CHAPTER 9

# FURTHER STATISTICAL RESULTS

## 9.1 Introduction

This chapter consists of three somewhat unrelated discussions. In § 9.2 serial correlation problems are discussed, and the residuals of eqs. (3.9)′ and (7.2)′ are examined for first-order serial correlation. In § 9.3 the question of the possible correlation between the residuals of eq. (3.9)′ and the residuals of eq. (7.2)′ for each industry is examined, and estimates using a technique developed by ZELLNER (1962) are presented. In this section the question of the possible correlation between the residuals of eq. (3.9)′ or eq. (7.2)′ for one industry and the residuals of eq. (3.9)′ or eq. (7.2)′ for another industry is also examined, and estimates using Zellner's technique are presented. Finally, in § 9.4 a brief comparison of the short-run demand for workers across industries is made.

## 9.2 Tests for first-order serial correlation

It is well known that the Durbin–Watson statistic is biased toward two when there is a lagged dependent variable among the regressors.[1] For equations with lagged dependent variables the DW statistic is thus not a reliable indicator of whether or not the residuals are serially correlated. It is also well known that the least squares technique yields inconsistent estimates when used to estimate the coefficients of an equation with a lagged dependent variable and serially correlated errors. GRILICHES (1961) in fact has shown that for positively correlated errors the least squares estimate of the coefficient of the lagged dependent variable is likely to be too large. It was mentioned in § 4.3.1 that the excess labor variable in the workers equation (3.9)′, $\log M_{2wt-1} - \log M^*_{2wt-1}H^*_{2wt-1}$, is of the nature of a lagged dependent variable, but that because $M^*_{2wt-1}H^*_{2wt-1}$ has a large short-run

[1] See NERLOVE and WALLIS (1966).

variance, the excess labor variable is by no means equivalent to a lagged dependent variable. It was also seen in § 4.3.1 that the excess labor variable definitely appears to be significant in its own right and not merely because it is of the nature of a lagged dependent variable. In the hours equation (7.2)′ log $HP_{2wt-1}$ enters the equation directly, and so for this equation there definitely is a lagged dependent variable among the regressors.

Fortunately, there are consistent and efficient methods of estimating equations with first-order serially correlated errors. Assume that the equation to be estimated is

$$y = X\beta + \mu, \tag{9.1}$$

where

$$\mu_t = \rho\mu_{t-1} + \varepsilon_t, \ |\rho| < 1, \ t = 2, 3, \ldots, T, \tag{9.2}$$

and where $\varepsilon_t$ is assumed to be distributed with zero mean and constant variance $\sigma^2$ and to be uncorrelated with the variables in $X$ and with its own past values. $y$ is a $T \times 1$ vector of observations on the dependent variable $y_t$, $X$ is a $T \times K$ matrix of observations on the explanatory variables $x_{it}$, $\beta$ is a $K \times 1$ vector of coefficients, $\mu$ is a $T \times 1$ vector of disturbances $\mu_t$, and $\rho$ is the serial correlation coefficient. The variance-covariance matrix for $\mu$ can be seen to be

$$E(\mu\mu') = \frac{\sigma^2}{1-\rho^2}\Omega, \tag{9.3}$$

where

$$\Omega = \begin{bmatrix} 1 & \rho & \rho^2 & . & . & . & \rho^{T-1} \\ \rho & 1 & \rho & . & . & . & \rho^{T-1} \\ . & . & . & & & & . \\ . & . & . & & & & . \\ . & . & . & & & & . \\ \rho^{T-1} & \rho^{T-2} & \rho^{T-3} & . & . & . & 1 \end{bmatrix}. \tag{9.4}$$

If $\rho$ (and thus $\Omega$) were known, then an efficient estimate of $\beta$ in eq. (9.1) could be obtained by the use of Aitken's generalized least squares method, but since $\rho$ is usually not known *a priori*, it must be estimated along with the coefficients in $\boldsymbol{\beta}$.

Let

$$P = \begin{bmatrix} \sqrt{1-\rho^2} & 0 & 0 & . & . & . & 0 & 0 & 0 \\ -\rho & 1 & 0 & . & . & . & 0 & 0 & 0 \\ . & . & . & & & & . & . & . \\ . & . & . & & & & . & . & . \\ . & . & . & & & & . & . & . \\ 0 & 0 & 0 & . & . & . & -\rho & 1 & 0 \\ 0 & 0 & 0 & . & . & . & 0 & -\rho & 1 \end{bmatrix}. \quad (9.5)$$

Then if eq. (9.1) is multiplied through by $\boldsymbol{P}$,

$$\boldsymbol{P y} = \boldsymbol{P\, X\, \beta} + \boldsymbol{P \mu} = \boldsymbol{P\, X\, \beta} + \boldsymbol{v}, \quad (9.6)$$

the variance-covariance matrix of the error term $\boldsymbol{v}$ in the resulting equation is seen to be

$$\begin{aligned} E(\boldsymbol{vv}') &= E(\boldsymbol{P\mu\mu}'\boldsymbol{P}') \\ &= \boldsymbol{P}E(\boldsymbol{\mu\mu}')\boldsymbol{P}' \\ &= \frac{\sigma^2}{1-\rho^2}\boldsymbol{P\Omega P}' \\ &= \sigma^2 \boldsymbol{I} \quad , \end{aligned} \quad (9.7)$$

where $\boldsymbol{I}$ is the $T \times T$ identity matrix. The error term $\boldsymbol{v}$ in the transformed equation (9.6) is thus seen to have a scalar variance-covariance matrix like the one assumed for the error term of the classical linear regression model.

Eq. (9.6) is non-linear in the coefficients $\boldsymbol{\beta}$ and $\rho$, but the coefficients can be estimated by minimizing the sum of squared residuals in the equation using a minimization technique like the quadratic hill-climbing technique of Goldfeld, Quandt, and Trotter which was used to estimate eq. (5.1) in § 5.4. If $\boldsymbol{v}$ is normally distributed, then the estimates obtained by the minimization procedure will be maximum likelihood estimates.[1] An estimate of the asymptotic variance-covariance matrix of the parameter estimates can thus be obtained as $(-\partial^2 \log L/\partial\boldsymbol{\phi}^2)^{-1}$, where $L$ is the likelihood function, $\boldsymbol{\phi}'$ is the $1 \times K+2$ vector $(\boldsymbol{\beta}' \; \rho \; \sigma^2)$, and where the derivatives are evaluated at the coefficient estimates.[2] In the present context the asymptotic variance-covariance matrix of the parameters other than $\sigma^2$ is $2\sigma^2(\partial^2\boldsymbol{v}'\boldsymbol{v}/\partial\boldsymbol{\theta}^2)^{-1}$, where $\boldsymbol{\theta}^1$ is the $1 \times K+1$ vector $(\boldsymbol{\beta}' \; \rho)$. The maximum likelihood estimate of $\sigma^2$ is $\hat{\boldsymbol{v}}'\hat{\boldsymbol{v}}/T$, where $\hat{\boldsymbol{v}}$ is the vector of calculated residuals. As was done in § 5.4, however, the estimates presented below were adjusted for degrees of freedom to make them more comparable with the ordinary least squares estimates presented in chs. 4 and 7. The estimate of $\sigma^2$ was thus taken to be $\hat{\boldsymbol{v}}'\hat{\boldsymbol{v}}/(T-K-1)$, and the estimate of the asymptotic variance-covariance matrix which was calculated was $2\,\hat{\boldsymbol{v}}'\hat{\boldsymbol{v}}/(T-K-1)[\partial^2\boldsymbol{v}'\boldsymbol{v}/\partial\boldsymbol{\theta}^2]^{-1}$, where the derivations are evaluated at $\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}$. Notice that from this matrix an estimate of the standard error of the estimate of the serial correlation coefficient $\rho$ is available, as well as the estimates of the standard errors of the other coefficient estimates.

The technique just described can be applied to eqs. (3.9)′ and (7.2)′ to test the hypothesis implicitly made in the previous chapters that $\rho$ is zero. There is one difficulty which arises in using this technique, however, which is due to the fact that there are gaps in the periods of estimation. For most industries the July observations were omitted; for some industries the December observations were omitted as well; and for seven of the industries observations were omitted because of strikes. The DW statistics presented

[1] If there are no lagged dependent variables in $\boldsymbol{X}$, then the maximum likelihood estimates have the desirable properties of consistency and asymptotic efficiency. (DHRYMES, 1966, has further proved that in the present case the estimates obtained by choosing various values of $\varrho$ between minus one and plus one in eq. (9.6), estimating the resulting equations, which are then linear in the parameters $\boldsymbol{\beta}$, by ordinary least squares, and then choosing that value of $\varrho$ and the corresponding estimate of $\boldsymbol{\beta}$ which yield the smallest sum of squared residuals are maximum likelihood estimates and possess the properties of consistency and asymptotic efficiency.) As mentioned in ch. 5, footnote 2, page 97, the properties of the maximum likelihood estimates are less well established when there are lagged dependent variables among the "independent" variables, but that the results which have been achieved indicate that the desirable properties are likely to be retained. In the present context, MALINVAUD (1966, p. 469, footnote ††), has outlined a proof of the statement that the technique of minimizing the sum of squared residuals of eq. (9.6) yields consistent estimates even when there is a lagged dependent variable in the $\boldsymbol{X}$ matrix.

[2] See ch. 5, footnote 3, page 97.

TABLE 9.1

*Parameter estimates for eq. (3.9)′ under the assumption of first-order serial correlation of the residuals*

| Industry | No. of obs. | $\widehat{\alpha_1 \log \bar{H}}$ | $\hat{\alpha}_1$ | $1000\widehat{\alpha_1 \mu}$ | $\hat{\beta}_4$ | $\hat{\beta}_3$ | $\hat{\beta}_2$ | $\hat{\beta}_1$ | $\hat{\gamma}_0$ |
|---|---|---|---|---|---|---|---|---|---|
| 201 | 192 | −1.041 | −.178 | −.077 | | | .074 | .067 | .265 |
| | | (3.93) | (3.93) | (3.62) | | | (3.73) | (3.45) | (9.26) |
| 207 | 136 | 0.653 | −.113 | .098 | | | .068 | .107 | .243 |
| | | (2.75) | (2.74) | (2.98) | | | (5.91) | (5.49) | (11.01) |
| 211 | 136 | −.797 | −.136 | −.051 | | | | | .087 |
| | | (5.46) | (5.47) | (3.08) | | | | | (4.36) |
| 212 | 136 | −.638 | −.109 | −.038 | | | | .053 | .154 |
| | | (4.68) | (4.66) | (1.50) | | | | (4.53) | (7.72) |
| 231 | 136 | −.892 | −.152 | .086 | | | .026 | .064 | .108 |
| | | (4.05) | (3.94) | (3.43) | | | (2.33) | (4.08) | (3.40) |
| 232 | 136 | −.564 | −.100 | −.069 | | | | .018 | .127 |
| | | (4.91) | (4.96) | (3.17) | | | | (2.44) | (8.26) |
| 233 | 136 | −.178 | −.028 | .019 | | | | .124 | .167 |
| | | (0.84) | (0.74) | (0.31) | | | | (5.79) | (7.12) |
| 242 | 154 | −.278 | −.047 | −.121 | .060 | .104 | .145 | .148 | .219 |
| | | (1.41) | (1.40) | (0.57) | (4.55) | (7.09) | (8.29) | (6.52) | (13.52) |
| 271 | 166 | −.223 | −.038 | .000 | | | | | .112 |
| | | (2.48) | (2.51) | (0.32) | | | | | (7.53) |
| 301 | 134 | −.634 | −.109 | −.063 | | | | | .056 |
| | | (6.81) | (6.79) | (2.62) | | | | | (2.84) |
| 311 | 170 | −1.004 | −.171 | −.055 | | | | | .190 |
| | | (6.65) | (6.63) | (3.34) | | | | | (8.22) |
| 314 | 136 | −.429 | −.073 | .043 | | | | | .309 |
| | | (1.60) | (1.59) | (2.44) | | | | | (10.79) |
| 324 | 187 | −.655 | −.110 | .060 | | | | | .224 |
| | | (6.13) | (6.10) | (2.41) | | | | | (16.31) |
| 331 | 128 | −.215 | −.036 | .017 | .044 | .067 | .037 | .121 | .181 |
| | | (2.93) | (2.87) | (1.01) | (3.37) | (4.71) | (2.48) | (6.30) | (9.56) |
| 332 | 170 | −.727 | −.122 | .045 | | | | | .172 |
| | | (8.73) | (8.70) | (2.11) | | | | | (8.31) |
| 336 | 170 | −.729 | −.124 | −.017 | | | | .083 | .171 |
| | | (5.11) | (5.09) | (0.62) | | | | (4.20) | (6.75) |
| 341 | 191 | −.370 | −.066 | −.060 | | | | .038 | .181 |
| | | (3.56) | (3.58) | (2.38) | | | | (3.84) | (15.26) |

$t$-statistics are in parentheses.

[a] $\hat{\delta}$ is the coefficient estimate of $\log Y_{a t-1} - \log Y_{a t-13}$ under the non-perfect expectational hypothesis.

| $\hat{\gamma}_1$ | $\hat{\gamma}_2$ | $\hat{\gamma}_3$ | $\hat{\gamma}_4$ | $\hat{\gamma}_5$ | $\hat{\gamma}_6$ | [a]$\hat{\hat{\delta}}$ | $\hat{\rho}$ | SE |
|---|---|---|---|---|---|---|---|---|
| .171 | .118 | .138 | .159 | .087 | .073 | .039 | .016 | .0120 |
| (6.64) | (4.80) | (6.65) | (9.29) | (5.57) | (4.04) | (2.19) | (0.18) | |
| .168 | .120 | .076 | .034 | | | .056 | −.202 | .0178 |
| (7.25) | (7.13) | (8.85) | (3.76) | | | (2.83) | (2.09) | |
| .025 | .043 | | | | | −.011 | .046 | .0102 |
| (1.90) | (4.97) | | | | | (0.65) | (0.48) | |
| | | | | | | | .011 | .0159 |
| | | | | | | | (0.12) | |
| .010 | .055 | .032 | | | | −.014 | −.179 | .0192 |
| (0.52) | (3.70) | (3.25) | | | | (0.89) | (1.78) | |
| .098 | .066 | .021 | | | | | .140 | .0106 |
| (6.23) | (5.95) | (2.77) | | | | | (1.28) | |
| | | | | | | | .150 | .0291 |
| | | | | | | | (1.48) | |
| .076 | .066 | | | | | | .028 | .0127 |
| (5.23) | (5.20) | | | | | | (0.31) | |
| .020 | .040 | .046 | .010 | .039 | | | −.137 | .0048 |
| (1.39) | (3.20) | (4.13) | (0.97) | (4.21) | | | (1.61) | |
| .062 | .033 | .035 | | | | | .072 | .0142 |
| (3.41) | (1.92) | (2.19) | | | | | (0.76) | |
| .081 | .114 | .084 | .056 | .039 | | | −.033 | .0115 |
| (4.61) | (7.57) | (6.06) | (4.56) | (3.51) | | | (0.39) | |
| .085 | .126 | .041 | | | | .068 | −.183 | .0142 |
| (3.18) | (6.91) | (2.64) | | | | (3.02) | (1.63) | |
| .039 | .026 | .052 | .051 | | | .008 | .005 | .0177 |
| (2.37) | (1.59) | (3.35) | (3.34) | | | (0.47) | (0.06) | |
| | | | | | | | .058 | .0101 |
| | | | | | | | (0.58) | |
| .049 | .057 | .042 | .034 | | | | −.038 | .0167 |
| (3.39) | (4.56) | (3.49) | (2.85) | | | | (0.46) | |
| .093 | .095 | .078 | .044 | .029 | | | .156 | .0174 |
| (4.76) | (5.86) | (5.67) | (3.32) | (2.28) | | | (1.86) | |
| .067 | .044 | .035 | .022 | | | | −.016 | .0181 |
| (5.98) | (4.59) | (3.85) | (2.41) | | | | (0.20) | |

TABLE 9.2

*Parameter estimates for eq. (7.2)′ under the assumption of first-order serial correlation of the residuals*

| Industry | No. of obs. | $\widehat{(\alpha_1-\alpha_2)\log \bar{H}-\psi_1\log \bar{U}}$ | $\hat{\alpha}_1$ | $\hat{\alpha}_2$ | $1000\widehat{(\alpha_1-\alpha_2)\mu}$ | $\hat{\beta}_4$ | $\hat{\beta}_3$ | $\hat{\beta}_2$ | $\hat{\beta}_1$ | $\hat{\gamma}_0$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 201 | 192 | 2.079 | −.071 | −.410 | −.012 | | | .128 | .052 | .236 |
| | | (4.21) | (1.79) | (5.67) | (0.58) | | | (5.45) | (2.22) | (7.93) |
| 207 | 136 | 2.144 | −.042 | −.398 | .028 | | | | | .090 |
| | | (4.66) | (2.20) | (4.47) | (1.29) | | | | | (9.37) |
| 211 | 136 | .602 | −.202 | −.308 | .020 | | | | | .550 |
| | | (1.73) | (3.58) | (4.19) | (0.52) | | | | | (10.42) |
| 212 | 136 | 1.837 | −.094 | −.397 | .091 | | | | | .276 |
| | | (5.35) | (2.45) | (5.04) | (3.01) | | | | | (8.67) |
| 231 | 136 | 1.092 | −.260 | −.430 | .022 | | | | | .182 |
| | | (2.50) | (6.49) | (5.98) | (0.70) | | | | | (5.54) |
| 232 | 136 | 2.660 | −.151 | −.583 | .054 | | | | | .146 |
| | | (2.32) | (5.69) | (2.98) | (0.87) | | | | | (7.14) |
| 233 | 136 | 5.656 | −.122 | −1.079 | −.088 | | | | | .079 |
| | | (10.30) | (3.74) | (10.44) | (1.23) | | | | | (4.35) |
| 242 | 154 | 1.462 | −.031 | −.273 | .002 | .037 | .021 | .066 | .019 | .123 |
| | | (3.46) | (1.06) | (4.03) | (0.08) | (2.19) | (1.27) | (3.51) | (0.82) | (6.46) |
| 271 | 166 | 1.952 | −.065 | −.392 | −.106 | | | | | .097 |
| | | (2.96) | (2.37) | (3.16) | (2.89) | | | | | (3.61) |
| 301 | 134 | 1.358 | −.173 | −.384 | .056 | | | | | .151 |
| | | (3.46) | (5.17) | (4.85) | (1.24) | | | | | (4.33) |
| 311 | 170 | 1.348 | −.103 | −.322 | .031 | | | | | .117 |
| | | (3.83) | (4.02) | (4.59) | (1.94) | | | | | (5.96) |
| 314 | 136 | 1.195 | −.193 | −.395 | .061 | | | | | .417 |
| | | (2.35) | (2.95) | (4.45) | (2.21) | | | | | (11.16) |
| 324 | 187 | 1.612 | −.017 | −.283 | −.013 | | | | | .044 |
| | | (4.67) | (3.86) | (4.72) | (1.47) | | | | | (10.91) |
| 331 | 128 | 2.505 | −.167 | −.576 | .104 | | | | | .189 |
| | | (5.36) | (5.21) | (5.56) | (4.13) | | | | | (9.18) |
| 332 | 170 | .751 | −.096 | −.214 | .052 | | | | | .124 |
| | | (3.59) | (6.18) | (5.13) | (2.96) | | | | | (7.57) |
| 336 | 170 | 1.399 | −.035 | −.260 | .033 | | | | | .070 |
| | | (4.21) | (3.11) | (4.50) | (2.23) | | | | | (5.00) |
| 341 | 191 | 4.318 | −.085 | −.791 | .110 | | | | | .101 |
| | | (5.50) | (4.66) | (5.49) | (3.35) | | | | | (11.01) |

$t$-statistics are in parentheses.

[a] $\hat{\delta}$ is the coefficient estimate of $\log Y_{dt-1} - \log Y_{dt-13}$ under the non-perfect expectational hypothesis.

| $\hat{\gamma}_1$ | $\hat{\gamma}_2$ | $\hat{\gamma}_3$ | $\hat{\gamma}_4$ | $\hat{\gamma}_5$ | $\hat{\gamma}_6$ | $^{a}\hat{\delta}$ | $\hat{\psi}_1$ | $\hat{\rho}$ | SE |
|---|---|---|---|---|---|---|---|---|---|
| .048 | .054 | .041 | .145 | .016 | .073 | .009 | −.0076 | −.228 | .0143 |
| (1.83) | (2.02) | (1.79) | (6.62) | (0.77) | (2.84) | (0.49) | (2.30) | (2.43) | |
| .022 | .009 | .038 | | | | .006 | −.0025 | −.106 | .0116 |
| (1.44) | (0.90) | (7.09) | | | | (0.47) | (0.78) | (0.80) | |
| | | | | | | | .0083 | −.428 | .0327 |
| | | | | | | | (1.16) | (4.52) | |
| | | | | | | | −.0135 | −.359 | .0227 |
| | | | | | | | (2.41) | (3.25) | |
| .045 | .047 | .038 | | | | −.003 | −.0127 | −.030 | .0201 |
| (2.08) | (2.92) | (3.63) | | | | (0.14) | (1.92) | (0.28) | |
| .106 | .083 | .045 | | | | | −.0245 | .375 | .0145 |
| (4.96) | (5.46) | (4.47) | | | | | (1.85) | (1.43) | |
| | | | | | | | −.0118 | .450 | .0245 |
| | | | | | | | (1.05) | (4.41) | |
| | | | | | | | −.0038 | −.264 | .0151 |
| | | | | | | | (1.47) | (2.48) | |
| .071 | .038 | .060 | .031 | .055 | | | −.0004 | .200 | .0051 |
| (2.60) | (1.55) | (2.75) | (1.55) | (3.23) | | | (0.25) | (1.01) | |
| .070 | .035 | .068 | | | | | −.0208 | .034 | .0233 |
| (2.29) | (1.23) | (2.60) | | | | | (2.28) | (0.30) | |
| .039 | .060 | .032 | .021 | .022 | | | −.0083 | −.103 | .0097 |
| (2.67) | (4.96) | (2.85) | (2.10) | (2.10) | | | (2.97) | (0.91) | |
| .143 | .150 | .092 | | | | .134 | .0018 | .013 | .0169 |
| (4.23) | (6.07) | (4.20) | | | | (4.48) | (0.31) | (0.12) | |
| | | | | | | | −.0027 | −.435 | .0084 |
| | | | | | | | (2.27) | (5.53) | |
| | | | | | | | −.0140 | −.095 | .0138 |
| | | | | | | | (3.20) | (0.76) | |
| .027 | .043 | .039 | .024 | | | | −.0098 | −.173 | .0132 |
| (2.29) | (4.32) | (4.24) | (2.59) | | | | (2.85) | (1.87) | |
| .027 | .027 | .038 | | | | | −.0116 | −.242 | .0109 |
| (2.54) | (2.90) | (4.68) | | | | | (3.81) | (2.48) | |
| .022 | | | | | | | −.0103 | .170 | .0159 |
| (3.01) | | | | | | | (2.99) | (1.10) | |

in the tables above were adjusted for these gaps, which means that in the formula for the DW statistic (where $\hat{\mu}_t$ is the calculated residual for month $t$)

$$\mathrm{DW} = \frac{\sum_{t=2}^{T} (\hat{\mu}_t - \hat{\mu}_{t-1})^2}{\sum_{t=1}^{T} \hat{\mu}_t^2} \tag{9.8}$$

if month 13, for example, were omitted from the sample period, then neither $(\hat{\mu}_{13} - \hat{\mu}_{12})^2$ nor $(\hat{\mu}_{14} - \hat{\mu}_{13})^2$ was included in the summation for the numerator and $\hat{\mu}_{13}^2$ was not included in the summation for the denominator.[1] If this gap adjustment procedure were used in estimating eq. (3.9)′ by the above technique, it would necessitate, for example, omitting the log $M_{2w\mathrm{Sept.}}$ − log $M_{2w\mathrm{Aug.}}$ observations from the period of estimation in addition to the already omitted log $M_{2w\mathrm{July}}$ − log $M_{2w\mathrm{June}}$ and log $M_{2w\mathrm{Aug.}}$ − log $M_{2w\mathrm{July}}$ observations for those industries in which shutdowns occurred in July. Likewise, for the December shutdowns an extra observation would be lost each year, and an extra observation would be lost for every strike period.

Instead of losing all of these observations, a slightly different procedure was used when applying the above technique to eqs. (3.9)′ and (7.2)′. The first-order serial correlation of the residuals was assumed to be of such a nature that the residual for each included observation was correlated with the residual from the previous included observation instead of necessarily with the previous chronological observation. If, for example, the July–June and August–July observations were omitted, the residual for the September–August observation was assumed to be correlated with the residual for the June–May observation. For those industries in which shutdowns occur in July and December this assumption saved two observations per year from having to be omitted, and for those industries in which no shutdowns occurred the assumption is equivalent to the normal assumption that the residuals are correlated chronologically. Since there was good reason for omitting the observations when shutdowns occurred, the assumption that the included residuals are correlated in the manner just described is not completely unrealistic, although because of the necessity of making this assumption, the

[1] $\hat{\mu}_{13}$, of course, does not really exist if observation 13 has been omitted.

tests for first-order serial correlation performed here are somewhat crude.

Eqs. (3.9)′ and (7.2)′ were thus estimated using the above techniques under the assumption that the included residuals are first-order serially correlated. The quadratic hill-climbing technique was used to minimize the sum of squared residuals, and the asymptotic variance-covariance matrix was estimated in the manner described above. The results of estimating eq. (3.9)′ are presented in table 9.1 and the results of estimating eq. (7.2)′ are presented in table 9.2. The same period of estimation and the same expectational variables were used here for each industry as were used for the results presented in tables 4.3 and 7.2, and so the results presented in tables 9.1 and 9.2 are directly comparable with the results in tables 4.3 and 7.2.

Looking at the results of estimating eq. (3.9)′ in table 9.1 first, it is seen that the estimate of the serial correlation coefficient $\rho$ is significant ($t$-statistic greater than two) for only one of the seventeen industries – industry 207. The estimate of $\rho$ ranges from $-.202$ for industry 207 to .156 for industry 336. Seven of the seventeen estimates are negative (negative first-order serial correlation). As expected from the work of GRILICHES (1961) mentioned above, when the estimate of $\rho$ is positive, the estimate of the coefficient $\alpha_1$ of the excess labor variable increases in absolute value from what it was in table 4.3, and when the estimate of $\rho$ is negative, the estimate $\alpha_1$ decreases in absolute value. Only for industry 314 has the estimate of $\alpha_1$ lost its significance from table 4.3. The over-all results clearly indicate that first-order serial correlation of the residuals is not a serious problem for eq. (3.9)′: the estimates of $\rho$ are small and insignificant, and the other coefficient estimates have been changed only slightly from what they were in table 4.3.

Looking next at the results of estimating eq. (7.2)′ in table 9.2, it is seen that the estimate of $\rho$ is significant for seven of the seventeen industries, and for six of these seven industries it is negative. The estimate ranges from $-.435$ in industry 324 to .450 in industry 233. When the estimate of $\rho$ is positive, both the estimate of the coefficient $\alpha_1$ of the excess labor variable and the estimate of the coefficient $\alpha_2$ of log $HP_{2wt-1}$ increase in absolute value from what they were in table 7.2, and when the estimate of $\rho$ is negative, both of the estimates decrease in absolute value. For some of the industries the estimate of $\alpha_2$ has been changed considerably. For industry 211, where the estimate of $\rho$ is $-.428$, the estimate of $\alpha_2$ changes from $-.612$ in table 7.2 to $-.308$ in table 9.2; for industry 212, where the estimate of $\rho$ is $-.359$, the estimate of $\alpha_2$ changes from $-.583$ to $-.397$; for industry 233, where the estimate of $\rho$ is $+.450$, the estimate of $\alpha_2$ changes from $-.733$ to $-1.079$;

and for industry 324, where the estimate of $\rho$ is $-.435$, the estimate of $\alpha_2$ changes from $-.574$ to $-.283$. For none of the industries has the estimate of $\alpha_2$ lost its significance, however, and only for industry 201 has the estimate of $\alpha_1$ lost its significance from table 7.2.

For at least seven of the industries, therefore, serial correlation of the residuals of eq. (7.2)′ does appear to be a problem, with negative serial correlation being more pronounced than positive serial correlation. The conclusion reached in ch. 7 that the amount of excess labor on hand is a significant determinant of the change in the number of hours paid-for per worker does not appear to have been modified by the results presented in table 9.2, however, nor does the conclusion that the difference between the number of hours paid-for per worker and the standard number of hours of work per worker is also a significant determinant of this change. Only the size of the estimates of $\alpha_1$ and $\alpha_2$ appears to have been changed for any of the industries. This change in size, however, could have an affect on the conclusion reached in ch. 8 for the total man-hours paid-for equation that the sum of the estimates of $\alpha_1$ in eqs. (3.9)′ and (7.2)′ is less in absolute value than the estimate of $\alpha_2$ in eq. (7.2)′ (i.e., that firms react more strongly in changing total

TABLE 9.3

*Sum of the estimates of $\alpha_1$ in table 4.3 and table 9.2 and the estimate of $\alpha_2$ in table 9.2: derived coefficient estimates for the total man-hours paid-for equation*

| Industry | $\hat{\alpha}_1$ | $\hat{\alpha}_2$ |
|---|---|---|
| 201 | $-.249$ | $-.410$ |
| 207 | $-.193$ | $-.398$ |
| 211 | $-.335$ | $-.308$ |
| 212 | $-.202$ | $-.397$ |
| 231 | $-.441$ | $-.430$ |
| 232 | $-.241$ | $-.583$ |
| 233 | $-.127$ | $-1.079$ |
| 242 | $-.075$ | $-.273$ |
| 271 | $-.109$ | $-.392$ |
| 301 | $-.281$ | $-.384$ |
| 311 | $-.277$ | $-.322$ |
| 314 | $-.308$ | $-.395$ |
| 324 | $-.127$ | $-.283$ |
| 331 | $-.202$ | $-.576$ |
| 332 | $-.219$ | $-.214$ |
| 336 | $-.148$ | $-.260$ |
| 341 | $-.152$ | $-.791$ |

man hours paid-for when the number of hours paid-for per worker differs from the standard level of hours than when the number of workers employed differs from the desired number). To see if this conclusion has been modified, the results of adding for each industry the estimate of $\alpha_1$ for eq. (7.2)′ in table 9.2 and the estimate of $\alpha_1$ for eq. (3.9)′ in table 4.3 are presented in table 9.3, along with the estimate of $\alpha_2$ for eq. (7.2)′ in table 9.2. Comparing the results in tables 9.3 and 8.1, it is seen that only for industries 211 and 332 has the estimate of $\alpha_1$ in the man-hours equation been changed from being less than the estimate of $\alpha_2$ in absolute value to being greater in absolute value. (For industry 231 the estimate of $\alpha_1$ is slightly greater in absolute value in both tables.) For the remaining fourteen industries the estimate of $\alpha_1$ remains smaller in absolute value, and the general conclusion reached in ch. 8 that the estimate of $\alpha_1$ is less in absolute value than the estimate of $\alpha_2$ in the total man-hours paid-for equation does not appear to have been modified.

In summary, then, some evidence has been found that the residuals of eq. (7.2)′ are serially correlated for a few industries, with negative serial correlation predominating, but none of the conclusions reached in chs. 7 and 8 regarding the hours paid-for per worker equation or the total man-hours paid-for equation appears to have been changed for these industries. For the majority of the industries the estimate of $\rho$ for eq. (7.2)′ is not significant. For eq. (3.9)′ there is almost no evidence at all that $\rho$ is different from zero.

### 9.3 More efficient estimates

In the previous chapters two basic equations were estimated for each industry, one determining the short-run demand for workers and the other determining the short-run demand for hours paid-for per worker. It seems likely that for each industry the residuals from these two equations will be positively correlated, that a random disturbance for a given month which affects the residual of one of the equations in a certain way will also affect the residual of the other equation in a similar way. If these residuals are in fact correlated, then the two-stage Aitken estimator proposed by ZELLNER (1962) will yield more efficient estimates than the ordinary least squares method used in the previous chapters. The gain in efficiency is greater to the extent that the residuals are highly correlated and to the extent that the independent variables in the different equations are highly uncorrelated. The gain in efficiency is zero if the residuals of the different equations are not correlated or if the independent variables in the different equations are all the same. Basically, the two-stage method consists in first estimating the variance-covariance matrix of the residuals from the ordinary least squares estimates of each equation and then using this matrix to estimate all of the equations simultaneously by Aitken's generalized least squares method.

Assuming that the residuals from the workers and hours equations are not serially correlated but are contemporaneously correlated with each other, the two-stage Aitken estimator can be used to estimate the two equations simultaneously.[1] With respect to the independent variables in eqs. (3.9)′ and (7.2)′, the hours equation (7.2)′ includes the log $HP_{2wt-1}$

TABLE 9.4

*Correlation between the residuals of eq. (3.9)′ and the residuals of eq. (7.2)′ for each industry*

| Industry | No. of obs. | Correlation coefficient |
|---|---|---|
| 201 | 192 | .32 |
| 207 | 136 | .19 |
| 211 | 136 | .16 |
| 212 | 136 | .22 |
| 231 | 136 | .32 |
| 232 | 136 | .56 |
| 233 | 136 | .09 |
| 242 | 154 | .33 |
| 271 | 166 | .07 |
| 301 | 134 | .19 |
| 311 | 170 | .30 |
| 314 | 136 | .62 |
| 324 | 187 | .07 |
| 331 | 128 | .16 |
| 332 | 170 | .24 |
| 336 | 170 | .11 |
| 341 | 191 | .38 |

[1] Zellner actually developed the two-stage Aitken estimator under the assumption that the "independent" variables are non-stochastic. This assumption is not met for the work here since there is a lagged dependent variable in eq. (7.2)′ and the excess labor variable, which is of the nature of a lagged dependent variable, in eq. (3.9)′. If there is no serial correlation nor cross serial correlation of the residuals in the equations, however, the two-stage Aitken estimator proposed by Zellner can be used for equations with lagged dependent variables as well. As was seen in the previous section, the residuals of eq. (3.9)′ do not appear to be serially correlated, and the residuals of eq. (7.2)′ appear to be serially correlated only for a few industries.

TABLE 9.5

*Parameter estimates for eq. (3.9)′ using the two-stage Aitken estimator for each industry*[b]

| Industry | No. of obs. | $\widehat{\alpha_1 \log \bar{H}}$ | $\hat{\alpha}_1$ | $\widehat{1000\,\alpha_1\mu}$ | $\hat{\beta}_4$ | $\hat{\beta}_8$ | $\hat{\beta}_2$ | $\hat{\beta}_1$ | $\hat{\gamma}_0$ | $\hat{\gamma}_1$ | $\hat{\gamma}_2$ | $\hat{\gamma}_3$ | $\hat{\gamma}_4$ | $\hat{\gamma}_5$ | [a]$\hat{\delta}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 207 | 136 | −.898 | −.156 | .122 | | | .062 | .090 | .262 | .182 | .126 | .080 | .036 | | .055 |
| | | (3.54) | (3.53) | (3.27) | | | (5.29) | (4.77) | (11.80) | (7.22) | (7.37) | (9.15) | (4.03) | | (2.60) |
| 211 | 136 | −.792 | −.136 | −.051 | | | | | .088 | .029 | .042 | | | | −.006 |
| | | (5.95) | (5.96) | (3.25) | | | | | (4.59) | (2.34) | (4.97) | | | | (0.39) |
| 212 | 136 | −.636 | −.108 | −.038 | | | | .054 | .154 | | | | | | |
| | | (4.77) | (4.74) | (1.52) | | | | (4.68) | (7.77) | | | | | | |
| 231 | 136 | −1.079 | −.185 | .086 | | | .032 | .063 | .127 | .021 | .061 | .035 | | | −.018 |
| | | (4.68) | (4.58) | (2.93) | | | (3.22) | (3.98) | (4.01) | (1.00) | (3.95) | (3.57) | | | (1.08) |
| 232 | 136 | −.505 | −.090 | −.062 | | | | .021 | .118 | .091 | .061 | .018 | | | |
| | | (5.58) | (5.66) | (3.42) | | | | (3.62) | (9.59) | (6.82) | (6.33) | (2.54) | | | |
| 233 | 136 | −.056 | −.006 | .041 | | | | .126 | .165 | | | | | | |
| | | (0.33) | (0.21) | (0.77) | | | | (5.97) | (6.73) | | | | | | |
| 242 | 154 | −.283 | −.048 | −.013 | .060 | .106 | .146 | .149 | .219 | .081 | .068 | | | | |
| | | (1.55) | (1.54) | (0.64) | (4.58) | (7.42) | (8.63) | (6.83) | (13.72) | (5.94) | (5.68) | | | | |
| 324 | 187 | −.644 | −.108 | .059 | | | | | .224 | .038 | .024 | .052 | .049 | | .007 |
| | | (6.29) | (6.26) | (2.39) | | | | | (16.51) | (2.30) | (1.52) | (3.40) | (3.32) | | (0.41) |
| 331 | 128 | −.206 | −.034 | .015 | .043 | .062 | .041 | .125 | .182 | | | | | | |
| | | (3.01) | (2.95) | (0.99) | (3.32) | (4.48) | (2.80) | (6.56) | (9.81) | | | | | | |
| 336 | 170 | −.660 | −.112 | −.014 | | | | .093 | .162 | .085 | .091 | .076 | .043 | .026 | |
| | | (5.56) | (5.54) | (0.62) | | | | (4.82) | (6.46) | (4.77) | (5.97) | (5.76) | (3.37) | (2.02) | |
| 341 | 191 | −.352 | −.063 | −.058 | | | | .046 | .180 | .067 | .041 | .038 | .027 | | |
| | | (3.58) | (3.60) | (2.30) | | | | (5.02) | (15.78) | (6.17) | (4.65) | (4.43) | (3.24) | | |

*t*-statistics are in parentheses.

[a] $\hat{\delta}$ is the coefficient estimate of $\log Y_{dt-1} - \log Y_{dt-13}$ under the non-perfect expectational hypothesis.

[b] For industries 201, 271, 301, 311, 314, and 332 the two-stage Aitken estimates were the same as the ordinary least squares estimates presented in table 4.3.

TABLE 9.6

*Parameter estimates for eq. (7.2)′ using the two-stage Aitken estimator for each industry*

| Industry | No. of obs. | $\widehat{(\alpha_1-\alpha_2)\log \bar{H}-\psi_1\log \bar{U}}$ | $\hat{\alpha}_1$ | $\hat{\alpha}_2$ | $\widehat{1000(\alpha_1-\alpha_2)\mu}$ | $\hat{\beta}_4$ | $\hat{\beta}_3$ | $\hat{\beta}_2$ | $\hat{\beta}_1$ |
|---|---|---|---|---|---|---|---|---|---|
| 201 | 192 | 1.982 | −.110 | −.434 | −.030 | | | .118 | .050 |
| | | (4.33) | (2.31) | (7.15) | (1.26) | | | (5.13) | (2.16) |
| 207 | 136 | 2.452 | −.052 | −.460 | .033 | | | | |
| | | (6.99) | (3.11) | (6.97) | (1.50) | | | | |
| 211 | 136 | 1.343 | −.392 | −.620 | .023 | | | | |
| | | (3.10) | (5.78) | (8.06) | (0.41) | | | | |
| 212 | 136 | 2.380 | −.173 | −.566 | .098 | | | | |
| | | (6.18) | (4.03) | (6.96) | (2.41) | | | | |
| 231 | 136 | 1.097 | −.263 | −.433 | .022 | | | | |
| | | (2.70) | (6.83) | (7.06) | (0.68) | | | | |
| 232 | 136 | 1.179 | −.125 | −.315 | −.002 | | | | |
| | | (4.26) | (5.98) | (6.99) | (0.07) | | | | |
| 233 | 136 | 3.682 | −.078 | −.702 | −.055 | | | | |
| | | (7.95) | (2.66) | (8.30) | (1.17) | | | | |
| 242 | 154 | 2.233 | −.044 | −.412 | .007 | .051 | .030 | .065 | .021 |
| | | (5.65) | (1.20) | (7.09) | (0.25) | (3.21) | (1.73) | (3.18) | (0.80) |
| 271 | 166 | 1.484 | −.054 | −.302 | −.080 | | | | |
| | | (5.70) | (2.71) | (6.29) | (5.27) | | | | |
| 301 | 134 | 1.383 | −.176 | −.392 | .053 | | | | |
| | | (4.42) | (5.82) | (6.38) | (1.27) | | | | |
| 311 | 170 | 1.628 | −.116 | −.380 | .036 | | | | |
| | | (5.68) | (4.79) | (7.26) | (2.24) | | | | |
| 314 | 136 | 1.016 | −.171 | −.342 | .060 | | | | |
| | | (2.30) | (2.92) | (5.06) | (2.27) | | | | |
| 324 | 187 | 3.273 | −.032 | −.571 | −.034 | | | | |
| | | (8.59) | (5.68) | (8.65) | (2.62) | | | | |
| 331 | 128 | 2.631 | −.171 | −.601 | .109 | | | | |
| | | (7.58) | (6.48) | (7.84) | (4.50) | | | | |
| 332 | 170 | .955 | −.108 | −.258 | .062 | | | | |
| | | (4.71) | (6.76) | (6.62) | (3.13) | | | | |
| 336 | 170 | 2.053 | −.044 | −.375 | .048 | | | | |
| | | (6.60) | (3.27) | (6.90) | (2.82) | | | | |
| 341 | 191 | 3.426 | −.067 | −.627 | .087 | | | | |
| | | (10.22) | (6.22) | (10.19) | (3.72) | | | | |

$t$-statistics are in parentheses.

[a] $\hat{\delta}$ is the coefficient estimate of $\log Y_{dt-1} - \log Y_{dt-13}$ under the non-perfect expectational hypothesis.

| $\hat{\gamma}_0$ | $\hat{\gamma}_1$ | $\hat{\gamma}_2$ | $\hat{\gamma}_3$ | $\hat{\gamma}_4$ | $\hat{\gamma}_5$ | $\hat{\gamma}_6$ | [a]$\hat{\delta}$ | $\hat{\psi}_1$ |
|---|---|---|---|---|---|---|---|---|
| .253 | .068 | .070 | .057 | .146 | .018 | .072 | −.000 | −.0045 |
| (7.87) | (2.32) | (2.44) | (2.43) | (6.98) | (0.88) | (3.00) | (0.01) | (1.20) |
| .094 | .023 | .010 | .039 | | | | .007 | −.0022 |
| (10.37) | (1.47) | (0.97) | (7.44) | | | | (0.54) | (0.64) |
| .501 | | | | | | | | .0082 |
| (8.91) | | | | | | | | (0.82) |
| .231 | | | | | | | | −.0157 |
| (7.92) | | | | | | | | (2.19) |
| .183 | .045 | .047 | .038 | | | | −.004 | −.0128 |
| (5.58) | (2.05) | (2.90) | (3.68) | | | | (0.19) | (2.04) |
| .126 | .094 | .080 | .047 | | | | | −.0102 |
| (7.51) | (5.32) | (6.20) | (4.75) | | | | | (2.47) |
| .095 | | | | | | | | −.0074 |
| (4.55) | | | | | | | | (0.90) |
| .123 | | | | | | | | −.0072 |
| (6.33) | | | | | | | | (2.53) |
| .080 | .055 | .023 | .045 | .016 | .044 | | | −.0006 |
| (4.44) | (3.01) | (1.44) | (3.32) | (1.37) | (3.94) | | | (0.43) |
| .151 | .068 | .033 | .067 | | | | | −.0193 |
| (4.55) | (2.30) | (1.19) | (2.58) | | | | | (2.35) |
| .120 | .043 | .061 | .032 | .020 | .018 | | | −.0096 |
| (5.85) | (2.82) | (4.68) | (2.68) | (1.88) | (1.91) | | | (3.51) |
| .410 | .143 | .156 | .099 | | | | .127 | .0010 |
| (11.39) | (4.69) | (6.77) | (5.00) | | | | (4.48) | (0.22) |
| .042 | | | | | | | | −.0057 |
| (7.55) | | | | | | | | (3.51) |
| .191 | | | | | | | | −.0153 |
| (9.05) | | | | | | | | (3.82) |
| .125 | .032 | .045 | .040 | .023 | | | | −.0124 |
| (7.26) | (2.71) | (4.35) | (4.18) | (2.46) | | | | (3.61) |
| .079 | .034 | .030 | .035 | | | | | −.0161 |
| (5.36) | (3.07) | (3.03) | (4.32) | | | | | (5.30) |
| .094 | .022 | | | | | | | −.0083 |
| (13.01) | (2.98) | | | | | | | (3.28) |

variable and the log $U_{2wt}$ variable which the workers equation (3.9)′ does not, and sometimes the workers equation includes expected future output change variables which the hours equation does not. In general, the number of different independent variables in the two equations is not large. With respect to the correlation of the residuals in the two equations, the correlation coefficient for each industry is presented in table 9.4. Notice that all of the coefficients in the table are positive, as expected, with a range of .07 to .62.

In tables 9.5 and 9.6 the results of estimating eqs. (3.9)′ and (7.2)′ using the two-stage Aitken estimator are presented for each industry. The same period of estimation and same expectational variables were used for these estimates as were used for the ordinary least squares estimates presented in tables 4.3 and 7.2 above. In table 9.5 the estimates for eq. (3.9)′ are presented and in table 9.6 the estimates for eq. (7.2)′ are presented. For industries 201, 271, 301, 311, 314, and 332, eq. (3.9)′ included no independent variables which eq. (7.2)′ did not also include, and so for these industries the two-stage Aitken estimates for eq. (3.9)′ were the same as the ordinary least squares estimates. In table 9.5 the Aitken estimates are not presented for these six industries since the estimates are the same as those presented in table 4.3.

Comparing the results in tables 9.5 and 9.6 with those in tables 4.3 and 7.2, it is seen that the coefficient estimates are only slightly changed and that very little efficiency has been gained. The estimates in table 9.6 for the hours equation (7.2)′ have been changed more than the estimates in table 9.5 for the workers equation (3.9)′, but even in table 9.6 the results are only slightly changed from the results in table 7.2. It is a property of the two-stage Aitken estimator that the estimates of the standard errors of the coefficient estimates are never greater than the ordinary least squares estimates of the standard errors. Some of the $t$-statistics (ratios of the coefficient estimates to their standard errors) in tables 9.5 and 9.6 are less than the corresponding statistics in tables 4.3 and 7.2, however, and in these cases the two-stage Aitken coefficient estimates decreased by a larger percentage than did the estimates of the standard errors. From the over-all results it is quite obvious that very little efficiency has been gained using the two-stage procedure.

The two-stage Aitken estimates can also be used to estimate equations of different industries simultaneously. It may be, for example, that a random disturbance for a given month which affects the residual of eq. (3.9)′ or (7.2)′ in a specific way for one industry will also affect the residual of eq. (3.9)′ or (7.2)′ for another industry in a similar way. Economy-wide disturbances, for example, may affect different industries in a similar manner.

For the work here not all of the equations of the seventeen industries could be estimated simultaneously because different periods of estimation were used for different industries, but three sets of industry equations were estimated using the two-stage Aitken estimator. In the first set eqs. (3.9)′ and (7.2)′ for the Tobacco industries 211 and 212 (giving a total of four equations) were estimated simultaneously; in the second set eqs. (3.9)′ and (7.2)′ for the Apparel industries 231, 232, and 233 (giving a total of six equations) were estimated simultaneously, and in the third set eqs. (3.9)′ and (7.2)′ for the Primary Metals industries 332 and 336 (giving a total of four equations) were estimated simultaneously. The gain in efficiency should be greater for these estimates than for the ones presented in tables 9.5 and 9.6 since the independent variables in the different industry equations are different except for the time-trend and unemployment-rate variable.

TABLE 9.7

*Correlation between the residuals of eq. (3.9)′ or eq. (7.2)′ for one industry and the residuals of eq. (3.9)′ or eq. (7.2)′ for another industry*

| | | 211 eq. (7.2)′ | 212 eq. (3.9)′ | 212 eq. (7.2)′ |
|---|---|---|---|---|
| | 211 eq. (3.9)′ | .16[a] | .07 | .04 |
| 136 obs. | 211 eq. (7.2)′ | | .02 | .28 |
| | 212 eq. (3.9)′ | | | .22[a] |

| | | 231 eq. (7.2)′ | 232 eq. (3.9)′ | 232 eq. (7.2)′ | 233 eq. (3.9)′ | 233 eq. (7.2)′ |
|---|---|---|---|---|---|---|
| | 231 eq. (3.9)′ | .32[a] | .26 | .03 | .07 | −.01 |
| | 231 eq. (7.2)′ | | .17 | .32 | .20 | .57 |
| 136 obs. | 232 eq. (3.9)′ | | | .56[a] | .22 | .00 |
| | 232 eq. (7.2)′ | | | | .16 | .19 |
| | 233 eq. (3.9)′ | | | | | .09[a] |

| | | 332 eq. (7.2)′ | 336 eq. (3.9)′ | 336 eq. (7.2)′ |
|---|---|---|---|---|
| | 332 eq. (3.9)′ | .24[a] | .41 | .06 |
| 170 obs. | 332 eq. (7.2)′ | | .14 | .60 |
| | 336 eq. (3.9)′ | | | .11[a] |

[a] Same as that given in table 9.4.

With respect to the correlation of the residuals in the different equations, the correlation coefficients are presented in table 9.7. All but one of the coefficients are positive, as expected. For the Tobacco industries the corre-

TABLE 9.8

*Parameter estimates for eq. (3.9)′ using the two-stage Aitken estimator for three sets of industry groups*

| Industry | No. of obs. | $\widehat{\alpha_1 \log \bar{H}}$ | $\hat{\alpha}_1$ | $1000\,\widehat{\alpha_1 \mu}$ | $\hat{\beta}_2$ | $\hat{\beta}_1$ | $\hat{\gamma}_0$ | $\hat{\gamma}_1$ | $\hat{\gamma}_2$ | $\hat{\gamma}_3$ | $\hat{\gamma}_4$ | $\hat{\gamma}_5$ | [a] $\hat{\delta}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 211 | 136 | −.777 | −.133 | −.051 | | | .089 | .030 | .043 | | | | −.005 |
| | | (5.85) | (5.87) | (3.22) | | | (4.60) | (2.37) | (5.00) | | | | (0.34) |
| 212 | 136 | −.635 | −.108 | −.038 | | .054 | .155 | | | | | | |
| | | (4.77) | (4.75) | (1.52) | | (4.72) | (7.80) | | | | | | |
| 231 | 136 | −.999 | −.171 | .081 | .023 | .052 | .124 | .021 | .055 | .032 | | | −.008 |
| | | (4.53) | (4.43) | (2.78) | (2.49) | (3.48) | (4.07) | (1.00) | (3.65) | (3.30) | | | (0.48) |
| 232 | 136 | −.529 | −.094 | −.060 | | .021 | .113 | .076 | .047 | .012 | | | |
| | | (6.15) | (6.22) | (3.34) | | (3.59) | (9.64) | (6.09) | (5.11) | (1.76) | | | |
| 233 | 136 | .078 | .018 | .064 | | .137 | .144 | | | | | | |
| | | (0.47) | (0.60) | (1.23) | | (6.68) | (6.08) | | | | | | |
| 332 | 170 | −.666 | −.112 | .044 | | | .158 | .050 | .059 | .041 | .033 | | |
| | | (8.28) | (8.25) | (1.98) | | | (7.92) | (3.61) | (4.81) | (3.54) | (2.92) | | |
| 336 | 170 | −.597 | −.102 | −.011 | | .085 | .146 | .084 | .089 | .074 | .045 | .020 | |
| | | (5.29) | (5.26) | (0.46) | | (4.80) | (6.13) | (4.86) | (6.07) | (5.82) | (3.63) | (1.75) | |

*t*-statistics are in parentheses.

[a] $\hat{\delta}$ is the coefficient estimate of $\log Y_{dt-1} - \log Y_{dt-13}$ under the non-perfect expectational hypothesis.

TABLE 9.9

*Parameter estimates for eq. (7.2)′ using the two-stage Aitken estimator for three sets of industry groups*

| Industry | No. of obs. | $\widehat{(\alpha_1-\alpha_2)\log \bar{H}-\psi_1\log \bar{U}}$ | $\hat{\alpha}_1$ | $\hat{\alpha}_2$ | $1000\widehat{(\alpha_1-\alpha_2)\mu}$ | $\hat{\gamma}_0$ | $\hat{\gamma}_1$ | $\hat{\gamma}_2$ | $\hat{\gamma}_3$ | $\hat{\gamma}_4$ | [a]$\hat{\delta}$ | $\hat{\psi}_1$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 211 | 136 | 1.612 | −.354 | −.627 | .034 | .489 | | | | | | .0069 |
| | | (3.86) | (5.43) | (8.45) | (0.63) | (9.04) | | | | | | (0.68) |
| 212 | 136 | 2.621 | −.159 | −.591 | .113 | .243 | | | | | | −.0177 |
| | | (7.06) | (3.84) | (7.54) | (2.81) | (8.61) | | | | | | (2.48) |
| 231 | 136 | .909 | −.219 | −.362 | .030 | .127 | .030 | .036 | .027 | | −.013 | −.0083 |
| | | (2.84) | (7.14) | (7.59) | (0.96) | (4.83) | (1.70) | (2.72) | (3.19) | | (0.90) | (1.40) |
| 232 | 136 | 1.255 | −.111 | −.314 | .010 | .106 | .076 | .062 | .038 | | | −.0097 |
| | | (4.79) | (5.60) | (7.40) | (0.38) | (6.65) | (4.57) | (5.05) | (4.01) | | | (2.39) |
| 233 | 136 | 3.291 | −.068 | −.627 | −.043 | .079 | | | | | | −.0052 |
| | | (8.42) | (2.85) | (8.87) | (0.97) | (4.52) | | | | | | (0.65) |
| 332 | 170 | .977 | −.095 | −.249 | .059 | .116 | .025 | .038 | .037 | .020 | | −.0120 |
| | | (5.53) | (6.99) | (7.58) | (3.07) | (7.58) | (2.28) | (4.03) | (4.14) | (2.67) | | (3.76) |
| 336 | 170 | 1.869 | −.024 | −.326 | .045 | .076 | .027 | .021 | .029 | | | −.0139 |
| | | (7.02) | (2.04) | (7.17) | (2.72) | (5.83) | (2.65) | (2.36) | (3.94) | | | (5.06) |

*t*-statistics are in parentheses.

[a] $\hat{\delta}$ is the coefficient estimate of $\log Y_{dt-1} - \log Y_{dt-13}$ under the non-perfect expectational hypothesis.

lation between the residuals of eq. (3.9)′ for 211 and the residuals of eq. (3.9)′ for 212 is .07, and for eq. (7.2)′ the correlation is .28. For the Apparel industries the correlation is .26, .07, and .22 respectively between the residuals of eq. (3.9)′ for 231 and 232, 231 and 233, and 232 and 233; and the correlation is .32, .57, and .19 respectively between the residuals of eq. (7.2)′ for 231 and 232, 231 and 233, and 232 and 233. For the Primary Metals industries the correlation between the residuals of eq. (3.9)′ for 332 and 336 is .41, and for eq. (7.2)′ the correlation is .60. It appears from table 9.7 that there is more correlation among the residuals in the Primary Metals industries than in the other two industry groups.

In tables 9.8 and 9.9 the results of estimating the three sets of industry equations using the two-stage Aitken method are presented: in table 9.8 the results for eq. (3.9)′ and in table 9.9 the results for eq. (7.2)′. These results are directly comparable with the ordinary least squares results in tables 4.3 and 7.2. Comparing the results in tables 9.8 and 9.9 with those in tables 4.3 and 7.2, three conclusions seem to emerge. The estimates are changed more here than they were in tables 9.5 and 9.6, which is as expected, although the extent of the change for either the coefficient estimates or the standard errors is not very great; the estimates for the hours equation have been changed more than the estimates for the workers equation; and there is a tendency, especially in the Apparel and Primary Metals industries, for the size of the coefficient estimates to decrease in absolute value. From the over-all results the gain in efficiency does not appear to have been very large using the two-stage Aitken estimator, and none of the conclusions reached in the previous chapters appears to need changing from the results achieved here.

## 9.4 A comparison of the short-run demand for workers across industries

So far very few across industry comparisons have been made from the results presented above. The model of the short-run demand for workers developed in ch. 3 was estimated for seventeen three-digit manufacturing industries, and the results were consistently good for all of the industries. The size of the parameter estimates do differ from industry to industry, however, and the purpose of this section is to examine whether any of these differences across industries can be explained. Attention will be concentrated on the estimate of the coefficient $\gamma_0$ of $\log Y_{dt} - \log Y_{dt-1}$ presented in table 4.3 for each industry. This coefficient is a measure of how strongly firms react, other things being equal, to current changes in output: the larger $\gamma_0$ is the larger is the percentage change in the number of workers employed corresponding to a given percentage change in output. Three hypotheses will be tested regarding the size of $\gamma_0$ for an industry. The first hypothesis is that the size of $\gamma_0$ for an industry is related to the amount of specific training required in the industry; the second hypothesis is that the size of $\gamma_0$ is related to the degree of unionization in the industry; and the third hypothesis is that the size of $\gamma_0$ is related to the average wage level in the industry.

With respect to the first hypothesis that the size of $\gamma_0$ for an industry is related to the amount of specific training required in the industry, one would expect the size of $\gamma_0$ to be inversely related to the amount of specific training required. If the amount of specific training is high, for example, one would expect the short-run employment reaction to be smaller than otherwise since firms will presumably be more reluctant to lay off workers for fear of not being able to hire them back when they are needed and of having to train new workers. From the work of ECKHAUS (1964) data are available for 1950 on specific industry training requirements in number of years required for most of the industries considered in this study.[1] In order to use these data the Apparel industries 231, 232, and 233 had to be grouped together, as did the Tobacco industries 211 and 212. Some of the other data on training requirements were for industries slightly more aggregated than the three-digit industries considered in this study, but these data were used as proxies for the unavailable three-digit industry data. For the industries which were grouped together, a weighted average of the estimates of $\gamma_0$ was taken to represent the grouped industry reaction, the weights being the number of production workers employed in each industry in 1958 as a percent of the total number of production workers employed in the group in 1958. There were a total of fourteen observations. The data for these fourteen industries are presented in table 9.10.

The Kendall Tau rank correlation coefficient was calculated using the fourteen observations presented in table 9.10. The coefficient was $-.30$, which is of the expected negative sign (the larger the amount of specific training required the smaller the employment reaction to current output changes) and which is significant at the ten-percent confidence level but not at the five-percent level. There is thus some slight indication from this

[1] HAMERMESH (1967) has also used these data and the data on unionization described below in a comparison of industry behavior.

TABLE 9.10

*Estimates of $\gamma_0$ and of the amount of specific training required for fourteen industry groups*

| Industry or industry group | Estimate or weighted estimate of $\gamma_0$ (from table 4.3) | Specific training required in years |
|---|---|---|
| 201 | .265 | .73 |
| 207 | .262 | .70 |
| 211 and 212 | .118 | .63 |
| 231, 232 and 233 | .141 | .64 |
| 242 | .218 | .78 |
| 271 | .120 | 2.79 |
| 301 | .055 | .97 |
| 311 | .190 | .79 |
| 314 | .322 | .55 |
| 324 | .224 | 1.05 |
| 331 | .184 | 1.23 |
| 332 | .172 | 1.15 |
| 336 | .164 | 1.24 |
| 341 | .182 | 1.26 |

rather small sample that those industries which have higher specific training requirements have lower employment reactions.

With respect to the second hypothesis that the size of $\gamma_0$ for an industry is related to the degree of unionization in the industry, one would expect the size of $\gamma_0$ to be inversely related to the degree of unionization. Highly unionized industries may have less freedom of action regarding short-run employment decisions, and they may thus react less to current output changes than industries which have less union pressure. From a study by DOUTY (1960) data are available at the two-digit industry level for 1958 on the percent of workers employed in establishments in which the majority of workers are unionized. In order to use these data, the three-digit industries considered in this study had to be grouped into their respective two-digit industries by weighting the estimates of $\gamma_0$ in the manner described above. This meant grouping the Food industries 201 and 207 together, the Tobacco industries 211 and 212 together, the Apparel industries 231, 232, and 233 together, the Leather industries 311 and 314 together, and the Primary Metals industries 331, 332, and 336 together. This gave a total of ten groups for which weighted estimates of $\gamma_0$ and figures on the percent of workers in

TABLE 9.11

*Estimates of $\gamma_0$ and of the percent of workers employed in establishments in which the majority of workers are unionized for ten industry groups*

| Industry or industry group | Estimate or weighted estimate of $\gamma_0$ (from table 4.3) | Percent |
|---|---|---|
| 201 and 207 | .264 | 68.1 |
| 211 and 212 | .118 | 62.6 |
| 231, 232 and 233 | .141 | 59.7 |
| 242 | .218 | 43.8 |
| 271 | .120 | 65.3 |
| 301 | .055 | 80.6 |
| 311 and 314 | .304 | 49.3 |
| 324 | .224 | 77.9 |
| 331, 332 and 336 | .180 | 88.6 |
| 341 | .182 | 70.6 |

establishments in which the majority of workers are unionized were available. The data for these ten industry groups are presented in table 9.11.

The Kendall Tau rank correlation coefficient was calculated using the ten observations presented in table 9.11. The coefficient was −.20, which is of the right sign (the larger the degree of union pressure the smaller the employment reaction to current output changes) but which is not significant at even the ten-percent level. The hypothesis that the degree of unionization and the size of the employment reaction are inversely correlated, therefore, is not confirmed from the test. The test is based on only a small number of observations, however, and any conclusion must remain tentative.

With respect to the third hypothesis that the size of $\gamma_0$ for an industry is related to the average wage level in the industry, the expectation as to whether the size of $\gamma_0$ should be positively or negatively related to the average wage level is not unambiguous. On the one hand, a high wage level means that it is expensive to hold excess labor, and this may lead to a larger reaction to current output changes. On the other hand, a high wage level means that the workers are likely to be more skilled and perhaps more specifically trained, and this may lead to a smaller reaction to current output change since firms may be reluctant to lay off these workers for fear of not being able to get them back when they are needed again.

Average yearly wage levels are available for the seventeen three-digit industries considered in this study from the US DEPARTMENT OF COMMERCE (1967b), and these data were collected for the year 1958. There were thus a total of seventeen industries for which observations on the size of $\gamma_0$ and on the average wage level were available. These observations are presented in table 9.12.

TABLE 9.12

*Estimates of $\gamma_0$ and of the average wage level for the seventeen industries*

| Industry | Estimate of $\gamma_0$ (from table 4.3) | Average wage level for 1958 |
|---|---|---|
| 201 | .265 | 2.18 |
| 207 | .262 | 1.68 |
| 211 | .086 | 2.08 |
| 212 | .154 | 1.40 |
| 231 | .127 | 1.74 |
| 232 | .118 | 1.29 |
| 233 | .164 | 1.61 |
| 242 | .218 | 1.59 |
| 271 | .120 | 2.80 |
| 301 | .055 | 2.92 |
| 311 | .190 | 2.10 |
| 314 | .322 | 1.56 |
| 324 | .224 | 2.45 |
| 331 | .184 | 3.10 |
| 332 | .172 | 2.41 |
| 336 | .164 | 2.39 |
| 341 | .182 | 2.65 |

The Kendall Tau rank correlation coefficient was calculated using the seventeen observations presented in table 9.12. The coefficient was $-.07$, the sign of which implies that a high wage level corresponds to a smaller employment reaction. The coefficient is not significant at even the ten-percent confidence level, however, and there seems to be little relationship between the average wage level in an industry and the employment reaction in the industry.

In summary, then, the size of an industry's employment reaction to current output changes appears to be inversely related to the amount of specific training required in the industry, but does not appear to be related to the degree of union pressure nor the average wage level in the industry. Since the results were based on very small samples, however, and the tests using simple rank correlations were rather crude, the conclusions reached here must remain very tentative.