Natural Concepts in Macroeconomics

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Abstract

Ragnar Frisch proposed in 1936 a procedure for estimating natural variable values by modifying what are now called structural macroeconometric models. This paper shows that Frisch's procedure can be used to illuminate natural concepts using today's models. The procedure also forces one to be precise regarding the assumptions used in moving from a short-run model to a medium-run or long-run model.

1 Introduction

Natural concepts play an important role in macroeconomics. Wicksell (1898) originated the idea of a natural rate of interest, and recently there has been renewed interest in this concept.[1] Friedman (1968) and Phelps (1968) originated the idea of a natural rate of unemployment, and a huge literature developed from this work. From early on economists have struggled with defining and measuring natural values. An early attempt at this is in an important paper by Frisch (1936). At the

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[1]See Bomfim (1997), Orphanides and Williams (2002), Laubach and Williams (2003), and Woodford (2003).

1935 meeting of the Econometric Society, Frisch, Breit, F.G. Koopmans, Marschak, and Tinbergen had discussions of Wicksell's concept of the natural interest rate and more generally of what was to be "understood by a 'natural' or 'equilibrium' position of a certain set of economic variables."[2] Frisch's paper is an outcome of this discussion. This is a fascinating paper for its time, and I argue in this paper that Frisch's basic idea can be used to illuminate natural concepts in today's structural macroeconometric models. Using his procedure makes clear the assumptions that are behind the measurement of natural variable values.

It will be seen that Frisch's procedure requires more theory than does the time-series approach to measuring natural values, where various time-series processes are postulated for the variables of interest, from which natural values are estimated. For examples of the time-series approach, see Watson (1986), Clark (1987), and Kuttner (1994) for estimates of the natural rate of output, see Staiger et al. (1997), Gordon (1998), and Laubach (2001) for estimates of the natural rate of unemployment, and see Laubach and Williams (2003) for estimates of the natural rate of interest.

Frisch's idea is also relevant for the short-run, medium-run, long-run debate in macroeconomics. Tobin (1980) drew a distinction between the long run, where Friedman and Phelps may be relevant, and the short run, where Keynesian ideas may be relevant. Lucas (1981) sharply criticized this distinction, arguing that the long run is just a sequence of short runs and the two must be consistent. Solow (2000), in his discussion of the medium run, addresses this question in his usual pragmatic way. While conceding Lucas's point ("How does someone who is being

[2]Frisch (1936), p. 100.

Keynesian from quarter to quarter ever stop?”—p. 157), he argues that research may best progress at this point by being practical:

> “I can easily imagine that there is a ‘true’ macrodynamics, valid at every time scale. But it is fearfully complicated, and nobody has a very good grip on it. At short time scales, I think, something sort of ‘Keynesian’ is a good approximation, and surely better than anything straight ‘neoclassical.’ At very long time scales, the interesting questions are best studied in a neoclassical framework, and attention to the Keynesian side of things would be a minor distraction. At the five-to-ten-year scale, we have to piece things together as best we can, and look for a hybrid model that will do the job” (p. 158).

It will be seen that Frisch’s procedure is a way of dealing with this short-run, medium-run, long-run issue.

Section 2 presents Frisch’s procedure using Wicksell’s model as an example. Section 3 then applies the procedure to a macroeconomic model. A numerical example using Frisch’s procedure and this model is presented in Section 4. Section 5 concludes with a brief discussion of an alternative approach to policy that does not use natural concepts, namely the optimal control procedure. A numerical example using this procedure is presented in Section 5.

2 Frisch’s Procedure

Frisch begins with a set of n independent dynamic structural relations, which may be nonlinear. They can be in discrete or continuous time. Using discrete time, the model is a set of nonlinear structural difference equations. To represent Wicksell’s theory, Frisch uses the two equations:

$$S_t = F(\rho_t, etc.) \qquad [1]$$

$$I_t = G(\rho_t, etc.) \qquad [2]$$

where S_t is saving, I_t is investment, and ρ_t is the actual interest rate. "etc." refers to all the other variables in the equations, which can differ from equation to equation.

In today's notation one can think of Frisch's n independent dynamic structural relations as a structural macroeconometric model—the kind of model that Tinbergen (1939) pioneered and that was the main focus of the Cowles Commission—Koopmans (1950), Hood and Koopmans (1953). My multicountry (MC) econometric model—Fair (2004)—is of this type, and it is used as the example in Section 3. These models consist of estimated structural equations and identities. Because they are designed to try to fit the short-run fluctuations in the data well and possibly to make real-time forecasts, they are usually referred to as short-run models. This is not to say that long-run issues are completely ignored in the specification and estimation of the equations, but if there is a trade-off between short-run explanatory power and long-run issues, the short-run specification may dominate. Ideally, of course, if one had the macrodynamics specified correctly, there should be no trade-off, but, as Solow notes, we are probably not there yet. In the following discussion Frisch's n independent dynamic structural relations will be called the "estimated" model. I could have called it the "short-run" model, but I prefer "estimated" because there is nothing that rules out (in the long run?) an estimated model having good medium-run and long-run explanatory power as well as good short-run explanatory power.

Frisch first points out that one obvious concept of equilibrium values in a model are values that would exist in a stationary state[3] if the model had one and if the system were stable around at least small disturbances. Regarding this concept he then states that "...as the tendency to formulate the economic reasoning in exact dynamic mathematical terms gains ground, it is probably that this concept of 'normality' will prevail more and more. But at present the notion of 'normal' values is in economics most frequently used in a different sense" (p. 102). This is an interesting statement of Frisch's. He clearly thought that dynamic economic modeling would improve over time to the point where one would have confidence in a model's long-run properties, i.e., (in the present notation) in an *estimated* model's long-run properties. If Solow is right, we are still not there after 69 years. So Frisch's idea that we can't simply stop with the estimated model may still be relevant. In other words, we may not be able with any confidence to use an estimated model to derived normal or natural values.

Frisch's idea of deriving normal or natural values that are different from steady state values involves three steps. The first step is pick a set of m variables ($m \leq n$) to be the "equilibrium analyzed" variables. These are variables that will have "normal" values. In the Wicksell example, Frisch takes S_t, I_t, and ρ_t to be these types of variables, so m is 3. Although Frisch does not discuss this, to make sense of this example, an equation for ρ_t must be postulated:

$$\rho_t = H(etc.) \qquad [3]$$

With this third equation, n is now 3. Otherwise m would be greater than n.[4] The

[3]Frisch discusses stationary states, but his discussion could easily be extended to steady states. For the rest of this paper I will use the phrase "steady states" instead of "stationary states."

[4]In fact, Frisch probably had in mind for this example a much larger model within which

structural model thus consists of equations [1], [2], and [3].

The second step is to add k "supplementary hypothetical equations." These equations will usually be in the normal values of the variables. Normal values are distinguished from actual values by having bars over them. In the Wicksell example, one supplementary hypothetical equation is added:

$$\bar{S}_t = \bar{I}_t \tag{4}$$

Each supplementary hypothetical equation replaces an equation in the structural model. In this case equation [4] replaces equation [3].

The third step is to select $h = m - k$ of the structural equations and put bars over the variables in the equations.[5] Frisch calls this the "barring process." In the Wicksell example equations [1] and [2] are selected:

$$\bar{S}_t = F(\bar{\rho}_t, etc.) \tag{5}$$

$$\bar{I}_t = G(\bar{\rho}_t, etc.) \tag{6}$$

The new model, which will be called the "barred" model, consists of equations [4], [5], and [6]. Solving this model yields:

$$F(\bar{\rho}_t, etc.) = G(\bar{\rho}_t, etc.) \tag{7}$$

The natural rate of interest, $\bar{\rho}_t$, is the solution of equation [7]. This is not, of course, the solution for ρ_t from the estimated model, namely, equations [1], [2], and [3]. In

equations [1] and [2] were imbedded. For present purposes it is sufficient just to add equation [3]. Although Frisch allowed m to be less than n, it seems in most cases that m can simply be taken to be n, which means that all the endogenous variables will be "equilibrium analyzed" variables.

[5] If m is equal to n (see footnote 4), then bars are put over the current values of all the endogenous variables in the remaining equations, i.e., in the equations not replaced by the supplementary hypothetical equations.

the estimated model, saving does not necessarily equal investment,[6] and the actual interest rate is not determined by an equation like [7].

The "supplementary hypothetical equations" are, of course, key to this analysis. Frisch points out that there is no formal rule for choosing these equations; it is the decision of the theorist. If the theorist makes "a happy choice, he may get a tool of great value in describing and explaining the forces that produce the change from one moment to the next" (p. 104). The supplementary hypothetical equations are not estimated, and so the choice for these equations must be made on some criterion other than fitting the short-run fluctuations in the data well. In other words, the barred model is not completely estimated, and its worth depends on how good the supplementary hypothetical equations are in capturing the true equilibrium or long-run nature of the economy.

Frisch was clear in pointing out that the normal values, i.e., the solution values from the barred model, change over time because they depend on the variables in "etc." In modern notation, the normal values depend on the initial conditions as reflected in lagged variable values and on any current exogenous-variable values.

Returning to the Wicksell example, one can consider two possible estimates of the natural rate of interest. One is the steady state value (assuming it exists) from the estimated model, and one is the solution of equation [7] from the barred model. These values differ from the actual (current) value of the interest rate. Frisch argued in the last paragraph of his paper that studying the difference between the actual rate and the natural rate "and the way in which it influences the behaviour of

[6] In the national income and product accounts actual investment, of course, always equals actual saving. In the present example I_t is probably best thought of as some measure of *planned* investment, where planned investment can differ from saving.

entrepreneurs and the functioning of the banking mechanism, etc. throws a flood of light on what goes on during a business cycle" (p. 105). It is thus clear that Frisch does not mean that the barred model necessarily provides a better explanation of the actual interest rate than does the estimated model; one would not want to use the barred model to predict the current value of the actual interest rate. Instead, the barred model provides an alternative way of estimating natural values from simply computing steady state values from the estimated model. However, if the choice of the supplementary hypothetical equations is a "happy" one, Frisch seems to have in mind, given the state of modeling at the time, that a barred model would provide more accurate estimates of natural values than would the steady state (if it exists) of an estimated model. As discussed above, this may still be true 69 years later.

The quote from Frisch in the previous paragraph shows that he did not think of equation [4] as holding every period, i.e., that saving always equals investment. But if this equilibrium condition does hold for a given period, the value of the equilibrium interest rate for that period is as computed from the barred model. In the estimated model, on the other hand, equilibrium may never be forced to hold, depending on the specification of equation [3]. Equilibrium theory has thus been used to guide the choice of the supplementary hypothetical equation [4]. It is the case, of course, that much of macro theory since the early 1970s has been based on the assumption of equilibrium holding every period, contrary to Frisch's view. Under this assumption, equation [4] would hold every period, and the barred model might be the better model every period, not just better for computing long-run equilibrium values.

It may thus be the case if equilibrium holds every period that a barred model is a better approximation of the economy than is an estimated model. The possibly better fit of the estimated model of the short-run fluctuations in the data may be misleading. Speaking loosely, the estimated model may be misspecified by not imposing various equilibrium conditions and may have a better fit simply from data mining. If data mining has led to over fitting of the short-run fluctuations in the data, this may lead to poor explanations of the long-run features of the economy. The barred model would be an improvement because its specification would be based on the correct macro equilibrium theory.

If the economy is not always in equilibrium, a practical problem may arise when applying the barring process, which concerns the initial conditions. Say that the economy has been in recession and that there is considerable slack in the economy—machines that are not being fully utilized. The capital stock is thus large relative to current output, which is likely to effect future investment decisions. If equation [4] is postulated for the next period, say period t, this would be a large shock to the economy—a rapid change to equilibrium. No time would be allowed to have the capital stock adjust toward equilibrium. It may thus be desirable to phase the barring process in. An example of how this might be done is the following.

Say that one wants to phase in the barred model over 16 periods, between t and $t + 15$. Let θ_i be 1/16 for $i = t$, 2/16 for $i = t + 1$, through 1 for $i = t + 15$. Consider the solution of the model for period t, where values for period $t - 1$ and back are known. Let $\hat{\hat{\rho}}_t$ denote the solution for the interest rate from equation [3], and let $\tilde{\rho}_t$ denote the solution from equation [7]. Define

$$\hat{\rho}_t = \theta_1 \tilde{\rho}_t + (1 - \theta_1)\hat{\hat{\rho}}_t$$

$\hat{\rho}_t$ is the solution value used for period t, a weighted average of the other two solutions. The solution for period $t+1$ is:

$$\hat{\rho}_{t+1} = \theta_2 \tilde{\rho}_{t+1} + (1 - \theta_2)\hat{\hat{\rho}}_{t+1}$$

The solution for the last period, $t+15$, is

$$\hat{\rho}_{t+15} = \tilde{\rho}_{t+15}$$

At period $t+15$ the solution value is the solution value from the barred model only. It should be understood that in this solution process if an endogenous variable like ρ_t is on the right hand side of an equation, the value used in the iterative solution process is the value with one hat. Also, the solution values carried to the next period are always the values with one hat.

An alternative to this phasing in process is to change the specification of equation [4] to have there be an adjustment to equilibrium over time. In other words, equation [4] would have imbedded in it some adjustment process. In this case the barred model would have to be solved for enough periods to reach equilibrium, at which point the solution values would be the natural values. This approach is not pursued in this paper, but the above phasing in process is used for the numerical example in Section 4.[7]

[7] Bomfim (1997) uses part of the MPS econometric model to estimate an equilibrium nominal federal funds rate. He works with the IS block of the MPS model plus an interest rate reaction function that targets a full-employment value of output. He takes all stock variables like capital

3 The MC Model and Two Barred Versions

The following is an outline of a two-country structural macro model. It consists of 67 equations determining 67 endogenous variables. It is an attempt to capture the key equations of the MC model mentioned above. Once the model is outlined, Frisch's procedure will be applied to it to determine the normal or natural values. The overall MC model is fully estimated (no calibration), and it incorporates the main macroeconomic links within and among countries. It is structural in that economic theory has been used to guide the specification of the equations. The estimated equations are meant to be approximations of decision equations. The method of estimation is two stage least squares. Expectations are not taken to be rational (model consistent) because in the empirical tests for the MC model there was little empirical support for the rational expectations hypothesis. If expectations are not rational, the Lucas (1976) critique is not likely to be a problem.[8] Also, time inconsistency is not likely to be a problem when solving optimal control problems. The MC model has been tested in many ways, and it appears to be a good approximation of the economy. A complete discussion of the model is in

stocks, wealth, and government debt to be constant and exogenous; he sets all lagged exogenous-variable values equal to their current values; and he sets all lagged endogenous-variable values equal to their current solution values. Inflation expectations, which depend on lagged inflation, are also taken to be exogenous. He solves this "static" model and takes the solution value for the nominal federal funds rate to be the equilibrium rate. This approach uses more theory than the time-series approach mentioned in Section 1, since it is using part of the MPS model, but it differs considerably from Frisch's procedure. Frisch's procedure works with a complete model, does not change lagged values, does not take stock variables to be exogenous, and changes the model before solution by adding supplementary hypothetical equations.

[8] Evans and Ramey (2003) have shown that in some cases the Lucas critique is a problem even if expectations are not rational. These cases are specific to the Evans and Ramey framework, and it is unclear how much they can be generalized.

Fair (2004), and this discussion is not repeated here.

Although the model presented below is a highly simplified or stylized version of the overall MC model, I have tried to incorporate all the main variables. The variables that are listed in parentheses after the functions are empirically significant and economically important explanatory variables. Lagged values are heavily used in the MC model to capture expectational and partial adjustment effects. For simplicity these values are not included in the list of explanatory variables—they are in "...". Similarly, a number of other, generally more minor, variables are not included. Also, the following model is much more aggregated than the MC model. The disaggregation below is just the minimum needed to make the points. Finally, population is ignored even though population variables play an important role in the MC model. It should be stressed that the specifications that are outlined below are what appear to be supported by the data. The final specification chosen for each stochastic equation is one that did well in the various tests. These test results are in Fair (2004, Chapter 2).

A t subscript denotes period t, and an f superscript denotes that the variable is for country 2 (the "foreign" country). For any variable Z_t, ΔZ_t denotes $Z_t - Z_{t-1}$ and $\dot{Z}_t$ denotes the percentage change in Z_t at an annual rate. The currency of country 1 is the \$. The currency of country 2 is denoted *fc* (for "foreign currency"). The exchange rate, e_t, is in units of *fc* per \$. Net international reserve holdings, Q_t and Q_t^f, are in \$. The money, bonds, and stocks of one country are not held by the other country: any nonzero value of the current account results only in a change in Q_t.[9] The base year for computing real values is taken to be 2000. Table 1 presents

[9]In the actual theoretical model that was used to guide the specification of the MC model, each

the notation in alphabetical order. The variable Y, real GDP, can be thought of as total output or total income. All the flows of funds among the four sectors—the private and government sectors in each of the two countries—are accounted for.

The first 30 equations are for country 1. The equations for country 2 are the same with the superscript f added except for three equations. These three equations are presented below for country 2—equations (39), (40), and (53)—but none of the others are. After these 60 equations, there are three more for each country plus an exchange rate equation, giving a total of 67 equations. Table 1 lists 33 endogenous variables for country 1 plus the exchange rate, e_t. There are thus a total of 67 endogenous variables.

In the MC model there are both short-term and long-term interest rates, where long-term rates are linked to short-term rates through estimated term structure equations. For simplicity it is assumed in the following outline that all bonds are

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country holds the bonds of the other country. Let BF_t denote the bonds of country 1 held by country 2 (so that BF_t^f denotes the bonds of country 2 held by country 1), let R_t and R_t^f denote the interest rates, and let e_{t+1}^e denote the expected exchange rate for period $t+1$ made at the beginning of period t. The demand for country 2's bond by country 1, BF_t^f, was postulated to be determined as:

$$BF_t^f = f[R_t, \frac{e_{t+1}^e}{e_t}(1 + R_t^f) - 1]$$

where the second term in brackets is the expected return on country 2's bond. [mention one period somewhere] A similar equation was postulated for BF_t. Also, interest rate rules were postulated for R_t and R_t^f, and an exchange rate equation was postulated for e_t. Postulating these three equations implicitly assumes that uncovered interest rate parity does not hold. If it does hold, then $R_t = \frac{e_{t+1}^e}{e_t}(1 + R_t^f) - 1$, and so given a value for e_{t+1}^e, however determined, only two of the three equations can be postulated. Also, if it does hold, BF_t^f cannot be determined by the above equation (and similarly for BF_t). Although it is assumed that uncovered interest rate parity does not hold, covered interest parity does hold in the data. In other words, the value of F_t is very close to the value of $e_t\frac{1+R_t^f}{1+R_t}$ for all t, where F_t is the period-t market-determined (observed) forward exchange rate for period $t+1$. For purposes of this paper nothing is lost by assuming that the two countries don't hold each other's bonds as long as one is aware that the specification requires that uncovered interest parity not hold.

Table 1
Notation
Variables are real unless stated otherwise

A	wealth
B	value of government bonds, nominal
C	consumption
CA	current account, nominal
CG	capital gains or losses on stocks, nominal
DIV	dividends, nominal
e	exchange rate
EX	exports
EXK	excess capital
EXL	excess labor
G	government spending (exogenous)
I	investment
IM	imports
$INTG$	government interest payments, nominal
J	employment
J'	required employment to produce the output
K	capital stock
K'	required capital to produce the output
L	labor force
M	money supply, nominal
P	price level
PIM	price of imports
Q	government international reserve holdings, nominal
R	nominal interest rate
S	value of stocks, nominal
SG	saving, government, nominal
SH	saving, households, nominal
TR	government transfer payments, nominal (exogenous)
UR	unemployment rate
V	stock of inventories
W	nominal wage rate
X	sales
Y	output
YD	disposable income
δ	depreciation rate (exogenous)
λ	production function parameter (exogenous)
μ	production function parameter (exogenous)
ρ	real interest rate
Π	profits, nominal
τ_1	personal income tax rate (exogenous)
τ_2	profit tax rate (exogenous)

one-period securities and thus that there is only one interest rate per country. More will be said about this below.

The reader may wonder whether it is necessary to wade through 67 equations to see an application of Frisch's procedure. The answer is yes because of the supplementary hypothetical equations. These are essentially equilibrium conditions, and one needs to have a complete model to think about what equilibrium conditions to impose. This is in fact one of the main advantages of Frisch's procedure: it forces one to be precise about what equations to replace and with what.

An Outline of the MC Model

The first two equations are decision equations of households:

$$C_t = f_1(YD_t, R_t, A_{t-1}, \ldots) \tag{1}$$

$$L_t = f_2[(1-\tau_{1t})W_t/P_t, A_{t-1}, UR_t, \ldots] \tag{2}$$

Equations (1) and (2) represent the consumption and labor supply decisions of households. Real consumption (C_t) depends on real disposable income (YD_t), the nominal interest rate (R_t), and the initial value of real wealth (A_{t-1}). Labor supply (L_t) depends on the after-tax real wage rate $[(1-\tau_{1t})W_t/P_t]$, the initial value of real wealth, and the unemployment rate (UR_t). τ_{1t} is the personal income tax rate. In the MC model[10] consumption is disaggregated into services, nondurables, and durables, and labor supply is disaggregated into the labor force of men 25-54, women 25-54, all others 16 and over, and the number of people holding two jobs. Remember that "..." in general includes lagged values (to pick up partial

[10] In the following discussion "in the MC model" refers to the equations for the United States part of the model. The specification for the other countries is somewhat simpler.

adjustment and expectational effects) and some other variables. An important explanatory variable, omitted above, is the stock of durable goods in the durable consumption equation. The tests of the MC model suggest that consumption responds to the nominal interest rate rather than the real rate, and so the nominal rate is used in equation (1). This is an important issue for the specification of the supplementary hypothetical equations, and it is discussed further below. The results estimating the MC model also suggest that a variable like the unemployment rate is important in explaining labor force participation. It is picking up discouraged worker effects.

The next three equations represent decisions of firms:

$$Y_t = f_3(X_t, V_{t-1}, \ldots) \tag{3}$$

$$I_t = f_4(Y_t, \rho_t, CG_t, EXK_{t-1}, \ldots) \tag{4}$$

$$J_t = f_5(Y_t, EXL_{t-1}, \ldots) \tag{5}$$

Equation (3) is in effect an inventory investment equation. Production (Y_t) depends on sales (X_t) and the initial stock of inventories (V_{t-1}). Investment (other than inventory investment) (I_t) depends on production, the real interest rate (ρ_t),[11] capital gains or losses on stocks (CG_t), and the initial amount of excess capital (EXK_{t-1}). In the MC model housing investment, a decision variable of households, is treated separately, but this disaggregation is ignored here.[12] The CG_t variable represents part of the cost of capital. In the actual estimation it is normalized by nominal output. The excess capital variable is discussed below. Excess capital has a negative

[11] For the U.S. investment equation the data support the use of the real interest rate over the nominal rate, although this is not in general the case for the investment equations of the other countries.

[12] An important explanatory variable in the housing investment equation is the stock of housing. For this equation the data support the use of the nominal interest rate over the real rate.

effect on investment. Employment (labor demand) (J_t) depends on production and the initial amount of excess labor (EXL_{t-1}). J stands for jobs. In the MC model there is also an equation for hours paid per worker, but this is ignored here. The excess labor variable is discussed below.

In the MC model the dynamic specifications of equations (3), (4), and (5) are such that there is an adjustment over time toward equilibrium-type values. In equation (3) there is an adjustment toward having the stock of inventories be some desired fraction of sales; in equation (4) there is an adjustment toward zero excess capital; and in equation (5) there is an adjustment toward zero excess labor. More will be said about this below when comparing the MC model to a barred version.

The next two equations determine the demand for money and CG_t:

$$M_t/P_t = f_6(Y_t, R_t, \ldots) \tag{6}$$

$$CG_t = f_7(\Delta DIV_t, \Delta R_t, \ldots) \tag{7}$$

In equation (6) the real demand for money (M_t/P_t) depends on real income and the interest rate. This is a standard demand for money equation. In the estimation of the demand for money equations for the various countries, the interest rate is usually highly significant. In equation (7) CG_t depends on the change in dividends (ΔDIV_t) and the change in the interest rate. This is the "stock market" equation in the MC model. Very little of the variance of CG_t is explained by the estimated equation (as expected).

The next equation determines the demand for imports:

$$IM_t = f_8(C_t + I_t, P_t/PIM_t, \ldots) \tag{8}$$

Imports (IM_t) depends on total demand as represented by consumption plus investment and on the ratio of the price of domestic goods (P_t) to the price of imports (PIM_t). PIM_t is defined next.

Equations (9) through (30) are definitions or identities. The price of imports in \$ is equal to the price of country 2's good in *fc* times the exchange rate in the base year divided by the current exchange rate:

$$PIM_t = P_t^f (e_{2000}/e_t) \tag{9}$$

Remember that 2000 is taken to be the base year. Exports in 2000 \$ (EX_t) equals imports of country 2 in 2000 *fc* divided by the exchange rate in 2000:

$$EX_t = IM_t^f / e_{2000} \tag{10}$$

Total sales equals consumption plus investment plus government spending (G_t) plus exports minus imports:

$$X_t = C_t + I_t + G_t + EX_t - IM_t \tag{11}$$

The stock of inventories at the end of period t equals the stock at the end of the previous period plus production minus sales:

$$V_t = V_{t-1} + Y_t - X_t \tag{12}$$

Inventory investment is $V_t - V_{t-1}$. The capital stock at the end of period t is equal to the stock at the end of period $t - 1$ plus gross investment:

$$K_t = (1 - \delta)K_{t-1} + I_t \tag{13}$$

Depreciation of capital is assumed to be proportional, where δ is the depreciation rate.

The next four equations concern the production technology:

$$K'_t = Y_t/\mu_t \tag{14}$$

$$J'_t = Y_t/\lambda_t \tag{15}$$

$$EXK_t = K_t - K'_t \tag{16}$$

$$EXL_t = J_t - J'_t \tag{17}$$

Results estimating the MC model suggest that firms at times have excess capacity—both excess capital and excess labor. If this is true, then some way must be found to estimate excess capital and excess labor. Given that there can be substitution between capital and labor, this estimation is not straightforward. The above equations are based on the assumption that the production function in the short run is one of fixed proportions:

$$Y_t = \min(\mu_t K'_t, \lambda_t J'_t), \tag{i}$$

where μ_t and λ_t change as technology changes. In this setup K'_t is the minimum amount of capital required to produce Y_t and J'_t is the minimum amount of labor required to product Y_t. If K_t is the actual amount of capital on hand, equation (16) defines excess capital. Similarly, if J_t is the actual amount of labor employed, equation (17) defines excess labor. μ_t and λ_t are taken to be exogenous. In practice excess capital and excess labor cannot be negative. More will be said about the production technology later.

The unemployment rate is:

$$UR_t = (L_t - J_t)/L_t \tag{18}$$

UR_t will never be zero if there is frictional unemployment.

The real interest rate is defined by the equation:

$$1 + \rho_t = (1 + R_t)/[1 + f_{19}(\dot{P}_{t-1}, \ldots)] \tag{19}$$

$f_{19}(\dot{P}_{t-1}, \ldots)$ represents the expected rate of inflation for period t, where the expected value depends on the actual rate of inflation in period $t-1$ and other lagged values (represented by $\ldots$).

The level of profits (Π_t) is defined to be:

$$\Pi_t = P_t(C_t - IM_t) + P_tG_t + P_tEX_t - W_tJ_t \tag{20}$$

This equation is a simplification in that Π_t as just defined is really cash flow rather than profits as defined in the national income and product accounts (and as defined in the MC model). For present purposes it is unnecessary to deal with the difference between cash flow and profits, and it is simply assumed that the level of "profit" taxes paid to the government equals $\tau_{2t}\Pi_t$, where τ_{2t} is the profit tax rate. It is also assumed that what is left over is paid out in dividends (DIV_t):

$$DIV_t = \Pi_t(1 - \tau_{2t}) \tag{21}$$

The saving of the government is equal to tax revenue minus transfer payments (TR_t), purchases of goods (P_tG_t), and interest payments ($INTG_t$):

$$SG_t = (W_tJ_t + INTG_t + DIV_t)\tau_{1t} + \Pi_t\tau_{2t} - TR_t - P_tG_t - INTG_t \tag{22}$$

The next equation is the balance sheet constraint of the government:

$$Q_t = Q_{t-1} + (B_t - B_{t-1}) + (M_t - M_{t-1}) + SG_t \tag{23}$$

M_t is the money supply, and B_t is the value of government bonds. They are liabilities of the government. Q_t is the value of international reserve holdings of the government. Aside from international reserves and stocks, there are two financial instruments per country in the model: money and bonds. As noted above, it is assumed that the countries do not hold each other's money and bonds. For simplicity it is assumed that the government consists of both the fiscal and monetary side, and equation (23) states that any nonzero value of government saving results in a change in at least one of Q_t, B_t, and M_t.

The level of interest payments of the government is determined as:

$$INTG_t = f_{24}(R_t, B_t) \tag{24}$$

If all bonds were one-period bonds, the level of interest payments would simply be $R_t B_t$. In practice the situation is more complicated. $INTG_t$ depends on the maturities of the bonds and on the interest rates of the different maturities. This is taken into account in the estimation of the MC model, but for present purposes the issue of different maturities is ignored.

The real value of disposable income of households is equal to after-tax wage, interest, and dividend income plus transfer payments, all divided by the price level:

$$YD_t = [(W_t J_t + INTG_t + DIV_t)(1 - \tau_{1t}) + TR_t]/P_t \tag{25}$$

The saving of households (SH_t) is equal to nominal disposable income minus consumption expenditures:

$$SH_t = P_t YD_t - P_t(C_t - IM_t) - PIM_t IM_t \tag{26}$$

The balance sheet constraint for households is:

$$B_t = B_{t-1} - (M_t - M_{t-1}) + SH_t \tag{27}$$

B_t is the value of net bond holdings of households. This equation states that any nonzero value of SH_t results in a change in at least B_t or M_t.

The financial assets of households include M_t, B_t, and stocks. Assume that households own the firms, and let S_t denote the nominal value of stocks in period t. From above, CG_t is the change in the value of stocks in period t, and so:

$$S_t = S_{t-1} + CG_t \tag{28}$$

The real wealth variable that is used in equations (1) and (2) is:

$$A_t = (B_t + M_t + S_t)/P_t \tag{29}$$

Equation (29) is an important equation in the MC model. In practice much of the fluctuation in household wealth is from fluctuations in the stock market, which is picked up by CG_t. In the MC model the stock market has a large effect on aggregate demand through A_{t-1} in equation (1) and CG_t in equation (4).[13] It should be noted that equation (29) excludes capital gains or losses on bonds. Although this is justified in the present outline because the bonds are one-period securities, even in the MC model, where B_t includes bonds of many maturities, capital gains or losses on bonds are not accounted for. Sufficient data are not available to allow this to be done.

The current account of country 1 is equal to export revenue minus import cost:

$$CA_t = P_t EX_t - PIM_t IM_t \tag{30}$$

[13]In the MC model A also includes the real value of the housing stock.

The same 30 equations hold for country 2 with superscripts f added everywhere except for the three equations in which the exchange rate or Q_t appears: equations (9), (10), and (23). For country 2 these three equations are:

$$PIM_t^f = P_t(e_t/e_{2000}) \tag{39}$$

$$EX_t^f = e_{2000}IM_t \tag{40}$$

$$Q_t^f = Q_{t-1}^f + (1/e_t)[(B_t^f - B_{t-1}^f) + (M_t^f - M_{t-1}^f) + SG_t^f] \tag{53}$$

Equation (53) reflects the fact that international reserves are denominated in \$.

These 60 equations have the feature that all the flows of funds among the four sectors are accounted for, something noted above. Because of this, the equations imply that:

$$Q_t = Q_{t-1} - (Q_t^f - Q_{t-1}^f)$$

In other words, the sum of the changes in international reserve holdings across the two countries is zero. This equation is not numbered because it is not an independent equation. It is, however, a useful check that the accounting has been done properly.

So far nothing has been said about how prices, wages, interest rates, and the exchange rate are determined. As will be seen below, this has been saved for last to make Frisch's procedure clearer. In a structural macroeconometric model like the MC model, stochastic equations are postulated for these variables. The equations explaining the price level, the nominal wage rate, and the interest rate for country 1 are represented here as:

$$P_t = f_{62}(W_t/\lambda_t, UR_t, PIM_t, \ldots) \tag{61}$$

$$W_t/\lambda_t = f_{61}(P_t, UR_t, \ldots) \tag{62}$$

$$R_t = f_{63}(UR_t, \dot{P}_t, \ldots) \tag{63}$$

Equation (61) states that the price level depends on two cost variables—the nominal wage rate and the price of imports—and a demand pressure variable—the unemployment rate. Equation (62) states that the nominal wage rate depends on the price level and the unemployment rate. In both equations (61) and (62) the nominal wage rate is divided by labor productivity, λ_t.[14]

Equation (63) is an interest rate rule of the monetary authority, where the interest rate depends on the unemployment rate and the rate of inflation. The estimation of interest rate rules goes back to Dewald and Johnson (1963), although they are usually called "Taylor rules" from Taylor (1993). The output gap is usually used in place of the unemployment rate in the equation, but I have found better results using the unemployment rate. I first added an estimated interest rate rule to my macroeconometric model in Fair (1978).

In practice the specification of the dynamics in equations (61), (62), and (63) is important. For present purposes the dynamics can be subsumed in "...". In the MC model a restriction is put on the estimation of equations (61) and (62) to insure that W_t/P_t has reasonable long-run properties. This restriction is discussed below when comparing the MC model to a barred version. The estimated price equations for the various countries in the MC model are not NAIRU equations, where the change in the inflation rate depends on the difference between the unemployment rate and the natural rate. The NAIRU dynamics are tested in Fair (2004, Chapter 4) and are generally rejected. The functional form of the price equation is discussed

[14] The price equation is identified in the MC model because W_{t-1}/λ_{t-1} appears in the wage equation but not in the price equation.

further below.

Number the equivalent three equations for country 2 as equations (64), (65), and (66). The final equation determines the exchange rate:

$$e_t = f_{67}(P_t/P_t^f, R_t/R_t^f, \ldots) \tag{67}$$

The exchange rate is taken to be a function of the relative price levels and the relative interest rates. In the model one can think about e_t being control by the two governments through their control of Q_t, and so if equation (67) is postulated, Q_t becomes endogenous.

These 67 equations determine the 67 endogenous variables in Table 1 (counting country 1 and country 2).

Supplementary Hypothetical Equations

The fully estimated MC model, of which the above is an outline, is Frisch's set of n independent dynamic structural relations, called in Section 2 the "estimated" model. We are now ready to modify the model by adding supplementary hypothetical equations. In the following discussion, two additions are outlined. The first is a fairly modest change in the MC model and the second is fairly extreme. The first will be called "Barred One" and the second "Barred Two." An example of solving Barred One is presented in Section 4. In what follows bars are put over the period t values of the variables to denote that the equations are part of the barred model.

Barred One

First, the inventory investment equation (3) is replaced by

$$\bar{Y}_t = \bar{X}_t + \alpha_t^* \bar{X}_t - V_{t-1} \tag{3$'$}$$

where α_t^* is the "normal" inventory-sales ratio. Second, under the assumption that the production function is equation (i), the investment and labor demand equations (4) and (5) are replaced by

$$\bar{K}_t = \bar{K}_t' \tag{4$'$}$$

$$\bar{J}_t = \bar{J}_t' \tag{5$'$}$$

These two equations state that there is no excess capital and no excess labor. Given that the capital stock is determined by equation (4)$'$, investment is determined by equation (13). As noted in the discussion of equations (3), (4), and (5) above, in the MC model there is specified to be an adjustment toward the equilibrium values in these three equations.

Third, the wage equation (62) is replaced by an equation that states that the growth rate of the real wage equals the growth rate of labor productivity:

$$(\dot{\bar{W}/\bar{P}})_t = \dot{\lambda}_t \tag{62$'$}$$

This equation reflects the assumption of a constant labor share. The restriction imposed in the MC model on the long-run properties of the real wage mentioned above is similar to the constraint in equation (62)$'$.

Fourth, the exchange rate equation (67) is replaced by:

$$\bar{e}_t = e_t^* \tag{67$'$}$$

or

$$\bar{CA}_t = CA_t^* \tag{67$'$}$$

where e_t^* is the desired (normal) value for the exchange rate and CA_t^* is the desired (normal) value for country 1's current account. Regarding this replacement, one can think of the two governments agreeing on either e_t^* or CA_t^* and choosing $\bar{Q}_t$ to obtain this value.

The equivalents of equations (3), (4), (5), and (62) for country 2 are also replaced as above.

The last change concerns the interest rate rule (63) (and the equivalent for country 2). This could be replaced by one of the following two equations:

$$\bar{UR}_t = UR_t^* \tag{63$'$}$$

or

$$\dot{\bar{P}}_t = \dot{P}_t^* \tag{63$'$}$$

where UR_t^* is the desired (normal) value of the unemployment rate and $\dot{P}_t^*$ is the desired (normal) value of the inflation rate. In other words, the monetary authority could pick the nominal interest rate to achieve some target value of the unemployment rate or some target value of the inflation rate. (The same applies to country 2.) The value of the interest rate that achieves the target value is the natural (normal) value of the interest rate.

The above changes are fairly modest. In Barred One, unlike in the estimated model, 1) the stock of inventories is as desired, 2) no excess capital and no excess labor are being held, 3) the labor share is constant, 4) the exchange rate or the current account is as desired by the two countries, and 5) the unemployment rate or the inflation rate in each country is as desired by the country's monetary authority.

The changes are modest because items 1), 2), and 3) are already specified in the MC model to hold in the long run.

Note that the price equation (61) has not been changed, although it could be if one wanted to impose a NAIRU specification on the barred model. If equation (61) is a NAIRU equation, then the only sensible choice for the monetary authority would be to choose the interest rate so that the unemployment rate equaled the natural rate as implied by the price equation. In this case the natural interest rate is simply the interest rate that achieves the natural unemployment rate.[15]

Barred Two

In Barred One the nominal interest rate, R_t, affects real output and the unemployment rate because it affects consumption through equation (1) and (possibly) the real value of the stock market through equation (7). Money is not neutral. Barred Two is an example in which money is neutral. The changes are as follows.

First, the above changes in equations (3), (4), (5), (62), (63), the equivalent equations for country 2, and equation (67) are made. Second, the demand for money equation (6) is replaced by:

$$1 + \dot{\bar{P}}_t = (1 + \dot{\bar{M}}_t)/(1 + \dot{\bar{Y}}_t) \tag{6$'$}$$

This equation reflects the assumption that velocity is constant, where velocity equals $(\bar{P}_t \bar{Y}_t)/\bar{M}_t$. Third, the price equation (61) is taken to be a NAIRU equation,

[15]As noted above, the results estimating the MC model suggest that the NAIRU dynamics are not accurate. An alternative price equation is one in which the inflation rate is a nonlinear function of the unemployment rate, where at some low value of the unemployment rate the inflation rate begins to increase substantially. In this case the aim of the monetary authority might be to target an unemployment rate near the bend.

where the change in the inflation rate is constant at $\bar{UR}_t = UR^*_t$:

$$\Delta \dot{\bar{P}}_t = f_{61'}(\bar{UR}_t - UR^*_t) \tag{61$'$}$$

Fourth, the expected inflation rate in equation (19) is taken to be the actual rate, so that the real interest rate equation is:

$$1 + \bar{\rho}_t = (1 + \bar{R}_t)/(1 + \dot{\bar{P}}_t) \tag{19$'$}$$

Fifth, the equivalent changes for country 2 are made.

The changes for Barred Two so far are not sufficient for money to be neutral. The household decision equations for consumption and labor supply—equations (1) and (2)—also have to be changed (and the equivalent for country 2).[16] For starters, assume that the nominal interest rate in equation (1) is replaced by the real rate:

$$\bar{C}_t = f_1(\bar{YD}_t, \bar{\rho}_t, A_{t-1}, \ldots) \tag{1$'$}$$

It will be easiest to see what further changes are needed for Barred Two to make money neutral by considering the following experiment. Assume that Barred Two has been solved for period t, where the solution is based on a particular value of $\dot{\bar{M}}_t$ chosen by the monetary authority. Now say that $\dot{\bar{M}}_t$ is increased by enough to make $\dot{\bar{P}}_t$ increase by 0.01 under the assumption that $\dot{\bar{Y}}_t$ is unchanged—see equation (6)$'$. The monetary authority does this by buying $\bar{B}_t$ with $\bar{M}_t$.[17] If the real interest rate remains unchanged, then the nominal rate, $\bar{R}_t$, increases by roughly 0.01 from equation (19)$'$.

[16] In the following discussion, everything that is done for country 1 is also assumed to be done for country 2. For simplicity, only country 1 will be discussed.

[17] Remember that the same changes are made for country 2.

When the model is solved for the new value of $\dot{\bar{M}}_t$, will in fact $\dot{\bar{Y}}_t$ and the real interest rate remain unchanged? To take a special case, assume that the initial solution values of $\bar{SH}_t$ and $\bar{SG}_t$ are zero. Regarding fiscal policy, assume that in response to the monetary policy change the tax rates remain unchanged, $\bar{G}_t$ remains unchanged, and $\bar{TR}_t$, which is in nominal terms, increases by one percent. One requirement for $\bar{SH}_t$ and $\bar{SG}_t$ to remain zero is that nominal interest payments, $\bar{INTG}_t$, increase by one percent. So equation (24) has to be such that this happens.

A second requirement is that $\bar{C}_t$ in equation (1)$'$ remains unchanged. If real wealth remains an explanatory variable in equation (1)$'$, then real wealth must remain unchanged.[18] The real value of stocks will remain unchanged if the change in $\bar{CG}_t$ is such that $\bar{S}_t$ increases by one percent. So equation (7) might be changed to have this happen. The situation regarding $\bar{B}_t$ and $\bar{M}_t$, however, is more complicated. In practice there are bonds of many maturities, and so capital gains and losses from the inflation rate change must be taken into account. The situation is further complicated if $\bar{SH}_t$ and $\bar{SG}_t$ are not zero. A perhaps easier approach is to impose directly on the specification of equation (1)$'$ the constraint that $\bar{C}_t$ is unaffected by the inflation rate. Similarly, the specification of equation (2) could be changed to impose directly that $\bar{L}_t$ is unaffected by the inflation rate.

If equations (1)$'$ and (2) are changed so that $\bar{C}_t$ and $\bar{L}_t$ do not depend on the inflation rate, then Barred Two can be solved as follows. On the real side, the model can be solved for the real interest rate, $\bar{\rho}_t$, at the point $\bar{UR}_t = UR^*_t$. Speaking

[18]Although real wealth enters with a lag of one period in equations (1)$'$ and (2), it is easiest in the present context to think of the money supply change occurring at the beginning of the period and affecting nominal wealth also at the beginning of the period. In other words, change A_{t-1} to $\bar{A}_t$ in equations (1)$'$ and (2).

loosely, the NAIRU price equation (61)$'$ ties down the real interest rate and thus the real side of the economy. The rate of inflation is then determined from equation (6)$'$, and the nominal interest rate, $\bar{R}_t$, is determined from equation (19)$'$. Since neither $\bar{R}_t$ nor the rate of inflation affects $\bar{C}_t$ and $\bar{L}_t$, the real side is not affected. Changes in $\bar{M}_t$ only affect the rate of inflation and the nominal interest rate.

Other Barring Issues

Expectations

Expectations, however formed, must be based on known values at the time they are formed. In an estimated model like the MC model, expectations are assumed to depend on lagged values with at most fairly modest restrictions on the expectational process. If expectations are rational, agents form their expectations by solving the model. Expectations are still based on lagged values, since this is what the solution of the model is based on, but there are in effect many restrictions on the expectational process.[19] If the estimated model does not assume rational expectations, but one wants to impose this assumption in the barred model, then Frisch's supplementary hypothetical equations should be considered as including the addition of the rational expectations hypothesis.

[19]The solution of a model in real time is also based on guessed values of current and future exogenous variables. These guesses for the most part will also be based on lagged values, although some may be based on announced future policy actions. So some of the "known" values under the rational expectations hypothesis may be announced values.

The Production Technology

If the production function is changed from equation (i) to one in which there are substitution possibilities between capital, K_t, and labor, J_t, then the values of capital and labor will depend on the real wage rate and the interest rate. If there are substitution possibilities, the equivalent of no excess capital and no excess labor is the requirement that the economy be "on" the production function. This means that substituting the actual values of K_t and J_t into the production function yields the actual value of Y_t. Under this technology money would still be neutral in Barred Two if money has no effect on the real wage rate and the real interest rate.

4 Barred One: Numerical Example

There are 39 countries in the complete MC model for which stochastic equations are estimated. There are 31 stochastic equations for the United States and up to 15 each for the other countries. In addition, there are about 1,200 estimated trade share equations. Trade share data were collected for 59 countries, and so the trade share matrix is 59×59. The estimation periods begin in 1954 for the United States and as soon after 1960 as data permit for the other countries. The model is discussed in Fair (2004), and the exact version that was used for the results in this section is on the website listed in the introductory footnote.

For present purposes only the equations for the United States were changed to construct "Barred One." The changes represented by equations $(3)'$, $(4)'$, $(5)'$, and $(62)'$ were made. In addition, there is a demand equation for hours paid per worker in the model, and this equation was replaced with one that set hours paid per worker equal to a "normal" value. No changes were made to the exchange rate equations.[20]

The 16 quarter period 2005:1–2008:4 was examined. At the time of this work this was a real-time future period, and the base path of the economy was a path predicted by the MC model. This prediction was based on actual values from 2004:4 back and guessed values of the exogenous variables from 2005:1 on. All future error terms were taken to be zero. The solution values for Barred One were phased in over the 16 quarters using the process discussed at the end of Section 2. Therefore, the predicted values for 2008:4 are the predicted values from the barred model alone. The nominal interest rate path was chosen to yield an inflation rate of about 3 percent. The results are presented in Table 2. The first set of results is from the base path, i.e., the path predicted by the MC model, and the second set is from the path predicted by Barred One with the phase in. (The third set of results in Table 2 is discussed in the next section.)

Before discussing the results, it should be stressed that they are meant for illustration only. They are conditional on the particular set of guessed exogenous-variable values, and the prediction paths would obviously differ if other values were used. However, the differences in the paths are much less affected by a change in exogenous variables than are the levels (because exogenous-variable changes affect both sets of paths similarly), and so more weight should be placed on the

[20]The matching of the equation numbers in the MC model and the numbers in this paper is: 11 to (3), 12 to (4), 13 to (5), and 16 to (62). The hours paid per worker equation is 14. α_t^* was taken to be 0.7 for all t. Not changing the exchange rate equations means that the governments are assumed to be happy with whatever exchange rate values are predicted by the equations.

Table 2
Solution Values for 2005:1–2008:4
Values are percentage points

	MC Model Solution				**Barred One Solution**				**MC Model Optimal**			
	R	$\dot{P}$	UR	$\dot{Y}$	R	$\dot{P}$	UR	$\dot{Y}$	R	$\dot{P}$	UR	$\dot{Y}$
2005:1	2.54	3.25	5.26	3.09	2.75	3.18	5.24	3.82	2.94	3.20	5.27	3.01
2005:2	2.60	3.68	5.33	1.77	3.00	3.64	5.36	1.76	3.23	3.61	5.35	1.64
2005:3	2.63	3.59	5.34	2.16	3.25	3.45	5.51	1.97	3.35	3.50	5.38	1.97
2005:4	2.84	3.53	5.32	2.40	3.50	3.25	5.70	2.16	3.58	3.43	5.38	2.20
2006:1	3.09	3.52	5.29	2.26	3.75	3.04	5.91	2.15	3.84	3.37	5.38	2.06
2006:2	3.26	3.48	5.27	2.36	4.00	2.92	6.08	2.33	3.89	3.34	5.37	2.21
2006:3	3.37	3.45	5.25	2.40	4.25	2.84	6.19	2.39	3.91	3.31	5.37	2.32
2006:4	3.49	3.43	5.24	2.43	4.50	2.81	6.24	2.41	3.95	3.29	5.37	2.40
2007:1	3.61	3.40	5.23	2.46	4.50	2.80	6.25	2.48	4.04	3.27	5.37	2.46
2007:2	3.72	3.38	5.23	2.49	4.50	2.85	6.23	2.59	4.15	3.25	5.37	2.50
2007:3	3.81	3.36	5.23	2.53	4.50	2.90	6.20	2.67	4.27	3.23	5.36	2.54
2007:4	3.89	3.33	5.24	2.57	4.50	2.95	6.17	2.71	4.39	3.21	5.36	2.58
2008:1	3.96	3.31	5.24	2.62	4.50	2.99	6.14	2.75	4.53	3.17	5.36	2.59
2008:2	4.04	3.30	5.24	2.67	4.50	3.05	6.10	2.80	4.69	3.16	5.36	2.63
2008:3	4.11	3.29	5.23	2.71	4.50	3.09	6.06	2.85	4.82	3.15	5.36	2.66
2008:4	4.18	3.28	5.22	2.75	4.50	3.14	6.01	2.89	4.92	3.14	5.36	2.71

R = nominal three-month Treasury bill rate, $\dot{P}$ = percentage change in the GDP deflator,
UR = unemployment rate, $\dot{Y}$ = percentage change in real GDP.

differences than on the levels.

The predicted path of the interest rate for the MC model uses the estimated interest rate rule of the Fed. Table 2 shows that the path chosen (exogenously) for Barred One has slightly larger values, which was done to bring the inflation rate closer to 3 percent. The values of the unemployment rate are larger for Barred One. This is primarily because of the excess labor differences. For Barred One there is no excess labor by the end of the period (equation (5)′), whereas (not shown) there is excess labor for the MC model, including at the end of the period. In general, however, the paths are similar, and so the differences between the regular MC model and Barred One seem modest. In other words, the MC model is not too different from a model in which there is no excess capital, no excess labor, no excess inventories, and the real wage rate growing at the rate of productivity.

For Barred One for 2008:4, the nominal interest rate is 4.5 percent, the inflation rate is 3.1 percent, and the growth rate of output is 2.9 percent. The natural or neutral nominal interest rate is thus 4.5 percent and the natural or neutral real rate is 1.4 percent. Using just the MC model and taking the predictions for 2008:4 to be natural values, the natural nominal interest rate is 4.2 percent and the natural real rate is 0.9 percent.

5 Optimal Control

As mentioned in Section 1, an alternative to using natural values for policy purposes is to use optimal control techniques. These require that a welfare or loss function be postulated. Consider the period 2005:1–2008:4, and let t be 2005:1. Assume that the nominal interest rate is the control variable (of the monetary authority) and that the loss function is:

$$L = \sum_{i=t}^{t+15}[0.5(\dot{P}_i - 3.0)^2 + 0.5(UR_i - 5.0)^2 + 0.1(\Delta R_i - \Delta R_i^*)^2 \\ +0.1/(R_i - 0.999) + 0.1/(16.001 - R_i)]$$

This loss function targets an inflation rate of 3 percent and an unemployment rate of 5 percent, with equal weights on the two variables. (The units of all the variables in the loss function are in percentage points, contrary to the case in Section 3, where the units are percents.) The last two terms insure that the optimal values of R will be between 1 percent and 16 percent. The middle term penalizes large changes in the interest rate, which is designed to lessen the chances of instrument instability.

ΔR_i^* is the actual change in the interest rate for period i in the base path (the first set of results in Table 2).

The loss function was minimized using the same exogenous-variable values as were used for the first two sets of results in Table 2 and also zero values for the future error terms. The general optimization method is discussed in Fair (2004, Section 1.7). The estimated interest rate rule of the Fed was dropped from the model, and the interest rate was taken to be control variable. The optimization problem is to find the 16 values of the interest rate that minimize L subject to the MC model. The results are the third set of results in Table 2. Remember that the model used for this purpose is the regular MC model, not Barred One.

Even given the middle term in the loss function, the optimal values of the interest rate near the end of the control period may be extreme because there is no tomorrow after the end of the period. It may thus be best to focus, say, on the results for 2008:1 in Table 2. For this quarter the optimal value of the interest rate is 4.53 percent, which results in values of 3.17 percent for the inflation rate and 5.36 percent for the unemployment rate. Again, these are not that different from the other two sets of values. From this exercise one would conclude that it is optimal (given the loss function) to have the nominal interest rate rise to about 4.5 percent in three years. Here, the concept of an optimal value has replaced the concept of a natural or neutral value. Since the optimal interest rate path in Table 2 is slightly higher than the path predicted by the MC model, which uses the estimated interest rate rule of the Fed, this says that the rule is predicting more expansive Fed behavior than is optimal for the given loss function above.

6 Conclusion

Frisch's procedure provides a way of modifying an estimated structural macroeconometric model to meet equilibrium conditions. The resulting model, a "barred" model, can be solved to yield natural values. The procedure has the advantage of forcing one to be clear on the additional theory used to move from the estimated model to the barred model.

If the economy is always in equilibrium, a barred model may be a better representation of the economy than is the estimated model from which it is derived. The estimated model may be seriously misspecified from failing to account for various equilibrium conditions, and if it fits better than the barred model, this may simply be because of data mining. On the other hand, if the economy is not always in equilibrium, the barred model is not likely to be a better representation because of its equilibrium nature, but it may still yield accurate estimates of natural values. Also, if the economy is not always in equilibrium, a barred model can possibly be specified to allow a gradual adjustment to equilibrium, which could make it realistic in the transition to equilibrium in addition to at equilibrium.

A competitor to a barred model is simply the estimated model itself. It may be that an estimated model captures both the short-run and long-run properties of the economy well. There is nothing in the procedure of specifying and estimating a structural macroeconometric model that rules out accurately accounting for the macro dynamics. If an estimated model accurately represents the dynamic features of the economy, then Frisch's procedure is not needed and Solow's call for a hybrid model is not necessary. Also, the concept of natural values is not needed either, since policy experiments—perhaps optimal control experiments—can be done directly using the estimated model.

The tests in Fair (2004) suggest that the MC model is a fairly good approximation of the economy, and if this is true, natural values may not be needed and one can rely only on optimal control experiments like the one in Section 5. The results in Section 4 show that the MC model and phased-in Barred One have similar properties. Given the supplementary hypothetical equations that were added for Barred One, this means that the MC model has long-run properties similar to a model in which there no excess capital and labor, no undesired inventories, and the real wage rate growing at the rate of productivity. In this sense, the MC model seems to have reasonable long-run properties.

Barred Two, on the other hand, is a different story, one where future research may be interesting. A key addition for Barred Two is the replacement of the money demand equation with equation (6)$'$. This allows (after other specification changes) the price equation (61)$'$ to tie down the real side of the economy, with the inflation rate depending only on the rate of growth of the money supply. The specification of Barred Two is, of course, related to the literature on whether money is neutral, superneutral, or neither (see, for example, Orphanides and Solow (1990) for a review). Again, in the present context Frisch's procedure has the advantage of showing what is needed to move from an estimated model like the MC model to a model in which money is neutral. In this case the changes are substantial, contrary to the changes in moving from the MC model to Barred One.

Barred Two also brings up the question of whether it can be phased in, as was done for Barred One in Section 4, or whether the equilibrium conditions must hold every period. If, for example, equation (6)′ does not hold for some period, a change in the growth rate of the money supply will not result in the same percentage point change in inflation, and so output growth will change. Money will not be neutral. Exactly how Barred Two might be specified to allow for a gradual adjustment to equilibrium is left for future research.

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