6 Inventory Investment

6.1 Introduction

In this chapter the inventory investment equation will be discussed. Inventory investment has much in common with consumption in the sense that it is extremely important in the determination of short-run fluctuations in GNP and at the same time difficult to explain. An eclectic point of view was taken in developing the inventory investment equation in this chapter. Essentially four basic approaches were tried before arriving at the final version. In Section 6.2 the basic theoretical model will be presented and the four approaches will be described. In Section 6.3 the results of following the four approaches will be discussed and the final equation will be presented. A summary of the results of the chapter will be presented in Section 6.4

6.2 The Four Approaches

The Basic Theoretical Model

Let V_t denote the aggregate stock of inventories at the end of period t and let V_t^* denote the desired stock for the end of period t. A basic model of inventory investment that has been widely used is the following simple stock adjustment model:

$$V_t - V_{t-1} = q(V_t^* - V_{t-1}), \qquad 0 \leqq q \leqq 1, \tag{6.1}$$

where

$$V_t^* = a_0 + a_1 SALES_t. \tag{6.2}$$

$SALES_t$ in (6.2) denotes the level of aggregate sales during period t. Equation (6.1) states that the change in the stock of inventories during period t is a function of the difference between the desired stock for the end of period t and the actual stock on hand at the end of period $t - 1$, and equation (6.2) states that the desired stock for the end of period t is a function of the level of sales for the period.

Combining equations (6.1) and (6.2) yields

$$V_t - V_{t-1} = qa_0 + qa_1 SALES_t - qV_{t-1}. \tag{6.3}$$

In other words, inventory investment in period t is a function of the level of sales in period t and of the stock of inventories on hand at the end of period $t-1$. The variable that has been used to measure sales in this study will be discussed in Section 6.3 below.

There are a number of directions in which one can go from this simple model to more complicated and perhaps more realistic models. The four approaches that have been tried in this study are the following.

The First Approach: Disaggregation

V_t as defined above is an aggregate of inventories from manufacturing, retail trade, wholesale trade, construction, and others. Each of these in turn is an aggregate of many dissimilar firms; and, for the manufacturing sector, finished goods inventories, work in progress, and materials and supplies are aggregated together as well. One would not expect the determinants of inventory investment to be the same for all firms and types of inventories, and so disaggregating may prove to be quite helpful.

The Second Approach: The Effect of Expectations

It is well known that expectations play an important role in the determination of inventory investment. The $SALES_t$ variable in equation (6.2) really should be expected sales, since decisions on inventory investment are presumably made before the sales of period t are known. A simple model, which is in the spirit of the work of Lovell [34] and others, is the following.

Let $PROD_t$ denote the aggregate amount produced during period t (as opposed to the amount sold, $SALES_t$). By definition

$$V_t - V_{t-1} = PROD_t - SALES_t, \tag{6.4}$$

i.e., the change in inventories during period t is the difference between production of that period and sales. Planned production, $PROD_t^e$ (the plans being made at the beginning of period t) is assumed to be

$$PROD_t^e = SALES_t^e + b_0(V_t^* - V_{t-1}), \qquad 0 \leqq b_0 \leqq 1, \tag{6.5}$$

where $SALES_t^e$ denotes the expected level of sales for period t, the expectations also being made at the beginning of period t. Equation (6.5) states that planned production is equal to expected level of sales plus an amount that reflects the partial adjustment of the stock of inventories to its desired level.

As the period progresses, actual sales deviate from expected sales, and firms may have enough flexibility in their production plans to change them as a result of the unexpected change in sales. It is thus assumed that

$$PROD_t - PROD_t^e = b_1(SALES_t - SALES_t^e), \qquad 0 \leqq b_1 \leqq 1. \tag{6.6}$$

If b_1 is equal to 1 in equation (6.6), then firms have complete flexibility in their production plans and never produce more or less than they would like to given the level of sales that actually occurs during period t. If b_1 is equal to 0, then firms have no flexibility and produce what they decide to produce at the beginning of the period regardless of what happens to sales. Most firms, of course, are probably somewhere between these two extremes.

Adding equations (6.5) and (6.6), solving equation (6.4) for $PROD_t$, and substituting the resulting expression for $PROD_t$ into the sum of (6.5) and (6.6) yields

$$V_t - V_{t-1} = (1 - b_1)(SALES_t^e - SALES_t) + b_0(V_t^* - V_{t-1}). \tag{6.7}$$

If desired inventories are then taken to be a function of expected sales (which is similar to the assumption made in (6.2)),

$$V_t^* = c_0 + c_1 SALES_t^e, \tag{6.8}$$

then equation (6.7) gives inventory investment as a function of expected sales, the stock of inventories at the end of the previous period, and the difference between expected and actual sales:

$$V_t - V_{t-1} = b_0 c_0 + b_0 c_1 SALES_t^e - b_0 V_{t-1} + (1 - b_1)(SALES_t^e - SALES_t). \tag{6.9}$$

Equation (6.9) cannot be estimated directly because expected sales are not directly observed. A simple assumption that can be made about how expectations are formed is the following:

$$SALES_t^e = SALES_{t-1} + \bar{S}, \tag{6.10}$$

where $\bar{S}$ is a constant. Equation (6.10) states that the level of sales expected for period t is equal to the observed level of sales for period $t - 1$ plus some constant amount. In other words, the *change* in sales is expected to be constant from quarter to quarter. The assumption in (6.10) has been used in the empirical work below. While the assumption is quite simple, it is unlikely that the aggregate data used in this study are capable of distinguishing among more complicated expectational hypotheses. Indeed, even the concept of an aggregate level of expected sales is somewhat vague.

The expression for $SALES_t^e$ in equation (6.10) can be substituted into equation (6.9) to eliminate $SALES_t^e$ from the equation. This yields:

$$\begin{aligned} V_t - V_{t-1} = {} & [b_0 c_0 + b_0 c_1 \bar{S} + (1 - b_1)\bar{S}] + b_0 c_1 SALES_{t-1} \\ & - b_0 V_{t-1} + (1 - b_1)(SALES_{t-1} - SALES_t). \end{aligned} \tag{6.11}$$

Equation (6.11) is now in a form that can be estimated, given the measure of sales that is to be used.

The Third Approach: A More Complicated Adjustment Process

A third way in which the model introduced at the beginning of this section can be expanded is by assuming a more complicated adjustment process than (6.1) for inventory investment. Assume first of all that desired inventory investment for period t, denoted as $(V_t - V_{t-1})^d$, is

$$(V_t - V_{t-1})^d = q_0(V_t^* - V_{t-1}), \qquad 0 \leqq q_0 \leqq 1, \tag{6.12}$$

where V_t^* still denotes the desired stock of inventories for the end of period t. V_t^* is assumed to be a function of $SALES_t$ as postulated in equation (6.2). It is now further assumed that desired inventory investment is subject to an adjustment process:

$$\begin{aligned} (V_t - V_{t-1}) - (V_{t-1} - V_{t-2}) = q_1[(V_t - V_{t-1})^d - (V_{t-1} - V_{t-2})], \\ 0 \leqq q_1 \leqq 1. \end{aligned} \tag{6.13}$$

In other words, it is assumed that, due to adjustment costs and the like, only part of the desired inventory investment is actually achieved during any one period.

Combining equations (6.2), (6.12), and (6.13) yields

$$V_t - V_{t-1} = q_1 q_0 a_0 + q_1 q_0 a_1 SALES_t - q_1 q_0 V_{t-1} + (1 - q_1)(V_{t-1} - V_{t-2}), \tag{6.14}$$

which is equivalent to adding the lagged dependent variable, $V_{t-1} - V_{t-2}$, to the basic equation (6.3). Equation (6.14) can be further complicated by making the above assumptions about how expectations effect inventory investment. Doing this results in the variable $V_{t-1} - V_{t-2}$ being added to equation (6.9).

The Fourth Approach: Adding Other Variables

A fourth way of trying to improve the explanatory power of equation (6.3), especially for forecasting purposes, is to add various expectational variables. One of the more successful attempts in this area has been the work of Friend and Taubman [23]. They add the plant and equipment investment expectation variable, $PE2_t$, to an equation like (6.3) and find that this variable is highly significant and improves the fit of the equation considerably. This is probably due to the fact that capital goods require a relatively long time to complete, so that large plant and equipment expenditures require that large stocks of inventories be held during the construction period. $PE2_t$ is a particularly desirable variable to use in a forecasting model because data or proxies on it are available ahead of the prediction period. In other studies, variables like unfilled orders, the change in unfilled orders, and Department of Defense obligations (either current or lagged values) have been added to equations like (6.3), with partial success in some cases. (See, for example, Darling and Lovell [7].) These variables are of limited use in a forecasting model, however, because of the difficulties involved in trying to explain them within the model or else forecast them exogenously.

Two new series that may prove to be useful for forecasting purposes have recently become available from a quarterly survey of manufacturing firms conducted by the OBE. The survey is conducted in February, May, August, and November of each year, and firms are asked to estimate the level of inventories they expect for the current quarter and the forthcoming quarter. In addition, they are asked to evaluate the condition of their inventories (high, about right, or low) relative to their sales and their unfilled orders position as of the last day of the previous quarter (December 31, March 31, June 30, and September 30, respectively). The inventory expectations series are adjusted for "systematic tendencies," and the figures are published in March, June, September, and December issues of the *Survey of Current Business*. Also published in these issues are series on the percent of firms (weighted by inventory book values) reporting their inventory conditions as high, about right, and low. The two series on inventory expectations are available from the third quarter of 1961 to the present, and the series on inventory conditions are available from the first quarter of 1959 to the present.

For purposes of the discussion below, $VE1_t$ will denote the one-quarter-ahead expectation of the stock of inventories for quarter t for all of manufacturing, $VE2_t$ will denote the two-quarter-ahead expectation for quarter t, and VH_t will denote a variable which is defined as the percent of firms reporting their inventory conditions as high minus the percent reporting their conditions as low (for all manufacturing). For VH_t the t refers to the quarter for which the evaluation was made. (For example, for the evaluation concerning inventory conditions as of 31 December 1967, the t is 674.) VH_t is meant to be a measure of how dissatisfied manufacturing firms are with their stock of inventories.

6.3 The Results

In developing the inventory equation for the present model, essentially all four of the above approaches were tried. With respect to the disaggregation question (the first approach), an attempt was made in the initial phases of this study to disaggregate total inventory investment into that for durable manufacturing, nondurable manufacturing, retail trade, wholesale trade, and all other. This attempt failed. The estimates of the individual equations were of dubious quality, and when tests like those described in Chapter 11 were performed, the versions of the model that included the disaggregated inventory investment equations yielded poorer results than the versions that included only one aggregated inventory equation. The results of this attempt will not be presented here.

There are two probable reasons why this attempt failed. In the first place, the disaggregation was not a true disaggregation, since an aggregate sales variable was used for each equation rather than the sales of the individual sector. This is admittedly a questionable procedure, but attempting to forecast or explain sales of individual sectors of the economy is beyond the scope of the present model. Secondly, the "all other" category of necessity included the inventory valuation adjustment figures of the OBE. This variable is subject to large short-run fluctuations and is difficult to explain, at least within the context of this model. The failure here, therefore, does not necessarily indicate that it is undesirable to disaggregate inventory investment, but only that to do so requires a considerably larger model than the one developed here.

The second approach, determining the effect of sales expectations on inventory investment, did meet with some success. In following the approach, it was necessary to decide which variable to use for the aggregate sales variable in equation (6.11). A number of variables were tried, and the one that gave the best results was the sum of durable and nondurable consumption, $CD_t + CN_t$. Adding variables such as consumption of services, CS_t, plant and equipment investment, IP_t, and the federal government defense component of G_t to $CD_t + CN_t$ did not improve the results. The sales variable defined as total GNP less inventory investment, $GNP_t - (V_t - V_{t-1})$, was also tried in place of $CD_t + CN_t$, and again the results were not as good. It definitely appeared to be the case that the sum of durable and nondurable consumption was the primary sales variable affecting aggregate inventory investment.

For all of the estimates, the simple assumption in equation (6.10) about expectations was made: the level of expected sales for period t was assumed to be equal to the actual level of sales in period $t-1$ plus a constant amount. Equation (6.11) was thus the basic equation estimated. Using $CD_t + CN_t$ as the sales variable, the results of estimating equation (6.11) over the larger sample period were:

$$\begin{aligned} V_t - V_{t-1} = &\underset{(6.09)}{-114.76} + \underset{(4.27)}{.728}\,(CD_{t-1} + CN_{t-1}) - \underset{(3.94)}{.357}\,V_{t-1} \\ &+ \underset{(0.42)}{.095}(CD_{t-1} + CN_{t-1} - \widehat{CD}_t - \widehat{CN}_t) \\ &\hat{r} = \underset{(9.15)}{.791} \\ &\mathrm{SE} = 2.540 \\ &R\Delta^2 = .589 \\ &50 \text{ observ.} \\ [1, GNP_{t-1}, CD_{t-1}, &CD_{t-2}, CN_{t-1}, CN_{t-2}, CS_{t-1}, CS_{t-2}, V_{t-1}, \\ &V_{t-2}, G_t, MOOD_{t-2}, PE2_t, PE2_{t-1}]. \end{aligned} \tag{6.15}$$

$V_t - V_{t-1}$ is the change in total business inventories during quarter t seasonally adjusted at annual rates in billions of current dollars, and V_{t-1} is the sum of past inventory investment (the origin being arbitrary[1]). The results in (6.15) definitely indicate that one-quarter-lagged sales are more important in determining inventory investment than are current sales. The coefficient for lagged sales is $.728 + .095$, while the coefficient for current sales is $-.095$. The coefficient for current sales is negative, as expected, but it is small and not significant. This implies from equation (6.11) that b_1 is close to 1, which implies from equation (6.6) that firms have considerable flexibility in changing their short-run production plans. This conclusion is, of course, dependent on the validity of the assumption about expectations in equation (6.10).

The other coefficient estimates in (6.15) are of the expected sign (the constant term is expected to be negative because of the zero origin chosen for

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[1] The arbitrary value for the origin is merely reflected in the estimate of the constant term in the equation. For the work in this study V_t was assumed to be zero in 534.

the V_t series). The standard error of 2.540 billion dollars in (6.15) is larger than the standard errors for any of the other expenditure equations of the model, which reflects the volatile nature of the inventory investment series. The $R\Delta^2$ is .589 in (6.15), which means that 58.9 percent of the variance of the change in inventory investment (i.e., of $(V_t - V_{t-1}) - (V_{t-1} - V_{t-2})$) has been explained. Note that serial correlation is quite pronounced in (6.15): the estimate of the serial correlation coefficient is .791.

Using equation (6.15) as a base, the third and fourth approaches were then tried. With respect to the third approach, the variable $V_{t-1} - V_{t-2}$ was added to equation (6.15) to test for the more complicated adjustment process specified in equation (6.13). The results were:

$$\begin{aligned} V_t - V_{t-1} = {} & \underset{(4.37)}{-118.33} + \underset{(4.56)}{.751}\,(CD_{t-1} + CN_{t-1}) - \underset{(4.23)}{.372}\,V_{t-1} \\ & + \underset{(0.55)}{.126}\,(CD_{t-1} + CN_{t-1} - \widehat{CD}_t - \widehat{CN}_t) \\ & + \underset{(0.72)}{.093}\,(V_{t-1} - V_{t-2}) \\ & \hat{\rho} = \underset{(9.04)}{.788} \\ & \text{SE} = 2.541 \\ & R\Delta^2 = .597 \\ & \text{50 observ.} \\ & [\text{variables same as for (6.15) plus } V_{t-3}]. \end{aligned} \tag{6.16}$$

The $V_{t-1} - V_{t-2}$ variable is not significant in equation (6.16), and the fit of the equation has not been improved from the fit in (6.15). Also, using $V_{t-1} - V_{t-2}$ in (6.16) has not decreased the estimated amount of serial correlation in the equation to any extent. There is thus little evidence of a more complicated adjustment process than the one specified in (6.1), and so equation (6.16) was dropped from further consideration.

With respect to the fourth approach, a number of equations were estimated using $VE1_t$ or $VE2_t$ or the change in these variables, and the results were not very good.[2] The variables did not appear to have any independent

[2] Data on the VE series were available only from 614 on, and the period of estimation included only 27 observations (622 through 694, excluding the three strike quarters). The series was revised in 634, but since the effect of the revision for total manufacturing was slight, the prerevised and revised figures were taken here as one continuous series. The data for $VE1_t$ and $VE2_t$ are presented in Appendix A.

explanatory power in equations like (6.15). Not too much emphasis should be put on these results, however, since the period of estimation was short and since the VE series is actually relevant only for manufacturing inventory investment and not for the aggregate inventory investment series considered here. What can be concluded from these results is that for purposes of forecasting aggregate inventory investment the VE series at present appears to be of little use.

The use of the inventory condition series (the VH series) also did not produce good results. The current and various lagged values of VH were added to equations like (6.15), and none of the results appeared to be an improvement over the results in (6.15).[3] Again, however, the VH series pertains only to manufacturing inventory investment, so that the negative results here should be interpreted with some caution.

The use of the plant and equipment investment expectation series (the PE2 series) produced somewhat better results. When $PE2_t$ was added to equation (6.15), the results were:

$$\begin{aligned} V_t - V_{t-1} = &\underset{(4.52)}{-122.28} + \underset{(4.45)}{.695}\,(CD_{t-1} + CN_{t-1}) - \underset{(4.33)}{.403}\,V_{t-1} \\ &+ \underset{(0.55)}{.121}\,(CD_{t-1} + CN_{t-1} - \widehat{CD}_t - \widehat{CN}_t) \\ &+ \underset{(1.93)}{.470}\,PE2_t \\ &\hat{r} = \underset{(7.38)}{.722} \qquad\qquad (6.17) \\ &\text{SE} = 2.470 \\ &R\Delta^2 = .620 \\ &50 \text{ observ.} \end{aligned}$$

[variables same as for (6.15)].

$PE2_t$ is nearly significant in equation (6.17), and adding it to the equation has had only slight effect on the coefficient estimates of the other variables. The estimate of the serial correlation coefficient has dropped slightly from .791 in (6.15) to .722 in (6.17). The fit of equation (6.17) is only slightly better than the fit of equation (6.15), however, and in general adding $PE2_t$ to the inventory equation has been of only marginal benefit.

[3] The period of estimation used for these regressions was the basic period beginning in 602 (36 observations). The data for the VH_t series are presented in Appendix A.

Aside from its marginal significance in (6.17), there were two main reasons why $PE2_t$ was not included in the final equation explaining inventory investment. The first was that the estimate of the coefficient of $PE2_t$ was not very stable for changes in the sample period. For the sample period ending in 684, for example, the estimate was .696, whereas in equation (6.17) for the sample period ending in 694 the estimate is only .470. The importance of $PE2_t$ in the inventory investment equation clearly decreased throughout 1969. The second reason $PE2_t$ was not included in the final equation is that including $PE2_t$ in the inventory equation means that in the reduced form equation for GNP_t, the coefficient of $PE2_t$ is quite large (since $PE2_t$ also enters with a fairly large coefficient in the plant and equipment investment equation). For forecasting purposes, the model is then quite sensitive to errors made in forecasting $PE2_t$. Because of the marginal significance of $PE2_t$ in equation (6.17) anyway, this sensitivity did not appear to be particularly desirable (even though, as mentioned in Chapter 4, proxies for $PE2_t$ are sometimes available as far as four quarters ahead).

None of the variables considered in the fourth approach, therefore, were included in the final equation, and the basic equation determining inventory investment was taken to be equation (6.15). One other equation was also considered before equation (6.15) was finally chosen, however, and this equation is worth mentioning. Somewhat by accident, both current GNP_t and the current change in durable and nondurable consumption were included, along with V_{t-1}, in the inventory equation. The results were:

$$
\begin{aligned}
V_t - V_{t-1} = {} & \underset{(5.66)}{-94.48} + \underset{(6.25)}{.241}\, \widehat{GNP}_t - \underset{(5.88)}{.368}\, V_{t-1} \\
& \underset{(5.04)}{-.568}\, (\widehat{CD}_t + \widehat{CN}_t - CD_{t-1} - CN_{t-1}) \\
& \hat{r} = \underset{(13.24)}{.882} \qquad\qquad (6.18) \\
& \mathrm{SE} = 1.927 \\
& R\Delta^2 = .763 \\
& 50 \text{ observ.} \\
& \text{[variables same as for (6.15)].}
\end{aligned}
$$

The fit of equation (6.18) is much improved over the fit of equation (6.15). The standard error has changed from 2.540 in (6.15) to 1.927 in (6.18), and the $R\Delta^2$ has risen from .589 to .763. Equation (6.18) has little theoretical justification—presumably the GNP_t variable is reflecting expected sales of some kind and the change in consumption variable is reflecting unexpected sales—but the better fit is impressive. The better fit may, of course, reflect the fact that $V_t - V_{t-1}$ is part of GNP_t, but the two-stage estimation technique should have removed any simultaneous equation bias.

Both equations (6.15) and (6.18) were tested within the context of the overall model in Chapter 11, and somewhat surprisingly, equation (6.15) gave better results. These results will be discussed in Chapter 11. It is encouraging that equation (6.15) performed better, since it is based on much stronger theoretical grounds.

6.4 Summary

The approach taken in this chapter in explaining inventory investment has been an eclectic one. Building on the basic stock adjustment model, an attempt was made to disaggregate total inventory investment into five different components; an attempt was made to account for the effect of sales expectations on inventory investment; a more complicated lag adjustment model was tested; and an attempt was made to add other kinds of expectational results to the basic equation.

The attempt at disaggregation failed, and there was no evidence of a more complicated adjustment process than that specified by the basic model. The attempt to account for sales expectations was fairly successful, and the sales variable that gave the best results was the sum of durable and nondurable consumption. The attempt to add the three inventory expectational variable, $VE1_t$, $VE2_t$, and VH_t, was not successful, although the results were based on relatively few observations. The attempt to add the plant and equipment expectational variable, $PE2_t$, was marginally successful, but $PE2_t$ was not included in the final equation.