Chapter Five

The Government and the Bond Dealer

5.1 THE GOVERNMENT

In Table 5-1 the important symbols used in this chapter are listed in alphabetic order. With a few exceptions, the notation used for the government and the bond dealer is the same for both the non-condensed and condensed models. The government collects taxes from banks ($TAXB_{it}$), from firms ($TAXF_{it}$), from households ($TAXH_{it}$), and from the bond dealer ($TAXD_t$). It hires labor from households (HPG_t), buys goods from firms (XG_t), issues bills and bonds ($VBILLG_t$ and $BONDG_t$), and pays interest on its bills and bonds (r_tVBILLG$_t$ + $BONDG_t$). The government also sets the various tax parameters in the system and the reserve requirement ratio. It is subject to the following budget constraints:[a]

$$
\begin{aligned}
PG_tXG_t + WG_tHPG_t + r_tVBILLG_t + BONDG_t - \sum_{i=1}^{NB} TAXB_{it} - \sum_{i=1}^{NF} TAXF_{it} \\
- \sum_{i=1}^{NH} TAXH_{it} - TAXD_t = VBILLG_t - VBILLG_{t-1} \\
+ \frac{BONDG_t - BONDG_{t-1}}{R_t} + \sum_{i=1}^{NB} BR_{it} - \sum_{i=1}^{NB} BR_{it-1} \quad .
\end{aligned}
\tag{5.1}
$$

The first four terms in Equation (5.1) are government expenditures, and the next four terms are government tax collections. The left-hand side of Equation (5.1) is thus expenditures minus taxes, and this value must equal the change in the value of bills plus bonds plus bank reserves.

Table 5-1. Notation for the Government and the Bond Dealer in Alphabetic Order

Non-Condensed and Condensed Models	
The Government	
$BONDG_t$	= number of bonds issued
HPG_t	= hours paid for
PG_t	= price paid [$= P_t$ in the condensed model]
$TAXB_{it}$	= taxes paid by bank i [$=TAXB_t$ in the condensed model]
$TAXD_t$	= taxes paid by the bond dealer
$TAXF_{it}$	= taxes paid by firm i [$=TAXF_t$ in the condensed model]
$TAXH_{it}$	= taxes paid by household i
$VBILLG_t$	= value of bills issued
WG_t	= wage rate paid [$= W_t$ in the condensed model]
XG_t	= number of goods purchased
The Bond Dealer	
$BONDD_t$	= number of bonds held
DDD_t	= demand deposits
DIV_t	= total dividends paid and received in the economy
$DIVD_t$	= dividends paid
PS_t	= price of the aggregate share of stock
r_t	= bill rate
R_t	= bond rate
$TAXD_t$	= taxes paid
VBD^*	= value of bills and bonds that the bond dealer desires to hold
$VBILLD_t$	= value of bills held
ΠD_t	= before-tax profits

5.2 THE BOND DEALER

The bond dealer is taken in the model to represent the government bill and bond market and the stock market. The three decision variables of the bond dealer are the bill rate (r_t), the bond rate (R_t), and the stock price (PS_t). The assets of the bond dealer consist of bills ($VBILLD_t$), bonds ($BONDD_t$), and demand deposits (DDD_t). Households own the stock of the bond dealership. The profits of the bond dealer consist of the interest received on its bill and bond holdings ($r_t VBILLD_t + BONDD_t$), and capital gains or losses on its bond holdings ($BONDD_t/R_{t+1} - BONDD_t/R_t$). The bond dealer pays taxes to the government on its profits ($TAXD_t$). After-tax profits are paid to households in the form of dividends ($DIVD_t$). The basic equations for the bond dealer are the following:

$$\Pi D_t = r_t VBILLD_t + BONDD_t + \left(\frac{BONDD_t}{R_{t+1}} - \frac{BONDD_t}{R_t}\right), \text{ [before-tax profits]} \quad (5.2)$$

$$TAXD_t = d_1\, \Pi D_t\,, \quad \text{[taxes paid]} \tag{5.3}$$

$$DIVD_t = \Pi D_t - TAXD_t\,, \quad \text{[dividends paid]} \tag{5.4}$$

$$\begin{aligned} DDD_t &= DDD_{t-1} - (VBILLD_t - VBILLD_{t-1}) - \frac{(BONDD_t - BONDD_{t-1})}{R_t} \\ &\quad - \left(\frac{BONDD_t}{R_{t+1}} - \frac{BONDD_t}{R_t}\right) \quad \text{[demand deposits]} \\ &= DDD_{t-1} - (VBILLD_t - VBILLD_{t-1}) - \left(\frac{BONDD_t}{R_{t+1}} - \frac{BONDD_t}{R_t}\right) . \end{aligned} \tag{5.5}$$

Equation (5.5) states that the change in demand deposits of the bond dealer is equal to minus the change in the value of its bills and bonds and minus the capital gains or losses on its bond holdings. Since the bond dealer pays out in the form of taxes and dividends any capital gains made in the period (and conversely for capital losses), and yet does not receive any cash flow from the capital gains, capital gains take away from (and conversely capital losses add to) demand deposits.

In any period the bond dealer is assumed to absorb the difference between the supply of bills and bonds from the government and the demand for bills and bonds from the banks:

$$VBILLD_t = VBILLG_t - \sum_{i=1}^{NB} VBILLB_{it}\,, \tag{5.6}$$

$$BONDD_t = BONDG_t - \sum_{i=1}^{NB} BONDB_{it}\,. \tag{5.7}$$

The bond dealer is assumed to have a certain desired value of bills and bonds, denoted as VBD^*, that it aims to hold in inventories each period. Now, the total demand for bills and bonds from the banks in, say, period t-1 is $\sum_{i=1}^{NB} VBB_{it-1}$. Therefore, the total demand for bills and bonds from both the banks and the bond dealer in period $t-1$ is $\sum_{i=1}^{NB} VBB_{it-1} + VBD^*$. The total supply of bills and bonds from the government in period $t-1$ is, of course, $VBILLG_{t-1} + BONDG_{t-1}/R_{t-1}$. The bond dealer is assumed to have knowledge of $\sum_{i=1}^{NB} VBB_{it-1}$, $VBILLG_{t-1}$, and $BONDG_{t-1}$ near the end of period t-1, and it is assumed to set at this time the bill rate for period t according to the following formula

$$\frac{r_t - r_{t-1}}{r_{t-1}} = \lambda \left[\frac{(VBILLG_{t-1} + \frac{BONDG_{t-1}}{R_{t-1}}) - (\sum_{i=1}^{NB} VBB_{it-1} + VBD^*)}{\sum_{i=1}^{NB} VBB_{it-1} + VBD^*} \right], \lambda > 0. \tag{5.8}$$

The numerator of the term in brackets is the excess supply of bills and bonds in period t-1. Equation (5.8) thus states that the bond dealer raises the bill rate for period t if there was an excess supply of bills and bonds in period $t-1$ and lowers the bill rate for period t if there was an excess demand (negative excess supply).[b]

It was mentioned in Section 1.2 that banks are assumed to communicate to the bond dealer near the end of period $t-1$ their expectations of the future bill rates. All banks are assumed to have the same expectations. Let r^e_{t+k} denote the banks' expectation of the bill rate for period $t+k$ $(k=1, 2, \ldots)$. Then given the value of r_t and given these expectations, the bond dealer is assumed to set the bond rate, R_t, according to the formula

$$\frac{1}{R_t} = \frac{1}{(1+r_t)} + \frac{1}{(1+r_t)(1+r^e_{t+1})} + \frac{1}{(1+r_t)(1+r^e_{t+1})(1+r^e_{t+2})} + \ldots \tag{5.9}$$

The price of a bond, in other words, is set equal to the presented discounted value of a perpetual stream of one-dollar payments, the discount rates being the current and expected future bill rates. Equation (5.9) is consistent with Equation (2.19) in Chapter Two, which is the equation describing the way that banks *expect* the bond rate to be set. Since banks are assumed always to expect that the bill rate will remain unchanged from its last observed value, the bond rate that the bond dealer sets is always equal to the bill rate:[c]

$$R_t = r_t \,. \tag{5.10}$$

It was also mentioned in Section 1.2 that households are assumed to communicate to the bond dealer near the end of period $t-1$ their expectations of the future bill rates and dividend levels. All households are assumed to have the same expectations. Let r^e_{t+k} now denote the households' expectation of the bill rate for period $t+k$ $(k=1, 2, \ldots)$, and let DIV^e_{t+k} denote their expectation of the dividend level for period $t+k$ $(k=0, 1, 2, \ldots)$. Then given the value of r_t and given these expectations, the bond dealer is assumed to set the stock price, PS_t, according to the formula

$$PS_t = \frac{DIV_t^e}{(1+r_t)} + \frac{DIV_{t+1}^e}{(1+r_t)(1+r_{t+1}^e)} + \frac{DIV_{t+2}^e}{(1+r_t)(1+r_{t+1}^e)(1+r_{t+2}^e)} + \ldots \quad (5.11)$$

The stock price, in other words, is set equal to the present discounted value of the expected future dividend levels, the discount rates being the current and expected future bill rates. Equation (5.11) is consistent with Equation (4.16) in Chapter Four, which is the equation describing the way that households *expect* the stock price to be set. Since households are assumed always to expect that the bill rate will remain unchanged from its last observed value and are assumed always to expect that the dividend level will remain unchanged from the level expected for period t, the stock price that the bond dealer sets is merely[d]

$$PS_t = \frac{DIV_t^e}{r_t}. \quad (5.12)$$

DIV_t^e in (5.12) is determined in Equation (4.15) in Chapter Four as the average of the past five dividend levels:[e]

$$DIV_t^e = \frac{1}{5}(DIV_{t-1} + DIV_{t-2} + DIV_{t-3} + DIV_{t-4} + DIV_{t-5}). \quad (5.13)$$

This completes the discussion for the bond dealer. Although the bond dealer represents the bill and bond market and the stock market, it is important to note that the bond dealer is not an auctioneer. The bond dealer sets the bill rate for period t according to the excess supply or demand situation for bills and bonds that exists in period $t-1$. Any difference between the supply of bills and bonds from the government and the demand from the banks in a period is absorbed by the bond dealer. Although the bond dealer can be thought of as always trying to achieve a zero excess supply and demand for bills and bonds in the next period, it does not continually call out rates in the current period until a zero excess supply and demand for bills and bonds is reached in the current period.

5.3 THE CONDENSED MODEL FOR THE GOVERNMENT AND THE BOND DEALER

The condensed model for the government and the bond dealer is the same as the non-condensed model. The equations for the condensed model are presented in Table 5-2. The only difference between the equations in Table 5-2 and the equations for the non-condensed model is the change in notation for some of the variables.

Table 5-2 The Government and Bond Dealer Equations for the Condensed Model

The Government

(1) $P_tXG_t + W_tHPG_t + r_tVBILLG_t + BONDG_t - TAXB_t - TAXF_t - TAXH_{1t} - TAXH_{2t} - TAXD_t = VBILLG_t - VBILLG_{t-1} + \frac{BONDG_t - BONDG_{t-1}}{R_t} + BR_t - BR_{t-1}$, [government budget constraint]

The Bond Dealer

(2) $\frac{r_t - r_{t-1}}{r_{t-1}} = \lambda \left[\frac{(VBILLG_{t-1} + \frac{BONDG_{t-1}}{R_{t-1}}) - (VBB_{t-1} + VBD^*)}{(VBB_{t-1} + VBD^*)} \right]$, [equation determining the bill rate]

(3) $R_t = r_t$, [equation determining the bond rate]

(4) $PS_t = \frac{\frac{1}{5}(DIV_{t-1} + DIV_{t-2} + DIV_{t-3} + DIV_{t-4} + DIV_{t-5})}{r_t}$. [equation determining the stock price]

NOTES

[a] NB in (5.1) is the number of banks in existence, NF is the number of firms, and NH is the number of households.

[b] In the programming for the non-condensed model, the bond dealer was assumed to estimate the parameter λ in Equation (5.8) each period on the basis of its past observations of the correlation between percentage changes in the demand for bills and bonds from the banks and percentage changes in the bill rate. The exact procedure by which λ was assumed to be estimated is described in the Appendix.

[c] In order for the r^e_{t+k} $(k=1, 2, \ldots)$ in Equation (5.9) to be equal to r_t, so that (5.10) holds, it must be assumed that the banks know the value of r_t before they form their future expectations. Therefore, the bond dealer must be thought of as communicating the value of r_t (obtained from (5.8)) to the banks, who then in turn communicate their future expectations to the bond dealer. All this communication takes place near the end of period $t-1$.

[d] The same assumption regarding the communication flow to and from the bond dealer has to be made here as was made for banks in footnote c.

[e] In order for the value of DIV^e_t to be communicated to the bond dealer near the end of period $t-1$, the households must be assumed to know at this time the value of DIV_{t-1}. As will be seen in Tables 6-2 and A-2, this assumption introduces a slight degree of simultaneity into the model. This simultaneity could have been eliminated by assuming that DIV^e_t in (5.13) is the average of the past five dividend levels starting with period $t-2$, but because the degree of simultaneity was so slight, the assumption in (5.13) was retained.