table 2 (1) *UR*
- (5): Table 2 (2) $1/UR$
- (6): Table 2 (3) *GAP*
- (7): Table 2 (4) $1/(GAP + .07)$

absolute value in 2021.1.) In addition, *TR* is large from the ARPA. The predicted output growth rate is also large in 2021.3 and 2021.4 at 9.6 and 6.3 percent respectively. This is from the continuing wealth effects and the continuing large transfer payments. The output gap becomes negative in 2021.2, falling from 0.022 to -0.001. By 2021.4 it is -0.024. The unemployment rate falls from 6.2 percent in 2021.1 to 5.5 percent in 2021.2. By 2022.1 it is down to 3.6 percent. None of this is, of course, surprising. The U.S. economy has had a huge fiscal stimulus, a huge increase in financial and housing wealth, and an accommodating monetary policy.

The forecast details are on my website, but it is instructive to give a few more details here. Comparing 2021.1 to 2019.4, private jobs fell by 7.93 million, government jobs fell by 1.12 million, and the number of people holding two jobs (moonlighters) fell by 1.55 million. The number of people employed, which is jobs minus moonlighters, thus fell by 7.50 million. Had there been no change in the labor force, the number of people unemployed would have increased by 7.50 million. In fact it increased by only 4.09 million because the labor force fell by 3.41 million. The unemployment rate rose from 3.6 percent to 6.2 percent.

How fast is the economy forecasted to come back? Comparing the forecast values for 2022.1 to the actual values in 2021.1, private jobs rose by 5.78 million, government jobs rose by 0.24 million, and moonlighters rose by 0.97 million. The number of people employed thus rose by 5.05 million. The labor force rose by 0.96 million, so the number of people unemployed fell by 4.09 million. The unemployment rate fell from 6.2 percent to 3.6 percent. Had the labor force been forecast to come back to where it was, the fall in the unemployment would obviously been less. One of the reasons for the small forecasted rise in the labor force relative to how much it fell is that household wealth has a negative effect on labor supply in the labor force participation equations, and, as noted above, there are large increases in household wealth. The labor force is not back to its 2019.4 value until 2023.3. The number of private jobs is back by 2022.3.

5 Inflation forecasts

Given the unemployment rate values in Table 5, what are the inflation forecasts? The first three columns in Table 6 present the forecasts using the three equations in Table 1. Although the first two equations are rejected by the data, it is of interest to see what they imply. Equation (5) has an increasing inflation rate, from 2.7 percent in 2021.2 to 4.7 percent in 2023.4. Equation (5) with π_{t-1} added has a roughly constant inflation rate at about 2.5 percent. Equation (5) with π_{t-1} and p_{t-1} added has an inflation rate rising to 3.4 percent in 2022.1 and then leveling out at about 3.7 percent. The low inflation rate forecasts from equation (5) with π_{t-1} added are low in part because the coefficient on *UR* (Table 1) is fairly low in absolute value.

Presented next in Table 6 are four inflation forecasts from the US model, using the four price equations in Table 3. Each of the four forecasts corresponds to a slightly different estimated wage rate equation because the coefficient constraint uses the estimates from the price equation. Also, each forecast corresponds to slightly different unemployment rate and gap forecasts because the two variables are endogenous. However, these differences are small across the four forecasts. Column (4) contains the forecast using *UR* as the explanatory variable in the price equation. These forecast values are similar to those in column (3) since the two price equations are similar—both use the level of the unemployment rate and both are in level form. Column (5) is for $1/UR$ as the explanatory variable in the price equation. Remember that this is the best fitting equation. After the first two quarters the inflation forecasts in column (5) are larger than those in column (4), which uses *UR*. By the middle of 2022 they are about 1 percentage point higher, with an inflation rate of 3.7 percent. Given the low values of the unemployment rate, the nonlinearity is predicting more inflation.

Columns (6) and (7) use *GAP* and $1/(GAP + .07)$. The forecast values using *GAP* are slightly higher than those using *UR*, although the forecasts using $1/(GAP + .07)$ are much higher than those using $1/UR$. By the end of 2021 the inflation rate is up to 6.6 percent using $1/(GAP + .07)$. Probably less weight should be put on the *GAP* results since the equations do not fit quite as well. This does, however, show the fragility of macroeconometric research. While the fits are fairly close, the implications are quite different.

In Table 6 the most weight should probably be place on column (5), which uses $1/UR$ in the price equation. This gives the best fit, and $1/UR$ is better than *UR* when both are included in the equation. The reason for the low inflation forecasts for the first two quarters is that the unemployment rate is still fairly high. Once the unemployment rate gets down to about 3.5 percent, the inflation forecasts increase to over 4 percent. They are coming down at the end, but slowly. An interesting question is if this turns out to be the case, will the Fed step in and if so how effective will it be? This question is examined in Section 7. Another interesting question is how uncertain are these forecasts? What are the standard errors, and what is the probability of inflation getting much higher, like 6 percent? This question is examined next.

6 Stochastic Simulation

Stochastic simulation can be used to estimate the uncertainty of the above forecasts. The US model consists of 23 estimated equations, not counting the estimated Fed rule. It is estimated by 2SLS for the 1954.1–2019.4 period, 264 quarters. Thus for each estimated equation there are 264 estimated residuals.[11] In addition, two other estimated equations were added. In the model the price of imports (*PIM*) and the relative price of housing (*PSI*14) are exogenous. For the first equation the log change in *PIM* was regressed on a constant, and for the second equation the log change in *PSI*14 was regressed on a constant. Adding these two equations to the model allows the uncertainty from the two to affect the overall uncertainty estimates. *PIM* is like an asset price in that it is affected by oil prices and exchange rates. Similarly, the relative price of housing is an asset price. The expanded model thus has 25 estimated equations. Let $\hat{u}_t$ denote the 25-dimension vector of estimated residuals for quarter t, $t = 1, ..., 264$. The $\hat{u}_t$ error terms are after adjustment for any autoregressive properties, and they are taken to be *iid* for purposes of the draws.

The solution period is 2021.2–2023.4, 11 quarters. The model was solved 10,000 times for this period. Each trial is as follows. First, 11 error vectors are drawn with replacement from the 264 error vectors $\hat{u}_t$, $t = 1, ..., 264$. These errors are added to the equations and the model is solved dynamically for the 2021.2–2023.4 period. The predicted values are recorded. This is one trial. This procedure is then repeated 10,000 times, which gives 10,000 predicted values of each variable. The mean and standard error and other measures can then be computed for each variable. See Sections 2.6 and 2.7 in MM for more details. When this was done there were 80 solution errors, and in these cases the trial was skipped. There are thus 9,920 trials. This means that the uncertainty estimates are at least slightly too low since the solution errors are due to extreme draws.

Results are reported here for four variables: *UR* four and eight quarters ahead and the four-quarter percentage change in *PF* for the first and second four-quarter periods, 2022.1–2021.1 and 2023.1–2022.1. For *UR* to two predicted values are 3.63 and 3.24 with standard errors of 0.75 and 1.03. For the four-quarter ahead percentage changes in *PF* the two predicted values are 2.95 and 4.70 with standard errors of 1.29 and 3.20. There is thus more uncertainty in the inflation forecasts than in the unemployment rate forecasts.

[11] If the initial estimate of an equation suggests that the error term is serially correlated, the equation is reestimated under the assumption that the error term follows an autoregressive process (usually first order). The structural coefficients in the equation and the autoregressive coefficient or coefficients are jointly estimated (by 2SLS).

Table 7 Forecasts for 2021.2–2023.4 Using the Fed Rule

	Estimated Fed Rule				No Fed Rule			
	RS	%Δ*Y*	*UR*	%Δ*PF*	*RS*	%Δ*Y*	*UR*	%Δ*PF*
2021.1[a]	0.1	7.4	6.2	3.7	0.1	7.4	6.2	3.7
2021.2	0.8	12.6	5.5	1.6	0.1	12.8	5.5	1.6
2021.3	1.9	8.9	4.7	2.5	0.1	9.6	4.7	2.5
2021.4	2.6	5.1	4.2	3.3	0.1	6.3	4.0	3.5
2022.1	3.0	2.7	3.9	3.7	0.1	4.2	3.6	4.2
2022.2	3.2	1.8	3.8	3.8	0.1	3.4	3.4	4.6
2022.3	3.4	1.6	3.8	3.7	0.1	3.3	3.2	4.8
2022.4	3.5	1.1	3.9	3.4	0.1	2.6	3.2	4.8
2023.1	3.5	1.1	4.1	3.1	0.1	2.5	3.2	4.6
2023.2	3.5	1.5	4.3	2.8	0.1	2.7	3.3	4.3
2023.3	3.5	1.9	4.4	2.7	0.1	2.9	3.3	4.1
2023.4	3.6	2.1	4.5	2.6	0.1	2.9	3.3	4.0

- [a]Actual
- *RS* = three month Treasury bill rate
- %Δ*Y* = percentage change in real output, annual rate
- %Δ*PF* = percentage change in *PF*, annual rate

According to these results, how likely is it that inflation will be quite high. If one takes "quite high" as the four-quarter percentage change in *PF* in the second four-quarter period greater or equal to 6 percent, there were 3,131 trials in which this was true, or 0.316 percent. This reflects the fact that there is considerable uncertainty in the second four-quarter forecast of inflation.

7 Fed Response

For the above forecasts the Fed is assumed to keep the short term interest rate at essentially zero. There is an estimated Fed rule in the US model, which has been turned off. The estimated rule is a "leaning against the wind" rule, where the interest rate rises as inflation rises and unemployment falls. In practice if the inflation numbers are as in column (5) in Table 6, the Fed is likely to respond by raising the interest rate. How effective would this be in lowering inflation? This can be examined in the model by turning the rule back on. Table 7 presents a forecast in which the rule is added to the model from the beginning of the forecast period. . The price equation used is the one with $1/UR$ as the demand variable.

As expected, the results in Table 7 show that given the low values of the unemployment rate and the high values of inflation, the Fed rule calls for an increase in the interest rate. The rate is 0.8 percent in 2021.2, the 1.9 percent, 2.6 percent, and then 3.0 percent in 2022.1. The unemployment rate is higher and inflation is lower, but not by much. What these results show, which is a property of the model, is that the Fed has limited ability to affect the inflation rate. The Fed is currently saying that it has the tools needed to stop high inflation if it gets started, but not according to the model. It is clear in the model why this is true. If inflation expectations depend only on past inflation, the only way the Fed can change expectations over time is by changing actual inflation. Actual inflation is changed by changing the unemployment rate (or the output gap).

To get a sense of how effective monetary policy is in changing output, the unemployment rate, and inflation, I ran the following experiment. For the forecast period, 2021.2–2023.4, I increased *RS* from the base path by 1 percentage point (the Fed rule obviously dropped). The percentage decreases in real output for the 11 quarters are: 0.06, 0.18, 0.33, 0.47, 0.59, 0.69, 0.77, 0.83, 0.88, 0.92, and 0.96. There is thus about a half a percentage point decrease after 4 quarters and about a full percentage point after 11 quarters. The effects build slowly. The unemployment rate increases are (in percentage points): 0.01, 0.04, 0.10, 0.15, 0.21, 0.25, 0.29, 0.31, 0.33, 0.34, and 0.35. The unemployment rate thus rises by about a third of a percentage point for a 1 percentage point increase in *RS*, but it takes about two years to reach this. The percentage point decreases in inflation are: 0.01, 0.05, 0.15, 0.29, 0.47, 0.54, 0.59, 0.59, 0.56, 0.52, and 0.49. The effects on inflation are thus about a half percentage point fall for a 1 percentage point increase in *RS*, but it takes about 5 quarters to achieve this. The results in Table 7 are thus not surprisng given these effects.

8 Conclusion

The main results are:

1. The data suggest that price equations should be specified in level form rather than in first or second difference form (Table 1).
2. The is some slight evidence of nonlinear demand effects on prices in that $1/UR$ gives slightly better results than UR (Table 3).
3. There is no evidence that demand effects have gotten smaller over time.
4. The stimulus from the American Rescue Plan Act combined with large wealth effects from past household saving, rising stock prices, and rising housing prices is large and it is forecast to drive the unemployment rated down to below 3.5 percent by the middle of 2022 (Table 5).
5. Given this stimulus, the inflation rate is forecast to rise to slightly under 5 percent by the middle of 2022 and comes down slowly. If UR is used in the price equation rather than $1/UR$, the inflation rate rises to slightly under 4 percent (Table 6).
6. There is considerable uncertainty in the point forecasts, especially two years out. The probability that inflation will be larger than 6 percent next year is estimated to be 31.6 percent.
7. If the Fed were behaving as historically estimated by the Fed rule, it would raise the interest rate to about 3 percent by the end of 2021 and 3.5 percent by the end of 2022. This would lower output growth, raise the unemployment rate, and lower inflation, although lowering inflation takes time. By the middle of 2022 inflation is about 1 percentage point lower. By the end of 2023 it is 1.4 percentage points lower (Table 7). The only tool the Fed has to lower inflation according to the model is to increase the unemployment rate by raising interest rates. This effect is modest and takes time.

The estimated price equations do not take into account any special features of the pandemic. They are estimated through 2019.4 and then used to forecast 2021.2 and beyond. If there are unusual supply constraints, pandemic related, this might lead to the forecasts of inflation for, say, the second and third quarters of 2021 being too low. For example, the 1.6 and 2.5 inflation rates in column (5) in Table 6 for 2021.2 and 2021.3, could be too low. If one subjectively adjusted the price equations to have higher inflation rates in 2021.2 and 2021.3, the story in this paper would be the same except with higher future inflation rates.

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