Chapter Five

The Firm Sector

5.1 INTRODUCTION

The eleven stochastic equations that relate to the firm sector are discussed in this chapter. The equations explain the eleven variables that are listed on the right-hand side of Table 5-1. Table 5-1 contains for the firm sector a matching of the variables in the theoretical model to the related variables in the empirical model. The six most important variables explained in this chapter are: the price level (PF_t), production (Y_t), investment (INV_t), the number of jobs ($JOBF_t$), the average number of hours paid per job (HPF_t), and the wage rate (WF_t).

The treatment of the firm sector in the theoretical model was summarized in Chapter One, section 1.1. A firm's price, production, investment, employment, and wage rate decisions are determined simultaneously in the theoretical model through the solution of the firm's optimal control problem. The underlying technology of a firm is of the "putty-clay" type, and it may at times be optimal for a firm to plan to hold either excess labor or excess capital or both. Market share considerations and expectations play an important role in determining a firm's price and wage behavior. The two possible constraints on a firm are the loan constraint and the labor constraint.

Although a firm's decisions are determined simultaneously in the theoretical model, it is sometimes useful for descriptive purposes to consider the decisions as being made sequentially. This sequence is from the price decision, to the production decision, to the investment and employment decisions, to the wage rate decision. A firm should first be considered as having chosen its optimal price path. This path implies a certain expected sales path, from which the optimal production path can be chosen. Given the optimal production path, the optimal paths of investment and employment can be chosen. Finally, given the optimal employment path, the optimal wage rate path can be chosen. The optimal wage rate path is that path that the firm

Table 5–1. Matching of Dependent Variables in the Theoretical and Empirical Models for the Firm Sector

Decision Variable in the Theoretical Model (Notation for Condensed Model)	*Related Variables in the Empirical Model*
1. P_t (price level)	PF_t (deflator for $X_t - COM_t$)
2. Y_t (number of goods produced)	Y_t (production of the firm sector, B1958)
3. INV_t (number of goods purchased for investment purposes)	INV_t (nonresidential plant and equipment investment of the firm sector, B1958)
4. HPF_t (number of worker hours paid for)	$JOBF_t$ (number of jobs in the firm sector) HPF_t (average number of hours paid per job) $HPFO_t$ (average number of overtime hours paid per job)
5. W_t (wage rate)	WF_t (average hourly earnings, adjusted for overtime and interindustry employment shifts)
6. DDF_t (demand deposits)	DDF_t (demand deposits)
7. $DIVF_t$ (dividends paid)	$DIVF_t$ (dividends paid)
8. RL_tLF_t (interest paid)	$INTF_t$ (interest paid)
9. $(P_t - P_{t-1})V_{t-1}$ (inventory valuation adjustment)	IVA_t (inventory valuation adjustment)

expects is necessary to attract the amount of labor implied by its optimal employment path. It will be useful to keep this sequence in mind for the discussion in section 5.3.

Before discussing the stochastic equations, it is necessary to consider the measures of excess labor and excess capital that have been used. These measures are discussed in the next section. (This section can be skipped if desired without much loss of continuity.) The empirical model of the firm sector is outlined in section 5.3, and the equation estimates are explained in section 5.4. Section 5.5 contains a brief review of the model.

5.2 THE TECHNOLOGY OF THE FIRM SECTOR AND THE MEASUREMENT OF EXCESS LABOR AND EXCESS CAPITAL

Two possible ways of measuring the capital stock of the firm sector have been considered in this study. The first, more conventional way is to assume that the capital stock deteriorates at some rate δ_K each quarter and thus to postulate that:

$$K_t^a - K_{t-1}^a = INV_t - \delta_K K_{t-1}^a, \tag{5.1}$$

where K_t^a is the value of the capital stock (in real terms) in quarter t and INV_t is the value of plant and equipment investment of the firm sector (in real terms) in quarter t. For this measure of the capital stock, the production function of the firm sector is postulated to be:

$$Y_t = \min\{\lambda_t(M_t H_t^M), \mu_t(K_t^a H_t^K)\}, \tag{5.2}$$

where M_t is the number of workers employed, H_t^M is the number of hours worked per worker, H_t^K is the number of hours each unit of K_t^a is utilized, and λ_t and μ_t are coefficients that may change over time due to technical progress.

Equations (5.1) and (5.2) are not consistent with the putty-clay assumptions of the theoretical model. Each machine in the theoretical model wears out after m periods, but its productiveness does not lessen as it gets older. Machines do not change at all until age m, when they just fall apart completely. Consequently, even if there were only one type of machine ever in existence, Equation (5.1) would not be true. Rather, $K_t^a - K_{t-1}^a$ would equal $INV_t - INV_{t-m}$, where INV_{t-m} would be the number of machines that wear out at the end of period $t-1$. It is also the case that no technical change was postulated in the theoretical model, but even if technical change were postulated, it would not enter in the way specified in Equation (5.2). Technical change would take the form of machines having different λ and μ coefficients according to when they were purchased. One could not write down an equation like (5.2), but would instead have to keep track of when each machine was purchased and what the coefficients were for that machine in order to be able to calculate how much output could be produced with the existing stock of machines. Equations (5.1) and (5.2) are thus at best only approximations to the production technology in the theoretical model.

Since Equations (5.1) and (5.2) are only approximations, a slightly different way of approximating the technology was tried to see if this led to better results. Consider INV_t to be the number of machines purchased in period t, and assume that these machines are all alike. Let μ_t stand for the amount of output that can be produced per machine hour on one of these machines. Assume, finally, that all machines wear out after m periods, but do not deteriorate physically before that time. Then the amount of output that can be produced per hour with all of the machines running is:

$$\frac{Y_t}{H_t^K} = \mu_t INV_t + \mu_{t-1} INV_{t-1} + \cdots + \mu_{t-m+1} INV_{t-m+1}, \tag{5.3}$$

where Y_t/H_t^K is output per hour when all machines are running. Associated with each machine is a λ_t coefficient, which is the amount of output that can be produced per worker hour on machines purchased in period t. Assume that all machines are used H_t^K hours, so that Y_t in Equation (5.3) is the actual amount of output produced. The number of worker hours required to produce Y_t in this case is:

$$M_t H_t^M = \frac{\mu_t INV_t H_t^K}{\lambda_t} + \frac{\mu_{t-1} INV_{t-1} H_t^K}{\lambda_{t-1}} + \cdots + \frac{\mu_{t-m+1} INV_{t-m+1} H_t^K}{\lambda_{t-m+1}}. \tag{5.4}$$

This second technology, which will be considered below, is thus represented by Equations (5.3) and (5.4).

Two variables that are needed for the estimation work in the next section are a variable that measures the amount of excess labor on hand and a variable that measures the amount of excess capital on hand. For the technology represented by Equations (5.1) and (5.2), these two variables were constructed in the following way. For the measurement of excess labor, output per paid for worker hour ($Y_t/(JOBF_t HPF_t)$) was first plotted for the 1952I–1975I period. The peaks of this series were assumed to correspond to cases where the number of worker hours paid for ($JOBF_t HPF_t$) equals the number of worker hours actually worked ($M_t H_t^M$). This assumption implies that values of λ_t in Equation (5.2) are observed at the peaks. The values of λ_t other than those at the peaks were then assumed to lie on straight lines between the peaks. Values of λ_t, in other words, were estimated from a peak-to-peak interpolation of the output per paid for worker hour series.

Given data series on λ_t and Y_t, a series on the number of worker hours required to produce Y_t, $M_t H_t^M$, is then merely Y_t/λ_t from Equation (5.2). This series can then be compared to the observed series on worker hours paid for, $JOBF_t HPF_t$, to determine the amount of excess labor on hand in any period. The quarters that were used as peaks for the interpolation are 1952I, 1953II, 1955II, and 1966I. The line drawn between the 1955II and 1966I peaks was extended beyond 1966I in determining the values of λ_t between 1966I and 1975I.

This procedure of constructing a series on $M_t H_t^M$ is the same as that used in Fair [23] and [14], the first for monthly seasonally unadjusted three-digit industry data and the second for quarterly seasonally adjusted data on the private nonfarm sector of the economy. It was argued in [23] that seasonally adjusted data should not be used to estimate production function parameters and worker hour requirements series because technical relationships are not likely to be subject to much seasonal variation. Unfortunately, however, much of the NIA data are not available on a seasonally unadjusted basis, and it is beyond the scope of this study to try to piece together enough data to be able to estimate the empirical model on a nonseasonally adjusted basis. Consequently, seasonally adjusted data have been used here, as well as in [14], in constructing the worker hour requirements series.

For the measurement of excess capital for the technology represented by Equations (5.1) and (5.2), a capital stock series first had to be constructed. Given data on INV_t, a series on K_t^a can be constructed once a base period value and a value for the depreciation rate δ_K are chosen. In a recent study [40], the Bureau of Economic Analysis (BEA) has estimated on an annual basis the fixed nonresidential business capital in the United States for the 1925–1973 period. The results of the BEA study were used here to estimate a base period value for K_t^a and a value of δ_K. The net stocks series on page 1 in [40] was first multiplied by 0.7 to scale it down to a series that pertains to the firm sector. 0.7 is roughly the ratio of plant and equipment investment in the firm sector to total plant and equipment investment. The net stocks series on page 1 in [40] pertains to all plant and equipment investment (firm, household, and financial). It is based on the assumptions of straight line depreciation and service lives equal to 85.0 percent of Bulletin F.

The base period for K_t^a was taken to be 1952IV, and the base period value was taken to be 197.2 billion (1958) dollars. This latter figure is 0.7 times the value on page 1 in [40] for the end of 1952. From this base period, various values of δ_K were used to generate different capital stock series, using the formula:

$$K_t^a = (1 - \delta_K)K_{t-1}^a + INV_t. \tag{5.5}$$

These series were compared to the "actual" series derived from [40] to see which value of δ_K most closely reproduced the actual series. It was apparent from this exercise that one value of δ_K for the whole period was not adequate to approximate the actual series at all accurately. There appeared to be a shift around 1966 in the value needed for δ_K, a larger value being needed after 1966.

In the end, two values of δ_K were chosen, a value of 0.0255 before 1966I and a value of 0.0285 from 1966I on. The use of the value of 197.2 for K_t^a in 1952IV and the value of 0.0255 for δ_K resulted in a value of 308.9 for K_t^a in 1965IV, which compares quite closely to the actual value of 308.6. The value for K_t^a in 1965IV was taken to be 308.9, and from this base the rest of the K_t^a series was generated using the value of 0.0285 for δ_K. The generated value of K_t^a for 1971IV was 404.4 (see Table 2-1), which compares fairly closely to the actual value of 406.3. The actual series from the BEA could not be used directly here because it is annual and because of the necessity of having a link between the investment series (INV_t) and the capital stock series.

Regarding the measurement of excess capital, there are no data on hours paid for or worked per unit of K_t^a, and so, given a series on K_t^a, one must be content with plotting Y_t/K_t^a. This is, from Equation (5.2), a plot of $\mu_t H_t^K$, where H_t^K is the average number of hours that each machine is utilized. If it is assumed that at each peak of this series H_t^K is equal to the same constant, say $\overline{H}$, then one observes at the peaks $\mu_t\overline{H}$. Interpolation between peaks can then produce a complete series on $\mu_t\overline{H}$. If, finally, $\overline{H}$ is assumed to be the maximum number of hours per period that each unit of K_t^a can be utilized, then $Y_t/(\mu_t\bar{H})$ is the minimum amount of capital required to produce Y_t. This variable is denoted as $KMIN_t$ in Table 2–2.

The observations that were used for the peaks are 1953II, 1966I, and 1973I. The values of $\mu_t\bar{H}$ between 1973II and 1975I were all taken to be equal to the 1973I value. The line drawn between 1953II and 1966I had a positive slope, but the line drawn between 1966II–1973I had a slight negative slope. There seemed to be some evidence of a slight deterioration in output per machine hour after 1966I. It is true, however, that the plot of Y_t/K_t^a over the entire 1952I–1975I period showed little evidence of either a positive or a negative trend. The slopes of both of the interpolation lines were fairly small.

This takes care of the measurement of excess labor and excess capital for the technology represented by Equations (5.1) and (5.2). Consider next the measurement of excess capital for the technology represented by Equations (5.3) and (5.4). The BEA study [40] was first used to get an estimate of m, the length of life of one unit of capital. The BEA study presents estimates of both the gross and net capital stocks, and for purposes of estimating m the gross capital stock series on page 1 in [40] was used. If it is assumed that machines do not physically depreciate until age m, when they fall apart, an estimate of m can be obtained by summing past values of gross investment (also presented in [40]) until the sum is equal to the BEA estimate of the gross capital stock. The number of periods that one uses in this sum is an estimate of m. This procedure can be followed for each yearly estimate of the gross capital stock. One will not, of course, necessarily get the same estimate of m for each year. It was quite evident when carrying out this procedure for the 1952–1972 period that m began to get much smaller in the 1960s, a result that is consistent with having to use a larger value of δ_K beginning around 1966 to approximate the net capital stock series. There is nothing in the following analysis that requires m to be constant over time, and so instead of choosing only one or two values of m, an entire time series for m (denoted as m_t) was constructed from the BEA gross investment and gross capital stock data.

Given a series for m_t, the next step in the construction of an excess capital series was to get estimates of the μ_t series in Equation (5.3). (Equation (5.3) should now be modified by adding a t subscript to m.) To do this, it was first assumed that $\mu_t = \bar{\mu}(1 + \delta)^t$, where $\bar{\mu}$ and δ are parameters to be estimated. If δ is zero, then μ_t is constant over time; otherwise μ_t is changing at rate δ each period. Next, a few quarters were chosen where it seemed plausible to assume that all machines were utilized $\bar{H}$ hours. These quarters, in other words, were assumed to be quarters in which the amount of excess capital on hand was zero. If quarter s is one of these quarters, then it is the case from Equation (5.3), and the assumption just made about μ_t, that:

$$Y_s = \bar{\mu}\bar{H}[(1 + \delta)^s INV_s + (1 + \delta)^{s-1} INV_{s-1} + \cdots + (1 + \delta)^{s-m_s+1} INV_{s-m_s+1}]. \quad (5.6)$$

Given data on investment, output, and m_t and given two quarters for which Equation (5.6) holds, one has two equations in two unknowns, the unknowns being $\bar{\mu}\bar{H}$ and δ. The two equations are nonlinear, but they can easily be solved numerically. If one has more than two quarters for which Equation (5.6) is assumed to hold, then different pairs of equations can be solved to see, among other things, how sensitive the solution values are to alternative pairs.

Values of δ and $\bar{\mu}\bar{H}$ were computed in this way for alternative pairs of equations, and it turned out that a value of zero for δ seemed quite consistent with the data. There did not appear, in other words, to be any evidence of capital's getting either more efficient or less efficient over time in terms of output per unit of capital. This result is consistent with the observation made earlier that the plot of Y_t/K_t^a for the first technology showed little evidence of a trend.

If δ is zero, then one can merely sum up past values of investment to get a measure of the capital stock:

$$K_t^a = INV_t + INV_{t-1} + \cdots + INV_{t-m_t+1}. \tag{5.7}$$

An estimate of the minimum amount of capital required to produce Y_t can in this case be obtained as merely $Y_t/(\bar{\mu}\bar{H})$, where $\bar{\mu}\bar{H}$ is estimated from solving one of the pairs of equations discussed above. It turned out that the estimates of $\bar{\mu}\bar{H}$ were roughly the same for alternative pairs of equations (with estimates of δ of approximately zero), so that it did not matter very much which pair of equations was used to estimate $\bar{\mu}\bar{H}$. The value of $\bar{\mu}\bar{H}$ that was chosen for the work below is 0.2660.

For the measurement of excess labor for this technology, it was first assumed that $\lambda_t = \bar{\lambda}(1 + \delta_\lambda)^t$. A few quarters were then chosen where it seemed plausible to assume that all machines were utilized $\bar{H}$ hours (no excess capital) and that the number of worker hours paid for equals the number of worker hours actually worked (no excess labor). If quarter s is one of these quarters, then it is the case from Equation (5.4) and the assumption just made about λ_t that:

$$JOBF_sHPF_s = \frac{\bar{\mu}\bar{H}}{\bar{\lambda}}\left[\frac{INV_s}{(1+\delta_\lambda)^s} + \frac{INV_{s-1}}{(1+\delta_\lambda)^{s-1}} + \cdots + \frac{INV_{s-m_s+1}}{(1+\delta_\lambda)^{s-m_s+1}}\right]. \tag{5.8}$$

Given data on worker hours paid for, investment, and m_t and given two quarters for which Equation (5.8) holds, one has two equations in two unknowns, $\bar{\mu}\bar{H}/\bar{\lambda}$ and δ_λ. Again, the equations are nonlinear, but they can easily be solved numerically. It turned out that the estimates of $\bar{\mu}\bar{H}/\bar{\lambda}$ and δ_λ were not highly sensitive to the choice of alternative pairs of equations to solve, but in the end two sets of estimates were considered. The two quarters chosen for the first set were 1953II and 1966I, and for these quarters the solution values were 118894.4 for $\bar{\mu}\bar{H}/\bar{\lambda}$ and 0.005204 for δ_λ. The two quarters chosen for the second set were 1953II and 1968II, with solution values of 121927.8 and 0.005602.

From the above information it is now possible to compute a series on worker hour requirements. Since δ_λ is positive, it is always optimal for a firm to utilize the newer machines first. Therefore, given Y_t and the estimate of $\bar{\mu}\bar{H}$, it is possible to compute from Equation (5.6), using also data on investment and the estimate of zero for δ, the age of the oldest machine operating in quarter t in the production of Y_t. This age—call it $\bar{m}_t$—will not be equal to the age of the oldest machine in existence in quarter t (denoted above as m_t) except for those quarters for which there is no excess capital on hand. Now, given values for $\bar{m}_t$, $\bar{\mu}\bar{H}/\bar{\lambda}$, δ_λ, and investment, one can compute from Equation (5.8) the number of worker hours required in period t to produce Y_t. This procedure can be carried out for each quarter, and so a series on worker hour requirements, $M_tH_t^M$, can be constructed.

It turned out that the two different sets of assumptions about the technology of the firm sector led to similar results. Some of these results are presented and discussed in Appendix A to this volume. In the end, the first technology was chosen to be used in the model because of its simpler nature. The fact that the two sets of results were similar means that the aggregate data used in this study do not appear to be capable of discriminating among alternative assumptions about the technology. Both technologies are clearly approximations, and what the data seem to indicate is that both approximations are about as equally good or as equally bad. The purpose of presenting both technologies in this chapter is to show that the results of this study do not appear to be sensitive to the choice of the technology for the model.

In the theoretical model it was possible for a firm to substitute capital for labor (or vice versa) over time through the purchase of different types of machines with differing worker-machine ratios. The type of machine that it was optimal for a firm to purchase in any one period resulted from the solution of its optimal control problem in the period. With the aggregate data used here, it seems highly unlikely that one would be able to pick up substitution effects of this sort, especially considering the fact that the data do not even appear to be capable of discriminating between the two somewhat different technologies considered above.

Using three digit industry data, some evidence was found in Fair [15] for the existence of capital-labor substitution of the kind just outlined, but the aggregate data used in this study do not permit the kind of test that was performed in [15]. Consequently, no attempt was made here to try to estimate the effects of this type of capital-labor substitution. This does not mean, however, that the cost of capital has no effect on the investment of the firm sector in the present model. This issue is discussed in section 5.5.

5.3 AN OUTLINE OF THE EMPIRICAL MODEL OF THE FIRM SECTOR

As was the case for the household sector, it is necessary regarding the firm sector to distinguish between its unconstrained and constrained decisions. The loan constraint on the firm sector can be handled in the same way that it was handled for the household sector, namely by including log ZR_t as an explanatory variable in the estimated equations (the equations being in log form). Under the assumptions made in the last chapter, adding this variable to an equation converts the left-hand side variable from an unconstrained decision variable to a constrained decision variable.

The treatment of the labor constant on the firm sector requires considerably more explanation. The labor constraint relates to the fact that a firm may not get as much labor in a period as it expected that it would at the wage rate that it set and may thus be forced to produce in the period less than it planned to. In other words, although the variable $JOBF_tHPF_t$ is the number of worker hours actually paid for by the firm sector in quarter t (see Table 5–1), it is not necessarily the number the firm sector planned to pay for. $JOBF_tHPF_t$ will be less than the planned number if the labor constriant is binding. When the labor constraint is binding on the firm sector, $JOBF_tHPF_t$ is determined by the household sector. Otherwise, it is determined by the firm sector. There are thus two regimes to consider regarding the determination of $JOBF_tHPF_t$.

In order to consider the two regimes problem in more detail, government employment must first be taken into account. The total number of worker hours paid for by the government sector in quarter t is, using the notation in Table 2–1, $JOBGC_tHPGC_t + JOBGM_tHPGM_t$. It will be useful for the present discussion to denote this variable as $MHPG_t$. The total number of hours that the household sector is paid for is, therefore, $JOBF_tHPF_t + MHPG_t$, which is also denoted as $JOBH_tHPH_t$ in Table 4–1. If the household sector is not constrained in its work effort, then it determines $JOBH_tHPH_t$.

If it is assumed, as is done in the theoretical model, that the government sector always gets the amount of labor that it wants, then in those cases where the household sector is not constrained in its work effort, $JOBF_tHPF_t$ is determined as the difference between $JOBH_tHPH_t$ and $MHPG_t$. This amount of labor may, as just mentioned, be less than the amount of labor that the firm sector planned at the beginning of the period to hire. If, on the other hand, the household sector is constrained in its work effort, then $JOBF_tHPF_t$ is determined by the firm sector, and $JOBH_tHPH_t$ is determined as the sum of $JOBF_tHPF_t$ and $MHPG_t$.

One possible approach to the two regimes problem would be to break up the sample period some way into two regimes and estimate separate equations in the two regimes. In one regime an equation explaining $JOBH_tHPH_t$ would be estimated, with $JOBF_tHPF_t$ being determined as the residual; and in the other regime an equation explaining $JOBF_tHPF_t$ would be estimated, with $JOBH_tHPH_t$ being determined as the residual. The either/or nature of this approach, however, has the disadvantage of making the results more sensitive to the choice of regimes than one might want. Since any procedure of choosing regimes is not error free, one would like to design a model that is not highly sensitive to errors made in choosing regimes.

In order to see how the two regimes problems was handled here, it is necessary to consider first a rough outline of the equations explaining the main decision variables of the firm sector. The following outline is based on the assumption that the loan constraint is not binding on the firm sector and on the assumption that sales expectations for the current period are perfect. A superscript p on a variable denotes the planned value of the variable, the plans being made at the beginning of period t. Consider the following seven equations:

$$PF_t^p = f_1(\cdots), \tag{5.9}$$

$$X_t^p = f_2(PF_t^p, \ldots), \tag{5.10}$$

$$Y_t^p = f_3(X_t^p, \ldots), \tag{5.11}$$

$$INV_t^p = f_4(Y_t^p, \ldots), \tag{5.12}$$

$$JOBF_t^p = f_5(Y_t^p, \ldots), \tag{5.13}$$

$$HPF_t^p = f_6(Y_t^p, \ldots), \tag{5.14}$$

$$WF_t^p = f_7(PF_t^p, JOBF_t^p HPF_t^p, \ldots). \tag{5.15}$$

PF_t^p is the price the firm sector plans to set. The variables that explain PF_t^p will be discussed later. X_t^p is the number of goods the firm sector plans to sell in period t. It is a function of PF_t^p and other variables. Y_t^p is the number of goods the firm sector plans to produce in period t. It is a function of X_t^p and other variables. INV_t^p is the amount of investment the firm sector plans to make in period t. It is a function of Y_t^p and other variables. $JOBF_t^p HPF_t^p$ is the number of worker hours the firm sector plans to pay for in period t. $JOBF_t^p$ and HPF_t^p are explained separately in the model; both are functions of Y_t^p and other variables. Finally, WF_t^p is the wage rate that the firm sector expects it will have to pay to attract the planned amount of labor. It is a function of PF_t^p, $JOBF_t^p HPF_t^p$, and other variables. Equations (5.9)–(5.15) are consistent with the decision sequence discussed at the end of section 5.1.

Let $JOBF_t^s HPF_t^s$ denote the supply of labor to the firm sector from the household sector at the wage rate WF_t^p. If this supply is greater than or equal to $JOBF_t^p HPF_t^p$, then all the plans of the firm sector can be realized. The planned values in (5.9)–(5.15) can be taken to be the observed values. If, on the other hand, the supply is less than $JOBF_t^p HPF_t^p$, the firm sector has to adjust. One must thus decide when the firm sector has to adjust and how it adjusts when it has to. With respect to the question of when the firm sector adjusts, assume for now that one has found a variable ZJ_t' that takes on a value of one when the firm sector does not have to adjust and a value of less than one otherwise. The construction of ZJ_t' will be explained later.

Consider now the question of how the firm sector adjusts when it receives less labor than it expects. The firm sector is assumed to adjust in this case by raising its price, thus cutting sales, and lowering its production, investment, and employment. In particular, it is assumed that:

$$\frac{PF_t}{PF_t^p} = (ZJ_t')^{\gamma_3},\ \gamma_3 < 0, \tag{5.16}$$

where PF_t is the observed price. The price is assumed to be raised enough in the constrained case ($ZJ_t' < 1$) so as to lead to the new values of $JOBF_t^p$ and HPF_t^p chosen by the firm sector to be equal to the supply from the household sector. The final set of equations for the firm sector is then postulated to be:

$$PF_t = (ZJ_t')^{\gamma_3} PF_t^p = (ZJ_t')^{\gamma_3} f_1(\cdots), \tag*{(5.9)$'$}$$

$$X_t = f_2(PF_t, \ldots), \text{ [this equation stands for a number of equations in the model]} \tag*{(5.10)$'$}$$

$$Y_t = f_3(X_t, \ldots), \tag*{(5.11)$'$}$$

$$INV_t = f_4(Y_t, \ldots), \tag*{(5.12)$'$}$$

$$JOBF_t = f_5(Y_t, \ldots), \tag*{(5.13)$'$}$$

$$HPF_t = f_6(Y_t, \ldots), \tag*{(5.14)$'$}$$

$$WF_t = f_7(PF_t, JOBF_t HPF_t, \ldots), \tag*{(5.15)$'$}$$

where all the variables are now observed variables. The possible labor constraint on the firm sector was thus handled by adding to the price equation, which is in log form, the term $\gamma_3 \log ZJ_t'$.

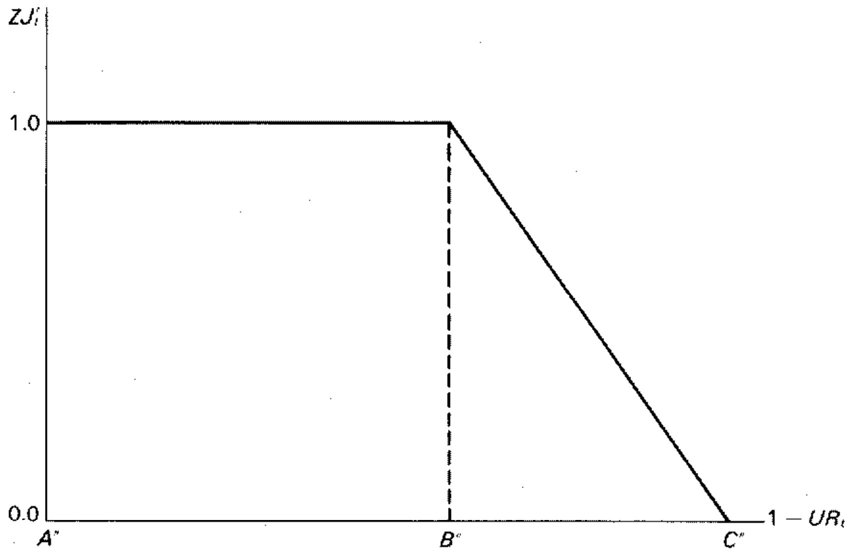

Figure 5-1. Desired Shape of ZJ'_t as a Function of $1 - UR_t$

The construction of ZJ'_t will now be explained. Although J_t^* was used as the measure of labor market tightness in the last chapter in computing ZJ_t, one minus the unemployment rate, $1 - UR_t$, is used as the measure of labor market tightness in this chapter in computing ZJ'_t. The reason for this difference is explained below. The desired shape of ZJ'_t as a function of $1 - UR_t$ is depicted in Figure 5-1. Point A'' is some value that is smaller than the smallest value of $1 - UR_t$ observed in the sample period; point B'' is the value of $1 - UR_t$ below which it seems reasonable to assume that the firm sector always gets as much labor as it expected; and point C'' is some value that is larger than the largest value of $1 - UR_t$ observed in the sample period.

If one wants ZJ'_t to equal 0.0 when $1 - UR_t$ equals C'', which, as explained in the next paragraph, is wanted here, then the right half of the normal density function cannot be used to approximate the curve in Figure 5-1. Instead, the following equation was used for the approximation:

$$ZJ'_t = \alpha_3 + \frac{1}{1 - UR_t - \alpha_4}, \tag{5.17}$$

where α_3 and α_4 are chosen so that ZJ'_t equals 1.0 when $1 - UR_t$ equals A'' and 0.0 when $1 - UR_t$ equals C''. The value chosen for A'' was 0.910 (a 9.0 percent unemployment rate), and the value chosen for C'' was 0.975 (a 2.5 percent unemployment rate). These values are slightly outside the range of observed values of $1 - UR_t$ in the sample period. For these two values, the values of α_3 and α_4 that lead to the above requirements being met are 4.454062 and 1.199514, respectively.

The procedure just described for constructing ZJ'_t constrains the unemployment rate always to lie above 2.5 percent in the model. When $1 - UR_t$ is equal to 0.975, ZJ'_t is equal to zero, which from Equation (5.9)′ implies (for $\gamma_3 < 0$) a value of PF_t of infinity. It turned out that the single equation fit of the price equation and the fit of the overall model were not very sensitive to the use of alternative values of the minimum unemployment rate. This result is not particularly surprising since during most of the sample period the economy was operating considerably above an unemployment rate of 2.5 percent. As a general rule, one would not expect the fit of a model to be sensitive to the imposition of a constraint on the behavior of the model regarding values that lie outside the range of values observed in the sample period. The constraint was imposed here not for any goodness-of-fit reasons, but to guarantee that the unemployment rate would never be driven below 2.5 percent in the optimal control experiments in Chapter Ten. It does seem unlikely that the unemployment rate in the United States could be driven much below 2.5 percent, and so the lower bound of 2.5 percent was imposed on the model.

The desire to impose this constraint on the model is the reason for the use of $1 - UR_t$ rather than J_t^* as the measure of labor market tightness for the construction of ZJ'_t. For the solution of the model, the predicted value of ZJ'_t was set equal to one whenever the predicted value of UR_t was greater than or equal to 9.0 percent. Otherwise, Equation (5.17) (Equation 78 in Table 2–2) was used to determine the predicted value of ZJ'_t. This procedure is similar to the procedures followed for ZJ_t and ZR_t, which were discussed in the last chapter.

As was the case for the treatment of the hours constraint on the household sector, the treatment of the labor constraint on the firm sector has the advantage of allowing the data some flexibility in estimating the effects of the constraint. No a priori constraints of a zero-one type are imposed on the data. The procedure followed here does have the disadvantage, however, of not necessarily using all the information on the labor market that is available. Only equations for $JOBF_t^p$ and HPF_t^p have been estimated; no attempt has been made to estimate also equations explaining $JOBH_t^s$ and HPH_t^s. When the hours constraint is not binding on the household sector, $JOBH_t^s$ and HPH_t^s are equal to the observed values ($JOBH_t$ and HPH_t). Since it is known from the theoretical model what variables affect $JOBH_t^s$ and HPH_t^s (the variables listed in Table 4–2), one has two potential equations to estimate that have not been estimated.

In order to put the present treatment of the labor constraint on the firm sector in a somewhat better perspective, it will be useful to review briefly three other ways that one could consider dealing with the demand for and supply of labor in the model. The present approach converts the two demand equations (explaining $JOBF_t^p$ and HPF_t^p) into equations with observed left-hand side variables. No explicit equations are postulated for $JOBH_t^s$ and HPH_t^s. Another approach would be to postulate also equations explaining $JOBH_t^s$ and HPH_t^s, convert the equations in some manner into equations with observed left-hand side variables, and then estimate the two resulting equations. One could perhaps convert the equations into equations with observed left-hand side variables by the use of some sort of a Z_t variable, as has been done in this study for other equations. Taking the government employment variables to be exogenous and using the definitions, $JOBF_t = JOBH_t - JOBGC_t - JOBGM_t$ and $JOBF_tHPF_t = JOBH_tHPH_t - JOBGC_t HPGC_t - JOBGM_tHPGM_t$, this approach would result in two equations explaining $JOBF_t$ (one in the firm sector and one in the household sector) and two explaining HPF_t. In solving the model the predicted values of $JOBF_t$ and HPF_t could be taken to be some weighted average of the predictions from the two equations for each variable.

A second alternative approach would be to postulate explicitly that the observed $JOBF_t$ is equal to the minimum of $JOBF_t^p$ and $JOBH_t^s - JOBGC_t - JOBGM_t$ (and similarly for HPF_t) and to use some of the recent econometric techniques that have been developed for estimating markets in disequilibrium to estimate the equations. (For a discussion of these techniques, see Fair and Jaffee [34], Fair and Kelejian [25], Ameniya [1], and Maddala and Nelson [32].)

A third alternative approach, but one that is not consistent with the specification of the theoretical model, would be to assume that the wage rate adjusts each quarter to clear the labor market, drop the equation explaining the wage rate from the model, and estimate separate equations explaining $JOBF_t^pHPF_t^p$ and $JOBH_t^sHPH_t^s$. In this case both $JOBF_t^pHPF_t^p$ and $JOBH_t^sHPH_t - JOBGC_tHPGC_t - JOBGM_tHPGM_t$ would always be equal to the observed value ($JOBF_tHPF_t$).

To summarize, there are clearly a number of other ways of dealing with the labor market than the approach taken here. The present approach has the advantages of flexibility and computational ease, but it does throw away some potentially important information. In future work it would be of interest to consider alternative approaches.

Before considering the other variables that appear in f_1 through f_7 in equations (5.9)′–(5.15)′, it should be noted that the inclusion of ZJ_t' in the price equation has introduced simultaneity into the model where there did not exist any before. PF_t affects X_t, which affects Y_t, which in turn affects $JOBF_t$. $JOBF_t$ affects UR_t, which in turn affects ZJ_t'. Consequently, PF_t has an effect on ZJ_t', as well as vice versa. Since in theory (i.e., not considering the approximation for ZJ_t' that has been used) ZJ_t' only enters the price equation in the case of a binding labor constraint on the firm sector, there is only simultaneity of the kind just described in the constrained case. The simultaneity takes the form in the constrained case of links between the expenditure equations in the household sector and the price equation in the firm sector.

One should think of the simultaneity in the constrained case as reflecting the outcome of a number of interactions between the household and firm sectors within the quarter. It is important to think this way to justify the rather strong assumption made following Equation (5.16) that the price is always raised enough in the constrained case so as to lead to the new values of $JOBF_t^p$ and HPF_t^p being equal to the supply from the household sector. It should also be noted that as the price is raised during this within-quarter adjustment process, the wage rate is also likely to be raised. Increasing the wage rate increases, of course, the supply of labor from the household sector. It is not, however, theoretically unambiguous that the wage rate will rise in this case. The effect on the wage rate is not unambiguous, because while the higher price level has a positive effect on the wage rate, the decrease in employment demand from the contraction of the firm sector has a negative effect.

5.4 THE ESTIMATES OF THE EQUATIONS FOR THE FIRM SECTOR

The eleven stochastic equations for the firm sector are Equations 9–19 in Table 2–3. The most important equations are 9, 10, 11, 12, 13, and 15, explaining, respectively, PF_t, Y_t, INV_t, $JOBF_t$, HPF_t, and WF_t. The following is a discussion of each of the eleven equations.

The PF_t Equation

Equation 9, explaining PF_t, is in log form and includes as explanatory variables the price of imports (PIM_t), the wage rate lagged one period (WF_{t-1}), the bond rate ($RAAA_t$), an investment tax credit variable ($DTAXCR_t$), the labor constraint variable (ZJ_t'), and PF_{t-1}.

The bond rate has a positive effect on PF_t, and the investment tax credit variable has a negative effect. $DTAXCR_t$ is defined in Table 2–1. It takes on a value of 1.0 when the credit of 7 percent is in full force, a value of 0.0 when the credit is not in force, and a value of 0.5 when the credit is estimated to about half in force. The value of 0.5 was used for the 1962III–1963IV, when the Long amendment was in effect, and for 1971III, when the credit was in effect for only about half of the quarter.

The inclusion of $RAAA_t$ and $DTAXCR_t$ in the price equation was guided by the results for the theoretical model. In the theoretical model the interest rate had a positive effect on the price that a firm sets. An increase in the interest rate, for example, caused a firm to contract, and the way that a firms contracts in the theoretical model is to raise its price, thus lowering expected sales, and to decrease its production, investment, and employment. The inclusion of $RAAA_t$ and $DTAXCR_t$ in Equation 9 should thus be considered an attempt to pick up this effect. Both variables are taken to measure part of the cost of capital to the firm sector.

The inclusion of the price of imports, the wage rate lagged one period, and the price level itself lagged one period in the price equation is designed to try to pick up expectational effects. As mentioned in Chapter One, section 1.1, a firm's expectations of other firms' prices plays an important role in the theoretical model in determining the price that the firm sets for the period. After some experimentation, the three variables just mentioned were chosen to represent expectational effects in the empirical model. Any choice of this sort is, of course, only a rough approximation to the actual way that expectations are formed.

Four variables that had some influence in the theoretical model on the price that a firm set were not found to be important in the experimentation that was done here. These four variables are the ratio of the stock of inventories to the level of sales, the level of sales itself, the amount of excess labor on hand, and the amount of excess capital on hand, all of the previous period.

Some experimentation was done, primarily through the use of dummy variables, to see if the effects of price controls should be taken into account in the estimation of the price equation. The only two quarters in which there appeared to be any important effects were 1971IV (the quarter affected by the first price freeze), where the price level seemed to change less than it otherwise would have, and 1972I (the quarter following the lifting of the freeze), where the price level seemed to change more than it otherwise would have. The smaller change in 1971IV seemed to be offset by the larger change in 1972I. When, for example, the dummy variables $D714_t$ and $D721_t$ were added to the PF_t equation, the coefficient estimates for the two variables were -0.00352 and 0.00684, respectively, with t-statistics of -1.14 and 2.28. The other coefficient estimates changed very little from the addition of the two dummy variables to the equation. Because of the small changes in the other coefficient estimates, and because there were no other quarters in which the effects of price controls seemed to be important, it was decided to ignore price controls altogether in the model and lump any effects from the controls into the error term in the equation. Price and wage controls may have some effects on the aggregate variables considered in the model, but the effects seem small enough to be able to be ignored with little harm.

The Y_t Equation

Equation 10 explains the production of the firm sector, Y_t. The equation is in log form and includes as explanatory variables the level of production of the previous period (Y_{t-1}), the current level of sales (X_t), the stock of inventories at the end of the previous period (V_{t-1}), and three dummy variables to account for the effects of the steel strike in 1959. This equation is based on the assumption that the firm sector first sets its price, knows then what its sales for the current period will be, and from this latter information decides on what its production for the current period will be. In practice, information on sales is available and decisions on production are made more than once during a quarter, and so firms have some flexibility within a quarter to adjust their production to unexpected changes in sales.

The assumption just made that sales expectations for the current quarter are perfect implies that firms can adjust their production completely during a quarter to any unexpected change in sales. This does not mean that firms are always assumed to plan to produce what they expect to sell, only that, given their plans and sales expectations at the beginning of the quarter, they can adjust their plans as actual sales deviate from expected sales. Some experimentation was done, using alternative assumptions about the formation of sales expectations, to see if production could be better explained under some other assumption than the assumption that sales expectations within a quarter are perfect, but this did not appear to be the case.

In the theoretical model production is smoothed relative to sales—i.e., the optimal production path of a firm generally has less variance than its expected sales path. This is because of various costs of adjustment that have been postulated in the model. The two most important adjustment costs are costs of changing employment and costs of changing the capital stock. There are also costs included in having the stock of inventories deviate from β_1 times sales, where $\beta_1 > 0$. If a firm were only interested in minimizing these latter costs, it would produce in period t according to the following equation (assuming perfect sales expectations for period t):

$$Y_t = X_t + \beta_1 X_t - V_{t-1}. \tag{5.18}$$

Since by definition $V_t - V_{t-1} = Y_t - X_t$, producing according to Equation (5.18) would insure that $V_t = \beta_1 X_t$.

Since there are other adjustment costs, it is generally not optimal for a firm to produce according to Equation (5.18). In the theoretical model there was no need to postulate explicitly how a firm's production plan deviated from Equation (5.18) because its optimal production path just resulted, along with the other optimal paths, from the direct solution of its optimal control problem. In the present case, however, it is necessary to postulate an explicit equation explaining the firm sector's production decision, an equation that can be considered to be an approximation to the way the firm sector actually makes its production decision. The standard assumption to make regarding the effects of adjustment costs on behavior is, in the present context, the following:

$$Y_t - Y_{t-1} = \lambda(Y_t^* - Y_{t-1}), 0 < \lambda \leqslant 1, \tag{5.19}$$

where, say, Y_t^* is the Y_t in Equation (5.18):

$$Y_t^* = X_t + \beta_1 X_t - V_{t-1}. \tag{5.18)'$$

Equation (5.19) states that the actual change in production in period t is some proportion of the change that the firm would make if it were only interested in minimizing the costs of having V_t deviate from $\beta_1 X_t$. Substituting Equation (5.18)′ into (5.19) yields:

$$Y_t = (1 - \lambda)Y_{t-1} + \lambda(1 + \beta_1)X_t - \lambda V_{t-1}. \tag{5.20}$$

Equation 10 in Table 2–3 is similar to Equation (5.20) except that it is in log form and has added to it a constant term and three dummy variables. It is also the case that the restrictions on the coefficients in Equation (5.20) have not been imposed in Equation 10. There are three variables on the right-hand side of Equation (5.20), but only two coefficients. Since Equations (5.19) and (5.18)′ are considered to be only a rough approximation to the production decision of the firm sector, the imposition of the restrictions in (5.20) did not seem warranted. The lagged output term in Equation 10 should be considered as picking up only in some rough way the effects of adjustment costs on the current production decision of the firm sector.

The production equation estimated here is consistent with the equation estimated for the lagged adjustment model in Fair [21]. The data used in [21] were monthly, seasonally unadjusted, three digit industry data, and for these data significant effects of future sales expectations were obtained. One would expect, if firms smooth production relative to sales, that the current production decision would depend in part upon expected future sales. This certainly appeared to be the case for the data used in [21], where significant effects of up to six months ahead were obtained. For the aggregate data used in this study, however, it did not appear to be possible to pick up any significant effects of future sales expectations on current production.

The INV_t Equation

Equation 11 explains the investment in plant and equipment of the firm sector, INV_t. The equation is in linear form, with the left-hand side variable being the change in investment. The explanatory variables include the amount of excess capital on hand at the end of the previous period ($K^a_{t-1} - KMIN_{t-1}$), the current change in output, the change in output lagged one, two, and three periods, the difference between gross investment and depreciation of the previous period ($INV_{t-1} - \delta_K K^a_{t-1}$), and two dummy variables to account for the effects of the automobile strike in 1970. The equation is based on the assumption that the firm sector decides on its level of production before deciding on its level of investment.

As was the case for the production decision, it is necessary with respect to the investment decision to postulate in the empirical model an explicit equation explaining this decision. The investment decision of a firm in the theoretical model results from the solution of its optimal control problem, and some approximation to this decision must be made here. Adjustment costs play an important role in the theoretical model in influencing a firm's investment decision, and because of these costs, it is sometimes optimal for a firm to hold excess capital. It is also the case, not surprisingly, that the amount of excess capital on hand at the beginning of a firm's decision period has an important effect on the solution values obtained in that period, especially on the solution values for investment.

Equation 11 is based on the following three equations:

$$(K^a_t - K^a_{t-1})^* = \alpha_0(K^a_{t-1} - KMIN_{t-1}) + \alpha_1(Y_t - Y_{t-1}) + \alpha_2(Y_{t-1} - Y_{t-2}) + \alpha_3(Y_{t-2} - Y_{t-3}) + \alpha_4(Y_{t-3} - Y_{t-4}), \tag{5.21}$$

$$INV^*_t = (K^a_t - K^a_{t-1})^* + \delta_K K^a_{t-1}, \tag{5.22}$$

$$INV_t - INV_{t-1} = \lambda(INV^*_t - INV_{t-1}),\ 0 < \lambda \leqslant 1. \tag{5.23}$$

For sake of the current discussion, call $(K^a_t - K^a_{t-1})^*$ in (5.21) "desired net investment" and call INV^*_t in (5.22) "desired gross investment." Equation (5.21) states that desired net investment is a function of the amount of excess capital on hand and of four change-in-output terms. If output is not changing and has not changed for the past four periods, and if there is no excess capital on hand, then desired net investment is zero. The past change-in-output terms in Equation (5.21) can best be thought of as being proxies for expected future output terms.

Equation (5.22) relates desired gross investment to desired net investment. $\delta_K K^a_{t-1}$ is the physical depreciation of the capital stock during period $t - 1$, δ_K being the estimated depreciation rate of the capital stock. By definition $INV_t = K^a_t - K^a_{t-1} + \delta_K K^a_{t-1}$, and Equation (5.22) is merely this same equation for the desired values. Equation (5.23) is a stock-adjustment equation relating the desired change in gross investment to the actual change. Equation (5.23) is meant to approximate cost-of-adjustment effects.

Combining Equations (5.21)–(5.23) yields:

$$\begin{aligned} INV_t - INV_{t-1} = {} & \lambda\alpha_0(K^a_{t-1} - KMIN_{t-1}) + \lambda\alpha_1(Y_t - Y_{t-1}) \\ & + \lambda\alpha_2(Y_{t-1} - Y_{t-2}) + \lambda\alpha_3(Y_{t-2} - Y_{t-3}) \\ & + \lambda\alpha_4(Y_{t-3} - Y_{t-4}) - \lambda(INV_{t-1} - \delta_K K^a_{t-1}), \end{aligned} \tag{5.24}$$

which is Equation 11 in Table 2–3 except for the dummy variables. The coefficient estimates in Equation 11 are of the expected signs, but the estimate of λ of 0.0155 is unreasonably small. Surely the actual change in gross investment in any one period is greater than 1.55 percent of the desired change. What the results appear to indicate is that the appropriate left-hand side variable in Equation (5.21) is not desired net investment, but rather the change in gross investment.

A number of investment equations were estimated in this study using different functional forms and different measures of excess capital, and invariably results were obtained for the change in gross investment that one would instead have expected to be true for net investment. One difficulty may be that depreciation has not been measured very precisely. The data on net investment depend on the particular measure of depreciation used, whereas the data on gross investment are direct NIA data.

It may be, for example, that a more accurate measure of depreciation would result in a larger coefficient estimate for the last term in Equation (5.24) (i.e., in a larger estimate of λ) and thus in a more reasonable set of results. It is also likely that the past change-in-output terms in the equation are picking up some cost-of-adjustment effects as well as expected future output effects, so that all cost-of-adjustment effects are not necessarily reflected in λ.

Whatever the case, the decision was made to take Equation 11 as the investment equation even though the estimate of λ seems too small. It should be noted that the long run properties of Equation (5.21) are still reasonable even if the change in gross investment is the left-hand side variable. One does expect the change in gross investment to be zero if there is no excess capital on hand and no recent changes in output.

The $JOBF_t$ and HPF_t Equations

Equation 12 explains the number of jobs in the firm sector, $JOBF_t$. The equation is in log form, with the left-hand side variable being $\log JOBF_t - \log JOBF_{t-1}$. The explanatory variables include a variable measuring the amount of excess labor on hand during the previous period ($\log JOBF_{t-1} - \log M_{t-1}H^M_{t-1}$), a time trend, three change-in-output terms, and two dummy variables to account for the effects of the steel strike in 1959. Equation 13 contains one less change-in-output term than does Equation 12, no dummy variables, and one added variable, $\log HPF_{t-1}$. Equations 12 and

13 are based on the assumption that the firm sector decides on its level of production before it decides on the number of jobs and the number of hours paid per job.

Equations 12 and 13 are meant to represent in an approximate sense the employment decisions of firms that result from the solutions of their optimal control problems. As was the case for a firm's investment decision, adjustment costs play an important role in the theoretical model in influencing a firm's employment decision. Because of these costs, it is sometimes optimal for a firm to hold excess labor. The amount of excess labor on hand at the beginning of the period has an important effect on the decisions made in that period, especially on the employment decision.

The excess labor variable in Equations 12 and 13 is explained as follows. $M_{t-1}H_{t-1}^M$ is, from the discussion in section 5.2, the number of worker hours required to produce Y_{t-1}. Let HS_{t-1} denote the average number of hours per job that a firm would like to be worked in period $t-1$ if there were no adjustment costs to contend with. $M_{t-1}H_{t-1}^M/HS_{t-1}$ is then the number of jobs required to produce Y_{t-1} if the average number of hours worked per job were HS_{t-1}. For the sake of the following discussion, this number will be referred to as the "desired" number of jobs for period $t-1$.

A measure of excess labor for period $t-1$ is the ratio of the actual number of jobs in the period to the desired number. The log form of this measure is $\log JOBF_{t-1} - \log(M_{t-1}H_{t-1}^M/HS_{t-1})$, or $\log JOBF_{t-1} - \log M_{t-1}H_{t-1}^M + \log HS_{t-1}$. If it is finally assumed that HS_{t-1} is a smoothly trending variable, namely $\overline{H}e^{\delta t}$, then the measure of excess labor is $\log JOBF_{t-1} - \log M_{t-1}H_{t-1}^M + \log \overline{H} + \delta t$. In Equations 12 and 13 the $\log JOBF_{t-1} - \log M_{t-1}H_{t-1}^M$ terms enters separately, and the equations include a constant term and time trend to pick up the effects of $\log HS_{t-1}$.

In the theoretical model, employment is generally smoothed relative to production because of the adjustment costs. The specification of Equations 12 and 13 and the coefficient estimates reflect this fact. The change in jobs times hours paid per job ($JOBF_tHPF_t$) is less than proportional to the current change in output. The past change-in-output terms in the two equations can be interpreted either as representing the effects of past output behavior on current employment decisions that are not captured in the excess labor terms or as being proxies for expected future output changes (or as both).

The $\log HPF_{t-1}$ term in Equation 13 reflects the fact that, unlike $JOBF_t$, which can move steadily upward or downward over time, HPF_t fluctuates around a relatively constant level of hours (such as 40 hours per week). If HPF_t is not equal to this level, this should, other things being equal, bring forces into play causing it to return to this level. Therefore, a term like $\log HPF_{t-1} - \log HS_{t-1}$ should be added to Equation 13, which, given the assumption made about HS_{t-1} above, is equivalent to adding $\log HPF_{t-1}$, a constant, and a time trend to the equation.

Equations 12 and 13 are similar to the equations estimated in Fair [23]. The data used in [23] are monthly, seasonally unadjusted, three digit industry data. For these data, significant effects (of up to six months ahead) of future output expectations on current employment decisions were obtained. For the aggregate data used in this study it is not possible to obtain such precise effects, although, as mentioned above, the past change-in-output terms in Equations 12 and 13 can be considered to be picking up in part expected future output effects.

Equations 12 and 13 are explained in detail in [23]. See in particular the discussion in Chapter 8 regarding the reasons for estimating separate equations for $JOBF_t$ and HPF_t, rather than just one equation explaining $JOBF_t HPF_t$. Although the study in [23] was completed before the theoretical model in Volume I was developed, the basic equations in [23] are consistent with the theoretical model if they are interpreted as representing approximations to the employment decisions of a firm that result from the solution of its optimal control problem. Equation 12 is also similar to the employment equation estimated in [14] for the private nonfarm sector, the only main difference here being the inclusion of one more change-in-output term.

The WF_t Equation

The last major equation estimated for the firm sector is Equation 15, explaining WF_t. The equation is in log form and includes as explanatory variables the price of firm sales (PX_t), a measure of labor market tightness (J_t^*), a time trend, and the wage rate lagged one period.

In the theoretical model a firm's optimal wage path is that path the firm expects it needs to set to attract the amount of labor implied by its optimal employment path. Two important factors influencing a firm's wage rate decision, in addition to the amount of labor that it wants, are its expectations of other firms' wage rates and its expectations of the labor supply curve facing it. It is thus necessary in empirical work to attempt to account for these expectations in some way.

The condensed model for the firm sector in Volume I is an approximation to the way a firm actually behaves in solving its optimal control problem each period. Equation 15 in Table 2–3 is similar to the equation representing the firm sector's wage rate decision in the condensed model. In the condensed model the current wage rate of the firm sector is a function of the wage rate of the previous period, the current price level, and two terms representing general labor market conditions and the firm sector's demand for labor (see statement [15] on page 66 in Volume I). The WF_{t-1}, PX_t, and J_t^* terms in Equation 15 can be considered to be accounting for the effects of these variables.

Equation 15 can also be considered, at least in a loose sense, as reflecting the outcome of bargaining over time between the firm and household sectors over the real wage rate. In the theoretical model bargaining takes the form of the firm sector adjusting over time, i.e., over more than one period, to changes in the labor supply curve facing it, the labor supply curve being determined each period by the household sector. If an equation like Equation 15 is interpreted in this way, an important question is which wage rate variable and which price variable are relevant for the bargaining process. The choice for the price variable here is PX_t, the price of total firms sales. PX_t excludes import prices and indirect business taxes. Neither an increase in import prices nor an increase in indirect business taxes benefits the firm sector, although both increases do hurt the household sector. The relevant price variable for the household sector is PH_t, which is inclusive of import prices and indirect business taxes. The use of PX_t in Equation 15 thus reflects the assumption that the household sector is aware that some price increases benefit the foreign sector and the government sector rather than the firm sector and considers only the prices that benefit the firm sector in its bargaining process with the firm sector.

The main question regarding which wage rate variable to use in an equation like 15 is whether the wage rate should be inclusive or exclusive of employer social security taxes. WF_t is exclusive of these taxes, whereas $(1 + d_{5t})WF_t$ is inclusive of the taxes. d_{5t} is the employer social security tax rate. If the firm sector effectively pays the taxes, then the appropriate wage rate variable in Equation 15 is WF_t. In this case, an increase in d_{5t} does not affect the bargaining process. If, on the other hand, the household sector effectively pays the taxes, then the appropriate wage rate variable is $(1 + d_{5t})WF_t$.

The procedure followed here regarding this question was to attempt to let the data tell how much of the tax is paid by each sector. Assume that the appropriate wage rate variable is $(1 + d_{5t})^{\gamma} WF_t$, where $0 \leqslant \gamma \leqslant 1$. The log of this variable is $\gamma \log(1 + d_{5t}) + \log WF_t$, so that this specification introduces the term $-\gamma \log(1 + d_{5t})$ on the right-hand side of Equation 15, with $\log WF_t$ being the left-hand side variable. γ is a parameter that can be estimated. The estimates of γ that were obtained in the experimentation with the wage equation were generally close to zero and not significant.

In the end the decision was made to constrain γ to be zero and so to drop the term $-\gamma \log(1 + d_{5t})$ from the equation. The results obtained in this study thus indicate that the firm sector pays the taxes. This conclusion should, however, be interpreted with a certain amount of caution because of the crude nature of the test and the highly aggregative nature of the data. See Brittain [6] for a more detailed study of the incidence of social security taxes.

One final question about the wage equation that was considered in this study is whether any long run constraints should be imposed on the equation. Ignore for now the effects of J_t^*, which can be considered to be short run in nature, and take WF_t/PX_t to be the real wage rate. If productivity is growing at roughly a constant rate (g) over time, then one might want to postulate that WF_t/PX_t also grows on average at rate g:

$$\frac{WF_t}{PX_t} = Ae^{gt}, \tag{5.25}$$

where A is a constant. Equation (5.25) in log form is:

$$\log WF_t = \log PX_t + \log A + gt. \tag{5.26}$$

Imposing this long run constraint on Equation 15 requires constraining the coefficients of $\log WF_{t-1}$ and $\log PX_t$ to sum to one. The TSLS and FIML estimates in Equation 15, which are not constrained, both sum to 1.031. This sum is close enough to one that it would make little difference regarding the properties of the model whether the sum was constrained to be one or not. In the final analysis the decision was made not to impose the constraint, primarily because of the feeling that the equation is too approximate to warrant this kind of refinement.

The Equations Explaining $HPFO_t$, DDF_t, $DIVF_t$, $INTF_t$ and IVA_t

The remaining five stochastic equations for the firm sector are not nearly as important as the others. Equation 14 explains the average number of overtime hours paid per quarter by the firm sector, $HPFO_t$. This variable, as explained in section 2.3, is needed for three definitions in the model. $HPFO_t$ is explained as a function of HPF_t, the total average number of hours paid per quarter by the firm sector. One would expect $HPFO_t$ to be related to HPF_t in roughly the manner indicated in Figure 5-2. Up to some point A (e.g., 40 hours per week), $HPFO_t$ should be zero or some small constant number, and after point A, increases in $HPFO_t$ and HPF_t should be one for one. An approximation to the curve in Figure 5-2 is:

$$HPFO_t = e^{\alpha_1 + \alpha_2 HPF_t}, \tag{5.27}$$

which in log form is:

$$\log HPFO_t = \alpha_1 + \alpha_2 HPF_t. \tag{5.28}$$

Equation 14 in Table 2-3 is the same as Equation (5.28) ,with two exceptions. First, HPF_t has a negative trend, and it was detrended before being used in the equation. The 0.5482 coefficient in Equation 14 was obtained from a regression of HPF_t on a constant and t for the 1952I–1974II period. Second, there appeared to be an important shift in the relationship

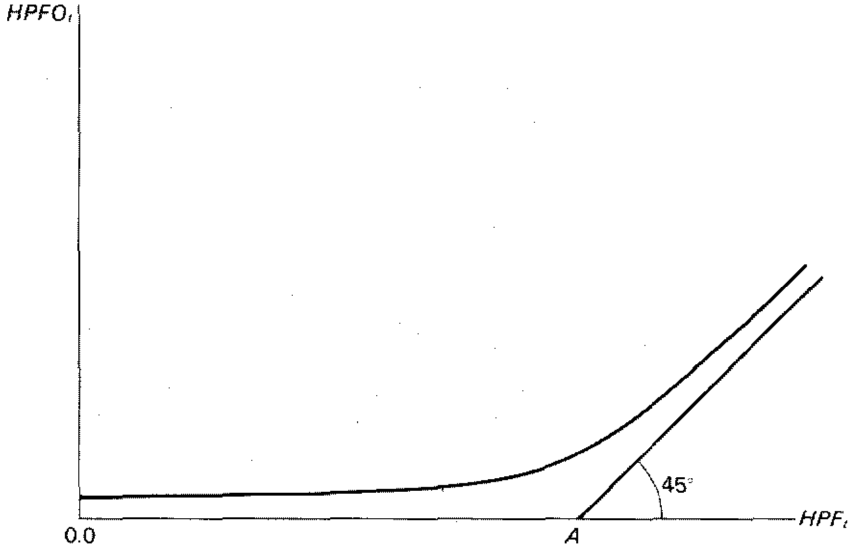

Figure 5-2. Expected Relationship Between *$HPFO_t$ and HPF_t*

between $HPFO_t$ and HPF_t beginning in 1966I, and so a dummy variable, $DD661_t$, was added to the equation to account for this shift. $DD661_t$ takes on a value of 0.0 before 1966I and a value of 1.0 from 1966I on. The sample period began in 1956I for this equation because data on $HPFO_t$ do not exist before this time.

Equation 16 explains demand deposits and currency of the firm sector, DDF_t. The equation is in log form, and the explanatory variables include the value of sales of the firm sector in current dollars, XX_t, the bill rate, and DDF_{t-1}. In the theoretical model there is a difference between aimed-for demand deposits and actual demand deposits. The former is a function of the firm's wage bill. The latter is determined residually and may differ from the former if the firm's expectations of its cash flow do not turn out to be correct. Actual demand deposits act in part as a buffer to absorb in a period any difference between actual and expected cash flow.

In the empirical model, DDF_t does not act as a buffer, but is rather assumed to be a direct decision variable of the firm sector and determined according to Equation 16. What is residually determined in the empirical model is the value of the loans of the firm sector, LF_t (see Equation 55 in Table 2-2). In the theoretical model, LF_t is not residually determined, but is itself a direct decision variable of a firm. It was useful in the theoretical model to take LF_t to be a direct decision variable because of the way that the loan constraint was treated. In the empirical model, however, it is useful *to take DDF_t to be a direct decision variable because of the different treatment* of the loan constraint. Although the firm's aimed-for demand deposits were tied to the firm's wage bill in the theoretical model, slightly better results were obtained here by the use of the value of sales of the firm sector in Equation 16.

Equation 17 explains the value of dividends paid by the firm sector, $DIVF_t$. The equation is in log form and includes as explanatory variables profits after taxes ($\pi F_t - TAXF_t$), the loan constraint variable (ZR_t), and $DIVF_{t-1}$. This is a typical kind of dividend equation that is estimated in the literature, except for the loan constraint variable. What the results indicate is that the firm sector pays out less of its after tax profits in dividends when the loan constraint is binding than otherwise. In the theoretical model a firm each period pays out all its after-tax profits in dividends, so that there is nothing in the model that can be used to guide the specification here.

Equation 18 explains the value of interest paid by the firm sector, $INTF_t$. The equation is in linear form and includes as explanatory variables the value of loans of the firm sector (LF_t), the bond rate ($RAAA_t$), and $INTF_{t-1}$. In the theoretical model $INTF_t$ equals RL_tLF_t, where RL_t is the loan rate, and Equation 18 is an attempt to approximate this.

The final stochastic equation for the firm sector, Equation 19, explains the inventory valuation adjustment, IVA_t. The equation is linear and includes as explanatory variables the current price of firm sales (PX_t), the price of firm sales lagged one period (PX_{t-1}), and the stock of inventories at the end of period $t-1$ (V_{t-1}). In the theoretical model IVA_t equals $-(P_t - P_{t-1})V_{t-1}$, and Equation 19 is an attempt to approximate this equation. The coefficient estimates for PX_t and PX_{t-1} in Equation 19 are almost exactly equal in absolute value, which is as expected.

A Brief Comparison of the TSLS and FIML Estimates

FIML estimates were obtained for Equations 9, 10, 12, 13, 15, and 16 in the firm sector, which explain, respectively, PF_t, Y_t, $JOBF_t$, HPF_t, WF_t, and DDF_t. The TSLS and FIML estimates of these six equations are generally quite close. The largest differences between the estimates occur for the coefficient of the labor constraint variable in the PF_t equation and for the coefficient of $\log Y_t - \log Y_{t-1}$ in the $JOBF_t$ equation. Otherwise, the differences are very small.

5.5 A REVIEW OF THE MODEL OF FIRM SECTOR

Before concluding this chapter, it will be useful to review briefly some of the important properties of the empirical model of the firm sector. One good way of reviewing the model is to consider the effects that the bond rate have on the firm sector. The bond rate has a positive effect on the price that the firm sector sets (Equation 9). The household sector responds negatively to higher prices, so that a higher price leads to lower consumption by the household sector and thus a lower level of sales of the firm sector. A lower level of sales has a negative effect on the production of the firm sector (Equation 10).

A lower level of production in turn has a negative effect on the investment of the firm sector (Equation 11) and on the number of jobs and the average number of hours paid per job in the firm sector (Equations 12 and 13). The bond rate thus has a negative effect on investment and employment in the firm sector. The bond rate does not appear directly as an explanatory variable in the investment and employment equations, but instead affects investment and employment through its effect on the price that the firm sector sets. In a similar fashion, the investment tax credit affects investment and employment through its effect on the firm sector's price. A higher credit leads to a lower price, which then leads to more investment and employment.

In the theoretical model a binding loan constraint has a positive effect on the price set by a firm and thus a negative effect on its production, investment, and employment. In the empirical work here, however, no important effect of the loan constraint variable on PF_t could be found. The only place where the loan constraint variable did appear to have an effect on the firm sector was in the dividend equation (Equation 17), where a more restrictive loan constraint implies fewer dividends paid by the firm sector than otherwise.

The amount of excess capital on hand has a direct negative effect on investment (Equation 11), and the amount of excess labor on hand has a direct negative effect on employment (Equations 12 and 13). In the theoretical model the amounts of excess capital and labor on hand had negative effects on the price set by a firm, but no important effects of this sort could be found in the empirical work here. Likewise, no important effects of the ratio of inventories to sales of the previous period (V_{t-1}/X_{t-1}) could be found on PF_t, even though this ratio had a negative effect on the price set by a firm in the theoretical model.

The measure of labor market tightness J_t^* has a contemporaneous positive effect on the wage rate set by the firm sector (Equation 15). The wage rate in turn has an effect on the price level with a lag of one period. The inclusion of the lagged wage rate in the price equation is designed to pick up expectational effects. The other variables in the price equation that are assumed to be picking up expectational effects are the lagged price (PF_{t-1}) and the price of imports (PIM_t).

An important variable in the price equation is the labor constraint variable, ZJ_t'. In theory, this variable affects PF_t only when the labor constraint is binding on the firm sector, although in practice it actually has at least a small effect all the time because of the approximation for ZJ_t' that has been used. ZJ_t' is a nonlinear function of $1 - UR_t$. The use of ZJ_t' in the price equation is designed to try to pick up the effect of the labor constraint on the firm sector. When the firm sector receives less labor than it expected it would at the wage rate that it set, it is assumed (within the quarter) to raise its price and contract.

This completes for now the discussion of the equations for the firm sector. The relationship between the firm sector and the other sectors in the model is examined in more detail in Chapter Nine.