Oxford Economic Papers 45 (1993), 169–190

TESTING THE RATIONAL EXPECTATIONS HYPOTHESIS IN MACROECONOMETRIC MODELS

By RAY C. FAIR

1. Introduction

ALTHOUGH the assumption that expectations are rational is often made in macroeconometric model building, it is seldom tested. Hall and Henry (1988), for example, use this assumption in a number of equations of their model, but no tests are made against other possible expectational assumptions. The rational expectations (RE) hypothesis has come to be treated like the profit maximization hypothesis—a working hypothesis to be used without testing. This is, however, a risky strategy because the properties of models can be quite sensitive to the use of the RE assumption.[1] If this assumption is not a good approximation of the way that expectations are actually formed, models that use this assumption may not be good approximations of the economy.

The RE hypothesis is tested in this paper by nesting equations without rational expectations within those with rational expectations. The test simply comes down to whether certain variables in an equation are statistically significant. Also, the test does not require the econometrician to have all the variables that agents use in forming their expectations at her or his disposal—a partial list will do. In this sense the test differs in an important way from Hendry's (1988) test, where a complete list of the variables that affect expectations is needed (see Section 3 for more discussion of this).

The test is developed in Sections 2 and 3, and the results of applying the test to a number of macroeconomic equations are reported in Section 4. The setup of this paper also allows an easy way to examine the sensitivity of the properties of a model to the RE hypothesis, and this issue is examined in Section 5.

2. Macroeconomic modeling

To motivate the test, a brief discussion of macroeconomic methodology will be helpful. There are a number of ways in which theory is used in the specification of empirical relationships. A common procedure in macroeconomics is to begin by postulating a maximization problem for a representative

[1] For example, the results in Fair (1979b) show that the properties of a macroeconometric model are sensitive to the assumption of rational expectations in the bond and stock markets.

agent. Consider, for example, the following maximization problem for a representative household. Maximize

$$E_0U(C_1, \ldots, C_T, L_1, \ldots, L_T) \tag{1}$$

subject to

$$\begin{aligned} S_t &= W_t(H - L_t) + r_tA_{t-1} - P_tC_t \\ A_t &= A_{t-1} + S_t \\ A_T &= \tilde{A} \end{aligned} \tag{2}$$

where C is consumption, L is leisure, S is savings, W is the wage rate, H is the total number of hours in the period, r is the one-period interest rate, A is the level of assets, P is the price level, $\tilde{A}$ is the terminal value of assets, and $t = 1, \ldots, T$. E_0 is the expectations operator conditional on information available at time 0. Given A_0 and the conditional distributions of the future values of W, P, and r, it is possible in principle to solve for the optimal values of C and L for period 1, denoted C_1^* and L_1^*. In general, however, this problem is not analytically tractable. In other words, it is not generally possible to find analytic expressions for C_1^* and L_1^*.

At this point there are two main approaches that can be followed. One is to postulate a particular functional form for the utility function and estimate its parameters from the first order conditions. This is the basic approach taken, for example, by Hall (1978), Hansen and Singleton (1982), and Mankiw, Rotemberg, and Summers (1985).

The other approach is to estimate approximations of the decision equations. This approach in the context of the above example is the following. First, the random variables, W_t, P_t, and r_t, $t = 1, \ldots, T$, are replaced by their expected values, W_t^e, P_t^e, r_t^e, $t = 1, \ldots, T$. Given this replacement, one can write the expressions for C_1^* and L_1^* as

$$C_1^* = f_1(A_0, \tilde{A}, W_1^e, \ldots, W_T^e, P_1^e, \ldots, P_T^e, r_1^e, \ldots, r_T^e, \alpha) \tag{3}$$

$$L_1^* = f_2(A_0, \tilde{A}, W_1^e, \ldots, W_T^e, P_1^e, \ldots, P_T^e, r_1^e, \ldots, r_T^e, \alpha) \tag{4}$$

where α is the vector of parameters of the utility function. Equations (3) and (4) simply state that the optimal values for the first period are a function of (i) the initial and terminal values of assets, (ii) the expected future values of the wage rate, the price level, and the interest rate, and (iii) the parameters of the utility function.[2] The functional forms of equations (3) and (4) are not in general known. The aim of the empirical work is to try to estimate equations that are approximations of equations (3) and (4). Experimentation consists of trying different functional forms and of trying different assumptions about how expectations are formed. Because of the large number of expected values in equations (3) and (4), the expectational assumptions usually restrict the number

[2] If information for period 1 is available at the time the decisions are made, when W_1^e, P_1^e, and r_1^e should be replaced by the actual values in equations (3) and (4).

of free parameters to be estimated. For example, the parameters for $E_0 W_1, \ldots, E_0 W_T$ might be assumed to lie on a low order polynomial or to be geometrically declining.

There are pros and cons to both of these approaches. The first approach has the advantage of estimating 'deep structural parameters' such as the parameters of utility functions, but the results may be sensitive to errors made in specifying the functional form of the utility function. The results in Mankiw, Rotemberg, and Summers (1985), for example, are not supportive of this approach. It is also generally not possible from the estimated first order conditions to derive analytically the decision equations that correspond to the conditions.

The second approach does not estimate deep structural parameters, and so it is subject to the Lucas (1976) critique. It also uses the certainty equivalence procedure, which is only strictly valid in the linear-quadratic setup. On the other hand, it allows more flexibility in estimating functional forms, and it does end up with estimated decision equations.

The second approach will be used in this paper. As will be seen, this approach allows an easy way of nesting equations without RE within those with RE. This paper is thus based on the implicit assumptions that the Lucas critique is not quantitatively important and that the certainty equivalence assumption is a reasonable approximation. These assumptions can be tested,[3] but this is beyond the scope of this paper.

When decision equations like (3) and (4) are estimated, it is often the case that lagged dependent variables are used as explanatory variables. Since C_0 and L_0 do not appear in equations (3) and (4), how can one justify the use of lagged dependent variables? A common procedure is to assume that C_1^* in (3) and L_1^* in (4) are long-run 'desired' values. It is then assumed that because of adjustment costs, there is only a partial adjustment of actual to desired values. The usual adjustment equation for consumption would be

$$C_1 - C_0 = \lambda(C_1^* - C_0), \qquad 0 < \lambda < 1 \tag{5}$$

which adds C_0 to the estimated equation. This procedure is *ad hoc* in the sense that the adjustment equation is not explicitly derived from utility maximization. One can, however, assume that there are utility costs to large changes in consumption and leisure and thus put terms such as $(C_1 - C_0)^2$, $(C_2 - C_1)^2$, $(L_1 - L_0)^2$, $(L_2 - L_1)^2, \ldots$ in the utility function (1). This would add the variables C_0 and L_0 to the right-hand side of equations (3) and (4), which would justify the use of lagged dependent variables in the empirical approximating equations for (3) and (4).

[3] One way they can be tested is the following. If either or both of the two assumptions are false, a model based on them will be seriously misspecified. A method is proposed in Fair (1980) for comparing models that should reject seriously misspecified models. In fact, one of the byproducts of the method is an estimate of how badly a model is misspecified. The use of this method should thus weed out models that suffer seriously from the Lucas critique or the use of the certainty equivalent assumption. See Fair (1979a) and Litterman (1980) for uses of this method.

3. Expectational assumptions and the test

Within the context of the second approach, the assumption that expectations are rational can be tested as follows. Assume that the equation to be estimated is:

$$y_t = X_{1t}\alpha_1 + {}_tX^e_{2t+i}\alpha_2 + u_t, \qquad t = 1, \ldots, T \tag{6}$$

where X_{1t} is a vector of observed variables and ${}_tX^e_{2t+i}$ is the expectation of X_{2t+i} based on information as of the beginning of period t, $i > 0$ being some fixed integer. The following discussion can easily be generalized to cover more than one expectational variable and more than one value of i, but for simplicity it will focus on the one-variable, one-value-of-i case. It will also be assumed for now that X_{1t} contains only predetermined variables and that all these variables are known at the time that expectations are formed for period $t + i$.

A traditional assumption about expectations is that the expected future values of a variable are a function of its current and past values. One might postulate, for example, that ${}_tX^e_{2t+i}$ depends on X_{2t} and X_{2t-1}, where it is assumed that X_{2t} (as well as X_{2t-1}) is known at the time the expectation is made. The equation could then be estimated with X_{2t} and X_{2t-1} replacing ${}_tX^e_{2t+i}$ in (6). Note that this treatment, which is common to many macroeconomic models, is not inconsistent with the view that agents are 'forward looking'. Expected future values do affect current behavior. It is just that the expectations are formed in fairly simple ways—say by looking only at the current and lagged values of the variable itself.

If instead ${}_tX^e_{2t+i}$ is rational, equation (6) can be consistently estimated using Hansen's (1982) generalized method of moments estimator. This method requires that agents form expectations rationally and that there is an observed vector of variables (observed by the econometrician), denoted Z_t, that is used in part by agents in forming their (rational) expectations. The method does not require for consistent estimates that Z_t include all the variables used by agents in forming their expectations.

Hansen's method is general enough to require some explanation of how it is applied in the present case. The following is a brief outline; more details are given in Appendix A. First, let the expectation error for ${}_tX^e_{2t+i}$ be

$${}_t\varepsilon_{t+i} = X_{2t+i} - {}_tX^e_{2t+i} \tag{7}$$

where X_{2t+i} is the actual value of the variable. Substituting (7) into (6) yields

$$\begin{aligned} y_t &= X_{1t}\alpha_1 + X_{2t+i}\alpha_2 + u_t - {}_t\varepsilon_{t+1}\alpha_2 \\ &= X_t\alpha + v_t. \end{aligned} \tag{8}$$

where $X_t = (X_{1t}X_{2t+i})$, $\alpha = \begin{pmatrix}\alpha_1 \\ \alpha_2\end{pmatrix}$, and $v_t = u_t - {}_t\varepsilon_{t+i}\alpha_2$.

Consider first the two stage least squares (2SLS) estimation of equation (8), where the vector of first stage regressors is the vector Z_t used by agents in forming their expectations. A necessary condition for consistency is that Z_t and v_t be uncorrelated. This will be true if both u_t and ${}_t\varepsilon_{t+i}$ are uncorrelated with Z_t. The assumption that Z_t and u_t are uncorrelated is the usual 2SLS assumption. The assumption that Z_t and ${}_t\varepsilon_{t+i}$ are uncorrelated is the rational expectations assumption. If expectations are formed rationally and if the variables in Z_t are used (perhaps along with others) in forming the expectation of X_{2t+i}, then Z_t and ${}_t\varepsilon_{t+i}$ are uncorrelated. Given this assumption (and the other standard assumptions that are necessary for consistency), the 2SLS estimate of α in equation (8) is consistent. The 2SLS estimator does not, however, account for the fact that v_t is a moving average error of order $i - 1$, and so it loses some efficiency. Hansen's estimator in this context is simply 2SLS modified to account for this moving average process.

The test of the RE hypothesis can now be described. Consider in the context of the above example testing the RE hypothesis against the simpler alternative that ${}_tX^e_{2t+i}$ is only a function of X_{2t} and X_{2t-1}. Assume that X_{2t} and X_{2t-1} are in the X_{1t} vector in equation (6). The test then consists of estimating equation (8) using Hansen's method and testing the hypothesis that α_2 equals zero. The Z_t vector used for Hansen's method would include the variables in X_{1t} plus other variables assumed to be in the agents' information sets. The test is really whether these other variables matter. If agents do not use more information than that contained in X_{1t} in forming their expectations of X_{2t+i}, then α_2 in equation (8) is zero—the use of the variables in Z_t as first stage regressors for X_{2t+i} adds nothing that is not already contained in X_{1t}.

The test of the RE hypothesis is thus to add variable values led one or more periods to an equation with only current and lagged values and estimate the resulting equation using Hansen's method. If the led values are not significant, this is evidence against the RE hypothesis. It means essentially that the extra variables in Z_t do not contribute significantly to the explanatory power of the equation.

An implicit assumption behind this test is that Z_t contains variables other than those in X_{1t}. If, say, the optimal predictor of X_{2t+i} (and thus the agents' expected value under the RE hypothesis) were solely a function of X_{2t} and X_{2t-1}, then the above test would not be appropriate. In this case the traditional approach is consistent with the RE hypothesis, and there is nothing to test. The assumption that Z_t contains many variables is consistent with the specification of most macroeconometric models (hence the reason for the last three words in the title of this paper), where the implicit reduced form equations for the endogenous variables contain a large number of variables. This assumption will be maintained throughout this paper. The paper has nothing to say about cases in which there is a very small number of variables in Z_t.

As an example of the test, consider the wage rate W in the consumption equation (3) and assume that W_t is known, where t is period 1. The wage variables in equation (3) are W_t, W^e_{t+1}, W^e_{t+2}, etc. If agents use only current and lagged values of W in forming expectations of future values of W, then candidates for explanatory variables are W_t, W_{t-1}, W_{t-2}, etc. Under the RE hypothesis, on the other hand, agents use Z_t in forming their expectations for periods $t + 1$ and beyond, and candidates for explanatory variables are W_{t+1}, W_{t+2}, etc., with Hansen's method used for the estimation. The led values of W can thus be added to the equation. Likewise, led values of the price variable P can be added. The test is then to test for the joint significance of the added values.

It should be noted that the test proposed here is quite different from Hendry's (1988) test of expectational mechanisms. Hendry's test requires one to postulate the expectation generation process, which is then examined for its constancy across time. If the structural equation that contains the expectations is constant but the expectations equations are not, this refutes the expectations equations. As noted above, for the test in this paper Z_t need not contain all the variables used by agents in forming their expectations, and so the test does not require a complete specification of the expectations generation process. The two main requirements are only that Z_t be correlated with X_{2t+i} but not with ${}_t\varepsilon_{t+i}$.

4. Results of the tests

Sixteen macroeconomic equations were examined in this study. The basic equations are taken from the Fair (1984) model, but to guard against the danger of having the results depend on a particular model, additional variables were added to the equations for some of the tests to make the equations general enough to encompass several different specifications. The data set used for the present results began in 1952 I and ended in 1988 IV. The estimation period for all the equations was 1954 I–1986 IV, a total of 132 observations. Each equation was first estimated without the leads and then the equation was re-estimated with the leads added. From these two estimates the joint significance of the led values can be tested using a chi-squared test.[4]

Four tests were performed for each of the 16 equations. The first three tests used the Fair-model specification of the equation. For the first test only values led one quarter were added; for the second test values led one through four quarters were added; and for the third test values led one through six quarters were added with the six parameters per variable constrained to lie on a polynomial. The polynomial for the third test was second degree with an end point constraint of zero at lead 7. This meant that two unconstrained

4 The chi-square test is as follows. The objective function that Hansen's method minimizes is $v'ZM^{-1}Z'v$ (see equation (A15) in Appendix A). The elements of v are defined by equation (8) in the text in the case of no autoregressive structural errors and by equation (A10) in Appendix A in the case of autoregressive errors. Let S^* be the value of the objective function with the led values included, and let S^{**} be the value when the led values are not included. Then $(S^{**} - S^*)/T$ is asymptotically distributed as chi-square with r degrees of freedom, where r is the number of new variables added. A general proof of this is in Andrews and Fair (1988). In performing this test the value of M must be the same for both estimates. For the results in this paper M was computed using the residuals from the equation with the led values included.

parameters were estimated for each variable for the third test.[5] The polynomial constraint was used to lessen possible collinearity problems.

For the fourth test the values of all the explanatory variables in the equation lagged once and the value of the lagged dependent variable (if it were not already included) were added to the equation (with the exception of the age variables in the household expenditure equations). In addition, real per capita disposable personal income was added to the household expenditure equations. Adding disposable personal income incorporates the Keynesian specification. Adding the lagged values incorporates a richer dynamic structure. For example, Hendry, Pagan, and Sargan (1984) show that adding the lagged dependent variable and lagged values of all the explanatory variables is quite general in that it encompasses many different types of dynamic structures. For the fourth test the six leads with the polynomial restriction were used.

The estimates of the basic equations before the addition of the leads are presented in Appendix B. The chi-squared values are presented in Table 1. It will help in the following discussion to refer closely to the equations in Appendix B.

The variables for Z_t were chosen to be the main predetermined variables in the Fair (1984) model. (These variables are listed in Table 6-1 in Fair, 1984). The only exception to this was for equations with an autoregressive structural error (equations 2, 9, and 11), where the viewpoint date is $t - 2$ rather than $t - 1$. This means that lagged endogenous variables for period $t - 1$ cannot be used. (See the discussion of the autoregressive case in Appendix A.) The use of the main predetermined variables in the Fair model for Z_t means that the information set used by agents is assumed to include these variables.

All but one of the 16 equations has right-hand side endogenous variables, which means that the assumption in the previous section that X_{1t} contains only predetermined variables must be relaxed. In the regular estimation of the model the equations are estimated using 2SLS, with the variables in Z_t used for the first stage regressors. A variable like W_t, for example, is taken to be endogenous. If, as is done here, Z_t is used both as the vector of first stage regressors for the 2SLS method (actually, Hansen's method in the present context) and as the vector of variables used by agents in forming their expectations, it is not possible

[5] Consider adding the values of W led one through six quarters to the equation. The values enter as $\sum_{j=1}^{6} \beta_j W_{t+j}$. The polynomial constraint is $\beta_j = \gamma_0 + \gamma_1 j + \gamma_2 j^2, j = 1, \ldots, 6$. Also, $\beta_7 = 0$. The zero constraint for β_7 implies that

$$\gamma_0 = -7\gamma_1 - 49\gamma_2$$

The way in which the led values enter the equation is then

$$\gamma_1 F_{1t} + \gamma_2 F_{2t}$$

where

$$F_{1t} = \sum_{j=1}^{6} (j - 7) W_{t+j}$$

$$F_{2t} = \sum_{j=1}^{6} (j^2 - 49) W_{t+j}$$

TABLE 1

Results of adding led values to the equations in table B.1

		1		2		3		4	
		One lead		*Four leads*		*Six leads*		*Six leads*	
	Led values added for:	*df*	χ^2	*df*	χ^2	*df*	χ^2	*df*	χ^2
1. Consumption of services	After-tax wage; price level	2	5.22	8	9.09	4	9.52*	4	5.85
2. Consumption of nondurables	After-tax wage; price level	2	5.97	8	14.90	4	10.12*	4	9.21
3. Consumption of durables	After-tax wage; price level	2	4.69	8	16.24*	4	12.12*	4	8.64
4. Housing investment	After-tax wage; price level	2	3.44	8	12.68	4	7.26	4	6.03
5. Labor force—males 25–54	After-tax wage; price level	3	2.96	12	20.62	6	16.51*	6	13.47*
6. Labor force—females 25–54	After-tax wage; price level	3	4.83	12	23.27*	6	17.53**	6	13.85*
7. Labor force—all others	After-tax wage; price level	2	0.94	8	7.39	4	5.88	4	8.41
8. Private nonfarm price deflator	Wage	1	0.00	4	0.63	2	0.22	2	0.30
9. Output	Sales	1	0.00	4	2.75	2	1.42	2	0.03
10. Nonresidential fixed investment	Output	1	1.53	4	7.25	2	8.24*	2	3.81
11. Private jobs	Output	1	1.61	4	6.66	2	7.20*	2	6.85*
12. Hours paid per private job	Output	1	0.80	4	8.85	2	1.54	2	2.13
13. Bill rate	Inflation; real growth; labor market tightness	3	2.61	12	9.98	6	9.17	6	5.69
14. Bond rate	Bill rate	1	1.94	4	10.03*	2	1.69	2	0.38
15. Mortgage rate	Bill rate	1	1.22	4	6.47	2	0.37	2	2.67
16. Change in stock price	Change in after-tax cash flow	1	0.93	4	3.33	2	3.44	2	0.63

Notes:
* Significant at 5% level.
** Significant at 1% level.

to test the assumption that a variable like W_t is known against the assumption that agents only have a rational expectation of it. It would in principle be possible to do this if there were some contemporaneous exogenous variables in the model that agents forming rational expectations do not know but must forecast ahead of time. These variables are appropriate first stage regressors for 2SLS (since they are exogenous), but they are not used by agents. In practice, however, this is likely to be a small difference upon which to base a test, and no attempt was made to do so here. The key question of interest here is whether values dated $t + 1$ and beyond are significant.

The first seven equations in Table 1 are behavioral equations for the household sector. The explanatory variables in the consumption and housing investment equations (equations 1–4) include the real value of wealth, the after-tax nominal wage, the price level, the after-tax interest rate, the real level of transfer payments, and a 'labor constraint' variable, which is designed to pick up possible disequilibrium effects. The lagged dependent variable is also included to pick up partial adjustment effects. In addition, the lagged stock of durable goods is in the durable goods equation, and the lagged stock of housing is in the housing investment equation. There are also two 'age' variables in each equation, which are designed to pick up effects of the changing age distribution on consumption and housing investment. The theory behind these equations is that households choose consumption and labor supply to maximize a multiperiod utility function, possibly subject to a 'disequilibrium' constraint regarding the amount that they can work at the current set of wage rates. The estimated equations can be considered to be approximations to decision equations like (3).[6]

Led values of the after-tax wage rate and the price level were added to the basic version of each of the four equations. If the RE hypothesis is valid, the led values of the wage rate and the price level should be significant in an estimated version of an equation like (3), which the present equations are like. The most favorable results for the RE hypothesis in Table 1 are those using six leads with no additional explanatory variables added. The results for this (third) test show that for the three consumption equations the led values are

[6] Equations 1 through 6 are the same as in Fair and Dominguez (1991) except that the period of estimation ends in 1986 IV here rather than in 1988 IV. The estimation period had to end earlier here because of the use of the led values. The use of the age variables is explained in this other paper.

Two of the variables in equation 1 and two in equation 4 have coefficient estimates of the wrong expected signs. These variables were left in, however, with the realization that the estimates may change sign when the led values are added. Note in Table B that W and P are entered separately rather than as W/P. One does not necessarily expect W and P to enter as W/P. The P used for each equation is the own price of the good (the price deflator for services for the service equation, the price deflator for nondurables for the nondurables equation, and so on), and in principle the prices of the other goods should be included in the equation as well. There is, however, too much collinearity among the various price deflators to pick up sensible effects in the aggregate data. Therefore, only the own price was included, but included separately. Finally, note that the values of the explanatory variables in equation 4, the housing investment equation, are lagged once rather than unlagged. This is because housing investment depends in large part on housing starts decisions made at least one quarter earlier. When the leads were added to this equation, they began with period t rather than period $t + 1$.

significant at the 5% level. The values are not significant for the housing investment equation. The only other significant value is for the second test for consumption of durables.

Consider now the three labor force equations, equations 5, 6, and 7. Explanatory variables that are common to these three equations are the after-tax nominal wage, the price level, and the labor constraint variable. Each equation also includes the lagged dependent variable. In addition, the equations for males 25–54 and females 25–54 include W times $AGE2554$, the percent of the working age population aged 25–54. This latter variable is designed to account for the possibility, as Easterlin (1987) suggests, that the average wage facing a particular age cohort depends negatively on the size of the cohort. If this is so, the coefficient of W, the aggregate wage, and $W \cdot AGE2554$ should have coefficient estimates of opposite signs. The coefficient of W should be positive if the substitution effect dominates and negative if the income effect dominates.[7] Finally, the labor force equation for all others except those 25–54 includes the lagged value of the wealth variable.

Led values of the after-tax wage rate and the price level were also added to the basic version of each of the three labor force equations. Again, if the RE hypothesis is valid, the led values of the wage rate and the price level should be significant in an estimated version of an equation such as (4), like the present equations. The results in Table 1 show that the led values are significant at the 5% level for males 25–54 for the third and fourth tests, at the 1% level for females 25–54 for the third test, and at the 5% level for females 25–54 for the second and fourth tests. The led values are not significant for the other labor force equation for any of the tests.

Equations 8 through 12 in Table 1 are behavioral equations for the firm sector.[8] Equation 8 is the price equation. According to the theory upon which this equation is based, expected future wages should have a positive effect on current price decisions. In the basic version of the equation the current value of the wage rate is used to proxy for expected future values, and it is highly significant in Table B. The results in Table 1 show that the led values of the wage rate are not significant for any of the tests.

Equation 9 determines output given sales. According to the theory behind the equation, output is smoothed relative to sales, and in the basic version of the equation the current value of sales is used to proxy for expected future values. If the RE hypothesis is valid, one would expect the led values of sales to be significant. Again, the results in Table 1 show that led values are not significant.

Equations 10, 11, and 12 determine the demand for investment, jobs, and hours paid per job, respectively. In the investment equation the change in investment is a function of the amount of excess capital on hand and of current

[7] Again, see Fair and Dominguez (1991) for a discussion of the use of the age data.

[8] See Fair (1984, Chapters 3 and 4) for a discussion of the remaining equations considered in this section.

and lagged output changes. Similarly, in the jobs and hours equations the changes in jobs and hours are a function of the amount of excess labor on hand and of current and expected future output changes. In all three equations the current and lagged output changes are proxying for expected future output changes. These are obvious equations to which led values of output changes can be added to test the RE hypothesis. When the led values of output changes are added to the equations, the results in Table 1 show that they are significant at the 5% level for investment (for the third test) and jobs (for the third and fourth tests), but not for hours paid per job.

Equation 13 explains the three-month Treasury bill rate. The estimated equation is interpreted as an interest rate reaction function of the Federal Reserve. The explanatory variables include: (i) the rate of inflation, (ii) a measure of labor market tightness, (iii) the rate of growth of real output, and (iv) the lagged rate of growth of the money supply. The equation is a 'leaning against the wind' equation in the sense that as these four variables increase (decrease) the Fed is estimated to allow short term interest rates to rise (fall). Current values of the rate of inflation, labor market tightness, and real growth are used in the basic version of the equation. If the Fed has rational expectations, the led values of these variables should be significant. When the led values are added, however, the results in Table 1 show that they are not significant.

Equations 14 and 15 explain the long term bond rate and the mortgage rate, respectively. These are standard term structure equations in which the long rate is a function of current and lagged short rates. In this case the short rate is the three-month bill rate. The current and lagged values of the bill rate are meant to proxy for expected future values. When the led values of the bill rate are added to the equations, the results in Table 1 show that they are not significant except for the second test for the bond rate. The results for the term structure equations are thus not generally supportive of the RE hypothesis.

Equation 16 explains the change in aggregate stock prices. The explanatory variables in the basic equation include the change in the bond rate (to pick up changes in expected future short term rates) and the change in after-tax cash flow. When the led values of the change in after-tax cash flow are added to the equation, the results in Table 1 show that they are not significant. These results are thus also not supportive of the RE hypothesis. In this case, however, the results probably do not say much because not even the current value of after-tax cash flow is significant.

The overall results thus show a scattering of support for the RE hypothesis. The led values are significant for at least one of the tests for five of the seven household equations and for two of the five firm equations. They are not significant for any of the interest rate and stock price equations except for the second text for the bond rate. With one exception, when the led values are significant, they are significant at the 5% level but not at the 1% level. The led values thus add to the explanatory power of these equations, but the addition is only moderate. Adding them is roughly like adding variables with t-statistics around 2. Put another way, the results show for the household equations that the variables in Z_t matter somewhat. This is some evidence that agents use the variables in Z_t informing their expectations.

5. Sensitivity of policy effects to the RE hypothesis

The last section examined the statistical significance of the RE hypothesis. This section examines its economic significance. How much difference to the properties of a model does the addition of the led values make? Two versions of a model are examined here. The first consists of the equations in Appendix B plus the other equations that make up the model in Fair (1984). This version will be called Version 1. It has no led values in it. The second version replaces the equations in Appendix B with the equations estimated for the third test in Table 1, namely the equations with the six led values of the various variables added. This version will be called Version 2. The other equations in the Fair (1984) model are the same for both versions.

It should be noted that Version 2 has fewer restrictions imposed on it than are imposed on most RE models. The only restrictions imposed before estimation are that there are six leads and the coefficients of the led variables lie on a second degree polynomial. In many RE models at least some of the coefficients are chosen *a priori* rather than estimated. For example, the RE version of the model in Fair (1979b) simply imposes rational expectations in the bond and stock markets without estimation. It is thus quite possible for Version 2 to have properties similar to those of Version 1 and yet for other, more restricted, RE models to have very different properties from those of their non-RE versions.

Version 2 is solved under the assumption that expectations are rational in the Muth sense. In particular, it is assumed that agents use the model in solving for their expectations and that their expectations of the exogenous variables are equal to the actual values. These two assumptions imply that agents' expectations of the future values of the endogenous variables are equal to the model's predictions of them. Version 2 is solved using the solution method in Fair and Taylor (1983). Because future predicted values affect current predicted values, the standard way of solving models period-by-period cannot be used. One must iterate over solution paths, and this is what the solution method does.

Four policy experiments were performed. The first two are a sustained increase in federal government purchases of goods in real terms beginning in 1970 I. For experiment 1 the change is unanticipated, and for experiment 2 the change is anticipated as of 1968 I. The second two experiments are a sustained decrease in the bill rate beginning in 1970 I. For experiment 3 the change is unanticipated, and for experiment 4 the change is anticipated as of 1968 I. For the second two experiments the interest rate reaction function (equation 13) is dropped and the bill rate is taken to be the exogenous policy variable of the Fed. The results for real GNP and the private non-farm price deflator are presented in Table 2. Both the anticipated and unanticipated results are the

TABLE 2

Estimated Policy Effects

		Sustained goverment spending increase						Sustained bill rate decrease					
		Real GNP			Price deflator			Real GNP			Price deflator		
		Version			Version			Version			Version		
		1	2	2[a]	1	2	2[a]	1	2	2[a]	1	2	2[a]
1968	I			.01			.00			.01			.00
	II			−.01			.00			.01			.00
	III			−.10			.00			.01			.00
	IV			−.21			−.01			.00			−.01
1969	I			−.30			−.03			−.02			−.01
	II			−.31			−.05			−.06			.00
	III			−.21			−.08			−.10			.00
	IV			.02			−.09			−.14			.01
1970	I	.98	1.02	1.25	.00	.00	−.08	.10	.00	−.11	.00	.00	.02
	II	1.17	1.35	1.66	.05	.06	−.01	.29	.12	.03	.00	.00	.03
	III	1.18	1.39	1.71	.10	.13	.08	.49	.27	.19	.02	.00	.04
	IV	1.14	1.30	1.59	.15	.19	.18	.70	.42	.35	.04	.01	.06
1971	I	1.06	1.17	1.41	.19	.24	.26	.87	.56	.50	.06	.03	.08
	II	.97	1.01	1.19	.22	.28	.32	1.02	.66	.61	.10	.06	.09
	III	.90	.89	1.01	.24	.31	.37	1.12	.74	.69	.14	.09	.10
	IV	.85	.82	.88	.26	.33	.40	1.20	.79	.74	.18	.12	.12
1972	I	.81	.81	.82	.27	.34	.41	1.22	.80	.75	.22	.15	.12
	II	.80	.84	.81	.29	.35	.42	1.21	.79	.74	.27	.18	.13
	III	.81	.89	.84	.30	.36	.42	1.16	.75	.71	.31	.21	.13
	IV	.83	.95	.88	.31	.38	.44	1.09	.69	.65	.36	.24	.14

Notes:
1 = Version 1, unanticipated and anticipated changes.
2 = Version 2, unanticipated changes.
2[a] = Version 2, anticipated changes as of 1968 I.

The increase in real government spending was 1.0% of real GNP.
The decrease in the bill rate was 1.0 percentage point.
The changes began in 1970 I.
The numbers in the tables are percentage changes from the base values (in percentage points).

same for Version 1 because future predicted values do not affect current predicted values—Version 1 is not forward looking in this sense.

The results for each experiment were obtained as follows. The version was first solved using the actual values of all exogenous variables. These solution values are the 'base' values. The policy variable was then changed and the version was solved again. The difference between the predicted value of a variable from this solution and the predicted value from the first solution is the estimate of the response of the variable to the policy change.

Before discussing the results, it should be noted that the experiments were performed without concern about possible wrong signs of the coefficient estimates of the led values. Not all signs were what one might expect. For example, in the investment equation the coefficients of the output changes led three through six quarters were negative, which is not consistent with the theory behind the equation. Nevertheless, the negative values were left in. The aim of the exercise in this section is not to test theories, but to see how much difference the addition of led values makes to a model's properties, regardless of what their coefficient estimates might be. It may be that other theories would imply different signs, and so this section has remained agnostic about the signs. Likewise, no concern was given as to whether the led values were statistically significant or not. All the equations estimated for the third test in Table 1 were used regardless of the significance levels of the led values.

Consider first the results in Table 2 for the government spending increase. The effects on real GNP are fairly similar, with Version 2 having slightly higher multipliers. The negative effects on real GNP before the policy change for Version 2 in the anticipated case are primarily due to the above mentioned negative effects of future output changes on current investment changes. Firms know that future output changes will be larger after the policy change, and, other things being equal, this has a negative effect on investment before the change (according to the coefficient estimates). The effects of the price deflator are in general slightly higher for Version 2. This is due in large part to the higher GNP values for Version 2.

The differences between the two versions are in general larger for the bill rate decrease than for the government spending increase. The peak effect on real GNP is 1.22% for Version 1 compared to .80% or .75% for Version 2. The effects on the price deflator are in general lower for Version 2, which is due in large part to the lower GNP values for Version 2. The differences between the anticipated and unanticipated results for Version 2 are quite small for this policy change.

Overall, the results in Table 2 do not show large differences in the policy properties of the model from the addition of the led values. The differences for the bill rate experiment are probably large enough to make at least some difference to policy makers in deciding what policy to follow, but this is not the case for the government spending experiment.

In a way the results in this section are not surprising given the results in Section 4. The led values are at best only marginally significant, and so one would not expect them to contribute in important ways to the properties of the model. A good way to see that the led values are not very important is to compare cases 2 and 2[a] in Table 2. The anticipated and unanticipated results are fairly similar, and in the anticipated case there is not much action until the policy change is actually carried out.

6. Conclusion

The results in this paper show for the particular model considered that the RE hypothesis is of fairly minor quantitative importance. The led values do not in general contribute much to the explanatory power of the equations, and their use does not change the properties of a model very much. These results may, of course, be model specific, and in the future it would be useful to perform the tests using other models. Until more models are used, no general conclusions can be drawn.

The results so far do suggest, however, that one should be very careful in imposing rational expectations constraints on models without testing the constraints against alternative specifications. These constraints may not be supported by the data. For example, the results in this paper do not support the use of led values of the short term interest rate in the term structure equations for the long term rates. It is thus probably not a good idea to replace estimated term structure equations like 14 and 15 in Table B.1 with equations that are consistent with there being rational expectations in the bond market. This replacement is not likely to be consistent with the data. For another example, the multicountry model of Taylor (1989) is one in which many rational expectations constraints have been imposed without being tested, and the properties of the model are quite different from those of non rational expectations models (see Fair, 1989).

It should be noted for future work that there is a more efficient way of testing the RE hypothesis than using Hansen's method, namely using full information maximum likelihood (FIML). A method for computing FIML estimates for nonlinear models with rational expectations is discussed in Fair and Taylor (1990) and this appears to be computationally feasible for large models. Using this method one could perform likelihood ratio tests to test the significance of the led values. An entire model would be estimated by FIML first without and then with the led values. From these two estimates the likelihood ratio test could be performed. The FIML technique is the obvious technique to use in this context because it accounts for all the cross equation restrictions implied by the RE hypothesis.

Cowles Foundation, Yale University

ACKNOWLEDGEMENTS

I am indebted to the referees for helpful comments.

REFERENCES

AMEMIYA, T. (1974). 'The Nonlinear Two-Stage Least Squares Estimator', *Journal of Econometrics*, **2**, 105–10.

ANDREWS, D. W. K. and FAIR, R. C. (1988). 'Inference in Nonlinear Econometric Models with Structural Change', *Review of Economic Studies*, **55**, 615–39.

CUMBY, R. E., HUIZINGA, J., and OBSTFELD, M. (1983). 'Two-Steps Two-Stage Least Squares Estimation in Models with Rational Expectations', *Journal of Econometrics*, **21**, 333–55.

EASTERLIN, R. A. (1987). *Birth and Fortune: The Impact of Numbers on Personal Welfare*, 2nd edition, University of Chicago Press, Chicago.

FAIR, R. C. (1979a). 'An Analysis of the Accuracy of Four Macroeconometric Models', *Journal of Political Economy*, **87**, 701–18.

FAIR, R. C. (1979b). 'An Analysis of a Macro-Econometric Model with Rational Expectations in the Bond and Stock Markets', *The American Economic Review*, **69**, 539–52.

FAIR, R. C. (1980). 'Estimating the Expected Predictive Accuracy of Econometric Models', *International Economic Review*, **21**, 355–78.

FAIR, R. C. (1984). *Specification, Estimation, and Analysis of Macroeconometric Models*, Harvard University Press, Cambridge, MA.

FAIR, R. C. (1989). 'Commentary on John B. Taylor "The Current Account and Macroeconomic Policy: An Econometric Analysis"', in *The US Trade Deficit: Causes, Consequences, and Cures*, 12th Annual Economic Policy Conference Proceedings, Federal Research Bank of St. Louis, Kluwer Academic Publishing, Boston, 187–91.

FAIR, R. C. and DOMINGUEZ, K. M. (1991). 'Effects of the Changing US Age Distribution on Macroeconomic Equations', *The American Economic Review*, **81**, 1276–94.

FAIR, R. C. and PARKE, W. R. (1988). 'The Fair–Parke Program for the Estimation and Analysis of Nonlinear Econometric Models', mimeo.

FAIR, R. C. and TAYLOR, J. (1983). 'Solution and Maximum Likelihood Estimation of Dynamic Rational Expectations Models', *Econometrica*, **51**, 1169–85.

FAIR, R. C. and TAYLOR, J. (1990). 'Full Information and Stochastic Simulation of Models with Rational Expectations', *Journal of Applied Econometrics*, **5**, 381–92.

HALL, R. E. (1978). 'Stochastic Implications of the Life-Cycle Permanent Income Hypothesis: Theory and Evidence', *Journal of Political Economy*, **86**, 971–89.

HALL, S. G. and HENRY, S. G. B. (1988). *Macroeconomic Modeling*, North-Holland, Amsterdam.

HANSEN, L. (1982). 'Large Sample Properties of Generalized Method of Moments Estimators', *Econometrica*, **50**, 1029–54.

HANSEN, L. P. and SINGLETON, K. (1982). 'Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models', *Econometrica*, **50**, 1269–86.

HENDRY, D. F. (1988). 'The Encompassing Implications of Feedback Versus Feedforward Mechanisms in Econometrics', *Oxford Economic Papers*, **40**, 132–49.

HENDRY, D. F., PAGAN, A. R., and SARGAN, J. D. (1984). 'Dynamic Specifications', in Z. Griliches and M. D. Intriligator, eds, *Handbook of Econometrics*, North-Holland, Amsterdam, 1023–100.

LITTERMAN, R. B. (1979). 'Improving the Measurement of Predictive Accuracy', mimeo.

LUCAS JR, R. E. (1976). 'Econometric Policy Evaluation: A Critique', in K. Brunner and A. H. Melzer, eds, *The Phillips Curve and Labor Markets*, North-Holland, Amsterdam.

MANKIW, N. G., ROTEMBERG, J. J., and SUMMERS, L. H. (1985). 'Intertemporal Substitution in Macroeconomics', *Quarterly Journal of Economics*, **100**, 225–51.

MCCALLUM, B. T. (1976). 'Rational Expectations and the Natural Rate Hypothesis: Some Consistent Results', *Econometrica*, **44**, 43–52.

MUTH, J. F. (1961). 'Rational Expectations and the Theory of Price Movements', *Econometrica*, **29**, 315–35.

TAYLOR, J. B. (1989). 'The Current Account and Macroeconomic Policy: An Econometric Analysis', in *The US Trade Deficit: Causes, Consequences, and Cures*, 12th Annual Economic Policy Conference Proceedings, Federal Reserve Bank of St. Louis, Kluwer Academic Publishing, Boston, 131–85.

APPENDIX

The Estimation Technique

The estimation technique used in this paper is Hansen's (1982) method of moments estimator. As noted in Section 3, this estimator is general enough to require some explanation of how it was applied. Some of the equations estimated in this paper have autoregressive error terms, and so this case is also considered here. In this case Hansen's estimator is also the 2S2SLS estimator of Cumby, Huizinga, and Obstfeld (1983).

The equation to be estimated is (8) in the text, which is repeated here:

$$y_t = X_t\alpha + v_t, \tag{A1}$$

where say X_t is a p dimensional vector. Assumg that the vector Z_t is of dimension k. The 2SLS estimator of α (denoted α_{2SLS}) is

$$\alpha_{2SLS} = (X'Z(Z'Z)^{-1}Z'X)^{-1}X'Z(Z'Z)^{-1}Z'y, \tag{A2}$$

where α_{2SLS} is $p \times 1$, X is $T \times p$, Z is $T \times k$, and y is $T \times 1$. The application of the 2SLS estimator to models of this type is due to McCallum (1976).

The standard formula for the covariance matrix of α_{2SLS} is not correct for i in equation (6) of the text greater than 1 because in this case v_t is serially correlated. If, for example, i is 3, an unanticipated shock in period $t + 1$ will affect ${}_t\varepsilon_{t+3}$, ${}_{t-1}\varepsilon_{t+2}$, and ${}_{t-2}\varepsilon_{t+1}$, and so v_t will be a second order moving average. In general, v_t will be a moving average of order $i - 1$. Hansen's estimator (1982) accounts for this moving average process. The estimator in the present case (denoted α_H) is

$$\alpha_H = (X'ZM^{-1}Z'X)^{-1}X'ZM^{-1}Z'y \tag{A3}$$

where M is some consistent estimate of $\lim T^{-1}E[Z'vv'Z]$. The estimated covariance matrix of α_H is

$$T\cdot(X'ZM^{-1}Z'X)^{-1} \tag{A4}$$

There are different versions of α_H depending on how M is computed. To compute M, one first needs an estimate of v_t in (A1). The residuals v_t can be estimated using α_{2SLS}:

$$\hat{v}_t = y_t - X_t\alpha_{2SLS}, \qquad t = 1, \ldots, T \tag{A5}$$

A general way of computing M is as follows. Let $f_t = \hat{v}_t \otimes Z_t$, where $\hat{v}_t$ is computed from (A5). Let $R_j = (T-j)^{-1}\sum_{t=j}^{T} f_t f'_{t-j}$, $j = 0, 1, \ldots, J$, where J is the order of the moving average. M is then $(R_0 + R_1 + R'_1 + \cdots + R_J + R'_J)$. In many cases computing M in this way does not result in a positive definite matrix, and so α_H cannot be computed. For the empirical work for this paper most of the M matrices computed in this way were not positive definite.

Hansen (1982) and Cumby, Huisinga, and Obstfeld (1983) discuss the computation of M based on an estimate of the spectral density matrix of $Z'_t v_t$ evaluated at frequency zero. This idea was not pursued here. Instead, M was computed under the following assumption:

$$E[v_t v_s | Z_t, Z_{t-1}, \ldots] = E[v_t v_s] \quad \text{for} \quad t \geqq s \tag{A6}$$

which says that the contemporaneous and serial correlation in v do not depend on Z. This assumption is implied by the assumption that $E[v_t Z_s] = 0$ for $t \geqq s$ if normality is also assumed. Under this assumption M can be computed as follows. Let $a_j = (T-j)^{-1}\sum_{t=j}^{T} \hat{v}_t\hat{v}_{t-j}$ and $B_j = (T-j)^{-1}\sum_{t=j}^{T} Z_t Z'_{t-j}$, $j = 0, 1, \ldots, J$. M is then $(a_0B_0 + a_1B_1 + a_1B'_1 + \cdots + a_JB_J + a_JB'_J)$. This way of estimating M always resulted in a positive definite matrix for the empirical work for this paper.

The case of an autoregressive structure error

Assume that the error term u_t in equation (7) of the text follows a first order autoregressive process:

$$u_t = \rho u_{t-1} + \eta_t \tag{A7}$$

Lagging equation (7) in the text by one period, multiplying through by ρ, and subtracting the resulting expression from (7) yields

$$y_t = \rho y_{t-1} + X_{1t}\alpha_t - X_{1t-1}\alpha_1\rho + {}_tX^e_{2t+i}\alpha_2 - {}_{t-1}X^e_{2t+i-1}\alpha_2\rho + \eta_t \tag{A8}$$

Note that this transformation yields a new viewpoint date, $t-1$. Let the expectation error for ${}_{t-1}X^e_{2t+i-1}$ be

$${}_{t-1}\varepsilon_{t+i-1} = X_{2t+i-1} - {}_{t-1}X^e_{2t+i-1} \tag{A9}$$

Substituting (8) and (A9) into (A8) yields

$$\begin{aligned} y_t &= \rho y_{t-1} - X_{1t}\alpha_1 - X_{1t-1}\alpha_1\rho + X_{2t+i}\alpha_2 - X_{2t+i-1}\alpha_2\rho + \eta_t - {}_t\varepsilon_{t+i}\alpha_2 + {}_{t-1}\varepsilon_{t+i-1}\alpha_2\rho \\ &= \rho y_{t-1} + X_t\alpha - X_{t-1}\alpha\rho + v_t \end{aligned} \tag{A10}$$

where $X = (X_{1t} \quad X_{2t+i})$, $\alpha = \begin{pmatrix}\alpha_1 \\ \alpha_2\end{pmatrix}$, and $v_t = \eta_t - {}_t\varepsilon_{t+i}\alpha_2 + {}_{t-1}\varepsilon_{t+i-1}\alpha_2\rho$. Equation (A10) is nonlinear in coefficients because of the introduction of ρ.

Given a set of first stage regressors, equation (A10) can be estimated by 2SLS. The estimates are obtained by minimizing

$$v'Z(Z'Z)^{-1}Z'v = v'Dv, \tag{A11}$$

where v is $T \times 1$ and Z is $T \times k$. Z is the matrix of observations of the first stage regressors. A necessary condition for consistency is that Z_t and v_t be uncorrelated, which means that Z_t must be uncorrelated with η_t, ${}_t\varepsilon_{t+i}$, and ${}_{t-1}\varepsilon_{t+i-1}$. In order to ensure that Z_t and ${}_{t-1}\varepsilon_{t+i-1}$ are uncorrelated, Z_t must not include any variables that are not known as of period $t-1$. This is an important additional restriction in the autogressive case.[9]

The estimator that is based on the minimization of (A11) is Amemiya's (1974) nonlinear 2SLS estimator. In the general case (A11) can be minimized using a general purpose algorithm. In the particular case considered here a simple iterative procedure can be used, where one iterates between estimates of α and ρ. Minimizing $v'Dv$ with respect to α and ρ results in the following first-order conditions:

$$\hat{\alpha} = [(X - X_{-1}\hat{\rho})'D(X - X_{-1}\hat{\rho})]^{-1}(X - X_{-1}\hat{\rho})'D(y - y_{-1}\hat{\rho}) \tag{A12}$$

$$\hat{\rho} = \frac{(y_{-1} - X_{-1}\hat{\alpha})'D(y - X\hat{\alpha})}{(y_{-1} - X_{-1}\hat{\alpha})'D(y_{-1} - X_{-1}\hat{\alpha})} \tag{A13}$$

where the -1 subscript denotes the vector or matrix of observations lagged one period.[10] Equations (A12) and (A13) can be easily solved iteratively. Given the estimates $\hat{\alpha}$ and $\hat{\rho}$ that solve (A12) and (A13), one can compute the 2SLS estimates of v_t, which are

$$\hat{v}_t = y_t - \hat{\rho}y_{t-1} - X_t\hat{\alpha} + X_{t-1}\hat{\alpha}\hat{\rho}, \qquad t = 1, \ldots, T \tag{A14}$$

Now, given $\hat{v}_t$, one can compute M for Hansen's estimator in one of the many possible ways. These calculations simply involve $\hat{v}_t$ and Z_t, $t = 1, \ldots, T$. For the work for this paper M was computed under assumption (A6). Given M, Hansen's estimates of α and ρ are obtained by minimizing[11]

$$v'ZM^{-1}Z'v = v'Cv \tag{A15}$$

[9] There is a possibly confusing statement in Cumby, Huizinga, and Obstfeld (1983), p. 341, regarding the movement of the instrument set backward in time. The instrument set must be moved backward in time as the order of the autoregressive process increases. It need not be moved backward as the order of the moving average processes increases due to an increase in i.

[10] Data for period 0 are assumed to exist so that y_{-1} can be taken to be $T \times 1$ and X_{-1} can be taken to be $T \times p$.

[11] The estimator that is based on the minimization of (A15) is also the 2S2SLS estimator of Cumby, Huizinga, and Obstfeld (1983).

Minimizing (A15) with respect to α and ρ results in the first order conditions (A12) and (A13) with C replacing D. The estimated covariance matrix is

$$T\cdot(G'CG)^{-1} \tag{A16}$$

where $G = (X - X_{-1}\hat{\rho}y_{-1} - X_{-1}\hat{\alpha})$.

To summarize, Hansen's method in the case of a first order autoregressive structural error consists of: (i) choosing Z_t so that it does not include any variables not known as of period $t-1$; (ii) solving (A12) and (A13); (iii) computing $\hat{v}_t$ from (A14); (iv) computing M in some way using Z_t and $\hat{v}_t$, $t = 1, \ldots, T$; and (v) solving (A12) and (A13) with C in (A15) replacing D.

APPENDIX B

Estimates of the basic version of each of the sixteen equations are presented in Table B.1. The estimation period is 1954 I–1986 IV. The variables for which led values were added for the results in Table 1 are the following:

Equation	*Variables*
1–4	W, P
5–6	W, P, W·AGE2554
7	W, P
8	W
9	S
10	ΔY
11–12	Δ log Y
13	$\dot{PD}$, LMT, $GN\dot{P}R$
14–15	RS
16	ΔCF

The notation in alphabetic order is:

A	Real value of total net wealth of the household sector
AGE_1	Age distribution variable (see Fair and Dominguez, 1991)
AGE_2	Age distribution variable (see Fair and Dominguez, 1991)
AGE2554	Percent of the population aged 25–54
CD	Real value of durable consumption
CF	After-tax cash flow
CN	Real value of nondurable consumption
CNST	Constant term
CS	Real value of service consumption
D593	1 in 1959 III; 0 otherwise
D594	1 in 1959 IV; 0 otherwise
D601	1 in 1960 I; 0 otherwise
D794823	1 in 1979 IV–1982 III; 0 otherwise
GAP	Estimated percentage gap between potential and actual output
$GN\dot{P}R$	Percentage change in real GNP at an annual rate
H	Average number of hours paid per job
IH	Real value of residential investment
IK	Real value of nonresidential fixed investment
J	Number of jobs in the first sector
JHMIN	Minimum number of worker hours needed to produce the output of the period
K	Real value of the stock of capital of the firm sector
KMIN	Minimum amount of capital needed to produce the output of the period
LDV	Lagged dependent variable
LMT	Measure of labor market tightness

L1	Labor force of men 25–54
L2	Labor force of women 25–54
L3	Labor force of all others 16+
$\dot{M1}$	Percentage change in the money supply at an annual rate
P	Price deflator for CS for equation 1; price deflator for CN for equation 2; price deflator for CD for equation 3; price deflator for IH for equation 4; price deflator for total sales for equations 5–7; price deflator for private non-farm output for equation 8
$\dot{PD}$	Percentage change in the price deflator for domestic sales at an annual rate
PM	Import price deflator
POP	Population 16+
POP1	Population men 25–54
POP2	Population women 25–54
POP3	Population all others 16+
Q	Labor constraint variable
RB	AAA bond rate
RM	Mortgage rate
RS	Three-month Treasury bill rate
RMA	After-tax mortgage rate (appears in equations 3 and 4)
RSA	After-tax three-month Treasury bill rate (appears in equations 1 and 2)
S	Value of corporate stocks held by the household sector
t	Time trend
V	Stock of inventories at the end of the period
W	After-tax nominal wage
WS	Nominal wage inclusive of employer social security contributions
X	Real value of sales of the firm sector
Y	Real value of output of the firm sector
YTR	Real value of transfer payment
δ	Depreciation rate of the capital stock
$\hat{\rho}$	Estimate of the first order serial correlation coefficient for the error term

TABLE B.1
The basic version of each equation

	Household Expenditure Equations			
Explanatory Variables	1 *CS/POP*	2 *CN/POP*	3 *CD/POP*	4 *IH/POP*
CNST	.0941	.241	.0201	.432
	(3.47)	(2.56)	(0.27)	(5.73)
LDV	.816	.624	.583	.432
	(18.05)	(5.87)	(7.19)	(33.11)
Lagged stock	–	–	−.0191	−.0507
			(1.81)	(7.75)
$(A/POP)_{-1}$	.000660	.00254	.00274	.00215
	(1.35)	(3.18)	(4.08)	(4.49)
RSA or RMA	−.00290	−.000195	−.00716	−.00649[b]
	(5.79)	(0.17)	(5.73)	(6.96)
YTR/POP	.0533	.0315	.127	.149[b]
	(1.52)	(0.67)	(2.12)	(3.97)
W	−.0784[a]	.323	.396	.146[b]
	(0.61)	(2.50)	(2.61)	(2.55)

Continued

TABLE B.1—*continued*

Explanatory Variables	Household Expenditure Equations 1 CS/POP	2 CN/POP	3 CD/POP	4 IH/POP
P	.148[a]	−.208	−.139	.120[a,b]
	(1.89)	(7.06)	(1.21)	(3.24)
Q	.153	.123	.187	−.0358[a,b]
	(3.32)	(1.54)	(2.15)	(0.80)
AGE_1	−.0952	−.0631	−.0461	−.195
	(5.01)	(3.07)	(3.19)	(7.89)
AGE_2	.00188	.00114	.000565	.00433
	(4.88)	(2.71)	(1.85)	(7.32)
$\hat{\rho}$	–	.269	–	–
		(1.92)		
SE	.00536	.00674	.00883	.00654
DW	2.21	1.98	2.09	2.03

Explanatory Variables	Labour Force Equations 5 L1/POP1	6 L2/POP2	7 L3/POP3		Price Equation 8 log P		Output Equation 9 Y
CNST	.407	.0646	.113	CNST	.199	CNST	−1.86
	(5.58)	(3.97)	(4.25)		(11.57)		(0.65)
LDV	.581	.834	.824	LDV	.891	LDV	.166
	(7.74)	(19.02)	(20.05)		(117.05)		(2.88)
W	−.162	.243	.0329	log WS	.0454	X	.955
	(5.39)	(4.28)	(2.45)		(11.98)		(13.78)
P	.0334	−.0201	−.0327	log PM	.0459	V_{-1}	−.129
	(3.32)	(1.47)	(3.12)		(13.22)		(3.24)
Q	.0128	.0360	.0548	GAP	−.00478	D593	−4.21
	(1.63)	(3.06)	(4.00)		(3.97)		(1.35)
W*AGE2554	.187	−.279	–			D594	.55
	(4.40)	(3.81)					(0.16)
$(A/POP)_{-1}$	–	–	−.000436	SE	.00387	D601	3.93
			(3.02)	DW	1.93		(1.26)
SE	.00188	.00295	.00264			$\hat{\rho}$	.631
DW	2.04	2.25	1.95				(6.44)
						SE	3.15
						DW	2.27

(*Continued*)

TABLE B.1—*continued*

Explanatory Variables	*Investment* 10 ΔIK		*Employment* 11 $\Delta \log J$		*Hours* 12 $\Delta \log H$
CNST	.677	CNST	−1.05	CNST	1.19
	(1.34)		(3.76)		(4.86)
$(K\text{-}KMIN)_{-1}$	−.00777	$\log(J/JHMIN)_{-1}$	−.168	$\log(J/JHMIN)_{-1}$	−.0657
	(1.64)		(3.75)		(3.54)
ΔY	0.679	$\Delta \log Y$	.338	$\log H_{-1}$	−.255
	(2.72)		(9.19)		(5.18)
ΔY_{-1}	.0579	$\Delta \log Y_{-1}$	.111	$\Delta \log Y$	.156
	(2.75)		(2.85)		(5.47)
ΔY_{-2}	.0380	$\Delta \log Y_{-2}$	.00586	t	−.000211
	(1.85)		(0.19)		(4.80)
ΔY_{-3}	.0397	t	.000189		
	(1.92)		(4.32)		
IK_{-1}	−.0809	D593	−.00936	SE	.00294
	(2.10)		(2.58)	DW	2.24
$\delta \cdot K_{-1}$	.0959	D594	.00201		
	(1.82)		(0.57)		
		$\hat{\rho}$	.503		
			(5.04)		
SE	1.23				
DW	2.03				
		SE	.00344		
		DW	2.03		

Explanatory Variables	*Bill Rate* 13 *RS*		*Bond Rate* 14 *RB*	*Mortgage Rate* 15 *RM*		*Change in Stock Price* 16 ΔS
CNST	−9.11	CNST	.167	.421	CNST	19.1
	(3.35)		(3.08)	(4.16)		(3.72)
RS_{-1}	.882	LDV	.865	.821	ΔRB	−55.8
	(37.10)		(47.01)	(29.48)		(3.06)
$\dot{P}D$	.0473	RS	.307	.244	ΔCF	1.45
	(1.97)		(8.88)	(5.15)		(0.65)
LMT	9.43	RS_{-1}	−.199	.0217		
	(3.34)		(4.33)	(0.34)		
$G\dot{N}PR$	.0880	RS_{-2}	.0426	−.0680	SE	57.8
	(4.71)		(1.52)	(1.60)	DW	1.89
$\dot{M}1_{-1}$	.0314					
	(3.49)					
$D794823 \cdot \dot{M}1_{-1}$	.205	SE	.239	.330		
	(6.92)	DW	1.77	2.11		
SE	.635					
DW	1.78					

Notes:
t-statistics in absolute value are in parentheses.
[a] Wrong sign.
[b] Variable is lagged one quarter.