#### CHAPTER 3

# A THEORETICAL MODEL OF THE SHORT-RUN DEMAND FOR PRODUCTION WORKERS

# 3.1 Introduction

A necessary requirement of any theoretical model is that it explain to a reasonable degree of approximation empirical phenomena which are observed. One fact which has been observed so far is that the basic model introduced in ch. 2 leads to unrealistically large estimates of the production function parameter  $\alpha$ , even under the Ireland and Smyth interpretation of  $\alpha$  as a measure of short-run returns to scale. For this reason and for the others which were discussed in ch. 2, the basic model appears to be incorrectly specified.

One limitation of the previous studies of short-run employment demand is that the relationship between the number of workers employed and the number of hours worked per worker does not appear to have been carefully examined. It was seen in ch. 2 that some of the previous models are in fact inconsistent because no assumption about firms' cost-minimizing behavior with respect to the workers-hours mix was made. On the empirical side, Kuh has been the only one who has done any work at all on explaining short-run fluctuations in the number of hours worked per worker.

In this chapter some empirical evidence on short-run fluctuations in output per man hour is presented which indicates that output per man hour and output are positively correlated even at high rates of output. An explanation is then provided of why this phenomenon of increasing returns to labor is so often observed. The explanation is based on the idea that during much of the year firms hold too much labor for the amount of output produced and that during these times the observed number of hours paidfor per worker is greater than the unobserved number of hours actually worked per worker. If this is true, then the properties of the short-run production function cannot be estimated (because the true production function inputs are not observed), and in this study various properties of the short-run production function have been postulated as opposed to being estimated. After the concept of excess labor is discussed and the postulates made about the properties of the short-run production function are introduced, measurements of the amount of excess labor on hand are made for the 1947–1965 period for the seventeen industries considered in this study. The measurements are based on the assumptions made about the properties of the short-run production function.

A theoretical model of the short-run demand for workers is then developed. Basically the short-run demand for workers is taken to be a function of the amount of excess labor on hand and the time stream of expected future output changes. The distinctions among the number of workers employed, the number of hours worked per worker, and the number of hours paidfor per worker are central to the entire analysis, and the model developed in this chapter opens the way to the development of a model of the shortrun demand for hours paid-for per worker in ch. 7. The chapter concludes with a discussion of the expectational hypotheses which were tested in this study.

### 3.2 Some empirical evidence on short-run fluctuations in output per man hour

From table 2.2 in ch. 2 it can be seen that for most industries and years the percentage change from the trough month to the peak month of the year in output is substantially greater than the percentage change in total man hours. The number of man hours appears to fluctuate much less in the short run than does the amount of output produced. Since this is true and

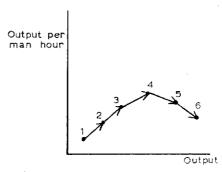


Fig. 3.1. Expected relationship between output per man hour and output for any one year.

since it is also true that the phases of the man-hours series and the output series are approximately the same, it is not too surprising that output per man hour is positively correlated with output and that estimates of increasing returns to labor services are obtained. If there is a production function which is at all observable in the short run, however, one should expect the properties of the function at least to become observable as output approaches peak rates of the year, since it is likely that there will be less slack at these high rates and thus that the production function constraint will be binding. Thus for any one year, where the stock of capital and the level of technical progress can be assumed to be fairly constant, one might expect to observe diminishing returns (or at least not

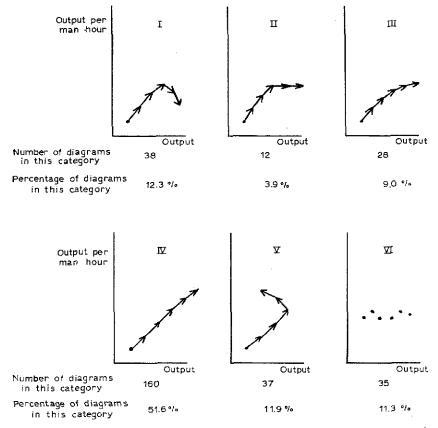


Fig. 3.2, Observed relationship between output per man hour and output for each year for each of the seventeen industries.

strongly increasing returns) to labor services at high rates of output. One thus might expect the relationship between output per man hour and output for any one year to look like that depicted in figure 3.1, provided perhaps that the year were not a recession year where even the rate of output in the peak month was low compared with past standards.<sup>1</sup> In figure 3.1 output moves from its trough in month one to its peak in month six. From month one to month four slack is being taken up as output rises, and after month four the true properties of the short-run production function are being observed.

These scatter diagrams were computed for each of the seventeen industries listed in table 2.2 for each of the years<sup>2</sup> 1947-1965. There were a total of 310 diagrams computed. Six basic types of diagrams resulted from this exercise, and they are depicted in figure 3.2. The arrows in these diagrams point in the direction of calendar time movements. If diminishing or constant returns to labor services are observed in the short run at high rates of output, then the scatter diagrams should look like those depicted in figures 3.2i, ii, and perhaps v. The number and percentage of diagrams which fell into each of the six categories are presented in figure 3.2.

Slightly over half of the diagrams (figure 3.2iv) showed no evidence that the growth in output per man hour even slowed down at high rates of output, let alone become negative. About twelve percent of the diagrams (figure 3.2i) showed a definite decline in output per man hour at high rates of output, and about twenty-five percent of the diagrams showed either a decline in output per man hour, a leveling off in output per man hour, or a slowing down in the growth rate of output per man hour (figures 3.2i, ii, and iii). Eleven percent of the diagrams (figure 3.2vi) showed a less clear-cut scatter, but perhaps can be interpreted as showing that the same output per man-hour ceiling was reached more than once during the year at different rates of output. The twelve percent of the diagrams depicted by figure 3.2v are also difficult to interpret since the time movements are odd,<sup>3</sup> but perhaps these diagrams can be interpreted as showing decreasing returns at high rates of output.

The general conclusion of this exercise is that there is some evidence that the growth in output per man hour at least slows down at high rates of output, but that for over half of the observations this is not the case and for only twelve to twenty-four percent of the cases (figure 3.2i and perhaps figure

<sup>&</sup>lt;sup>1</sup> If in fact technical progress and the stock of capital are growing smoothly over time, this will bias the scatter against a downward bend. Short-run fluctuations in output per man hour dominate the longer-run movements, however, and this bias is likely to be quite small.

<sup>&</sup>lt;sup>2</sup> A year being defined in this case as the (approximate) twelve-month period between troughs.

<sup>&</sup>lt;sup>3</sup> See the discussion in footnote 1 on page 45 for a further elaboration of this point.

3.2v) does output per man hour actually appear to decline. This seems to be rather conclusive evidence that a production function with the usual constant or diminishing returns property is only infrequently observed in the short run, even at high rates of output.

# 3.3 The notation

It was mentioned at the beginning of ch. 2 that for studies of short-run behavior it is important to make explicit the time periods to which the variables refer. For a monthly study such as this one, this can be quite important, and the theoretical model developed in this study is designed to be as consistent as possible with the available data. The Bureau of Labor Statistics data on the number of workers employed and the number of hours paid-for per worker, which are used in this study, are compiled from surveys

#### TABLE 3.1

#### Notation used in ch. 3

$Y_{2wt}$	the amount of output produced during the second week of month t.
$M_{2wt}$	the number of production workers employed during the second week of month t.
$M^{*_{2wt}}$	the number of production workers actually used during the second week of
141 2400	
	month t to produce $Y_{2wt}$ .
$H_{2wt}$	the average number of hours worked per employed worker $(M_{2wt})$ during the
	second week of month t.
$H^{*_{2wt}}$	the average number of hours worked per non-idle worker $(M^*_{2wt})$ during the
	second week of month t.
Kawt	the number of machines on hand during the second week of month t.
$K^*_{2wt}$	the number of machines actually used during the second week of month $t$ to
IN ZWU	
	produce $Y_{2wt}$ .
$H^{\kappa}_{2wt}$	the average number of hours each machine on hand $(K_{2wk})$ was used during the
	second week of month t.
$H^{*K}_{2wt}$	the average number of hours each non-idle machine $(K^*_{2wk})$ was used during
	the second week of month t.
$T_{2vot}$	the level of technical knowledge during the second week of month t.
HP <sub>2wt</sub>	the average number of hours paid-for per employed worker during the second
111 2Wt	
	week of month t.
$HS_{2wt}$	the standard number of hours of work per employed worker during the second
	week of month t.
$Y^{e_{2wt+i}}$	the amount of output expected to be produced during the second week of
	month $t+i$ (i=0,1,2), the expectation being made during the second week
	of month $t-1$ .

taken during the week which include the twelfth of the month, or approximately during the second week of each month. The second week of the month was thus taken to be the basic period considered in this study, and the variables which are considered below reflect this fact.

The symbols for the variables which are considered in this chapter, and which are to some extent considered throughout the rest of the text, are presented in table 3.1. The variables will be discussed as they are introduced below. With respect to the notation in table 3.1, for symmetry purposes the number of hours worked per employed worker should be denoted as  $H_{2wt}^M$ , instead of as  $H_{2wt}$ , but in order to simplify the notation slightly and to keep it reasonably consistent with that in ch. 2, the "M" was dropped from  $H_{2wt}^M$ . For similar reasons the "M" was also dropped from  $H_{2wt}^*$ ,  $HP_{2wt}^M$ , and  $HS_{2wt}^M$ .

One further comment regarding the variables listed in table 3.1. Except for  $M_{2wt}$  and  $HP_{2wt}$ , data on the variables listed in the table are not directly available. For the empirical work, therefore, either some empirical approximation has to be found for each of the variables or assumptions have to be made so that data on the variables are not needed. It will be seen in ch. 4, for example, that while data on  $Y_{2wt}$  are not available, data on the average daily rate of output for the month, denoted as  $Y_{dt}$ , are available, and that with a few modifications of the theoretical model  $Y_{dt}$  can be used in place of  $Y_{2wt}$  in the empirical work. For the most part in this chapter reference will be made to the variables listed in table 3.1, with discussion of the data problems being postponed until ch. 4.

### 3.4 The concept of excess labor

A theoretical model of the short-run demand for employment should provide an explanation of why increasing returns to labor are so often observed, even it appears at high rates of output. It is a major contention of this study that during much of the year firms hold too much labor for the amount of output produced, and that the observed number of hours paid-for per worker is a poor proxy for the number of hours actually worked per worker except during peak output periods. Let  $HP_{2wt}$  denote the number of hours paid-for per employed worker during the second week of month t, and let  $H_{2wt}$  denote the number of hours actually worked per employed worker during the second week of month t. When  $HP_{2wt}$  is greater than  $H_{2wt}$ , a firm can be considered to be holding too much labor in the sense that it is paying workers for more hours than they are actually working. On the other hand, during peak output periods when  $HP_{2wt}$  and  $H_{2wt}$  are likely to be equal to one another and overtime is being worked, a firm can be considered to be holding too little labor in the sense that if output were to remain at peak rates, more workers would probably be hired and fewer hours worked per worker in order to decrease high overtime costs. A measure of "excess labor on hand" should incompass both of these situations and should be positive when  $HP_{2wt}$  is greater than  $H_{2wt}$  and negative when  $HP_{2wt}$  is equal to  $H_{2wt}$  and overtime is being worked.

Let  $HS_{2wt}$  denote the standard (as opposed to overtime) number of hours of work per worker during the second week of month t. As was mentioned in § 2.3. HS may be subject to long-run trend influences and this is the reason for the time subscript. Generally,  $HS_{2wt}$  should be around 40 hours a week for most industries. Since  $HS_{2wt}$  is the dividing line between standard hours and more costly overtime hours, it can be considered to be the number of hours a firm would like each of its workers to work in the long run if there were no problems with fluctuating rates of output. In other words,  $HS_{2wt}$ can be considered to be the long-run equilibrium number of hours worked per worker. Using this concept, the measure of excess labor on hand during the second week of month t is taken to be  $\log HS_{2wt} - \log H_{2wt}$ , which is the difference between the long-run desired number of hours worked per worker and the actual number of hours worked per worker during the second week of month t.<sup>1</sup> If  $HS_{2wt}$  is greater than  $H_{2wt}$ , there is considered to be a positive amount of excess labor on hand, and if  $HS_{2wt}$  is less than  $H_{2wit}$ , there is considered to be a negative amount of excess labor on hand (i.e., too few workers on hand).

If in fact firms hold positive amounts of excess labor during at least part of the year, this provides an explanation of why estimates of increasing returns to labor have so often been obtained. The properties of the shortrun production function are not being estimated because of the slack situation which exists during much of the year, and the estimates merely show that output fluctuates more in the short run than does the number of workers employed or the number of man hours paid-for.

There are a number of reasons why firms may knowingly hold positive amounts of excess labor during part of the year. Given the large short-run fluctuations in output which occur during the year, large fluctuations in the

<sup>&</sup>lt;sup>1</sup> The functional form chosen for the model is the log-linear form, but to ease matters of exposition and where no ambiguity is involved, the difference of the logs of two variables (e.g., log  $HS_{2wt} - \log H_{2wt}$ ) will be referred to merely as the difference of the variables.

number of workers employed or in the number of hours paid-for per worker would be needed to keep  $HP_{2wt}$  always equal to  $H_{2wt}$ . Soligo<sup>1</sup> presents a comprehensive list of reasons why firms may be reluctant to allow large fluctuations in their work forces. The most important ones are: (1) Contractual commitments - such things as guaranteed annual wages, unemployment insurance compensation, severance pay, and seniority provisions where younger and perhaps more efficient workers must be laid off first. (2) Transactions costs - the size of the office space and the number of employees which must be used in the process of hiring and laying off workers will depend on the frequency and magnitude of lay-offs and rehirings. (3) Retraining costs and loss of acquired skills. (4) Morale and public relation factors - qualified workers may not be attracted to a firm which has a reputation of poor job security; large lay-offs may strain union-management relations and may affect the efficiency of the employees remaining on the job; and large lay-offs and rehirings may be harmful to its public image, which may be important to the firm. (5) Reorganization costs - large changes in the size of the work force may require considerable organizational changes, which may lower efficiency in the short run.

These reasons which Soligo lists pertain to fluctuations in the number of workers employed, but not necessarily to fluctuations in the number of hours paid-for per worker. Why do not firms allow larger fluctuations in the number of hours paid-for per worker corresponding to fluctuations in output? Here again firms may be reluctant to do this for some of the same reasons they are reluctant to allow large fluctuations in the number of workers employed, namely reasons (1) and (4) listed above. Workers may expect, for example, to be paid for a 40-hour work week, and firms may subject themselves to serious morale and public relation problems if they allowed this standard hourly work week to fluctuate very extensively.

It might be worthwhile at this point to discuss briefly how the concept of excess labor developed in this study relates to the concepts used in previous studies. The idea that firms may during any one period of time employ more workers than they actually need to produce the output of that period is, of course, not new. The lagged adjustment process (2.36) of the basic model, which is so widely used, implies that  $M_t$ , the number of workers employed, is not necessarily equal to  $M_t^d$ , the desired number of workers for the output  $Y_t$ . If  $M_t$  is greater than  $M_t^d$ , then there are, in effect, too many workers employed for the current amount of output produced. Solow, for

<sup>&</sup>lt;sup>1</sup> Soligo (1966, pp. 174-175).

example, uses the term "labor-hoarding" "as a catch-phrase to stand for all the frictions involved in meeting transitory variations in output with variations in employment".<sup>1</sup>

What is not clear in much of the previous work is what happens to hours paid-for per worker during the phases of adjustment. If the labor input variable in the production function is taken to be man hours, then an  $M_t$ greater than  $M_t^d$  need not imply any "man hours paid-for hoarding" if the number of hours paid-for per worker is reduced sufficiently. In the previous studies this aspect of the short-run adjustment process has not been carefully examined.

Ball and St Cyr, working not within the context of a lagged adjustment model, but with the production function directly, do postulate that measured man hours  $(M_tH_t)_m$  may differ from "productive man hours".<sup>2</sup> Specifically, they postulated (2.17), which is repeated here:

$$M_t H_t = (M_t H_t)_m (1 - U_t)^{\mu}.$$
(2.17)

 $U_t$  is a measure of labor market tightness. Using (2.17), they estimated the parameters of a Cobb-Douglas production function directly, assuming no lagged adjustment process, but assuming that true labor services differ from measured labor services in the manner depicted by (2.17). As stated in § 2.2.3, Ball and St Cyr remain agnostic as to whether this model or the lagged adjustment model is more realistic. The postulate made in this study that the number of hours paid-for per worker does not necessarily equal the number of hours actually worked per worker is essentially the same as Ball and St Cyr's postulate that measured man hours may differ from productive man hours. Ball and St Cyr, however, did not follow up their idea beyond specifying (2.17) and attempting to estimate the parameters of their Cobb-Douglas production function directly.

Since the number of hours actually worked per worker,  $H_{2wt}$ , is not observed except during peak output periods, where it probably equals  $HP_{2wt}$ , the amount of excess labor on hand,  $\log HS_{2wt} - \log H_{2wt}$ , cannot be computed directly, and some approximation to it must be found. In order for this to be done, however, more information is needed on the proporties of the short-run production function, and this is the subject of the next section.

<sup>&</sup>lt;sup>1</sup> Solow (1964, p. 8).

<sup>&</sup>lt;sup>2</sup> See the discussion in § 2.2.3.

#### 3.5 The short-run production function

Most of the studies discussed in ch. 2 postulated some kind of a short-run production function, the parameters of which were usually estimated by an equation similar to (2.37) of the basic model. A short-run production function is also postulated in the study, but it is assumed that the parameters of this function cannot be estimated in the usual fashion with the data which are available.

Notice from table 3.1 that a distinction is made between the number of workers employed during the second week of month t,  $M_{2wt}$ , and the number of workers actually used in the production process during the second week of month t,  $M_{2wt}^*$ . In other words, the possibility is allowed for that some workers may be completely idle during the period and contribute nothing toward the production of the period. The same possibility is allowed for with respect to the stock of capital: some machines may be completely idle during the period. It should also be noticed from the table that, by definition,  $M_{2wt}H_{2wt}$  equals  $M_{2wt}^*H_{2wt}^*$ , since both of these variables equal the total number of man hours worked during the second week of month t. The difference between  $H_{2wt}$  and  $H_{2wt}^*$  is merely whether in computing the average number of hours worked per worker the average is taken over all of the employed workers or only over the non-idle workers. For similar reasons,  $K_{2wt}H_{2wt}^K$  equals  $K_{2wt}^*H_{2wt}^{*K}$  in table 3.1.

The short-run production function is postulated to be:

$$Y_{2wt} = F(M_{2wt}^* H_{2wt}^*, K_{2wt}^* H_{2wt}^{*K}, T_{2wt}).$$
(3.1)

 $Y_{2wt}$  is the amount of output produced during the second week of month t,  $M_{2wt}^*H_{2wt}^*$  is the number of production worker hours used during the second week of month t to produce  $Y_{2wt}$ ,  $K_{2wt}^*H_{2wt}^{*K}$  is the number of machine hours used, and  $T_{2wt}$  is the level of technical knowledge during the second week of month t.

 $M_{2wt}^*$  by itself denotes the number of production workers used in the production process during the second week of month t, and  $H_{2wt}^*$  by itself denotes the average number of hours worked per non-idle worker during the second week of month t. Likewise,  $K_{2wt}^*$  by itself denotes the number of machines used during the second week of month t, and  $H_{2wt}^{*K}$  by itself denotes the average number of hours each of these non-idle machines was utilized during the second week of month t. The total number of production worker hours is thus taken to be the labor services variable in the production

function (3.1), and the total number of machine hours is taken to be the capital services variable.

Depending on the industry breakdown and the country, time series data are usually available on the number of workers employed. Time series estimates of the stock of capital are sometimes available as well, but data are seldom available on the utilization of the capital stock, especially for any kind of a detailed industry breakdown. For the United States, at least, rather detailed industry data are also available on the number of hours paid-for per production worker. For those studies described in ch. 2 which used an hours variable at all in their empirical work, the hours variable used was an hours paid-for variable. If the number of hours paid-for per worker is a poor estimate of the number of hours actually worked per worker, as is contended in this study, then good data on the number of hours worked per worker are not available; and since data on machine utilization are usually not available either, this means that the properties of the short-run production function (3.1) cannot be estimated using available data, even if the employment adjustment process can be correctly specified.

The approach taken in this study is to postulate certain properties about the short-run production function and to develop a theory of the short-run demand for workers using these postulates. Consequently, only indirect tests of the validity of these postulates will be available, which are the tests of how well the over-all model performs.

The first postulate relates to the short-run substitution possibilities between labor services and capital services. In the production function (3.1) the amount of labor services used,  $M_{2wl}^*H_{2wl}^*$ , can be changed either by changing the number of workers used,  $M_{2wt}^*$ , or by changing the number of hours worked per worker,  $H_{2wr}^*$ . In like manner, the amount of capital services used,  $K_{2wt}^* H_{2wt}^{*\kappa}$ , can be changed either by changing the number of machines used,  $K_{2wt}^*$ , or by changing the number of hours each machine is utilized,  $H_{2wt}^{*K}$ . Because of the different ways in which labor services and capital services can be changed, one must be careful when discussing substitution possibilities between capital services and labor services to specify exactly what he means. For example, increasing labor services by increasing the number of hours worked per worker and keeping the number of workers used constant need not require any additional machines used, since the existing machines can just be utilized more hours. Increasing labor services by increasing the number of workers used and keeping the number of hours worked per worker constant, however, is a different matter. Either the new workers hired work with the old workers on the same number of

machines, or the new workers hired work on machines which have previously been idle.

It is postulated in this study that the short-run production process is of such a nature that a fixed number of workers is required per machine. If the worker-machine ratio is greater than this number, it is assumed that no additional output can be produced, and if the ratio is smaller than this number, it is assumed that no output can be produced at all. This assumption implies that when new workers are hired, they work on previously idle machines.<sup>1</sup> Another implication of the assumption is that the average number of hours worked per non-idle worker,  $H_{2wt}^*$ , and the average number of hours each non-idle machine is utilized,  $H_{2wt}^{*K}$ , are the same. Machines, for example, cannot be run eight hours a day and have workers working on them only six hours a day.

The short-run production function is thus postulated to be (ignoring for the moment technical progress and the possibility of non-constant shortrun returns to scale):

$$Y_{2wt}/H_{2wt}^{*} = \min \left\{ \alpha M_{2wt}^{*}, \beta K_{2wt}^{*} \right\}.$$
 (3.2)

 $Y_{2wt}/H_{2wt}^*$  in eq. (3.2) is the amount of output produced per hour by the non-idle workers and machines.

This postulate of no short-run "substitution possibilities" between workers and machines may not be an unreasonable approximation of reality, but no direct empirical evidence is given here to confirm it. The postulate would be difficult to verify directly without a detailed examination of each production process, an examination which has not been undertaken here. It will be seen later to what extent the model developed in this study depends on this postulate.

The second postulate made about the properties of the short-run production function relates to the degree of increasing or decreasing short-run returns to scale. In (3.2) it is implicitly assumed that there are constant returns to scale. If it is assumed that there are short-run returns to scale of

<sup>&</sup>lt;sup>1</sup> This last statement is not quite true if the possibility of second and third shift work is allowed for. If there is more than one shift, then new workers hired need not work on previously idle machines if they work on a second or third shift; the same machines can be used on all three shifts. For present purposes the distinction between first and second or third shift work can largely be ignored by considering any machine which is used on, say, two shifts as two different machines. The number of physically different machines used in the production process may thus be less than  $K^*_{2wt}$  above.

$$Y_{2wt}/H_{2wt}^{*} = \min\left\{ \alpha M_{2wt}^{*\eta}, \beta K_{2wt}^{*\eta} \right\}.$$
(3.3)

If  $\eta$  is greater than one in (3.3), for example, then there are increasing returns to scale.

The way (3.3) is specified, there are constant returns to scale for changes in the number of hours worked per worker and machine,  $H_{2wt}^*$ . If it is assumed that there are also short-run returns to scale (of size v) with respect to the number of hours worked per worker and machine, then (3.3) becomes

$$Y_{2wt}/H_{2wt}^{*_{\nu}} = \min \left\{ \alpha M_{2wt}^{*_{\eta}}, \beta K_{2wt}^{*_{\eta}} \right\},$$
(3.4)

or

$$Y_{2wt} = \min \left\{ \alpha M_{2wt}^{*\eta} H_{2wt}^{*\nu}, \beta K_{2wt}^{*\eta} H_{2wt}^{*\nu} \right\}.$$
(3.5)

 $\eta$  and v in eq. (3.5), of course, do not necessarily have to be equal.

It is possible that empirical evidence on the extent of increasing or decreasing short-run returns to scale can be gleaned from the scatter diagrams discussed in § 3.2. In these diagrams the variable actually plotted against output was not output per worked man hour but output per paid-for man hour. If it is assumed that there are no completely idle workers so that  $M_{2wt}^*$  equals  $M_{2wt}$  and  $H_{2wt}^*$  equals  $H_{2wt}$ , and if it is assumed that there are constant short-run returns to scale both for changes in the number of workers employed and for changes in the number of hours worked per worker so that  $\eta$  and  $\nu$  are both equal to one in (3.5), then the scatter diagrams should look like the one depicted in figure 3.3.

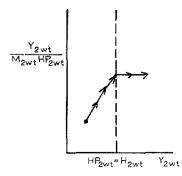


Fig. 3.3. Expected relationship between output per paid-for man hour and output on the assumption of constant returns to scale.

Up to the point where the number of hours paid-for per worker,  $HP_{2wt}$ , equals the number of hours actually worked per worker,  $H_{2wt}$ , one should observe an increasing output per paid-for man hour,  $Y_{2wt}/M_{2wt}HP_{2wt}$ , as output increases, because, while the number of hours worked per worker needs to increase when output increases, the number of hours paid-for per worker needs to increase much less, if at all. At the point where the number of hours paid-for per worker the production function constraint becomes binding on the number of hours paid-for per worker, and the scatter beyond this point should reveal properties about the production function, such as the returns to scale property.<sup>1</sup>

If there are constant returns to scale, then beyond the point where the number of hours paid-for per worker equals the number of hours worked per worker the scatter should be on a horizontal line, as in figure 3.3. If there are increasing returns to scale so that  $\eta$  and v in (3.5) are greater than one, the scatter should lie on an upward sloping line, and if there are decreasing returns to scale so that  $\eta$  and v are less than one, the scatter should lie on a downward sloping line. If  $\eta$  is less than one and v is greater than one or vice versa, the scatter can, of course, lie either on a upward or downward sloping line depending on the size of  $\eta$  and v and on the size of the short-run fluctuations in the number of workers employed and the number of hours worked per worker.

The results of the scatter diagrams were given in figure 3.2 and were discussed in § 3.2 above. It is the author's general impression that there is not enough evidence from these results to determine which is the most realistic assumption about short-run returns to scale to make. The main reason for this is that it is difficult to know where the point where the number of hours paid-for per worker equals the number of hours worked per

<sup>&</sup>lt;sup>1</sup> One should at least expect this to be true for a continually increasing output. For a decrease in output, even from a high level, it is difficult to know whether the number of hours paid-for per worker decreases as much as the number of hours actually worked per worker during the period, or whether the number of hours paid-for per worker is adjusted downward with a lag. For a continually increasing output the problem is likely to be less serious, since at points beyond the point where the number of hours paid-for per worker equals the number of hours worked per worker, the number of hours paid-for per worker must increase at least as fast as the number of hours worked per worker and is probably not likely to increase much faster. This is the reason why attention was concentrated in § 3.2 on the points of the scatter diagrams where output was increasing and why diagrams like figure 3.2v were difficult to interpret.

worker begins, and it may be that in many cases the point is reached only at the peak level of output for the year, so that no scatter is observed beyond this point. The results would tend to confirm the assumption of increasing short-run returns to scale if one had good reason for believing that scatter was actually observed beyond the point where the number of hours paid-for per worker equals the number of hours worked per worker, but since there is little evidence for believing this, the results are actually of little help.

The postulate made in this study is that both  $\eta$  and  $\nu$  in (3.5) are equal to one. In other words, it is assumed that the short-run production function is one of constant returns to scale, both with respect to changes in the number of workers and machines used and with respect to changes in the number of hours worked per worker and machine.

In eq. (3.5) it is implicitly assumed that all workers and all machines are of the same efficiency. In reality workers and machines are likely to differ in efficiency, and if workers are hired and machines utilized in order of their efficiency, this will have an effect similar to the existence of decreasing short-run returns to scale with respect to the number of workers and machines used (i.e., to  $\eta$  being less than one). Likewise, if the efficiency of workers decreases the more hours they work per week (due to such things as fatigue, boredom, etc.), this will have an effect similar to the existence of decreasing short-run returns to scale with respect to the number of hours worked per worker and machine per week (i.e., to v being less than one). It thus appears a priori that the assumption of  $\eta$  and v being less than one is more realistic than the assumption that they are equal to one or are greater than one. It was felt here, however, that it was better to make the assumption of constant returns to scale ( $\eta$  and v both equal to one) than to arbitrarily specify a certain degree of decreasing returns to scale. It will be shown later to what extent the model developed in this study depends on this assumption.

The assumption that both  $\eta$  and  $\nu$  are equal to one implies that there is no difference in the effect on output whether labor services are changed by changing the number of workers used or by changing the number of hours worked per worker. If  $M_{2wt}^*H_{2wt}^*$  is increased by, say, ten percent, then output will be increased by ten percent also, and it does not matter whether the ten percent increase in  $M_{2wt}^*H_{2wt}^*$  comes from increasing  $M_{2wt}^*$  or  $H_{2wt}^*$ or some combination of the two.

Except for the possibility of the existence of technical progress, the shortrun production function is thus postulated to be as in eq. (3.2): no short-run substitution possibilities between workers and machines and constant shortrun returns to scale both with respect to changes in the number of workers and machines used and with respect to changes in the number of hours worked per worker and machine per period. The assumption which is made about technical progress will be discussed in the next section.

### 3.6 The measurement of excess labor

Under the assumptions just made about the properties of the short-run production function, estimates of the amount excess labor on hand can be made. Estimates of the amount of excess labor on hand for each of the seventeen industries considered in this study were made in the following manner. For each of the industries, output per paid-for man hour,  $Y_{2wi}/M_{2wi}HP_{2wt}$ , was plotted monthly for the 1947–1965 period.<sup>1</sup> These points were then interpolated from peak to the next higher peak and so on for the nineteen-year period. The peaks in this output per paid-for man-hour series occurred at the corresponding peaks in the output series in most cases, as is implied by the results of the scatter diagrams above. Many yearly peaks were lower than the peaks of the previous years, and these were not used in the interpolations. For the beginning of each period the interpolation line was taken to be a horizontal line from the first month to the first peak, and for the end of each period the interpolation line was taken to be a horizontal line from the last month.

The "cyclical" movements in output per paid-for man hour were quite noticeable for most industries, corresponding roughly to the cyclical movements in output. The long-run trend in output per paid-for man hour was upward for nearly all industries. The procedure of going from peak to next higher peak was not strictly adhered to in every case. For a small fraction of the cases a particular peak seemed to be high relative to both past and future values, and these peaks were not used as interpolation peaks. In other words, an effort was made to smooth out the interpolation lines as much as seemed warranted by the nature of the plots, and a few peaks were rejected as aberrations in the basic data. The over-all procedure of choosing the peaks is, of course, a highly subjective one, although in most cases the choices were fairly

<sup>&</sup>lt;sup>1</sup> All data were seasonally unadjusted. See ch. 4 and the data appendix for a discussion of the data. It should be pointed out here that the output series actually used was not the (unobserved) series on the amount of output produced during the second week of the month, but the (observed) series on the average daily rate of output for the month. As discussed in ch. 4, this approximation should be reasonably good in most cases.

unambiguous. In the data appendix the months which were used as the peaks for each of the seventeen industries are presented.

The following two assumptions are made regarding these interpolations. The first assumption which is made is the assumption that at the interpolation peaks there are no completely idle workers, so that, in the notation of table 3.1,  $M_{2wt}$  equals  $M_{2wt}^*$ . As will be discussed in ch. 4, the BLS data on  $M_{2wt}$ include workers who are on vacation and on paid sick leave, and so this assumption that there are no completely idle workers at the peaks is not likely to be completely true. The assumption may not be too unrealistic, however, especially considering the fact that vacations are less likely to be scheduled during the peak output months of the year<sup>1</sup> than during the more slack months. If  $M_{2wt}$  equals  $M^*_{2wt}$ , then the number of hours worked per employed worker,  $H_{2wt}$ , and the number worked per non-idle worker,  $H_{2wt}^*$ , are the same. At the peaks, then, no distinction needs to be made between employed and non-idle workers. The second assumption which is made is the assumption that at the interpolation peaks the number of hours paidfor per worker equals the number of hours actually worked per worker, i.e., that  $HP_{2wt}$  equals  $H_{2wt}$  at the peaks. In other words, it is assumed that at the peaks firms are not paying workers for any more hours than they are actually working. This assumption appears to be fairly realistic, since it seems unlikely that firms will behave in such a way that they end up paying workers for unworked hours even at peak output rates.

These two assumptions imply that at the peaks the unobserved output per worked man hour,  $Y_{2wt}/M_{2wt}^*H_{2wt}^*$ , equals the observed output per paidfor man hour,  $Y_{2wt}/M_{2wt}/H_{2wt}$ . Therefore, from the postulated production function (3.2), an estimate of the parameter  $\alpha$  is available at each of the peaks. The final assumption which is made is the assumption that  $\alpha$  is a function of time and varies smoothly along the interpolation lines from peak to peak. In this way technical progress is introduced into the production function (3.2). This assumption is not equivalent to assuming that  $\alpha$  grows smoothly throughout the sample period, because the interpolation lines in general had kinks in them at the peaks. From these assumptions and the interpolation results, then, estimates of  $\alpha$  are available for each month throughout the sample period.

Notice that the assumptions of no short-run substitution possibilities between workers and machines and of constant returns to scale made in

<sup>&</sup>lt;sup>1</sup> Remember that the peaks in the output per paid-for man-hour series usually corresponded to the peaks in the output series.

§ 3.5 are necessary for the above procedure to be valid. Otherwise, the estimates of  $\alpha$  obtained above could not be considered to be estimates of a parameter of the short-run production function. The accuracy of the estimates of  $\alpha$  also depends on the assumption that at the peaks used in the interpolations  $M_{2wt}$  equals  $M_{2wt}^*$  and  $HP_{2wt}$  equals  $H_{2wt}^*$  and on the assumption that  $\alpha$  moves smoothly through time from peak to peak.

From the production function (3.2)

$$M_{2wt}^* H_{2wt}^* = \frac{1}{\alpha_{2wt}} Y_{2wt},$$
(3.6)

where the time subscript has been added to  $\alpha$  to indicate that it varies through time. Given the estimates of  $\alpha_{2wt}$  constructed above and the output data, estimates of  $M_{2wt}^*H_{2wt}^*$  can be constructed from eq. (3.6).  $M_{2wt}^*H_{2wt}^*$  is then the estimate of the number of man hours actually required to produce  $Y_{2wt}$ . When  $M_{2wt}^*H_{2wt}^*$  is divided by  $HS_{2wt}$ , the standard or long-run equilibrium number of hours of work per worker, the result, denoted as  $M_{2wt}^d$ , can be considered to be the desired number of workers employed for the second week of month t:

$$M_{2wt}^{d} = M_{2wt}^{*} H_{2wt}^{*} / HS_{2wt}.$$
(3.7)

 $M_{2wt}^d$  is the desired number of workers employed in the sense that if manhour requirements were to remain at the level  $M_{2wt}^*H_{2wt}^*$ ,  $M_{2wt}^d$  can be considered to be the number of workers the firm would want to employ in the long run. In the long run each worker would then be working the standard number of hours per week.

Using  $M_{2wt}^d$ , the amount of (positive or negative) excess labor on hand is taken to be  $\log M_{2wt} - \log M_{2wt}^d$ , which is the difference between the actual number of workers employed and the desired number. In the discussion in § 3.4 the amount of excess labor on hand was defined to be  $\log HS_{2wt} - \log H_{2wt}$ , which is the difference between the standard number of hours of work per worker and the actual number of hours worked per worker. It is easy to show that this measure and the measure just constructed are the same. Using (3.7) and remembering that  $M_{2wt}H_{2wt}$  equals  $M_{2wt}^*H_{2wt}^*$ , it follows that:

$$\log M_{2wt} - \log M_{2wt}^d = \log M_{2wt} - (\log M_{2wt}^* H_{2wt}^* - \log HS_{2wt})$$
  
= log  $M_{2wt} - \log M_{2wt}^* H_{2wt}^* + \log HS_{2wt}$   
+ log  $H_{2wt} - \log H_{2wt}$ 

#### MODEL OF SHORT-RUN DEMAND FOR PRODUCTION WORKERS

$$= \log M_{2wt} H_{2wt} - \log M_{2wt}^* H_{2wt}^* + \log HS_{2wt} - \log H_{2wt} = \log HS_{2wt} - \log H_{2wt}.$$
(3.8)

In the rest of the text, therefore, these two expressions for excess labor will be used interchangeably. What (3.8) says is that the amount of excess labor on hand can be looked upon either as the difference between the number of workers employed and the desired number employed or as the difference between the standard number of hours of work per worker and the actual number of hours worked per worker.

## 3.7 The short-run demand for production workers

The model developed here of the short-run demand for production workers is not rigorous in that the employment behavior of firms is not derived from the minimization of a particular short-run cost function. One possible justification of not basing the model on the minimization of a short-run cost function is that the behavior of firms may be sufficiently complex that it cannot accurately be described in terms of the minimization of a simple analytic cost function. The model of Holt, Modigliani, Muth, and Simon, for example, is based on the minimization of a cost function, and it is seen in ch. 6 that one of the approximations which is made in order to enable this to be done is unrealistic and leads to rather poor empirical results for the over-all model. For the model developed in this study there is certainly some kind of a complex cost function in the background, the minimization of which implies the behavior postulated by the model, but the behavior pestulated below is sufficiently complex that it is doubtful whether this underlying cost function could be easily derived.

 $M_{2wt}$  denotes the number of production workers on the payroll of the firm during the second week of month t. The problem is to explain the short-run fluctuations in log  $M_{2wt} - \log M_{2wt-1}$ , the (logarithmic) change in the number of production workers employed from the second week of month t - 1 to the second week of month t. In the model developed here production decisions are assumed to be made before employment decisions and are assumed not to be influenced by the number of workers on hand. A one-way causality is thus postulated from decisions on production to decisions are taken to be "exogenous" with respect to employment decisions.

It was mentioned above that large and rapid adjustments in the work force of a firm are likely to be costly (from the point of view of actual costs as well as of worker morale), and firms are likely to attempt to smooth out their work force fluctuations. If output is expected to increase over the next few months, firms may be reluctant to lay off workers they do not actually need at present, and they may begin to build up their work force in anticipation of higher future man-hour requirements. Conversely, if output is expected to decrease over the next few months, firms may be less reluctant to lay off workers, and they certainly have no need to build up their work force any further. Therefore, the expected current change in output (log  $Y_{2wt}^e - \log Y_{2wt-1}$ ) as well as expected future changes in output  $(\log Y_{2wt+i}^e - \log Y_{2wt+i-1}^e, i = 1, 2, ..., n)$  are likely to be significant factors in the determination of log  $M_{2wt} - \log M_{2wt-1}$ .  $(Y^e_{2wt+i})$  is the amount of output expected to be produced during the second week of month t + i, all expectations being made during the second week of month t - 1.) In other words, because of adjustment costs, the time stream of expected future changes in man-hour requirements (and thus, roughly, of expected future changes in output) is likely to be a significant factor affecting shortrun employment decisions.

The amount of excess labor on hand during the second week of month t - 1 would also be expected to have an effect on a firm's employment decisions. One would expect that, other things being equal, the larger the amount of positive excess labor on hand during the second week of month t - 1, the larger would be the number of workers who would be laid off during the monthly decision period. Holding positive amounts of excess labor is costly, and the firm can be considered in the short run to be continuously trying to eliminate this excess labor in the light of such things as worker morale problems and other adjustment costs. Conversely, if there is negative excess labor on hand (too few workers employed), the firm can be considered to be constantly trying to add workers to achieve a zero amount of excess labor.

The amount of excess labor on hand during the second week of month t-1 is measured as  $\log M_{2wt-1} - \log M_{2wt-1}^d$  and was constructed in the manner described above. Regarding the measurement of the amount of excess labor on hand, there is another set of variables which is worth considering as well. Since the number of man hours paid-for fluctuates much less than output in the short run and thus less than man-hour requirements, the past changes in output,  $\log Y_{2wt-i} - \log Y_{2wt-i-1}$  (i = 1, 2, ..., m), may be useful proxies for the amount of excess labor on hand in the

sense that if output has been declining in the past, there should be more excess labor on hand than if output has been rising in the past.<sup>1</sup> Of course, log  $M_{2wt-1} - \log M_{2wt-1}^d$  and the log  $Y_{2wt-i} - \log Y_{2wt-i-1}$  variables will be highly correlated, and to the extent that the assumptions made above are true, log  $M_{2wt-1} - \log M_{2wt-1}^d$  is the better measure of excess labor on hand.

It is not inconceivable, however, that both  $\log M_{2wt-1} - \log M_{2wt-1}^d$ and the past changes in output are significant in the determination of  $\log M_{2wt} - \log M_{2wt-1}$ . Even though the variables  $\log Y_{2wt-i} - \log Y_{2wt-i-1}$ (i = 1, 2, ..., m) are measuring part of  $\log M_{2wt-1} - \log M_{2wt-1}^d$ , the reaction of the firm to the two types of variables may be sufficiently different to make both types of variables significant. Even if it is assumed that  $\log M_{2wt-1} - \log M_{2wt-1}^d$  is a perfect measure of the amount of excess labor on hand and that a firm reacts in a specified way to this variable, the firm still may react more strongly (weakly) in eliminating this excess labor when the increase (decrease) in part of the excess labor comes in the immediate past month or two. In other words, the past two or three months' activities may have a stronger effect on a firm's employment decision than effects which have been cumulating over a longer period of time.

In the development of the model some assumption has to be made regarding the influence of wage rate fluctuations on the short-run demand for workers. As mentioned in § 2.3, there are two different kinds of short-run cost-minimizing assumptions which can be made – one concerned with the optimal short-run workers-hours worked per worker mix and the other concerned with the optimal short-run capital services-labor services mix. DHRYMES (1967) has been the only one who has been concerned with this second assumption.

If there are no short-run substitution possibilities between the number of workers and machines, short-run changes in the wage rate can have no effect on the short-run worker-machine ratio. Since a firm is assumed to hold positive and negative amounts of excess labor during much of the year, however, a change in the wage rate will change the cost of holding this excess labor. If the wage rate rises, for example, and if adjustments costs do not increase proportionately with the wage rate, a firm may decide to hold less excess labor, other things being equal, because of the increased relative cost of holding this labor. Thus, an increase in the wage rate may

[3.7

<sup>&</sup>lt;sup>1</sup>  $Y_{2wt-i}$  is the actual amount of output produced during the second week of month t-i.

have a negative effect on the change in employment, and a decrease in the wage rate a positive effect.

In the model developed here it is assumed that the short-run employment decisions of firms are not significantly affected by short-run wage rate changes.<sup>1</sup> This assumption does not appear too unreasonable, especially considering the fact that short-run wage rate fluctuations are likely to be rather small and that adjustment costs may increase nearly proportionately with the wage rate.

The long-run effects of the growth of technology on the number of production workers employed have already been accounted for in the construction of  $M_{2wt}^d$ . If  $\alpha$  in the production function (3.2) is increasing over time due to the growth of technology, then, other things being equal,  $M_{2wt}^d$  in eq. (3.7) will be falling, since man-hour requirements,  $M_{2wt}^*H_{2wt}^*$ , will be falling. The amount of excess labor on hand will thus be increasing. In the model developed here, therefore, the effects of the growth of technology on short-run employment decisions are taken care of by the firm's reaction to the amount of excess labor on hand.

The following equation is thus taken to be the basic equation determining  $\log M_{2wt} - \log M_{2wt-1}$ :

$$\log M_{2wt} - \log M_{2wt-1} = \alpha_1 (\log M_{2wt-1} - \log M_{2wt-1}^{e}) + \sum_{i=1}^{m} \beta_i (\log Y_{2wt-i} - \log Y_{2wt-i-1}) + \gamma_0 (\log Y_{2wt}^{e} - \log Y_{2wt-1}) + \sum_{i=1}^{n} \gamma_i (\log Y_{2wt+i}^{e} - \log Y_{2wt+i-1}^{e}).$$
(3.9)

In eq. (3.9)  $\alpha_1$  is the partial "reaction coefficient" to the amount of (positive or negative) excess labor on hand, and it is expected to be negative. The reasons for the inclusion of the various output variables in eq. (3.9) have been discussed above. One would expect that the  $\beta_i$  coefficients would decrease as *i* increases (the more distant the change in output the smaller the effect on current behavior) and that the  $\gamma_i$  coefficients would decrease

<sup>&</sup>lt;sup>1</sup> It would be better, of course, to test this assumption, but unfortunately data on standard hourly wage *rates* (as opposed to average hourly earnings, which reflect overtime earnings as well) are not available.

as *i* increases (the further in the future the expected change in output the smaller the effect on current behavior), with  $\gamma_0$  being the largest of the coefficients.

With respect to the excess labor variable in eq. (3.9),  $\log M_{2wt-1}^d$  is defined from eq. (3.7) to be

$$\log M_{2wt-1}^d = \log M_{2wt-1}^* H_{2wt-1}^* - \log HS_{2wt-1}.$$
(3.10)

The variable  $M_{2wt-1}^*H_{2wt-1}^*$  was constructed in the manner described in § 3.6, but as yet no assumption has been made regarding  $HS_{2wt-1}$ , the standard number of hours of work per worker for the second week of month t-1. The following assumption is made. It is assumed that HS is either a constant or a smoothly trending variable, and specifically that

$$HS_{2wt-1} = \overline{H}e^{\mu t},\tag{3.11}$$

where  $\overline{H}$  and  $\mu$  are constants. On this assumption log  $HS_{2wt-1}$  in eq. (3.10) equals log  $\overline{H} + \mu t$ , and so the excess labor variable in eq. (3.9) can be written

$$\alpha_1(\log M_{2wt-1} - \log M_{2wt-1}^d) = \alpha_1(\log M_{2wt-1} - \log M_{2wt-1}^* H_{2wt-1}^*) + \alpha_1 \log \overline{H} + \alpha_1 \mu t. \quad (3.12)$$

This introduces a constant term and a time trend in eq. (3.9).

There may also be an additional factor in the constant term of eq. (3.9) besides  $\alpha_1 \log \overline{H}$ . The specification of eq. (3.9) implies that the desired amount of excess labor on hand is zero. It may be, however, that a firm desires to hold a certain positive amount of excess labor at all times as insurance against, say, a sudden unexpected increase in demand or a sudden increase in absenteeism. If  $\log \overline{E}$  denotes this desired amount of excess labor, then the excess labor term in eq. (3.9) should be  $\alpha_1(\log M_{2wt-1} - \log M_{2wt-1}^d - \log \overline{E})$ , which adds the (constant) term  $-\alpha_1 \log \overline{E}$  to the equation. The possibility that  $\log \overline{E}$  is greater than zero will be ignored in the discussion which follows, but it should be kept in mind in the interpretation of the estimate of the constant term of eq. (3.9).<sup>1</sup>

Eq. (3.9) is not yet in estimatable form because the expected output

<sup>&</sup>lt;sup>1</sup> It should also be pointed out here that if the assumption made above that at the interpolation peaks output per paid-for man hour equals output per worked man hour is wrong, but that the percentage difference between output per paid-for man hour and output per worked man hour is the same at each one of the peaks, then this error will be absorbed in the constant term in eq. (3.9).

variables are not directly observed, and so in order for it to be estimated some assumption has to be made as to how expectations are formed. The expectational hypotheses which were tested in this study will be discussed in the next section.

### 3.8 The expectational hypotheses

Three basic expectational hypotheses were tested in this study. The first hypothesis which was tested is the hypothesis that expectations are perfect. In other words, the hypothesis states that

$$\log Y_{2wt+i}^{e} = \log Y_{2wt+i}, \quad i = 0, 1, ..., n.$$
(3.13)

The second hypothesis which was tested is the hypothesis that

$$\log Y_{2wt+i}^e = \log Y_{2wt+i-12} + \lambda_i (\log Y_{2wt-1} - \log Y_{2wt-13}),$$
  

$$i = 0, 1, ..., n.$$
(3.14)

What this hypothesis says is that firms during the second week of month t - 1 expect the amount of output to be produced during the second week of month t + i to be equal to the amount of output produced during the second week of the same month last year, plus a factor to take into account whether output has been increasing or decreasing in the current year over the previous year,  $\log Y_{2wt-1} - \log Y_{2wt-13}$ . If, for example, output has been increasing in the sense that  $\log Y_{2wt-1} - \log Y_{2wt-13}$  is positive, firms expect log  $Y_{2wt+i} - \log Y_{2wt+i-12}$  to be positive by a certain percentage, the expected percentage being based on the percentage increase of the past month. Similarly, if output has been declining,  $\log Y_{2wt+i} - \log Y_{2wt+i-12}$  is expected to be negative. The  $\lambda_i$  coefficients may conceivably be different for different *i*, since as the output to be predicted moves into the future, firms may put less reliance on immediate past behavior.

The third expectational hypothesis which was tested in this study is a combination of the first two. Specifically, it assumes that the hypothesis of perfect expectations holds for  $Y_{2wt}^e$  and that the second hypothesis holds for  $Y_{2wt+i}^e$ , i = 1, 2, ..., n. It seems likely that firms will have a rather good idea of the amount of output they are going to produce in the forthcoming month, but a less clear-cut idea for more distant periods. If in fact employment decisions are made on less than a monthly basis, the hypothesis of perfect expectations for the current month appears quite reasonable, since presumably the number of workers employed will be adjusted throughout the month as the amount of output produced changes.

The method which was used to test these hypotheses is as follows. For each expectational hypothesis the implied value of each  $Y_{2wt+i}^e$  was substituted into eq. (3.9), and the equation was estimated for each of the seventeen industries. The three equations for each industry were then compared with respect to the goodness of fit criterion and with respect to the significance of the  $\gamma_i$  coefficients, and that hypothesis was chosen for each industry which yielded the best results. Because the  $Y_{2wt+i}^e$  variables cannot be directly observed, only this indirect test of the three hypotheses is available. The validity of this test, of course, depends on the assumption that eq. (3.9) is specified correctly to begin with.

For the perfect expectational hypothesis the actual future values of output are used as measures of the expected future values. Under the second expectational hypothesis, the expectational part of eq. (3.9) becomes (assuming *n* to be three):

$$\gamma_{0}(\log Y_{2wt}^{e} - \log Y_{2wt-1}) + \sum_{i=1}^{3} \gamma_{i}(\log Y_{2wt+i}^{e} - \log Y_{2wt+i-1}^{e})$$

$$= \gamma_{0}(\log Y_{2wt-12} - \log Y_{2wt-1}) + \gamma_{1}(\log Y_{2wt-11} - \log Y_{2wt-12})$$

$$+ \gamma_{2}(\log Y_{2wt-10} - \log Y_{2wt-11}) + \gamma_{3}(\log Y_{2wt-9} - \log Y_{2wt-10})$$

$$+ (\gamma_{0}\lambda_{0} + \gamma_{1}\lambda_{1} - \gamma_{1}\lambda_{0} + \gamma_{2}\lambda_{2} - \gamma_{2}\lambda_{1} + \gamma_{3}\lambda_{3} - \gamma_{3}\lambda_{2})(\log Y_{2wt-1})$$

$$- \log Y_{2wt-13}). \quad (3.15)$$

For this second expectational hypothesis, if all of the  $\lambda_i$  coefficients are equal (to, say,  $\lambda$ ), then the coefficient of log  $Y_{2wt-1} - \log Y_{2wt-13}$  becomes  $\gamma_0 \lambda$ , and  $\lambda$  can be identified; otherwise the  $\lambda_i$  coefficients cannot be identified.

Under the third expectational hypothesis, the expectational part of eq. (3.9) becomes (again assuming *n* to be three):

$$\gamma_{0}(\log Y_{2wt}^{e} - \log Y_{2wt-1}) + \sum_{i=1}^{3} \gamma_{i}(\log Y_{2wt+i}^{e} - \log Y_{2wt+i-1}^{e})$$

$$= \gamma_{0}(\log Y_{2wt} - \log Y_{2wt-1}) + \gamma_{1}(\log Y_{2wt-11} - \log Y_{2wt})$$

$$+ \gamma_{2}(\log Y_{2wt-10} - \log Y_{2wt-11}) + \gamma_{3}(\log Y_{2wt-9} - \log Y_{2wt-10})$$

$$+ (\gamma_{1}\lambda_{1} + \gamma_{2}\lambda_{2} - \gamma_{2}\lambda_{1} + \gamma_{3}\lambda_{3} - \gamma_{3}\lambda_{2})(\log Y_{2wt-1} - \log Y_{2wt-13}).$$
(3.16)

Again, only if all of the  $\lambda_i$  coefficients are equal (to, say,  $\lambda$ ) can  $\lambda$  be identified.

The hypothesis that expectations are perfect may not be as unreasonable as it sounds. Firms are likely to have more information at their disposal regarding future demand conditions than merely information on the amount SUMMARY

of output they produced in the past or on past demand conditions. If firms do not use a naive equation like (3.14) to forecast and if the forecasting technique they do use is fairly accurate, then the perfect expectational hypothesis should be a better approximation of how expectations are formed than the other "non-perfect" expectational hypotheses. The hypothesis that expectations are perfect will, of course, be completely realistic if firms schedule production in advance and do not deviate from this schedule even if expected demand conditions change.

# 3.9 Summary

This completes the discussion of the theoretical model of the short-run demand for production workers developed in this study. It was seen that output per man hour and output appear to be positively related even during peak output periods and that there is little evidence of decreasing or constant returns to labor services. The explanation presented here of the widely observed phenomenon of increasing returns to labor is based on the idea that firms hold positive amounts of "excess labor" during much of the year and that the true production function inputs are not observed. A critical distinction is made between the (observed) number of hours paid-for per worker and the (unobserved) number of hours actually worked per worker, and it is contended that the former is a poor proxy for the latter except perhaps at peak rates of output. If this is true, then the properties of the short-run production function cannot be estimated because of lack of data, and in this study various properties of the short-run production function have been postulated. The short-run production process has been assumed to be of such a nature that a fixed number of workers is required per machine and that there are constant returns to scale both with respect to changes in the number of workers and machines used and with respect to changes in the number of hours worked per worker and machine.

The amount of excess labor on hand was defined to be the difference between the standard number of hours of work per worker and the actual number of hours worked per worker. A measurement of the amount of excess labor on hand was made for each industry for each month of the sample period by interpolating plots of output per paid-for man hour from peak to next higher peak; assuming that at the peaks output per paid-for man hour equals output per worked man hour so that an estimate of the production function parameter  $\alpha$  is available at each of the peaks; assuming that  $\alpha$  moves smoothly through time from peak to peak; using the estimates of  $\alpha$  and the output data to compute estimates of man-hour requirements; dividing man-hour requirements by the standard number of hours of work per worker to get the desired number of workers employed; and then taking the (logarithmic) difference between the actual number of workers employed and the desired number employed. This latter measure is the same as the (logarithmic) difference between the standard number of hours of work per worker and the actual number of hours worked per worker. The entire procedure is based on the assumptions made about the properties of the short-run production function.

A model of the short-run demand for workers was then developed in which the change in the number of workers employed was taken to be a function of the amount of excess labor on hand and the time stream of expected future changes in output. The past change in output variables were also added on the assumption that they may help depict the nature of the reaction to the amount of excess labor on hand. Three expectational hypotheses were proposed to be tested in this study, one in which expectations were assumed to be perfect and the other two in which expectations were assumed to be based on past output behavior.

In the next chapter the data which have been used in this study are discussed, and then eq. (3.9) is estimated for each of the seventeen industries considered in this study under the three proposed expectational hypotheses.