10

Analyzing Properties of Models

10.1 Introduction

This chapter discusses various methods for analyzing the properties of macroeconometric models. These methods are then applied in Chapter 11 to the US model and in Chapter 12 to the MC model. The methods discussed here are not tests of models. They are meant to be used after one has some confidence that the model being analyzed is a reasonable approximation of the economy. A model that does not do well in tests is not likely to have properties that accurately reflect the way the economy works.

It is sometimes argued with respect to the testing of models that if a particular model has properties that seem reasonable on *a priori* (i.e., theoretical) grounds, this is evidence in favor of the model. However, because of the back and forth movement between specification and results, including multiplier results, that occurs in macro model building, the final version of a model is likely to have multiplier properties that are similar to what one expects from the theory. Essentially one does not stop until this happens. Therefore, the fact that an econometric model has properties that are consistent with the theory is in no way a confirmation of the model. Models must be tested using methods like those in Chapters 4 and 7, not by examining the "reasonableness" of their multiplier properties.

10.2 Computing Multipliers and Their Standard Errors[1]

A useful way of examining the properties of a model is to consider how the predicted values of the endogenous variables change when one or more exogenous variables are changed. This exercise is usually called multiplier analysis, although the use of the word "multiplier" is somewhat misleading. The output that one examines from this exercise does not have to be the change in the endogenous variables *divided by* the change in the exogenous variable; it can merely be, for example, the change or percentage change in the endogenous variable itself. In fact, if more than one exogenous variable has been changed, there is no obvious thing to divide the change in the endogenous variable by. The form of the output that is examined depends on the nature of the problem, and thus the word "multiplier" should be interpreted in a very general way.

10.2.1 Deterministic Simulation

The procedure that is usually used to compute multipliers is based on deterministic simulation. Let x_t^a denote a "base" set of exogenous variable values for period t, and let x_t^b denote an alternative set. Assume that the prediction period begins in period t and is of length T. Given 1) the initial conditions as of the beginning of period t, 2) the coefficient estimates, 3) a set of exogenous variable values for the entire period, and 4) values of the error terms for the entire period, the predicted values of the endogenous variables can be computed. Let $\hat{y}_{itk}^a$ denote the k period ahead predicted value of endogenous variable i from the simulation that uses x_{t+k-1}^a $(k = 1, \ldots, T)$ for the exogenous variable values, and let $\hat{y}_{itk}^b$ denote the predicted value from the simulation that uses x_{t+k-1}^b $(k = 1, \ldots, T)$. The difference between the two predicted values, denoted $\hat{\delta}_{itk}$, is an estimate of the effect on the endogenous

[1]The original discussion of the procedure discussed in this section is in Fair (1980b). It was also discussed in Fair (1984), Section 9.3. The original procedure required that a stochastic simulation with respect to the error terms be done *within* a stochastic simulation with respect to the coefficients, although the first stochastic simulation could be avoided if one were willing to assume that predicted values from deterministic simulations are close to mean values from stochastic simulations, which is generally the case in practice. In private correspondence in 1984, S.G. Hall pointed out to me that a more straightforward procedure is simply to draw both error terms and coefficients at the same time. This avoids any stochastic simulations within stochastic simulations. The procedure described in the present section uses Hall's suggestion. This section is thus a replacement for Fair (1980b) and Fair (1984), Section 9.3.

variable of changing the exogenous variables:

$$\hat{\delta}_{itk} = \hat{y}^b_{itk} - \hat{y}^a_{itk} \tag{10.1}$$

Obvious values of the error terms to use in the deterministic simulations are their expected values, which are almost always zero. For linear models it makes no difference what values are used as long as the same values are used for both simulations. For nonlinear models the choice does make a difference, and in this case the choice of zero values has some problems. Consider, for example, a model in which inflation responds in a very nonlinear way to the difference between actual and potential output: inflation accelerates as output approaches potential. Consider now a period in which output is close to potential, and consider an experiment in which government spending is increased. This experiment should be quite inflationary, but this will not necessarily be the case if the model is predicting a much lower level of output than actually existed. In other words, if the model is predicting that output is not close to potential when in fact it is, the inflationary consequences of the policy change will not be predicted very well.

There is an easy answer to this problem if the simulation is within the period for which data exist, which is simply to use the actual (historical) values of the error terms rather than zero values. By "actual" in this case is meant the values of the estimated residuals that result from the estimation of the equations. If these values are used and if the actual values of the exogenous variables are used, the simulation will result in a perfect fit. This solution will be called the "perfect tracking" solution. Once the residuals are added to the equations, they are never changed. The same set of values is used for all experiments.

If the actual values of the error terms are used, the problem regarding the response of inflation to output does not exist. With the use of the actual residuals, the model predicts the actual data before any policy change is made. Note that this procedure is not inconsistent with the statistical assumptions of the model, since the error terms are assumed to be uncorrelated with the exogenous variables. The use of the actual values of the error terms has the advantage that only one simulation needs to be performed per policy experiment. $\hat{y}^a_{itk}$ is simply the actual value of the variable, and thus a simulation is only needed to get $\hat{y}^b_{itk}$.

10.2.2 Stochastic Simulation

For nonlinear models $\hat{\delta}_{itk}$ in 10.1 is not an unbiased estimate of the change because the predicted values are not equal to the expected values. This does not, however, seem to be an important problem in practice, since deterministic predictions are generally quite close to the mean values from stochastic simulations, and so if one were only interested in estimates of the changes, it seems unlikely that stochastic simulation would be needed. The main reason for using stochastic simulation is to compute standard errors of $\hat{\delta}_{itk}$.

The stochastic simulation procedure is as follows. The error terms are assumed to be drawn from the $N(0, \hat{\Sigma})$ distribution if the "base" values of the error terms are taken to be zero and from the $N(\hat{u}_t, \hat{\Sigma})$ distribution if the historical values of the error terms are used for the base values, where $\hat{u}_t$ is the vector of historical errors for period t. The coefficients are assumed to be drawn from the $N(\hat{\alpha}, \hat{V})$ distribution, where $\hat{\alpha}$ is the vector of coefficient estimates and $\hat{V}$ is the estimated covariance matrix of $\hat{\alpha}$. .

1. Draw a set of error terms and coefficients and solve the model using the base set of exogenous variables values ($x^a_{t+k-1}, k = 1, \ldots, T$). Let $\tilde{y}^{aj}_{itk}$ denote the k period ahead predicted value of variable i from this solution.

2. For the same set of error terms and coefficients as in step 1, solve the model again using the alternative set of exogenous variable values ($x^b_{t+k-1}, k = 1, \ldots, T$). Let $\tilde{y}^{bj}_{itk}$ denote the k period ahead predicted value of variable i from this solution.

3. Compute
$$\tilde{\delta}^j_{itk} = \tilde{y}^{bj}_{itk} - \tilde{y}^{aj}_{itk} \tag{10.2}$$

4. Repeat steps 1 through 3 J times, where J is the desired number of repetitions.

5. Given the values from the J repetitions, compute the mean, denoted $\bar{\delta}_{itk}$, and the variance, denoted $\tilde{s}^2_{itk}$, of $\bar{\delta}_{itk}$:

$$\bar{\delta}_{itk} = (1/J) \sum_{j=1}^{J} \tilde{\delta}^j_{itk} \tag{10.3}$$

$$\tilde{s}^2_{itk} = (1/J) \sum_{j=1}^{J} (\tilde{\delta}^j_{itk} - \bar{\delta}_{itk})^2 \tag{10.4}$$

$\bar{\delta}_{itk}$ is thus the multiplier, and the square root of $\tilde{s}^2_{itk}$ is its standard error.

10.3 Sources of Economic Fluctuations[2]

There has been considerable discussion in the literature about the ultimate sources of macroeconomic variability. Shiller (1987) surveys this work, where he points out that a number of authors attribute most of output or unemployment variability to only a few sources, sometimes only one. The sources vary from technology shocks for Kydland and Prescott (1982), to unanticipated changes in the money stock for Barro (1977), to "unusual structural shifts," such as changes in the demand for produced goods relative to services, for Lilien (1982), to oil price shocks for Hamilton (1983), to changes in desired consumption for Hall (1986). (See Shiller (1987) for more references.) Although it may be that there are only a few important sources of macroeconomic variability, this is far from obvious. Economies seem complicated, and it may be that there are many important sources. As discussed in this section, it is possible using stochastic simulation to estimate the quantitative importance of various sources of variability from a macroeconometric model.

Macroeconometric models provide an obvious vehicle for estimating the sources of variability of endogenous variables. There are two types of shocks that one needs to consider: shocks to the stochastic equations and shocks to the exogenous variables. Shocks to the stochastic equations can be handled by a straightforward application of stochastic simulation. Shocks to the exogenous variables are less straightforward to handle. Since by definition exogenous variables are not modeled, it is not unambiguous what one means by an exogenous variable shock. One approach is to estimate an autoregressive equation for each exogenous variable in the model and add these equations to the model. Shocks to the exogenous variables can then be handled by stochastic simulation of the expanded model. The US+ model is a model like this, and it is used in the next chapter in the application of the present approach.[3]

Assume, therefore, that one has a model like US+ to work with and assume that the variable of interest is real GDP. As discussed in Section 7.3, given the estimated covariance matrix of the error terms, one can estimate the variance of GDP by means of stochastic simulation. Let $\tilde{\sigma}_{it}^2$ denote the estimated variance of real GDP (endogenous variable i) for period t, where the estimated variance is based on draws of all the error terms in the model, including the error terms in the exogenous variable equations if such equations are added. Now consider

[2] The material in this section is taken from Fair (1988a).

[3] When using a model like US+, one may want to take the covariance matrix of the error terms to be block diagonal, as discussed in Section 8.2. This was done for the stochastic simulation work in the next chapter, as it was for the probability calculations in Section 8.8.

fixing one of the error terms at its expected value (usually zero) and computing the variance of GDP again. In this case the stochastic simulation is based on draws of all but one of the error terms. Let $\tilde{\sigma}_{it}^2(k)$ denote the estimated variance of real GDP based on fixing the error term in equation k at its expected value.

The difference between $\tilde{\sigma}_{it}^2$ and $\tilde{\sigma}_{it}^2(k)$ is an estimate of how much the error term in equation k contributes to the variance of GDP.[4] If, say, the variance of GDP falls by 5 percent when the error term for equation k is not drawn, one can say that equation k contributes 5 percent to the variance of GDP.

Another way to estimate this contribution would be to draw *only* the error term for equation k, compute the variance of GDP, and compare this variance to the variance when all the error terms are drawn. If the error term in equation k is correlated with the other error terms in the model, these two procedures are not the same. There is no right or wrong way of estimating this contribution, and because of the correlation, any procedure is somewhat crude. Fortunately, one can examine how sensitive the results are to the effects of the correlation of the error terms across equations to see how to weigh the results. This is done in Section 11.4, where it will be seen that the main conclusions using the US+ model are not sensitive to the effects of the correlation.

In the above discussion k need not refer to just one equation. One can fix the error terms in a subset k of the equations at their expected values and draw from the remaining equations. In this way one can examine the contribution that various sectors make to the variance of GDP. If the error terms across equations are correlated, then fixing, say, two error terms one at a time and summing the two differences is not the same as fixing the two error terms at the same time and computing the one difference. Again, however, one can examine the effects of the error term correlation on the results.

It is important to realize what is and what is not being estimated by this procedure. Consider an exogenous variable shock. What is being estimated is the contribution of the error term in the exogenous variable equation to the variance of GDP. This contribution is *not* the same as the multiplier effect of the exogenous variable on GDP. Two exogenous variables can have the same

[4]Regarding the use of this difference as an estimate of an error term's contribution to the variance of GDP, Robert Shiller has informed me that Pigou had the idea first. In the second edition of Industrial Fluctuations, Pigou (1929), after grouping sources of fluctuations into three basic categories, gave his estimate of how much the removal of each source would reduce the amplitude (i.e. the standard deviation) of industrial fluctuations. He thought that the removal of "autonomous monetary causes" would reduce the amplitude by about half. Likewise, the removal of "psychological causes" would reduce the amplitude by about half. Removal of "real causes," such as harvest variations, would reduce the amplitude by about a quarter. See Shiller (1987) for more discussion of this.

multiplier effects and yet make quite different contributions to the variance of GDP. If one exogenous variable fits its autoregressive equation better than does another (in the sense that its equation has a smaller estimated variance), then, other things being equal, it will contribute less to the variance of GDP. It is possible, of course, to use measures of exogenous variable shocks other than error terms from autoregressive equations, but whatever measure is used, it is not likely to be the same as the size of the multiplier.

The notation $\tilde{\sigma}_{it}^2$ will be used to denote the estimated variance of endogenous variable i for period t based on draws of all $m + q$ error terms. The notation $\tilde{\sigma}_{it}^2(k)$ will be used to denote the estimated variance when the error terms in subset k of the equations are fixed at their expected values, where subset k can simply be one equation. Let $\tilde{\delta}_{it}(k)$ be the difference between the two estimated variances:

$$\tilde{\delta}_{it}(k) = \tilde{\sigma}_{it}^2 - \tilde{\sigma}_{it}^2(k) \tag{10.5}$$

In the US application in the next chapter, values of $\tilde{\delta}_{it}(k)$ are computed for the one through eight quarter ahead predictions of real GDP and the private nonfarm price deflator for a number of different choices of k.

Because of the correlation of the error terms across equations, it can turn out that $\tilde{\delta}_{it}(k)$ is negative for some choices of k. Also, as noted above, it is not in general the case that $\tilde{\delta}_{it}(k)$ for, say, k equal to the first and second equations is the same as $\tilde{\delta}_{it}(k)$ for k equal to the first equation plus $\tilde{\delta}_{it}(k)$ for k equal to the second equation.

Computational Issues

For a number of reasons the stochastic simulation estimates of the variances are not exact. First, they are based on the use of estimated coefficients rather than the true values. Second, they are based on the use of an estimated covariance matrix of the error terms rather than the actual matrix. Third, they are based on a finite number of repetitions. Ignoring the first two reasons, it is possible to estimate the precision of the stochastic simulation estimates for a given number of repetitions. In other words, it is possible to estimate the variances of $\tilde{\sigma}_{it}^2$ and $\tilde{\sigma}_{it}^2(k)$. The formula for the variance of $\tilde{\sigma}_{it}^2$ is presented in equation 7.8 in Chapter 7. What is of more concern here, however, is the variance of $\tilde{\delta}_{it}(k)$, and this can also be estimated.

Let

$$\delta_{it}^j(k) = \sigma_{it}^{2j} - \sigma_{it}^{2j}(k) \tag{10.6}$$

where σ_{it}^{2j} is defined in equation 7.6. The estimated mean of $\delta_{it}^{j}(k)$ across the J repetitions is $\tilde{\delta}_{it}(k)$ in equation 10.5:

$$\tilde{\delta}_{it}(k) = \frac{1}{J}\sum_{j=1}^{J}\delta_{it}^{j}(k) \tag{10.7}$$

The estimated variance of $\tilde{\delta}_{it}(k)$, denoted $var[\tilde{\delta}_{it}(k)]$, is then

$$var[\tilde{\delta}_{it}(k)] = (\frac{1}{J})^2\sum_{j=1}^{J}[\delta_{it}^{j}(k) - \tilde{\delta}_{it}(k)]^2 \tag{10.8}$$

Given values of y_{it}^{j} and $y_{it}^{j}(k)$, $j = 1, \cdots, J$, from the stochastic simulations, all the above values can be computed.

Stochastic simulation error turned out to be a bigger problem than I originally thought it would be. One thousand repetitions was enough to make the variances of $\tilde{\sigma}_{it}^2$ and $\tilde{\sigma}_{it}^2(k)$ acceptably small, but without any tricks, it was not enough to make the variance of $\tilde{\delta}_{it}(k)$ anywhere close to being acceptably small. Fortunately, there is an easy trick available. The variance of $\tilde{\delta}_{it}(k)$ is equal to the variance of $\tilde{\sigma}_{it}^2$ plus the variance of $\tilde{\sigma}_{it}^2(k)$ minus twice the covariance. The trick is to make the covariance high, which can be done by using the same draws of the error terms for the computation of both $\tilde{\sigma}_{it}^2$ and $\tilde{\sigma}_{it}^2(k)$. Any one equation of a model, for example, requires 8000 draws of its error term for 1000 repetitions for a forecast horizon of 8 quarters. If these same 8000 numbers are used to compute both $\tilde{\sigma}_{it}^2$ and $\tilde{\sigma}_{it}^2(k)$, the covariance between them will be increased. When this trick is used, 1000 repetitions leads to variances of $\tilde{\delta}_{it}(k)$ that are acceptably small. This will be seen in Table 11.10 in the next chapter.

To conclude, estimating sources of economic fluctuations in macroeconometric models is an obvious application of stochastic simulation. The advent of inexpensive computing has made applications like this routine and thus has greatly expanded the questions that can be asked of such models.

10.4 Optimal Choice of Monetary-Policy Instruments[5]

Over twenty years ago today Poole (1970) wrote his classic article on the optimal choice of monetary-policy instruments in a stochastic IS–LM model.

[5]The material in this section is taken from Fair (1988b).

Poole assumed that the monetary authority (henceforth called the Fed) can control the interest rate (r) or the money supply (M) exactly. These are the two "instruments" of monetary policy. If the aim is to minimize the squared deviation of real output from its target value, Poole showed that the choice of the optimal instrument depends on the variance of the error term in the IS function, the variance of the error term in the LM function, the covariance of the two error terms, and the size of the parameters in the two functions.

Most people would probably agree that between about October 1979 and October 1982 the Fed put more emphasis on monetary aggregates than it did either before or after. Otherwise, the interest rate has seemed to be the Fed's primary instrument. It is interesting to ask if the use of the interest rate can be justified on the basis of the Poole analysis. Is the economy one in which the variances, covariances, and parameters are such as to lead, a la the Poole analysis, to the optimal instrument being the interest rate?

Stochastic simulation can be used to examine this question using a macroeconometric model. Are the variances, covariances, and parameters in the model such as to favor one instrument over the other, in particular the interest rate over the money supply? The purpose of this section is to show that stochastic simulation can be used to examine Poole like questions in large econometric models. Interestingly enough, Poole's analysis had not been tried on an actual econometric model prior to the work discussed here. The closest study before the present work was that of Tinsley and von zur Muehlen (1983), but they did not analyze the same question that Poole did.[6] Other studies that have extended Poole's work, such as those of Turnovsky (1975) and Yoshikawa (1981), have been primarily theoretical.

Poole also showed that there is a combination policy that is better than either the interest rate policy or the money supply policy. This is the policy where the Fed behaves according to the equation $M = \alpha + \beta r$, where the

[6] In their stochastic simulation experiments, Tinsley and von zur Muehlen always used the interest rate (the Federal Funds rate) as the policy instrument. They used this instrument to target a particular variable, called an "intermediate" target. The intermediate targets they tried are the monetary base, three definitions of the money supply, nominal GNP, and the Federal Funds rate itself. For each of these target choices, they examined how well the choice did in minimizing the squared deviations of the unemployment rate and the inflation rate from their target values. The unemployment rate and the inflation rate are the "ultimate" targets. In the present case the aim is to see how well the interest rate does when it is used as the policy instrument in minimizing the squared deviations of real output from its target value compared to how well the money supply does when it is used as the policy instrument. This is the question that Poole examined.

parameters α and β are chosen optimally.[7] It is possible through repeated stochastic simulation to find the optimal values of α and β for an econometric model, and this procedure is also done in Section 10.4 for the US model.

The Procedure

The procedure is a straightforward application of stochastic simulation. First, fix the interest rate path and perform a stochastic simulation to get the variance of real GDP. Second, fix the money supply path and perform a stochastic simulation to get the variance of real GDP. Finally, compare the two variances. The variance of real GDP for a given period corresponds to Poole's loss function if one takes the target value of GDP for that period to be the mean value from the stochastic simulation. If the variance is smaller when the interest rate is fixed, this is evidence in favor of the interest rate, and vice versa if the variance is smaller when the money supply is fixed.

If the horizon is more than one quarter ahead, then variances are computed for each quarter of the simulation period. The simulations are dynamic, so that, for example, the computed variance for the fourth quarter is the variance of the four quarter ahead prediction error.

Let $\tilde{\sigma}_{it}^2(r)$ denote the stochastic simulation estimate of the variance of endogenous variable i for period t when the interest rate is the policy instrument, and let $\tilde{\sigma}_{it}^2(M)$ denote the same thing when the money supply is the policy instrument. The issue is then to compare $\tilde{\sigma}_{it}^2(r)$ to $\tilde{\sigma}_{it}^2(M)$ for i equal to real GDP to see which is smaller.

10.5 Optimal Control

Optimal control techniques have not been widely used in macroeconometrics. Models may not yet be good enough to warrant the use of such techniques, but if they improve in the future, optimal control techniques are likely to become more popular. The following is a brief discussion of optimal control. A more complete discussion is in Fair (1984), Chapter 10.

The solution of optimal control problems using large scale models turns out to be fairly easy. The first step in setting up a problem is to postulate an objective function. Assume that the period of interest is $t = 1, \ldots, T$. A general specification of the objective function is

$$W = h(y_1, \ldots, y_T, x_1, \ldots, x_T) \tag{10.9}$$

[7] See also Tobin (1982) for a discussion of this.

where W, a scalar, is the value of the objective function corresponding to values of the endogenous and exogenous variables for $t = 1, \ldots, T$. In most applications the objective function is assumed to be additive across time, which means that 10.9 can be written

$$W = \sum_{t=1}^{T} h_t(y_t, x_t) \tag{10.10}$$

where $h_t(y_t, x_t)$ is the value of the objective function for period t. The model can be taken to be the model presented in equation 4.1 in Chapter 4.

Let z_t be a k–dimensional vector of control variables, where z_t is a subset of x_t, and let z be the $k \cdot T$–dimensional vector of all the control values: $z = (z_1, \ldots, z_T)$. Consider first the deterministic case where the error terms in 4.1 are all set to zero. For each value of z one can compute a value of W by first solving the model 4.1 for $y_1, \ldots, y_T$ and then using these values along with the values for $x_1, \ldots, x_T$ to compute W in 10.10. Stated this way, the optimal control problem is choosing variables (the elements of z) to maximize an *unconstrained* nonlinear function. By substitution, the constrained maximization problem is transformed into the problem of maximizing an unconstrained function of the control variables:

$$W = \Phi(z) \tag{10.11}$$

where Φ stands for the mapping $z \longrightarrow y_1, \ldots, y_T, x_1, \ldots, x_T \longrightarrow W$. For nonlinear models it is generally not possible to express y_t explicitly in terms of x_t, which means that it is generally not possible to write W in 10.11 explicitly as a function of $x_1, \ldots, x_T$. Nevertheless, given values for $x_1, \ldots, x_T$, values of W can be obtained numerically for different values of z.

Given this setup, the problem can be turned over to a nonlinear maximization algorithm like DFP. For each iteration, the derivatives of Φ with respect to the elements of z, which are needed by the algorithm, can be computed numerically. An algorithm like DFP is generally quite good at finding the optimum for a typical control problem.

Consider now the stochastic case, where the error terms in 4.1 are not zero. It is possible to convert this case into the deterministic case by simply setting the error terms to their expected values (usually zero). The problem can then be solved as above. In the nonlinear case this does not lead to the exact answer because the values of W that are computed numerically in the process of solving the problem are not the expected values. In order to compute the expected values correctly, stochastic simulation has to be done. In this case each function evaluation (i.e., each evaluation of the expected value of W for a given value of z) consists of the following:

1. A set of values of the error terms in 4.1 is drawn from an estimated distribution.
2. Given the values of the error terms, the model is solved for $y_1, \ldots, y_T$ and the value of W corresponding to this solution is computed from 10.10. Let $\tilde{W}^j$ denote this value.
3. Steps 1 and 2 are repeated J times, where J is the number of repetitions.
4. Given the J values of $\tilde{W}^j$ $(j = 1, \ldots, J)$, the expected value of W is the mean of these values:

$$\bar{W} = (1/J) \sum_{j=1}^{J} \tilde{W}^j \tag{10.12}$$

This procedure increases the cost of solving control problems by roughly a factor of J, and it is probably not worth the cost for most applications. The bias in predicting the endogenous variables that results from using deterministic rather than stochastic simulation is usually small, and thus the bias in computing the expected value of W using deterministic simulation is likely to be small.

The US model has the following problem regarding the application of optimal control techniques to it. If the aim is to minimize a loss function that has in it squared deviations of output from some target value and inflation from some target value, then the optimal policy for the US model will generally be to achieve the output target almost exactly unless the weight on the inflation loss is very high. The difficulty pertains to the demand pressure variable in the price equation 10, which was discussed in Chapter 5. Reliable estimates of the behavior of the price level at very high output levels cannot be obtained in the sense that the data do not appear to support any nonlinear functional forms. Without some nonlinearity in price behavior at high levels of output, the optimal control solution is likely to correspond to the output target being closely met unless the weight on the inflation loss is very high. In this sense the optimal control exercise is not very interesting because it all hinges on the form of the output variable in the price equation, about which the data tell us little. Because of this problem, no optimal control experiments were performed in the next chapter.

10.6 Counterfactual Multiplier Experiments

It is sometimes of interest to use a model to predict what an economy would have been like had something different happened historically. In Section 11.7, for example, the US model is used to predict what the U.S. economy would have been like in the 1980s had tax rates been higher and interest rates lower than they in fact were. The procedure for doing this is straightforward. One chooses the exogenous variable changes to make and solves the model for these changes. If one wants to use the shocks (error terms) that existed historically, then the estimated residuals are added to the model before solving it. From this base the model can then be solved using either deterministic or stochastic simulation. If stochastic simulation is used, the draws of the error terms are around their estimated historical values. If one is merely interested in the mean paths of the variables, then stochastic simulation is not likely to be necessary because mean values are usually quite close to predicted values from deterministic simulations.

Generating predictions in this manner is a way of answering counterfactual questions. One need not, however, stop with the predicted economies. These economies can be treated like the actual economy, and experiments like multiplier experiments performed. In other words, one can examine the properties of the predicted economies using methods like the ones discussed in this chapter. These properties can then be compared, if desired, to the estimated properties of the actual economy. An example of this is in Section 11.7, where the effectiveness of monetary policy is examined in the predicted economy with higher tax rates and lower interest rates. The monetary-policy properties in this economy are compared to those estimated for the actual economy. These kinds of experiments are useful ways of teaching students macroeconomics.