Chapter Four

Households

4.1 THE BASIC EQUATIONS

In Table 4-1 the important symbols used in this chapter are listed in alphabetic order. Each household receives wage income from firms and the government ($WH_{it}HPH_{it}$), purchases goods from firms (XH_{it}), and pays taxes to the government ($TAXH_{it}$). All goods that are purchased in a period are consumed in that period. A household either has a positive amount of savings or is in debt. If it has savings, the savings can take the form of demand deposits (DDH_{it}), savings deposits (SDH_{it}), or stocks (s_{it}). If it is in debt, the debt takes the form of loans from banks (LH_{it}). It is assumed that a household does not both borrow from banks and have savings deposits or stocks at the same time.

At the beginning of period t, each household receives information on the rate that it will be paid on its savings deposits in the period (the bill rate, r_t), on the aggregate stock price for the period (PS_t), on the loan rate that it will be charged (RH_{it}), on the maximum amount of money that it will be able to borrow ($LHMAX_{it}$), on the price that it will be charged for goods (PH_{it}), on the wage rate that it will be paid (WH_{it}), on the maximum number of goods that it will be able to purchase ($XHMAX_{it}$), and on the maximum number of hours that it will be able to be paid for ($HPHMAX_{it}$). The two main decision variables of a household are the number of hours to work (HPH_{it}), and the number of goods to purchase (XH_{it}).

Table 4–1. Notation for Households in Alphabetic Order

Non-Condensed Model

Subscript i denotes variable for household i. Subscript t denotes variable for period t. An e superscript in the text denotes an expected value of the variable.

A_{it}	= value of non-demand-deposit assets or liabilities
CG_{it}	= capital gains or losses on stocks
d_3	= personal tax rate
DDH_{it}	= demand deposits
DIV_t	= total dividends paid and received in the economy
$DIVH_{it}$	= dividends received by the household
HPH_{it}	= number of hours that the household is paid for
$HPHMAX_{it}$	= maximum number of hours that the household can be paid for
$HPHUN_{it}$	= unconstrained supply of hours of the household
LH_{it}	= value of loans taken out
$LHMAX_{it}$	= maximum value of loans that the household can take out
$LHUN_{it}$	= unconstrained demand for loans of the household
PH_{it}	= price paid for goods
PS_t	= price of the aggregate share of stock
r_t	= bill rate
RH_{it}	= loan rate paid
S_{it}	= fraction of the aggregate share of stock held
SAV_{it}	= savings net of capital gains or losses
SDH_{it}	= savings deposits
$TAXH_{it}$	= taxes paid
WH_{it}	= wage rate received
XH_{it}	= number of goods purchased
$XHMAX_{it}$	= maximum number of goods that the household can purchase
$XHUN_{it}$	= unconstrained demand for goods of the household
YG	= minimum guaranteed level of income (also can be thought of as the level of transfer payments to each household)
YH_{it}	= before-tax income excluding capital gains or losses

Condensed Model (For equations in Table 4–6 only.)

Subscript t denotes variable for period t. Superscript p in Table 4–6 denotes a planned value of the variable, and superscript e denotes an expected value of the variable. Asset variables pertain to household 1; liability variables pertain to household 2. Only the notation that differs from the notation for the non-condensed model is presented here.

CG_t	= capital gains or losses on stocks (household 1)
LH_t	= value of loans taken out (household 2)
$LHMAX_t$	= maximum value of loans that the household can take out (household 2)
$LHUN_t$	= unconstrained demand for loans of the household (household 2)
P_t	= price paid for goods.
RL_t	= loan rate paid
SD_t	= savings deposits (household 1)

$SDUN_t$ = unconstrained savings deposits of household 1 (corresponding to $HPHUN_{1t}$ and $XHUN_{1t}$)
W_t = wage rate received

The basic equations for household i for period t are the following:

$$CG_{it} = (PS_{t+1} - PS_t)S_{it}, \quad \text{[capital gains or losses on stocks]} \tag{4.1}$$

$$YH_{it} = WH_{it}HPH_{it} + r_tSDH_{it} + DIVH_{it}, \quad \text{[before-tax income net of capital gains or losses]} \tag{4.2}$$

$$TAXH_{it} = d_3(YH_{it} + CG_{it} - RH_{it}LH_{it}) - YG, \quad \text{[taxes paid]} \tag{4.3}$$

$$DDH_{it} = \gamma_1 PH_{it}XH_{it}, \quad \text{[demand deposits]} \tag{4.4}$$

$$SAV_{it} = YH_{it} - TAXH_{it} - RH_{it}LH_{it} - PH_{it}XH_{it}, \quad \text{[savings net of capital gains or losses]} \tag{4.5}$$

$$SDH_{it} - LH_{it} = SDH_{it-1} - LH_{it-1} - (DDH_{it} - DDH_{it-1}) + SAV_{it} - PS_t(S_{it} - S_{it-1}), \quad \text{[equation determining savings deposits or loans]} \tag{4.6}$$

$$A_{it} = SDH_{it} + PS_{t+1}S_{it} - LH_{it}, \quad \text{[total value of non-demand-deposit assets or liabilities at the end of period } t\text{]} \tag{4.7}$$

$$LH_{it} \leqslant LHMAX_{it}, \quad \text{[loan constraint]} \tag{4.8}$$

$$XH_{it} \leqslant XHMAX_{it}, \quad \text{[goods constraint]} \tag{4.9}$$

$$HPH_{it} \leqslant HPHMAX_{it}. \quad \text{[hours constraint]} \tag{4.10}$$

Equation (4.1) defines the capital gains or losses that are recorded for period t on the fraction of the aggregate share of stock held by household i in period t. PS_{t+1} is the value of the aggregate share of stock at the end of period t or the beginning of period $t+1$. S_{it} is the fraction of the aggregate share of stock held by household i in period t. Equation (4.2) defines before-tax income net of capital gains or losses. If the household is a debtor, then the last two terms are zero. Equation (4.3) defines taxes paid. d_3 is the (proportional) personal income tax rate, and YG is the minimum guaranteed level of income. Capital gains or losses are assumed to be taxed as regular income, and interest payments are assumed to be tax deductible. The tax parameter YG, which will be called the "minimum guaranteed level of income" in this study, can also be thought of as the level of transfer payments from the government to each household.

Equation (4.4) defines demand deposits. The demand deposit need of a household is assumed to be proportional to the value of goods purchased. Households are assumed to hold no demand deposits except those necessary for transactions purposes. Equation (4.5) defines savings net of capital gains or losses, and Equation (4.6) determines savings deposits or loans. The last term in Equation (4.6) is the amount of money that household i spends (receives) on stock purchases (sales) in period t. Equation (4.7) defines total non-demand-deposit assets or liabilities as of the end of period t or the beginning of period $t+1$. Household i is subject to the three constraints (4.8)-(4.10).

4.2 THE FORMATION OF EXPECTATIONS

Let $N+1$ denote the expected remaining length of household i's life. Household i is assumed to form the following expectations.

$$r^e_{t+k} = r_t, \quad \text{[expected bill rate for period } t+k \ (k=1,2,\ldots)] \tag{4.11}$$

$$RH^e_{it+k} = RH_{it}, \quad \text{[expected loan-rate for period } t+k \ (k=1,2,\ldots,N)] \tag{4.12}$$

$$PH^e_{it+k} = PH_{it}, \quad \text{[expected price for period } t+k \ (k=1,2,\ldots,N)] \tag{4.13}$$

$$WH^e_{it+k} = WH_{it}, \quad \text{[expected wage rate for period } t+k \ (k=1,2,\ldots,N)] \tag{4.14}$$

$$DIV^e_{t+k} = \tfrac{1}{5}(DIV_{t-1} + DIV_{t-2} + DIV_{t-3} + DIV_{t-4} + DIV_{t-5}), \quad \text{[expected aggregate level of dividends for period } t+k \ (k=0,1,\ldots)] \tag{4.15}$$

$$PS^e_{t+k} = \frac{DIV^e_{t+k}}{(1+r^e_{t+k})} + \frac{DIV^e_{t+k+1}}{(1+r^e_{t+k})(1+r^e_{t+k+1})} + \frac{DIV^e_{t+k+2}}{(1+r^e_{t+k})(1+r^e_{t+k+1})(1+r^e_{t+k+2})} + \ldots \quad \text{[expected stock price for period } t+k \ (k=1,2,\ldots,N)]$$

$$= \frac{DIV^e_t}{r_t} \quad . \qquad \text{[from (4.11) and (4.15)]} \tag{4.16}$$

Equations (4.11)-(4.14) state that household i expects that the future values of the bill rate, the loan rate that it will be charged, the price that it will be charged, and the wage rate that it will be paid will be equal to the last observed values of the variables. These assumptions of no change expected from the last observed value are consistent with the aim of keeping the expectational assumptions as simple as possible in the model. Since banks determine optimal loan rate paths and since firms determine optimal price and wage paths, it would have been possible to assume that banks and firms inform households of the planned future values in addition to the current values. As was the case for firms and the loan rates, it seemed more straightforward in this case just to assume that the households make the expectations themselves.

In Equation (4.15), household i is assumed to average the past five dividend levels and to expect that the future dividend levels will be equal to this average. The level of dividends is a fairly erratic variable (being a residual of sorts), and this is the reason for the averaging. In Equation (4.16) the expected stock price is assumed to be equal to the present discounted value of the expected future dividend levels, the discount rates being the expected future bill rates. As will be seen in the next chapter, this is the same formula that is used by the bond dealer to set the actual stock price. Because of (4.11) and (4.15), Equation (4.16) means that household i expects that all the future values of the stock price will be the same and will be equal to the expected dividend level for period t divided by the bill rate for period t.

One minor point regarding the expectations of future stock prices in (4.16) should be noted. Since firms are assumed in Chapter Three to maximize the present discounted value of expected future after-tax *cash flow* and since households are assumed in (4.16) to base their expectations of stock prices on expected future *dividends,* firms do not behave so as to maximize the value of their stocks outstanding. This is also true because firms are assumed to use the loan rate as their discount rate, whereas households are assumed in (4.16) to use the bill rate. These differences are, however, fairly minor, and it is easier to specify the model in this way than it is to have the objective function of firms be the value of their stocks outstanding.

4.3 BEHAVIORAL ASSUMPTIONS

The objective of a household is to maximize the present discounted value of its expected remaining lifetime utility. Utility in any period is assumed to be a negative function of hours worked in the period and a positive function of consumption. The form of the utility function is taken to be the log of the CES function:

$$U_{it+k} = \log\left[\eta_i(\overline{HPH} - HPH_{it+k})^{-\rho_i} + (1-\eta_i)\,XH_{it+k}^{-\rho_i}\right]^{-\frac{1}{\rho_i}}, \tag{4.17}$$

where U_{it+k} denotes the utility of household i for period $t{+}k$ and $\overline{HPH}$ is the total number of hours in a period. $\overline{HPH} - HPH_{it+k}$ is the amount of leisure time that household i has in period $t{+}k$. The objective function of household i at the beginning of period t is assumed to be

$$OBJH_{it} = \frac{U_{it}}{1+RDH_i} + \frac{U_{it+1}}{(1+RDH_i)^2} + \ldots + \frac{U_{it+N}}{(1+RDH_i)^{N+1}}, \tag{4.18}$$

where RDH_i is the discount rate of household i. All problems associated with the fact that the lengths of the remaining lives of households are uncertain have been ignored here. Each household is assumed to expect that the length is $N+1$ and to behave as if the actual length were exactly this.

The household chooses HPH_{it+k} and XH_{it+k} $(k=0,1,\ldots,N)$ so as to maximize $OBJH_{it}$. One of the constraints facing a household is a lifetime budget constraint. This constraint is handled by assuming that household i plans to end its life with a particular level of non-demand-deposit assets or liabilities:

$$A^e_{it+N} = \bar{A}_i \,, \tag{4.19}$$

where A^e_{it+N} is the expected level of non-demand-deposit assets or liabilities for the end of period $t+N$ and $\bar{A}_i$ is the target level.

A household with positive non-demand-deposit assets can either hold its assets in the form of stocks or savings deposits. Because of Equation (4.16), a household expects the before-tax, one-period rate of return on stocks (including capital gains and losses) for a given period to be the same as the expected bill rate for that period. Since the bill rate is the rate paid on savings deposits and since capital gains and losses are taxed at the same rate as other income, a household then expects that the after-tax rates of return on stocks and savings deposits are the same. A household can therefore be assumed to be indifferent between holding its assets in the form of stocks or savings deposits, and one need not distinguish between stocks and savings deposits for purposes of analyzing a household's decision. The expected one-period rate of return on the non-demand-deposit assets held during period $t+k$, A_{it+k}, is r^e_{t+k} for a creditor household. Using this fact, Equations (4.1)–(4.7) can be rewritten for purposes of analyzing a creditor household's decision as follows:

$$YYH^e_{it+k} = WH^e_{it+k}\,HPH_{it+k} + r^e_{t+k}\,A^e_{it+k} \,, \tag{4.20}$$

[expected before-tax income including capital gains or losses for period $t+k$ $(k=0,1,\ldots,N)$]

$$DDH^e_{it+k} = \gamma_1 PH^e_{it+k}\,XH_{it+k} \,, \tag{4.21}$$

[expected level of demand deposits for period $t+k$ $(k=0,1,\ldots,N)$]

$$A^e_{it+k} = A^e_{it+k-1} - (DDH^e_{it+k} - DDH^e_{it+k-1}) + (1-d_3)\,YYH^e_{it+k} + YG - PH^e_{it+k}\,XH_{it+k} \tag{4.22}$$

[expected value of non-demand-deposit assets for the end of period $t+k(k=0,1,\ldots,N)$]

Equations (4.20)–(4.22) have been written to hold for all periods of the decision horizon, and e superscripts have been added to the relevant variables to denote the fact that household i only has expectations of the variables for periods beyond $t\text{-}1$ or t. For purposes of analyzing a debtor household's decision, equations (4.2)–(4.7) can likewise be rewritten

$$YH^e_{it+k} = WH^e_{it+k}\,HPH_{it+k}, \quad \text{[expected before-tax income for period } t+k\ (k=0,1,\ldots,N)] \tag{4.23}$$

$$DDH^e_{it+k} = \gamma_1 PH^e_{it+k}\,XH_{it+k}, \quad \text{[expected level of demand deposits for period } t+k\ (k=0,1,\ldots,N)] \tag{4.24}$$

$$\begin{aligned} LH^e_{it+k} = {} & LH^e_{it+k-1} + (DDH^e_{it+k} - DDH^e_{it+k-1}) \\ & - (1-d_3)(YH^e_{it+k} - RH^e_{it+k}\,LH^e_{it+k}) \\ & - YG + PH^e_{it+k}\,XH_{it+k}. \end{aligned} \quad \text{[expected value of loans for the end of period } t+k\ (k=0,1,\ldots,N)] \tag{4.25}$$

The terminal condition (4.19) for debtor households is merely $LH^e_{it+N} = -\overline{A}_i$.

The maximization problem of a household is easy to describe. Given a path of hours worked, HPH_{it+k}, and a path of consumption, XH_{it+k}, $(k=0,1,\ldots,N)$, the objective function can be computed directly. The two paths must satisfy the terminal condition (4.19). Given the two paths and given the expectations from (4.11)–(4.14), Equations (4.20)–(4.22) and (4.23)–(4.25) each form a set of three linear equations in three unknowns for each period, which can be solved through time to obtain a terminal value of non-demand-deposit assets or liabilities. The hours and consumption paths must be chosen so that the resulting terminal value of non-demand-deposit assets or liabilities is equal to $\overline{A}_i$. The hours and consumption paths must also, of course, be chosen to satisfy the inequality constraints (4.8)–(4.10). Regarding the possibility of the loan, goods, and hours constraints existing for periods beyond t, households were assumed to expect that the constraints would *not* be binding for periods beyond t. As was the case for firms, having the constraints hold only for period t appeared to have an important enough influence on the households' decision values for period t so as to make further restrictions unnecessary.

4.4 THE SOLUTION OF THE CONTROL PROBLEM

Two algorithms were written to solve the control problem of a household; one to search over different hours paths and one to search over different consumption paths, given an hours path. For each hours path chosen by the first algorithm, a submaximization problem was solved using the second algorithm. Particular importance was attached to searching over values for the first two periods. The three constraints were handled by throwing out as infeasible those paths that failed to meet one or more of the constraints. Whenever a particular constraint was not met, an alternative path was always tried in which the value of the variable in question was set equal to the constraint. Given an hours path, the consumption paths tried by the second algorithm were always chosen so as to satisfy the terminal condition.

4.5 SOME EXAMPLES OF SOLVING THE CONTROL PROBLEMS OF THE HOUSEHOLDS

PARAMETER VALUES AND INITIAL CONDITIONS

For purposes of the simulation work, two different households were considered; a creditor household (household 1) and a debtor household (household 2). The parameter values and initial conditions used for the first example for each household are presented in Table 4-2. The values of prices and wages were set equal to 1.0, the bill rate was set equal to 0.0650, and the loan rate was set equal to 0.0750. The only values for period $t-1$ needed for household 1 were A_{1t-1} and DDH_{1t-1}, and the only values for period $t-1$ needed for household 2 were LH_{2t-1} and DDH_{2t-1}. The discount rates for the two households were chosen, after some experimentation, to yield fairly constant paths of hours and consumption over the life of the households. The terminal condition for household 1 was taken to be the level of wealth in period $t-1$, and the terminal condition for household 2 was taken to be the negative of the value of loans held in period $t-1$. Neither household, in other words, was taken to be a net saver or dissaver over its remaining life. Household 2, for example, was assumed to plan to end its life in debt to the same extent that it was in period $t-1$. As with banks and firms, this was done to make it easier to analyze the effects on the behavior of the households of changing various initial conditions. The values of ρ_1 and ρ_2 were chosen to make the supply of labor on the part of the two households a positive function of the wage rate.

THE RESULTS

The results of solving the control problems of households 1 and 2 are presented in Tables 4-3 and 4-4, respectively. Values of hours, consumption, and expected assets or liabilities are presented for the first two periods of the 30-period decision horizon.

Consider the behavior of household 1 in Table 4-3 first. The first set of results in the table is based on the assumption that no constraints were binding on the household. The results in the first row are based on the parameter values and initial conditions in Table 4-2. For this run the household essentially

Table 4-2. Parameter Values and Initial Conditions for the Control Problems of Households 1 and 2

	Parameter	*Value*
	$N+1$	30
	$\overline{HPH}$	1143.45
	η_1	0.5808
	η_2	0.5811
	ρ_1	−0.3
	ρ_2	−0.3
	γ_1	0.1609
	d_3	0.1934
	YG	0.0
	RDH_1	0.0603
	RDH_2	0.0695
	Variable	*Value*
Household 1		
	A_{1t-1}	2159.8
	DDH_{1t-1}	60.1
	r_t	0.0650
	PH_{1t}	1.0
	WH_{1t}	1.0
	$\overline{A}_1$	2159.8
Household 2		
	LH_{2t-1}	482.1
	DDH_{2t-1}	51.8
	RH_{2t}	0.0750
	PH_{2t}	1.0
	WH_{2t}	1.0
	$\overline{A}_2$	−482.1

chose a flat path of the variables throughout its remaining life, a result that reflects the way the parameter values and initial conditions were chosen in the first place.

An important set of reactions of a household is how it responds to changes in wages and prices. For the results in row 2 in Table 4-3, the wage rate of household 1 for period t was increased by 5.0 percent. This meant that the household expected its wage rates for periods $t+1$ and beyond to be higher by 5.0 percent as well. This change caused the household to work more (+3.4 percent) and consume more (+5.7 percent) in period t and likewise in future periods as well. The planned or expected level of wealth for period t decreased slightly.

The rest of the results in Table 4-3 are fairly self-explanatory. Decreasing the wage rate (row 3) caused the household to work less and consume less, as did increasing the price level (row 4). Decreasing the price level (row 5) caused the household to work more and consume more. It is interesting to note that with respect to its hours worked the household responded slightly more to changes in the wage rate than to changes in the price level, and with respect to its consumption slightly more to changes in the price level than to

Table 4–3. Results of Solving the Control Problem of Household 1

Initial Conditions from Table 4–2 except:	HPH_{1t}	HPH_{1t+1}	XH_{1t}	XH_{1t+1}	A^e_{1t}	A^e_{1t+1}
A. No Constraints Binding						
1. No exceptions	323.0	323.8	373.8	374.3	2160	2160
2. WH_{1t} = 1.05 (+5.0%)	334.0 (+3.4%)	333.8	395.0 (+5.7%)	395.5	2157	2158
3. WH_{1t} = 0.95 (−5.0%)	311.0 (−3.7%)	311.8	352.6 (−5.7%)	352.5	2162	2162
4. PH_{1t} = 1.05 (+5.0%)	317.0 (−1.9%)	316.8	352.0 (−5.8%)	351.3	2160	2160
5. PH_{1t} = 0.95 (−5.0%)	330.0 (+2.2%)	329.8	399.6 (+6.9%)	399.1	2159	2159
6. r_t = 0.0683 (+5.0%)	334.8 (+3.7%)	333.3	365.8 (−2.1%)	365.6	2186	2211
7. r_t = 0.0618 (−5.0%)	310.0 (−4.0%)	311.8	380.9 (+1.9%)	381.1	2134	2110
8. d_3 = 0.2031 (+5.0%)	317.0 (−1.9%)	317.8	370.8 (−0.8%)	370.2	2154	2148
9. d_3 = 0.1837 (−5.0%)	329.0 (+1.9%)	327.8	377.9 (+1.1%)	377.1	2165	2170
10. YG = 10.0 (+10.0)	316.0 (−2.2%)	315.8	377.5 (+1.0%)	377.9	2160	2160
11. YG = −10.0 (−10.0)	332.0 (+2.8%)	330.8	370.4 (−0.9%)	370.2	2161	2161
12. A_{1t-1} = 2268 (+5.0%)	317.0 (−1.9%)	316.8	376.4 (+0.7%)	376.6	2265	2263
13. A_{1t-1} = 2052 (−5.0%)	329.0 (+1.9%)	328.8	371.1 (−0.7%)	371.6	2054	2055
B. $HPHMAX_{1t}$ = 306.8						
1. No exceptions	306.8	322.8	368.6	374.0	2152	2151
2. WH_{1t} = 1.05 (+5.0%)	306.8 (+0.0%)	333.8	384.2 (+4.2%)	395.8	2146	2144
3. WH_{1t} = 0.95 (−5.0%)	306.8 (+0.0%)	311.8	351.9 (−4.5%)	352.9	2160	2160
4. PH_{1t} = 1.05 (+5.0%)	306.8 (+0.0%)	315.8	349.3 (−5.2%)	351.7	2155	2153
5. PH_{1t} = 0.95 (−5.0%)	306.8 (+0.0%)	329.8	390.6 (+6.0%)	399.7	2149	2147
C. $XHMAX_{1t}$ = 350.0						
1. No exceptions	319.0	321.8	350.0	374.5	2186	2181
2. WH_{1t} = 1.05 (+5.0%)	327.0 (+2.5%)	332.8	350.0 (+0.0%)	396.1	2206	2200
3. WH_{1t} 0.95 (−5.0%)	311.0 (−2.5%)	311.8	350.0 (+0.0%)	352.5	2166	2165
4. PH_{1t} = 1.05 (+5.0%)	317.0 (−0.6%)	316.8	350.0 (+0.0%)	351.3	2162	2162
5. PH_{1t} = 0.95 (−5.0%)	322.0 (+0.9%)	327.8	350.0 (+0.0%)	399.8	2210	2202
D. $HPHMAX_{1t}$ = 306.8, $XHMAX_{1t}$ = 350.0						
1. No exceptions	306.8	321.8	350.0	374.3	2175	2170
2. WH_{1t} = 1.05 (+5.0%)	306.8 (+0.0%)	332.8	350.0 (+0.0%)	397.0	2188	2180
3. WH_{1t} = 0.95 (−5.0%)	306.8 (+0.0%)	311.8	350.0 (+0.0%)	352.1	2162	2162
4. PH_{1t} = 1.05 (+5.0%)	306.8 (+0.0%)	315.8	349.3 (−0.2%)	351.7	2155	2153
5. PH_{1t} = 0.95 (−5.0%)	306.8 (+0.0%)	328.8	350.0 (+0.0%)	400.0	2197	2189

changes in the wage rate. Because of wealth holdings and income taxes, one would not necessarily expect the response of a household to be symmetric with respect to wage and price changes. It is also important to note that the value of ρ_1 was chosen to make the supply of labor a positive function of the wage rate. Different results would be obtained for different values of ρ_1, as will be seen in the Cobb-Douglas case of ρ_1 equal to zero below.

Two other important sets of reactions of a household are how the household responds to changes in interest rates and tax rates. Increasing the bill rate for household 1 (row 6) had a positive effect on hours worked and caused the household to save more and consume less. Decreasing the bill rate (row 7) caused the household to work less, save less, and consume more. Because the

Table 4–4. Results of Solving the Control Problem of Household 2

A. No Constraints Binding Initial Conditions from Table 4–2 except:	HPH_{2t}	HPH_{2t+1}	XH_{2t}	XH_{2t+1}	LH_{2t}	LH^e_{2t+1}
1. No exceptions	435.0	434.8	321.7	321.5	482.1	482.1
2. $WH_{2t} = 1.05$ (+5.0%)	440.0 (+1.1%)	438.8	342.5 (+6.5%)	343.2	484.6	485.6
3. $WH_{2t} = 0.95$ (−5.0%)	430.0 (−1.1%)	430.8	301.3 (−6.3%)	301.2	479.6	479.8
4. $PH_{2t} = 1.05$ (+5.0%)	429.0 (−1.4%)	429.8	302.4 (−6.0%)	302.4	482.1	482.1
5. $PH_{2t} = 0.95$ (−5.0%)	440.0 (+1.1%)	440.8	343.3 (+6.7%)	343.4	483.3	483.2
6. $RH_{2t} = 0.0788$ (+5.0%)	452.0 (+3.9%)	449.8	313.3 (−2.6%)	313.2	458.5	436.7
7. $RH_{2t} = 0.0713$ (−5.0%)	418.3 (−3.8%)	420.3	332.3 (+3.3%)	333.3	508.0	533.1
8. $d_3 = 0.2031$ (+5.0%)	430.0 (−1.1%)	429.8	319.0 (−0.8%)	318.8	487.0	492.8
9. $d_3 = 0.1837$ (−5.0%)	440.0 (+1.1%)	439.8	324.5 (+0.9%)	324.3	477.2	471.3
10. $YG = 10.0$ (+10.0)	428.0 (−1.6%)	426.8	325.5 (+1.2%)	325.1	482.1	482.1
11. $YG = -10.0$ (−10.0)	443.0 (+1.8%)	442.8	318.6 (−1.0%)	318.1	482.1	482.1
12. $LH_{2t-1} = 506.2$ (+5.0%)	437.0 (+0.5%)	436.8	321.2 (−0.2%)	321.0	505.4	504.6
13. $LH_{2t-1} = 458.0$ (−5.0%)	434.0 (−0.2%)	433.8	322.9 (+0.4%)	322.9	458.8	459.6
B. $HPHMAX_{2t} = 413.2$						
1. No exceptions	413.2	434.8	313.6	321.4	490.7	492.6
2. $WH_{2t} = 1.05$ (+5.0%)	413.2 (+0.0%)	440.8	330.9 (+5.5%)	342.9	494.5	496.0
3. $WH_{2t} = 0.95$ (−5.0%)	413.2 (+0.0%)	428.8	297.2 (−5.2%)	300.4	488.3	490.3
4. $PH_{2t} = 1.05$ (+5.0%)	413.2 (+0.0%)	429.8	297.0 (−5.3%)	301.9	488.6	489.4
5. $PH_{2t} = 0.95$ (−5.0%)	413.2 (+0.0%)	441.8	332.7 (+6.1%)	343.4	493.8	495.3
C. $XHMAX_{2t} = 300.0$						
1. No exceptions	431.0	433.8	300.0	322.5	458.7	462.9
2. $WH_{2t} = 1.05$ (+5.0%)	432.0 (+0.2%)	437.8	300.0 (+0.0%)	343.6	439.3	446.2
3. $WH_{2t} = 0.95$ (−5.0%)	431.0 (+0.0%)	430.8	300.0 (+0.0%)	300.3	477.2	476.3
4. $PH_{2t} = 1.05$ (+5.0%)	429.0 (−0.5%)	429.8	300.0 (+0.0%)	302.4	479.0	479.2
5. $PH_{2t} = 0.95$ (−5.0%)	434.0 (+0.7%)	438.8	300.0 (+0.0%)	344.3	437.6	444.4
D. $LHMAX_{2t} = 458.0$						
1. No exceptions	445.0	432.8	309.1	323.1	458.0	462.3
2. $WH_{2t} = 1.05$ (+5.0%)	450.0 (+1.1%)	436.8	328.3 (+6.2%)	343.7	458.0	462.3
3. $WH_{2t} = 0.95$ (−5.0%)	439.0 (−1.3%)	428.8	289.7 (−6.3%)	301.7	458.0	461.0
4. $PH_{2t} = 1.05$ (+5.0%)	439.0 (−1.3%)	426.8	290.5 (−6.0%)	303.3	458.0	462.4
5. $PH_{2t} = 0.95$ (−5.0%)	450.0 (+1.1%)	437.8	329.1 (+6.5%)	345.2	458.0	463.3
E. $HPHMAX_{2t} = 413.2$, $XHMAX_{2t} = 300.0$, $LHMAX_{2t} = 458.0$						
1. No exceptions	413.2	434.8	287.1	322.4	458.0	463.5
2. $WH_{2t} = 1.05$ (+5.0%)	413.2 (+0.0%)	438.8	300.0 (+4.5%)	343.4	456.2	463.1
3. $WH_{2t} = 0.95$ (−5.0%)	413.2 (+0.0%)	428.8	272.7 (−5.0%)	301.4	458.0	463.5
4. $PH_{2t} = 1.05$ (+5.0%)	413.2 (+0.0%)	438.8	273.4 (−4.8%)	303.3	458.0	463.7
5. $PH_{2t} = 0.95$ (−5.0%)	413.2 (+0.0%)	438.8	300.0 (+4.5%)	344.2	455.4	463.3

terminal condition states that household 1 must end its life with the same level of assets that it started with, changes in savings in the current period corresponding to changes in the bill rate eventually reversed themselves during the expected life of the household. Increasing the proportional tax rate (row 8) caused the household to work less and consume less, and decreasing the tax rate (row 9) had the opposite effect. Increasing the minimum guaranteed level of income (row 10) caused the household to work less and consume more, and decreasing the level (row 11) had the opposite effect. (When the tax parameters d_3 and YG were changed for these experiments, the changes were assumed to be permanent. In other words, the household was assumed to expect that the new value of the tax parameter would persist throughout its remaining lifetime.)

Tax decreases in the form of a decrease in the tax rate thus have a positive effect on work effort, while tax decreases in the form of an increase in the minimum guaranteed level of income have a negative effect. This is, of course, as expected because the parameter ρ_1 was chosen so that the (negative) income effect of a change in the wage rate on work effort was smaller in absolute value than the (positive) substitution effect. Capital gains and losses affect the wealth of creditor households, and so it is of interest to examine the effect of wealth changes on household 1. Increasing wealth of the previous period (row 12) caused household 1 to work less and consume more, and decreasing wealth (row 13) had the opposite effect.

For the second set of results in Table 4-3, household 1 was constrained in the number of hours it could work in period t. This constraint led it to work as much as it was allowed in period t--which was always less than the unconstrained amount--and to consume less. The values for period $t+1$ were much less affected. The household's responses in period t to changes in wages and prices were zero in terms of hours worked, but the household still responded in terms of the number of goods consumed. An increase in the wage rate of 5.0 percent, for example, led it to increase its consumption by 4.2 percent. This figure compares to 5.7 percent for the unconstrained case.

For the third set of results in Table 4-3, household 1 was constrained in the number of goods it could purchase in period t. For all five runs, this constraint led it to purchase the maximum number of goods it was allowed in period t. In two of the cases (rows 3 and 4) it worked the same as in the unconstrained case, but in the other three cases it worked less. Again, the values for period $t+1$ were much less affected.

For the fourth set of results in Table 4-3, both constraints were imposed on household 1. For all five runs this caused it to work the maximum number of hours allowed and to consume, with one exception, the maximum number of goods allowed. In this case, changing wages and prices merely changed how much the household saved in period t.

The results in Table 4-4 for household 2 are similar to the results in Table 4-3 for household 1 and require little further discussion. For household 2, an increase in savings means, of course, a decrease in loans. For the fourth set of results in Table 4-4, household 2 was constrained in the value of loans that it could take out for period t. For all five runs this caused it to work more and consume less in period t than in the corresponding unconstrained case. In all five cases the household chose to borrow the maximum amount of money that it was allowed for period t. For the fifth set of results in Table 4-4, all three constraints were imposed on household 2 for period t. When this happens, only two of the three constraints are really binding on the household, since given values for two of the three decision variables for period t the value of the other variable is automatically determined.

As mentioned above, the choice of the values of ρ_i in the CES utility function is important in determining how household i responds to wage and price changes and to changes in the other relevant variables. The case of $\rho_i = 0$ corresponds to the Cobb-Douglas function, and some results using this function are presented in Table 4-5. In this case the work effort of households is less responsive to changes in wages and prices. In the simple case of one time period and no nonlabor income, it can be easily shown that for a Cobb-Douglas utility function in consumption and leisure, work effort is not a function of the wage rate.[b] Although the present situation is more complicated, it is still true that for the Cobb-Douglas function work effort does not respond very much to the wage rate or the price level.

Table 4-5. Results of Solving the Control Problems of Households 1 and 2 Based on a Cobb-Douglas Utility Function

Household 1						
Initial Conditions from Table 4-2 except	HPH_{1t}	HPH_{1t+1}	XH_{1t}	XH_{1t+1}	A^e_{1t}	A^e_{1t+1}
1. No exceptions	322.0	321.8	374.1	373.8	2159	2157
2. WH_{1t} 1.05 (+5.0%)	327.0 (+1.5%)	326.8	388.9 (+4.0%)	389.7	2159	2159
3. WH_{1t} 0.95 (−5.0%)	318.0 (−1.2%)	317.8	357.3 (−4.5%)	357.1	2162	2162
4. PH_{1t} 1.05 (+5.0%)	322.0 (+0.0%)	321.8	355.3 (−5.0%)	356.1	2160	2159
5. PH_{1t} 0.95 (−5.0%)	322.0 (+0.0%)	321.8	393.7 (+5.2%)	393.4	2159	2158
Household 2						
	HPH_{2t}	HPH_{2t+1}	XH_{2t}	XH_{2t+1}	LH_{2t}	LH^e_{2t+1}
1. No exceptions	434.0	433.8	321.0	321.7	482.1	483.3
2. WH_{2t} 1.05 (+5.0%)	433.0 (−0.2%)	432.8	337.4 (+5.1%)	338.1	484.6	485.6
3. WH_{2t} 0.95 (−5.0%)	436.0 (+0.5%)	435.8	305.3 (−4.9%)	305.1	479.6	479.8
4. PH_{2t} 1.05 (+5.0%)	434.0 (+0.0%	433.8	305.7 (−4.8%)	306.5	482.1	483.4
5. PH_{2t} 0.95 (−5.0%)	434.0 (+0.0%)	433.8	338.9 (+5.6%)	338.6	483.3	484.4

For the results in this table $\eta_1 = 0.6375$, $RDH_1 = 0.0558$, $\eta_2 = 0.6380$, $RDH_2 = 0.0644$.

From the results in Tables 4-3 and 4-4, the behavior of the households can be summarized as follows. The main characteristic of households is that they maximize subject to constraints imposed on them by firms, banks, and the government. Unconstrained, their work effort (for $\rho_i = -0.3$) and consumption respond positively to the wage rate and negatively to the price level. Constrained, there may be no response at all or a much smaller response. Unconstrained, an increase in the bill rate or the loan rate causes households to work more, consume less, and save more, and conversely for a decrease in the rates. Increasing (decreasing) taxes has a negative (positive) effect on consumption, but may increase or decrease work effort depending on whether the proportional tax rate is changed or the minimum guaranteed level of income is changed. For a creditor household, an increase in initial wealth has a negative effect on work effort and a positive effect on consumption, and a decrease in initial wealth has the opposite effect.

4.6 THE CONDENSED MODEL FOR HOUSEHOLDS

The household behavioral equations for the condensed model are presented in Table 4-6. Subscript 1 refers to household 1 and subscript 2 to household 2. The subscript 1 was dropped from the asset variables, since only household 1 has assets, and the subscript 2 was dropped from the liability variables, since only household 2 has liabilities. Also, the loan rate for period t is denoted as RL_t rather than RH_{it}, the price is denoted as P_t rather than PH_{it}, the wage rate is denoted as W_t rather than WH_{it}, and the level of savings deposits is denoted as SD_t rather than SDH_{it}. Since household 1 owns all the stock, the variable S_{it} can be dropped completely. The value of stocks held by household 1 is merely the price of the aggregate share of stock, PS_t.

Table 4-6. Household Equations for the Condensed Model

Household 1

$$DIV_t^e = \frac{1}{5}\,(DIV_{t-1} + DIV_{t-2} + DIV_{t-3} + DIV_{t-4} + DIV_{t-5})\,, \tag{1}$$

$$\begin{aligned} HPHUN_{1t} = {} & e^{10.18}(P_t)^{-0.41}\,(W_t)^{0.71}\,(r_t)^{0.77}\,(d_3)^{-0.38}\,(SD_{t-1} + PS_t)^{-0.38} \\ & - 0.80YG\,, \end{aligned} \tag{2}$$

$$\begin{aligned} XHUN_{1t} = {} & e^{3.44}\,(P_t)^{-1.27}\,(W_t)^{1.14}\,(r_t)^{-0.40}\,(d_3)^{-0.19}\,(SD_{t-1}+PS_t)^{0.14} \\ & + 0.36YG\,, \end{aligned} \tag{3}$$

$$\begin{aligned} SDUN_t^p = {} & \frac{1}{1-(1-d_3)r_t}\;[SD_{t-1} - (\gamma_1 P_t\,XHUN_{1t} - DDH_{1t-1}) \\ & + (1-d_3)\,(W_t HPHUN_{1t} + DIV_t^e) + YG - P_t XHUN_{1t}]\,, \end{aligned} \tag{4}$$

$$\begin{aligned} HPH_{1t} & = HPHUN_{1t} \text{ if } HPHUN_{1t} \leqslant HPHMAX_{1t} \\ & = HPHMAX_{1t} \text{ if } HPHUN_{1t} > HPHMAX_{1t}, \end{aligned} \tag{5}$$

If $HPHUN_{1t} > HPHMAX_{1t}$, then (6)

Table 4-6. (continued)

Household 1

$$SD_t^p = SDUN_t^p - 0.074\ SDUN_t^p \left(\frac{HPHUN_{1t} - HPHMAX_{1t}}{HPHUN_{1t}} \right),$$

If $HPHUN_{1t} \leq HPHMAX_{1t}$, then $SD_t^p = SDUN_t^p$,

$XH_{1t} = XHUN_{1t}$ if $HPHUN_{1t} \leq HPHMAX_{1t}$ and $XHUN_{1t} \leq XHMAX_{1t}$ (7)

$$= \frac{1}{\gamma_1 P_t + P_t} \left[-(1-(1-d_3)r_t)SD_t^p + SD_{t-1} + DDH_{1t-1} + (1-d_3)(W_t HPH_{1t} + DIV_t^e) + YG\right]$$

if $HPHUN_{1t} > HPHMAX_{1t}$ and the computed value of XH_{1t} does not exceed $XHMAX_{1t}$

$= XHMAX_{1t}$ if $HPHUN_{1t} \leq HPHMAX_{1t}$ and $XHUN_{1t} > XHMAX_{1t}$ or if $HPHUN_{1t} > HPHMAX_{1t}$ and the computed value of XH_{1t} above exceeds $XHMAX_{1t}$; SD_t^p is then recomputed in these two cases.

Household 2

$$HPHUN_{2t} = e^{7.28}(P_t)^{-0.25}(W_t)^{0.22}(RL_t)^{0.77}(d_3)^{-0.22}(LH_{t-1})^{0.07} - 0.75\ YG, \quad (1)'$$

$$XHUN_{2t} = e^{4.34}(P_t)^{-1.27}(W_t)^{1.28}(RL_t)^{-0.59}(d_3)^{-0.17}(LH_{t-1})^{-0.06} + 0.35\ YG, \quad (2)'$$

$$LHUN_t = \frac{1}{1-(1-d_3)RL_t}\left[LH_{t-1} + (\gamma_1 P_t XHUN_{2t} - DDH_{2t-1}) - (1-d_3)W_t HPHUN_{2t} - YG + P_t XHUN_{2t}\right], \quad (3)'$$

If $HPHUN_{2t} \leq HPHMAX_{2t}$, $XHUN_{2t} \leq XHMAX_{2t}$, and $LHUN_{2t} \leq LHMAX_t$, then

$HPH_{2t} = HPHUN_{2t}$, (4)'

$XH_{2t} = XHUN_{2t}$, (5)'

$LH_t = LHUN_t$. (6)'

Otherwise, the actual values are determined by the following algorithm:

[1] If $HPHUN_{2t} \leq HPHMAX_{2t}$, then $HPH_{2t}^p = HPHUN_{2t}$ and go to statement [8],

[2] $HPH_{2t}^p = HPHMAX_{2t}$,

[3] $LH_t^p = LHUN_t + 0.36\ LHUN_t \left(\frac{HPHUN_{2t} - HPHMAX_{2t}}{HPHUN_{2t}} \right)$,

[4] If $LH_t^p > LHMAX_t$, then $LH_t^p = LHMAX_t$,

[5] $XH_{2t}^p = \frac{1}{-\gamma_1 P_t - P_t}\left[-(1-(1-d_3)RL_t)LH_t^p + LH_{t-1} - DDH_{2t-1} - (1-d_3)W_t HPH_{2t}^p - YG\right]$,

[6] If $XH_{2t}^p > XHMAX_{2t}$, then $XH_{2t}^p = XHMAX_{2t}$ and

Table 4-6. (continued)

Household 2

$$LH_t^p = \frac{1}{1-(1-d_3)RL_t}\,[LH_{t-1} + \gamma_1 P_t XH_{2t}^p - DDH_{2t-1} - (1-d_3)W_t HPH_{2t}^p - YG + P_t XH_{2t}^p\,] ,$$

[7] Go to statement [18],

[8] If $LHUN_t \leqslant LHMAX_t$, then $LH_t^p = LHUN_t$ and go to statement [15],

[9] $LH_t^p = LHMAX_t$,

[10] $HPH_{2t}^p = HPHUN_{2t} + 0.46\, HPHUN_{2t}\left(\dfrac{LHUN_t - LHMAX_t}{LHUN_t}\right)$,

[11] If $HPH_{2t}^p > HPHMAX_{2t}$, then $HPH_{2t}^p = HPHMAX_{2t}$,

[12] $XH_{2t}^p = \dfrac{1}{-\gamma_1 P_t - P_t}\,[-(1-(1-d_3)RL_t)\,LH_t^p + LH_{t-1} - DDH_{2t-1} - (1-d_3)\,W_t HPH_{2t}^p - YG]$,

[13] If $XH_{2t}^p > XHMAX_{2t}$, then $XH_{2t}^p = XHMAX_{2t}$ and

$$LH_t^p = \frac{1}{1-(1-d_3)RL_t}\,[LH_{t-1} + \gamma_1 P_t XH_{2t}^p - DDH_{2t-1} - (1-d_3)W_t HPH_t^p - YG + P_t XH_{2t}^p],$$

[14] Go to statement [18] ,

[15] If $XHUN_{2t} \leqslant XHMAX_{2t}$, then $XH_{2t}^p = XHUN_{2t}$ and go to statement [18],

[16] $XH_{2t}^p = XHMAX_{2t}$,

[17] $LH_t^p = \dfrac{1}{1-(1-d_3)RL_t}\,[LH_{t-1} + \gamma_1 P_t XH_{2t}^p - DDH_{2t-1} - (1-d_3)W_t HPH_{2t}^p - YG + P_t XH_{2t}^p]$,

[18] $HPH_{2t} = HPH_{2t}^p$,

[19] $XH_{2t} = XH_{2t}^p$,

[20] $LH_t = LH_t^p$.

Consider household 1 in Table 4-6 first. Equation (1) merely defines the expected level of dividends. It is the same as the equation in the non-condensed model. Equations (2) and (3) are based on the results in Table 4-3. The unconstrained number of hours worked is a positive function of the wage rate and the bill rate, and a negative function of the price level, the tax rate, the level of wealth of the previous period, and the minimum guaranteed level of income. The unconstrained number of goods purchased is a positive function of the wage rate, the level of wealth of the previous period, and the minimum guaranteed level of income, and a negative function of the price level, the bill rate, and the tax rate. The coefficients in Equations (2) and (3) were chosen to be consistent with the size of the reactions in Table 4-3.

Equation (4) defines the expected level of savings deposits, given the two unconstrained values and the expected level of dividends. The equation is derived as follows. Because of assumptions (4.11), (4.15), and (4.16), household 1 always expects the stock price to remain unchanged over time. Household 1 does not, therefore, expect to receive any capital gains or losses on its stocks, which means that CG_{it} in Equation (4.1) is expected to be zero. Now, Equations (4.2), (4.3), and (4.5) can be combined for household 1 to yield, using the notation for the condensed model:

$$SAV_{1t}^{e} = (1-d_3)(W_t HPH_{1t} + r_t SD_t^e + DIV_t^e) + YG - P_t XH_{1t}. \tag{4.26}$$

The e superscripts have been added to the appropriate variables to denote the fact that household 1 only has at the beginning of period t an expectation of these variables. Equations (4.4) and (4.6) can be similarly combined for household 1 to yield, again using the notation for the condensed model:

$$SD_t^e = SD_{t-1} - (\gamma_1 P_t XH_{1t} - DDH_{1t-1}) + SAV_{1t}^{e}. \tag{4.27}$$

The final term in Equation (4.6) is zero because household 1 always owns all the stock. Finally, Equations (4.26) and (4.27) can be solved to yield:

$$SD_t^e = \frac{1}{1-(1-d_3)r_t}\,[SD_{t-1} - (\gamma_1 P_t XH_{1t} - DDH_{1t-1})$$
$$+ (1-d_3)(W_t HPH_{1t} + DIV_t^e) + YG - P_t XH_{1t}]\,, \tag{4.28}$$

which is the same as Equation (4) in Table 4-6 with the appropriate change of notation.

Equation (5) in Table 4-6 determines the actual number of hours worked. If the unconstrained number is less than the maximum number allowed, then the actual number is the unconstrained number. Otherwise, the actual number is set equal to the maximum number. In Equation (6), the planned level of savings is lowered if the unconstrained number of hours worked is greater than the maximum number allowed. In row B.1 in Table 4-3 it can be seen that planned savings decreased slightly when household 1 was constrained in the number of hours that it could work, and this is the assumption reflected in Equation (6). The -0.074 coefficient is estimated from Table 4-3, ,where

$$-0.074 = \left(\frac{2152 - 2160}{2160}\right) / \left(\frac{323.0 - 306.8}{323.0}\right).$$

Equation (7) determines the actual number of goods purchased. If the unconstrained number of hours worked is less than the maximum number allowed and if the unconstrained number of goods purchased is less than the maximum number allowed, then the actual number of goods purchased is the unconstrained number. If the unconstrained number of hours worked is greater than the maximum number allowed, so that the actual number is set equal to the maximum, then the actual number of goods purchased is set equal to the number necessary to have the planned level of savings deposits be what it is in Equation (6), given the new lower level of hours worked. This is the expression following the second equal sign in (7). This expression is obtained by solving Equations (4.26) and (4.27) for XH_{1t}.

If the computed number of goods purchased from this exercise is greater than the maximum number allowed or if (in the unconstrained hours case) the unconstrained number of goods purchased is greater than the maximum number allowed, then the actual number of goods purchased is set equal to the maximum number. This is the expression following the third equal sign in (7). It should be noted that this procedure reflects the assumption that a binding goods constraint has no effect on hours worked. In row C.1 in Table 4-3 it can be seen that the goods constraint had a negative effect on the number of hours worked by household 1, but for simplicity this behavioral response was not incorporated into the condensed model.

The condensed model for household 1 is thus fairly simple. If the household is not constrained, then the number of hours worked and the number of goods purchased are determined from Equation (2) and (3). Otherwise, the household modifies its decisions according to Equations (4)-(7). Because of Equations (4.26) and (4.27), given two of the three values of HPH_{1t}, XH_{1t}, and SD_t^e, the other value is automatically determined, and this property was used in Equations (4) and (7) in determining how the household's decisions were modified.

The equations for household 2 in the condensed model are based on the results in Table 4-4. The problem is more complicated for household 2 because of the possibly binding loan constraint in addition to the hours and goods constraints. In Equation (1)′ the unconstrained number of hours worked is a positive function of the wage rate, the loan rate, and the value of loans of the previous period, and a negative function of the price level, the tax rate, and the minimum guaranteed level of income. In Equation (2)′ the unconstrained number of goods purchased is a positive function of the wage rate and the minimum guaranteed level of income, and a negative function of the price level, the loan rate, the tax rate, and the value of loans of the previous period.

Equation (3)′ defines the unconstrained value of loans, given the unconstrained number of hours worked and the unconstrained number of goods purchased. The equation is derived in a similar way that Equation (4.28) was derived above for household 1. Equations (4.2), (4.3), and (4.5) can be combined for household 2 to yield, using the notation for the condensed model:

$$SAV_{2t} = (1-d_3)\,(W_t HPH_{2t} - RL_t LH_t) + YG - P_t XH_{2t}\,. \tag{4.29}$$

Equations (4.4) and (4.6) can be combined to yield, again using the notation for the condensed model:

$$LH_t = LH_{t-1} + (\gamma_1 P_t XH_{2t} - DDH_{2t}) - SAV_{2t}. \tag{4.30}$$

Equations (4.29) and (4.30) can then be solved to yield:

$$LH_t = \frac{1}{1-(1-d_3)RL_t}\,[LH_{t-1} + (\gamma_1 P_t XH_{2t} - DDH_{2t-1}) - (1-d_3)W_t HPH_{2t} - YG + P_t XH_{2t}]\,, \tag{4.31}$$

which is the same as equation (3)′ in Table 4-6 with the appropriate change of notation.

If none of the unconstrained values in Equations (1)′–(3)′ is greater than the maximum values, then, as in Equations (4)′–(6)′, the actual values are the unconstrained values. Otherwise, the actual values are determined by the algorithm in Table 4-6. As was the case for the algorithm in Table 3-4 for the condensed model for the firm sector, the algorithm in Table 4-6 is written like a FORTRAN program. The following is a brief verbal description of the algorithm.

If the hours constraint is binding, then statements [2]-[6] hold. The actual number of hours worked is set equal to the maximum number in statement [2]. In statement [3] the planned value of loans is then increased. In row B.1 in Table 4-4 it can be seen that the planned value of loans increased when household 2 was constrained in the number of hours that it could work, and this is the assumption reflected in statement [3]. The 0.36 coefficient is estimated from Table 4-4, where

$$0.36 = \left(\frac{490.7 - 482.1}{482.1}\right) / \left(\frac{435.0 - 413.2}{435.0}\right)$$

If the new planned value of loans is greater than the maximum value, then in statement [4] the actual value is set equal to the maximum value.

Statement [5] determines the number of goods purchased, given the number of hours worked and the value of loans. The equation in statement [5] is derived by solving Equations (4.29) and (4.30) for XH_{2t}. If the new number of goods purchased is greater than the maximum value, then in statement [6] the actual number is set equal to the maximum value and a new value of loans is computed. The new value of loans is guaranteed to be less than the previous value because the new number of goods purchased is in this case less than the previous number. The equation for loans in statement [6] is, of course, the same as Equation (3)′ with the appropriate change of notation.

As was the case for household 1, the procedure in statement [6] reflects the assumption that a binding goods constraint has no effect on hours worked. In row C.1 in Table 4-4 it can be seen that the goods constraint had a negative effect on the number of hours worked by household 2, but for simplicity this behavioral response was not incorporated into the condensed model. Statement [6] ends the computations for household 2 in the case of an originally binding hours constraint, and the algorithm finishes off with statements [18]-[20], where the actual values are set equal to the planned values.

If the hours constraint is not binding but the loan constraint is, then statements [9]-[13] hold. The actual value of loans is set equal to the maximum value in statement [9]. In statement [10] the planned number of hours worked is then increased. In row D.1 in Table 4-4 it can be seen that the planned number of hours worked increased when household 2 was constrained in the value of loans that it could take out, and this is the assumption reflected in statement [10]. The 0.46 coefficient is estimated from Table 4-4, where

$$0.46 = \left(\frac{445.0 - 435.0}{435.0}\right) \Big/ \left(\frac{482.1 - 458.0}{482.1}\right) .$$

If the new planned number of hours worked is greater than the maximum number, then in statement [11] the actual number is set equal to the maximum number. Statements [12] and [13] are then exactly like statements [5] and [6]. Statement [13] then ends the computations for household 2 in this case.

If the hours and loan constraints are not binding but the goods constraint is, then statements [16] and [17] hold. The actual number of goods purchased is set equal to the maximum number in statement [16]. In statement [19] the value of loans is recomputed. Again, the new value of loans is guaranteed to be less than the previous value because the new number of goods purchased is less than the previous number. Statement [17] then ends the computation for household 2 in this case.

To summarize the condensed model for household 2, the number of hours worked, the number of goods purchased, and the value of loans taken out are determined by Equations (1)′–(3)′ if the household is not constrained. If the hours constraint is binding, then statements [2]-[6] hold. If the loan constraint is binding but the hours constraint is not, then statements [9]-[13] hold. If the goods constraint is binding but the hours and loan constraints are not, then statements [16]-[17] hold. As was the case for household 1, the algorithm for household 2 uses the fact that given two of the three values of HPH_{2t}, XH_{2t}, and LH_t, the other value is automatically determined (because of Equations (4.29) and (4.30)).

NOTES

[a]Since all expectations are made by household *i*, no *i* subscript or superscript has been added to the relevant symbols to denote the fact that it is household *i* making the expectation.

[b]See, for example, Henderson and Quandt [28], p. 24.