Chapter Three

Firms

3.1 THE BASIC EQUATIONS

INTRODUCTION

In Table 3-1 the important symbols used in this chapter are listed in alphabetic order. Each firm, say firm i, borrows money from banks (LF_{it}), hires labor from households (HPF_{it}), buys goods for investment purposes from other firms (INV_{it}), produces goods (Y_{it}), and sells goods to households and the government (XF_{it}). The seven main decision variables of a firm are its price (PF_{it}), its production, its investment, its wage rate (WF_{it}), the amount of money to borrow from the banks, the maximum number of hours that it will pay for ($HPFMAX_{it}$), and the maximum number of goods that it will sell ($XFMAX_{it}$). Firm i receives at the beginning of period t information from the banks on the loan rate it will be charged in the period (RF_{it}) and on the maximum amount of money that it will be able to borrow ($LFMAX_{it}$). The underlying technology of a firm is assumed to be of a "putty-clay" type, where at any one time different types of machines with differing worker-machine ratios can be purchased. The worker-machine ratio is assumed to be fixed for each type of machine.

Table 3-1. Notation for Firms in Alphabetic Order

Non-Condensed Model

Subscript i denotes variable for firm i. Subscript j denotes variable for firm j. Subscript t denotes variable for period t. A p superscript in the text denotes a planned value of the variable, and an e superscript denotes an expected value of the variable.

CF_{it}	= cash flow before taxes and dividends
$\overline{CF}_{it}$	= cash flow net of taxes and dividends
d_1	= profit tax rate
DDF_{it}	= actual demand deposits
DDF_{1it}	= demand deposits set aside for transaction purposes
DDF_{2i}	= demand deposits set aside to be used as a buffer to meet unexpected decreases in cash flow
DEP_{it}	= depreciation
$DIVF_{it}$	= dividends paid
$EMAXHP_i$	= largest error the firm expects to make in overestimating the supply of labor available to it for any period
$EMAXMH_i$	= largest error the firm expects to make in underestimating its worker hour requirements for any period
$\overline{H}$	= maximum number of hours that each machine can be used each period
HP_t	= total number of worker hours paid for in the economy
HPF_{it}	= number of worker hours paid for (by firm i)
HPF_{jt}	= number of worker hours paid for (by firm j)
$HPFMAX_{it}$	= maximum number of worker hours that the firm will pay for
$HPUN_t$	= total unconstrained supply of hours in the economy
I_{nit}	= number of machines of type n purchased (n=1,2)
INV_{it}	= number of goods purchased for investment purposes
$\overline{K}$	= minimum number of machines required to be held in each of the last m periods of the decision horizon
K^a_{nit}	= actual number of machines of type n held (n=1,2)
KH_{nit}	= actual number of machine hours worked on machines of type n (n=1,2)
$KMIN_{nit}$	= minimum number of machines of type n required to produce Y_{nit} (n=1,2)
LF_{it}	= value of loans taken out
$LFMAX_{it}$	= maximum value of loans that the firm can take out
m	= length of life of one machine
MH_{nit}	= number of worker hours worked on machines of type n (n=1,2)
MH_{3it}	= number of worker hours required to handle deviations of inventories from β_1 times sales
MH_{4it}	= number of worker hours required to handle fluctuations in sales
MH_{5it}	= number of worker hours required to handle fluctuations in worker hours paid for
MH_{6it}	= number of worker hours required to handle fluctuations in net investment
MH_{it}	= total number of worker hours required
PF_{it}	= price set (by firm i)
PF_{jt}	= price set (by firm j)
$\overline{PF}_t$	= average price level in the economy

Table 3–1. (continued)

PFF_{it}	= price paid for investment goods
RF_{it}	= loan rate paid
$T+1$	= length of decision horizon
$TAXF_{it}$	= taxes paid
V_{it}	= stock of inventories (of firm i)
V_{jt}	= stock of inventories (of firm j)
WF_{it}	= wage rate (of firm i)
WF_{jt}	= wage rate (of firm j)
$\overline{WF}_t$	= average wage rate in the economy
X_t	= total number of goods sold in the economy
XF_{it}	= number of goods sold (by firm i)
XF_{jt}	= number of goods sold (by firm j)
$XFMAX_{it}$	= maximum number of goods that the firm will sell
Y_{nit}	= number of goods produced on machines of type n ($n=1,2$)
Y_{it}	= total number of goods produced
δ_n	= number of goods it takes to create a machine of type n ($n=1,2$)
λ_n	= amount of output produced per worker hour on machines of type n ($n=1,2$)
μ_n	= amount of output produced per machine hour on machines of type n ($n=1,2$)
ΠF_{it}	= before-tax profits

Condensed Model (For equations in Table 3–4 only.)

Subscript t denotes variable for period t. Superscripts p and pp in Table 3–4 denote a planned value of the variable, and superscript e denotes an expected value of the variable. Unless otherwise stated, the variables refer to the firm sector. Only the notation that differs from the notation for the non-condensed model is presented here.

CF_t	= cash flow before taxes and dividends
$\overline{CF}_t$	= cash flow net of taxes and dividends
DDF_t	= actual demand deposits
DDF_{1t}	= demand deposits set aside for transactions purposes
DDF_2	= demand deposits set aside to be used as a buffer to meet unexpected decreases in cash flow
DEP_t	= depreciation
$EMAXHP$	= largest error the firm sector expects to make in overestimating the supply of labor available to it for any period
$EMAXMH$	= largest error the firm sector expects to make in underestimating its worker hour requirements for any period
HPF_t	= number of worker hours paid for by the firm sector
$HPFMAX_t$	= maximum number of worker hours that the firm sector will pay for
$HPFMAXUN_t$	= maximum number of worker hours that the firm sector would pay for if it were not constrained
INV_t	= number of goods purchased for investment purposes (one good = one machine)
$INVUN_t$	= unconstrained investment demand of the firm sector
K_t^a	= actual number of machines held
KH_t	= number of machine hours worked

Table 3-1. (continued)

$KMIN_t$	= minimum number of machines required to produce Y_t
LF_t	= value of loans taken out
$LFMAX_t$	= maximum value of loans that the firm sector can take out
$LFUN_t$	= unconstrained demand for loans of the firm sector
MH_{1t}	= number of worker hours worked on the machines
MH_{3t}	= number of worker hours required to handle deviations of inventories from β_1 times sales
MH_{4t}	= number of worker hours required to handle fluctuations in sales
MH_{5t}	= number of worker hours required to handle fluctuations in worker hours paid for
MH_{6t}	= number of worker hours required to handle fluctuations in net investment
MH_t	= total number of worker hours required
P_t	= price level
PUN_t	= price level that the firm sector would set if it were not constrained
RL_t	= loan rate
V_t	= stock of inventories
W_t	= wage rate
WUN_t	= wage rate that the firm sector would set if it were not constrained
$XFMAX_t$	= maximum number of goods that the firm sector will sell
Y_t	= total number of goods produced
Y^pUN_t	= number of goods that the firm sector would plan to produce if it were not constrained
λ_1	= amount of output produced per worker hour
μ_1	= amount of output produced per machine hour
ΠF_{it}	= before-tax profits

EQUATIONS REGARDING THE TECHNOLOGY AND CAPITAL AND LABOR REQUIREMENTS

$$MH_{nit} = \frac{Y_{nit}}{\lambda_n},\ n = 1, 2,\ \text{[worker hours required to produce } Y_{nit}\text{]} \tag{3.1}$$

$$KH_{nit} = \frac{Y_{nit}}{\mu_n},\ n = 1, 2,\ \text{[machine hours required to produce } Y_{nit}\text{]} \tag{3.2}$$

$$KMIN_{nit} = \frac{KH_{nit}}{\bar{H}},\ n = 1,2,\ \text{[minimum number of machines required to produce } Y_{nit}\text{]} \tag{3.3}$$

$$K^a_{nit} = K^a_{nit-1} + I_{nit} - I_{nit-m},\ n = 1,2,\ \text{[actual number of machines of type } n \text{ on hand]} \tag{3.4}$$

$$INV_{it} = \sum_{n=1}^{2} \delta_n I_{nit},\ n = 1,2,\ \text{[number of goods purchased for investment purposes]} \tag{3.5}$$

$$Y_{it} = \sum_{n=1}^{2} Y_{nit}, \text{ [total level of output]} \tag{3.6}$$

$$V_{it} = V_{it-1} + Y_{it} - XF_{it}, \text{ [level of inventories]} \tag{3.7}$$

$$MH_{3it} = \beta_2(V_{it} - \beta_1 XF_{it})^2, \beta_1 > 0, \beta_2 > 0, \text{ [worker hours required to maintain deviations of inventories from } \beta_1 \text{ times sales]} \tag{3.8}$$

$$MH_{4it} = \beta_3(XF_{it} - XF_{it-1})^2, \beta_3 > 0, \text{ [worker hours required to handle fluctuations in sales]} \tag{3.9}$$

$$MH_{5it} = \beta_4(HPF_{it-1} - HPF_{it-2})^2, \beta_4 > 0, \text{ [worker hours required to handle fluctuations in hours paid for]} \tag{3.10}$$

$$MH_{6it} = \beta_5 \left[\sum_{n=1}^{2} K^a_{nit} - \sum_{n=1}^{2} K^a_{nit-1}\right]^2, \beta_5 > 0, \text{ [worker hours required to handle fluctuations in net investment]} \tag{3.11}$$

$$MH_{it} = MH_{1it} + MH_{2it} + MH_{3it} + MH_{4it} + MH_{5it} + MH_{6it}, \text{ [total worker hours required]} \tag{3.12}$$

$$K^a_{nit} \geqslant KMIN_{nit}, n=1,2, \text{ [number of machines of type } n \text{ on hand must be greater than or equal to minimum number required]} \tag{3.13}$$

$$HPF_{it} \geqslant MH_{it}. \text{ [worker hours paid for must be greater than or equal to worker hours required]} \tag{3.14}$$

Equation (3.1) defines the number of worker hours required to product output Y_{nit} on machines of type n, and Equation (3.2) defines the number of machine hours required. These two equations reflect the putty-clay nature of the technology. Without loss of generality, the number of different types of machines is taken to be 2.[a] There is assumed to be no technical progress, so that λ_n and μ_n $(n = 1,2)$ are not functions of time. Machines are also assumed not to be subject to physical depreciation, so that λ_n and μ_n $(n = 1,2)$ are not a function of the age of the machines. The machines are assumed to wear out completely after m periods.

Equation (3.3) defines the minimum number of machines of type n required to produce Y_{nit}. It is assumed that $\overline{H}$, the maximum number of hours that each machine can be used each period, is constant over time. Equation (3.4) defines the actual number of machines of each type on hand in period t. Machines purchased in a period are assumed to be able to be used in the production process in that period. In Equation (3.4), I_{nit} is the number of machines of type n purchased in period t and I_{nit-m} is the number of machines of type n that wear out at the end of period $t-1$ and so cannot be used in the production process in period t. The firm is subject to the restriction (3.13), which says that the actual number of machines of type n on hand must be greater than or equal to the minimum number required.

There is assumed to be only one good in the system, which can be used either for consumption or investment purposes. δ_n is the number of goods it takes to create a machine of type n. In Equation (3.5) the number of machines purchased in period t is translated into the equivalent number of goods purchased. To rule out the possibility of one type of machine completely dominating the other in efficiency, it was assumed for the simulation work that $\mu_1 = \mu_2$, so that the types differ from each other only in terms of the λ coefficients. Machines of type 1 were assumed to have a lower worker-machine ratio, $\lambda_1 > \lambda_2$, and to require more goods to create one machine, $\delta_1 > \delta_2$. Equation (3.6) defines the total level of output, and equation (3.7) defines the stock of inventories.

Equations (3.8) through (3.11) define various adjustment costs facing the firm, the costs taking the form of increased worker hour requirements. Equation (3.8) reflects the assumption that there are costs involved in having inventories be either greater than or less than a certain proportion of sales. It is possible that inventory costs are asymmetrical in the sense that negative deviations may be more costly than positive deviations, but for simplicity this possibility was not incorporated into the model. Any positive stock of inventories is, of course, costly to the firm in the sense that the stock must be financed. Equations (3.9)–(3.11) reflect the assumptions that there are costs involved in having sales, worker hours paid for, and net investment fluctuate. The use of the lagged change in worker hours paid for in equation (3.10) is made for computational convenience and is not a critical assumption of the model. Equation (3.12) defines total worker hour requirements. The firm is subject to the restriction (3.14), which says that worker hours paid for must be greater than or equal to worker hour requirements.

EQUATIONS REGARDING FINANCIAL VARIABLES

$$DEP_{it} = \frac{1}{m}(PFF_{it}INV_{it} + PFF_{it-1}INV_{it-1} + \ldots + PFF_{it-m+1}INV_{it-m+1}),$$

[depreciation] (3.15)

$$\Pi F_{it} = PF_{it}Y_{it} - WF_{it}HPF_{it} - DEP_{it} - RF_{it}LF_{it} + (PF_{it} - PF_{it-1})V_{it-1},$$

[before-tax profits] (3.16)

$$TAXF_{it} = d_1\, \Pi F_{it}, \text{ [taxes paid]} \tag{3.17}$$

$$DIVF_{it} = \Pi F_{it} - TAXF_{it}, \text{ [dividends paid]} \tag{3.18}$$

$$CF_{it} = PF_{it}XF_{it} - WF_{it}HPF_{it} - PFF_{it}INV_{it} - RF_{it}LF_{it}, \text{ [cash flow before taxes and dividends]} \tag{3.19}$$

$$\begin{aligned}\overline{CF}_{it} &= CF_{it} - TAXF_{it} - DIVF_{it} \text{ [cash flow net of taxes and dividends]}\\ &= DEP_{it} - PFF_{it}INV_{it} + PF_{it-1}V_{it-1} - PF_{it}V_{it},\end{aligned} \tag{3.20}$$

$$DDF_{it} = DDF_{it-1} + LF_{it} - LF_{it-1} + \overline{CF}_{it}, \text{ [demand deposits]} \tag{3.21}$$

$$LF_{it} \leqslant LFMAX_{it}. \text{ [loan constraint]} \tag{3.22}$$

The government is assumed to allow for tax purposes straight line depreciation, which is reflected in Equation (3.15). Equation (3.16) defines before-tax profits on an accounting basis, which is equal to price times output less wage costs, depreciation, and interest costs and plus any gains or losses on the stock of inventories due to price changes. Taxes are defined in Equation (3.17), where d_1 is the profit tax rate.

The firm is assumed not to retain any earnings, so that the level of dividends, as defined in Equation (3.18), is merely the difference between before-tax profits and taxes. Equation (3.19) defines cash flow gross of taxes and dividends, and Equation (3.20) defines cash flow net of taxes and dividends. The level of demand deposits, defined in Equation (3.21), is a residual in the model, given the loans of the firm and its cash flow net of taxes and dividends. The firm's level of loans is a decision variable, and its determination is discussed in Section 3.3. The firm is subject to the loan constraint (3.22).

3.2 THE FORMATION OF EXPECTATIONS

As was the case for banks, let $T+1$ be the length of the decision horizon. In order for the firm to solve its control problem at the beginning of period t, it must form expectations of a number of variables for periods t through $t+T$. Firm i is assumed to form the following expectations:[b]

$$\frac{PF^e_{jt}}{PF_{jt-1}} = \left(\frac{PF_{it-1}}{PF_{jt-1}}\right)^{\beta_6} \left(\frac{V_{jt-1}}{\beta_1 XF_{jt-1}}\right)^{\beta_7}, \ \beta_6 > 0,\ \beta_7 < 0, \text{ [expected price of firm } j \text{ for period } t] \tag{3.23}$$

$$\frac{PF^e_{jt+k}}{PF^e_{jt+k-1}} = \left(\frac{PF_{it+k-1}}{PF^e_{jt+k-1}}\right)^{\beta_6}, \text{ [expected price of firm } j \text{ for period } t+k\ (k=1,2,\ldots,T)] \tag{3.24}$$

$$\overline{PF}^e_{t+k} = (PF_{it+k} \cdot PF^e_{jt+k})^{\frac{1}{2}}, \text{ [expected average price for period } t{+}k\ (k{=}0,1,\ldots,T)] \quad (3.25)$$

$$X^e_t = X_{t-1} \left(\frac{\overline{PF}^e_t}{\overline{PF}_{t-1}} \right)^{\beta_8}, \beta_8 < 0, \text{ [expected aggregate demand for goods for period } t] \quad (3.26)$$

$$X^e_{t+k} = X^e_{t+k-1} \left(\frac{\overline{PF}^e_{t+k}}{\overline{PF}^e_{t+k-1}} \right)^{\beta_8}, \text{ [expected aggregate demand for goods for period } t{+}k\ (k{=}1,2,\ldots,T)] \quad (3.27)$$

$$\frac{XF^e_{it}}{X^e_t} = \frac{XF_{it-1}}{X_{t-1}} \left(\frac{PF_{it}}{PF^e_{jt}} \right)^{\beta_9}, \beta_9 < 0, \text{ [expected market share of goods for period } t] \quad (3.28)$$

$$\frac{XF^e_{it+k}}{X^e_{t+k}} = \frac{XF^e_{it+k-1}}{X^e_{t+k-1}} \left(\frac{PF_{it+k}}{PF^e_{jt+k}} \right)^{\beta_9}, \text{ [expected market share of goods for period } t{+}k\ (k{=}1,2,\ldots,T)] \quad (3.29)$$

$$\frac{WF^e_{jt}}{WF_{jt-1}} = \left(\frac{WF_{it-1}}{WF_{jt-1}} \right)^{\beta_{10}}, \beta_{10} > 0, \text{ [expected wage rate of firm } j \text{ for period } t] \quad (3.30)$$

$$\frac{WF^e_{jt+k}}{WF^e_{jt+k-1}} = \left(\frac{WF_{it+k-1}}{WF^e_{jt+k-1}} \right)^{\beta_{10}}, \text{ [expected wage rate of firm } j \text{ for period } t{+}k\ (k{=}1,2,\ldots,T)] \quad (3.31)$$

$$\overline{WF}^e_{t+k} = (WF_{it+k} \cdot WF^e_{jt+k})^{\frac{1}{2}}, \text{ [expected average wage rate for period } t{+}k\ (k{=}0,1,\ldots,T)] \quad (3.32)$$

$$HPUN^e_t = HPUN_{t-1} \left(\frac{\overline{WF}^e_t}{\overline{WF}_{t-1}} \right)^{\beta_{11}} \left(\frac{\overline{PF}^e_t}{\overline{PF}_{t-1}} \right)^{\beta_{12}}, \beta_{11} > 0, \beta_{12} < 0, \text{ [expected aggregate unconstrained supply of labor for period } t] \quad (3.33)$$

$$HPUN^e_{t+k} = HPUN^e_{t+k-1} \left(\frac{\overline{WF}^e_{t+k}}{\overline{WF}^e_{t+k-1}} \right)^{\beta_{11}} \left(\frac{\overline{PF}^e_{t+k}}{\overline{PF}^e_{t+k-1}} \right)^{\beta_{12}}, \text{ [expected aggregate unconstrained supply of labor for period } t{+}k\ (k{=}1,2,\ldots,T)] \quad (3.34)$$

$$HP^e_{t+k} = HPUN^e_{t+k}, \text{ [expected aggregate constrained supply of labor for period } t{+}k\ (k{=}0,1,\ldots,T)] \quad (3.35)$$

$$\frac{HPF_{it}^{e}}{HP_{t}^{e}}=\frac{HPF_{it-1}}{HP_{t-1}}\left(\frac{WF_{it}}{WF_{jt}^{e}}\right)^{\beta_{13}},\ \beta_{13}>0,\ \text{[expected market share of labor for period } t\text{]} \tag{3.36}$$

$$\frac{HPF_{it+k}^{e}}{HP_{t+k}^{e}}=\frac{HPF_{it+k-1}^{e}}{HP_{t+k-1}^{e}}\left(\frac{WF_{it+k}}{WF_{jt+k}^{e}}\right)^{\beta_{13}},\ \text{[expected market share of labor for period } t+k\ (k=1,2,\ldots,T)\text{]} \tag{3.37}$$

$$PFF_{it+k}^{e}=\overline{PF}_{t+k}^{e},\ \text{[expected price of investment goods for period } t+k\ (k=0,1,\ldots,T)\text{]} \tag{3.38}$$

$$RF_{it+k}^{e}=RF_{it}.\ \text{[expected loan rate for period } t+k\ (k=1,2,\ldots,T)\text{]} \tag{3.39}$$

The first term on the right-hand side of Equation (3.23) reflects the fact that firm i expects its price setting behavior in period $t-1$ to have an effect on firm j's price setting behavior in period t. The second term is designed to represent the effect of market conditions on firm i's expectation of firm j's price. If, for example, firm j's stock of inventories at the end of period $t-1$ is greater than a certain proportion of sales, then firm i is assumed to expect that firm j will respond to this situation by lowering its price in period t in an effort to increase sales and draw down inventories.

Firm i must also form expectations of firm j's price for periods $t+1$ and beyond. These expectations are specified in Equation (3.24), which is the same as Equation (3.23) without the final term. Equation (3.24) means that firm i expects that firm j is always adjusting its price toward firm i's price. If firm i's price is constant over time, then firm i expects that firm j's price will gradually approach this value.

In Equation (3.25) firm i's expectation of the average price level is taken to be the geometric average of its price and its expectation of firm j's price. Without loss of generality, there is assumed to be only one other firm, firm j, in existence. (As was the case for banks, it should be obvious how the number of other firms in existence can be generalized to be more than one.) The geometric average is used in (3.25) rather than the arithmetic average to make the solution of the model easier. Firm i expects that the aggregate demand for goods is a function of the average price level, as specified in Equations (3.26) and (3.27).[c]

An important difference between Equations (3.26) and (3.27) for firms and Equations (2.11)–(2.13) for banks is that firms are assumed not to observe the unconstrained demand for goods, whereas banks are assumed to observe the unconstrained demand for loans. Equation (3.26), for example, is in terms of the actual (constrained) demand for goods, whereas Equation (2.11) is in terms of the unconstrained demand for loans. The rationale for this difference in assumptions has to do with the fact that the loan constraints on firms and households are likely to be binding more often than are the goods constraints on households.

The maximum number of goods that a firm will sell in a period is equal to the sum of the amount it knows it can produce in the period and the amount it has in inventories at the beginning of the period. Since firms usually hold a nonnegligible stock of inventories, the maximum number of goods that a firm will sell in a period is in most cases likely to be much larger than what it expects to sell and what it actually sells. Therefore, firms will not in general be turning customers away from buying their goods even if they set their prices of goods too low (in the sense that their actual sales exceed their expected sales), whereas banks will be turning customers away from taking out loans if they set their loan rates too low. It thus seems reasonable to assume that banks observe the unconstrained demand for loans because they turn customers away, and that firms do not observe the unconstrained demand for goods because they seldom turn customers away. On this same line of reasoning, it also seems reasonable to assume, as is done below, that firms observe the unconstrained supply of labor because they turn workers away when they set their wage rates too high.

Equations (3.28) and (3.29) determine firm *i*'s expectations of its market share of goods for periods *t* and beyond and are similar to Equations (2.14) and (2.15) for banks. The equations reflect the assumption that a firm expects that its market share of goods is a function of its price relative to the prices of other firms.

Firm *i*'s expectation of firm *j*'s wage rate is specified in Equations (3.30) and (3.31). Equation (3.30) for the wage rate is similar to Equation (3.23) for the price level, without the final term. Firm *i* is assumed to have no other basis upon which to base its expectation of firm *j*'s wage rate for period *t* than its and firm *j*'s wage rates for period $t-1$. Equation (3.32), defining firm *i*'s expectation of the average wage rate, is similar to Equation (3.24).

Firm *i* expects that the aggregate unconstrained supply of labor is a positive function of the average wage rate and a negative function of the average price level, as specified in Equations (3.33) and (3.34). Equations (3.33) and (3.34) for firms are similar to Equations (2.11) and (2.12) for banks. As mentioned above, firms are assumed to observe the unconstrained supply of labor.[d] Equation (3.35) states that firm *i* expects that households will not be constrained in their work behavior in periods *t* and beyond. The same justification for this equation can be made as was made for Equation (2.13) for banks. As will be seen below, firm *i* does not itself expect to turn any workers away, and so Equation (3.35) merely states that firm *i* also does not expect any workers in the aggregate to be turned away.

Equations (3.36) and (3.37) determine firm i's expectations of its market share of labor for periods t and beyond. The equations are similar to Equations (2.14)-(2.15) and (3.28)-(3.29) and require no further discussion here.

Aside from a few details, the symmetry of specifications among Equations (2.8)-(2.15) for banks and loans, Equations (3.23)-(3.29) for firms and goods, and Equations (3.30)-(3.37) for firms and labor should be obvious. Each set of equations is based on the assumption that a bank or firm expects that its behavior has an effect on the behavior of its competitors and that its market share is a function of the relationship of its price to the prices of its competitors.

Equation (3.38) states that firm i expects that the price that it must pay for investment goods each period is the expected average price level for that period. The firm is assumed not to be able to produce its own investment goods. Equation (3.39) states that firm i expects that the loan rate for all future periods is going to be the same as the loan rate for period t. Regarding this latter assumption, it would be possible, since banks determine optimal loan rate *paths,* to make the alternative assumption that banks inform firms of the planned future values of the loan rate in addition to the current value. It seemed more straightforward in this case, however, just to assume that firms make the expectations themselves.

3.3 BEHAVIORAL ASSUMPTIONS

The objective of the firm is to maximize the present discounted value of expected future after-tax cash flow. The discount rate is assumed to be the loan rate. The objective function of firm i at the beginning of period t is:

$$OBJF_{it} = \frac{CF^e_{it} - TAXF^e_{it}}{(1+RF_{it})} + \frac{CF^e_{it+1} - TAXF^e_{it+1}}{(1+RF_{it})\,(1+RF^e_{it+1})} + \ldots + \frac{CF^e_{it+T} - TAXF^e_{it+T}}{(1+RF_{it})\,(1+RF^e_{it+1}) \ldots (1+RF^e_{it+T})}, \tag{3.40}$$

where $CF^e_{it+k} - TAXE^e_{it+k}$ is the expected value of after-tax cash flow for period $t{+}k$ $(k = 0, 1, \ldots, T)$. The decision variables of the firm are its price, PF_{it+k}, its wage rate, WF_{it+k}, the number of each type of machine to buy, I_{1it+k} and I_{2it+k}, the planned number of goods to produce on each type of machine, Y^p_{1it+k} and Y^p_{2it+k}, the amount of money to borrow, LF_{it+k} $(k = 0, 1, \ldots, T)$, the maximum number of hours to pay for, $HPFMAX_{it}$, and the maximum number of goods to sell, $XFMAX_{it}$.

Given a set of paths of the decision variables, the corresponding value of the objective function can be computed as follows.

1. Given firm i's price path, firm i's expectation of firm j's price path can be computed from (3.23) and (3.24). The path of the expected average price level can then be computed from (3.25), followed by the path of expected aggregate demand from (3.26) and (3.27). Firm i's expectation of its own sales path can then be computed from (3.28) and (3.29). Given firm i's expected sales path, its expected path of inventories can be computed from (3.7).[e]
2. Given firm i's wage path, firm i's expectation of firm j's wage path can be computed from (3.30) and (3.31). The path of the expected average wage rate can then be computed from (3.32), followed by the path of the expected aggregate unconstrained supply of labor from (3.33) and (3.34), and then by the path of the expected aggregate constrained supply of labor from (3.35). Firm i's expectation of the supply of labor available to it can then be computed from (3.36) and (3.37).
3. Given paths of the number of each type of machine to buy, the path of investment denominated in goods can be computed from (3.5). The path of depreciation can then be computed from (3.15), given the path of the expected price of investment goods from (3.38).
4. Given the above paths and the path of the expected loan rate from (3.39), the paths of profits, taxes, and cash flow can be computed from (3.16), (3.17), and (3.19), which then means that the value of the objective function can be computed.

The firm is restricted in each period by (3.13) and (3.14) and by various nonegativity properties, such as the fact that the stock of inventories must be nonnegative. For any set of paths of the decision variables, these restrictions can be checked by solving Equations (3.1) through (3.12) and then making the appropriate checks. The firm is also constrained in the current period by the loan constraint (3.22). Regarding the possibility of the loan constraint existing for future periods as well, firm i is assumed to expect that the loan constraint will *not* be binding in periods beyond t. Banks, in other words, are assumed to communicate the maximum loan values to firms only for period t, and firms are assumed to expect that the maximum values in the future will be large enough so as not to be binding. This was the simplest assumption to make, and having the constraint hold only for period t appeared to have an important enough influence on the firm's decision values for period t so as to make further restrictions unnecessary.

The following two end-point constraints were also imposed on the firm.

$$V^e_{it+T} = \beta_1 XF^e_{it+T}\,, \tag{3.41}$$

$$K^a_{1it-k} + K^a_{2iT-k} \geqslant \bar{K},\ k = 0,1,\ldots,m-1. \tag{3.42}$$

The level of inventories at the end of the decision horizon was forced to be equal to β_1 times sales of the last period, and the number of machines held in each of the last m periods was required to be greater than or equal to a given number. These conditions were imposed to avoid quirks that would otherwise be likely to show up in the optimal paths near the end of the horizon.

A few general remarks can now be made regarding the control problem of the firm. The firm expects that it will gain customers by lowering its price relative to the expected prices of other firms. The main expected costs to the firm from lowering its price, in addition to the lower price it is charging per good, are the adjustment costs (3.9), (3.10), and (3.11) involved in increasing sales, employment, and investment. The firm also expects that other firms will follow it if it lowers its price, so that it does not expect to be able to capture an ever increasing share of the market without further and future price reductions.

The firm expects that it will lose customers by raising its price relative to the expected prices of other firms. The main costs from doing this, aside from the lost customers, are the adjustment costs. On the plus side, the firm expects that other firms will follow it if it raises its price, so that it does not expect to lose an ever increasing share of the market without further and further price increases.

The firm expects that it will gain workers if it raises its wage rate relative to the expected wage rates of other firms and lose workers if it lowers its wage rate relative to the expected wage rates of other firms. The firm also expects that other firms will follow it if it raises (lowers) its wage rate, so that it does not expect to capture (lose) an ever increasing share of the market without further and further wage rate increases (decreases).

Because of the various adjustment costs, the firm, if it chooses to lower its production, may choose in the current period not to lower its employment and capital stock to the minimum levels required. The firm may thus plan to hold either excess labor or excess capital or both during certain periods.

Before concluding this section, the determination of the three decision variables $HPFMAX_{it}$, $XFMAX_{it}$, and LF_{it+k} ($k = 0,1,\ldots,T$), must be described. Consider $HPFMAX_{it}$ first. As was the case for banks, a firm must prepare for the possibility that its expectations are incorrect. In the case of worker hours paid for, a firm must prepare for the possibility that it underestimates the supply of labor available to it at the wage rate that it has set.

A firm is assumed to prepare for this possibility by announcing to households not only the wage rate that it will pay in the period t, but also the maximum number of hours that it will pay for in the period, $HPFMAX_{it}$. This maximum is assumed to be set equal to the number of hours the firm expects to pay for from Equation (3.36), given its wage rate, its expectation of firm j's wage rate from Equation (3.30), and its expectations of the aggregate supply of labor from Equation (3.35).[f] Thus

$$HPFMAX_{it} = HPF^e_{it} \, . \tag{3.43}$$

By setting this maximum, a firm will never have to hire more labor than it expects to hire.

Regarding $XFMAX_{it}$, a firm must prepare for the possibility that the demand for its goods at the price it has set is greater than the amount that it can supply. A firm is assumed to prepare for this possibility by announcing to households not only the price that it will charge in period t, but also the maximum number of goods that it will sell in the period, $XFMAX_{it}$. This maximum is assumed to be

$$XFMAX_{it} = min \left\{ Y^p_{it}, \, [Y^p_{it} - \lambda_2 \, (EMAXHP_i + EMAXMH_i) + \lambda_2 (HPF^e_{it} - MH^e_{it})] \right\} + V_{it-1} , \tag{3.44}$$

where

Y^p_{it} = planned output for period t,

$EMAXHP_i$ = largest error the firm expects to make in overestimating the supply of labor available to it for any period,

$EMAXMH_i$ = largest error the firm expects to make in underestimating its worker hour requirements for any period,

$HPF^e_{it} - MH^e_{it}$ = expected amount of excess labor for period t.

What Equation (3.44) states is the following. If the firm were assured of being able to produce in period t all it had planned at the beginning of the period to produce, then it could sell in period t $Y^p_{it} + V_{it-1}$. It may, however, either overestimate the supply of labor available to it or underestimate its worker hour requirements,[g] or both, which will force it to produce less than it had planned unless it had planned to hold enough excess labor to make up the slack. Since machines of type 2 are less efficient absolutely than machines of type 1, if a firm has to cut back on its planned production, it will cut production on machines of type 2 first. Therefore, $\lambda_2(EMAXHP_i + EMAXMH_i)$ is the maximum amount of output the firm expects to have to cut back because of its expectation errors. (If the firm is not using any machines of type 2, then λ_1 replaces λ_2 in (3.44)).

If the firm had planned to hold excess labor, then amount $\lambda_2(HPF_t^e - MH_t^e)$ can be produced from taking up the excess labor slack. Therefore, the amount in square brackets in Equation (3.44) is the amount the firm knows it can produce in period t even if it overestimates its labor supply and underestimates its worker hour requirements by the maximum amounts. It is possible, if the firm plans to hold a lot of excess labor in period t, for $HPF_{it}^e - MH_{it}^e$ to be greater than $EMAXHP_i + EMAXMH_i$, in which case the term in square brackets in Equation (3.44) is greater than Y_{it}^p. It is assumed that a firm never produces more in period t than it originally planned, and since the term in square brackets can be greater than Y_{it}^p, the minimum expression is used in (3.44). $XFMAX_{it}$ as defined in (3.44) is thus the maximum number of goods the firm knows with certainty it can supply in period t.

As mentioned above, it is unlikely that goods constraints are very important in practice because of the fact that goods can be held in inventories. In the present case the goods constraints have been included in the model only for the sake of completeness, and the constraints do not play an important role in future discussion of the model and its properties.

Regarding the determination of the firm's demand for loans, consider first the demand deposit needs of the firm. The demand deposit needs are assumed to be of two kinds: the need for transactions purposes and the need to meet unexpected decreases in cash flow net of taxes and dividends. The need for transactions purposes is assumed to be proportional to the firm's wage bill. Let DDF_{1it} denote the value of demand deposits set aside by firm i for transactions purposes in period t. Then DDF_{1it} is assumed to be

$$DDF_{1it} = \beta_{14} WF_{it}\, HPFMAX_{it},\ \beta_{14} > 0. \tag{3.45}$$

Since it is assumed that the firm never hires more than $HPFMAX_{it}$ amount of labor, the firm's wage bill cannot exceed $WF_{it}\, HPFMAX_{it}$, and so the firm is assured by setting aside the value of demand deposits in (3.45) that it will always have enough demand deposits for transactions purposes.

With respect to the second need for demand deposits, firm i only has from Equation (3.20) an expectation of its cash flow net of taxes and dividends for period t because it only has an expectation of the price of investment goods, PFF_{it}^e, and of its level of inventories for the end of period t, V_{it}^e. The firm must prepare for the possibility that it underestimates the price of investment goods or its level of inventories and ends up with less cash flow net of taxes and dividends than it originally expected. The firm is assumed to prepare for this possibility by planning to hold more demand deposits than are needed for transactions purposes. The firm is assumed from past experience to have a good idea of the largest error it is likely to make in underestimating its cash flow, and this is the amount that the firm is assumed to plan to hold in demand deposits over and above its requirements for transactions purposes. Denote this amount as DDF_{2i}. For simplicity DDF_{2i} is assumed not to be a function of time.

Given its expectations, the firm is assumed to borrow money with the aim of holding amount $DDF_{1it} + DDF_{2i}$ in demand deposits in period t. The aimed-for change in demand deposits is $DDF_{1it} + DDF_{2i} - DDF_{it-1}$, where DDF_{it-1} is the actual value of demand deposits held by firm i in period $t-1$. The firm will need to increase its loans over and above any increase in aimed-for demand deposits if its expected cash flow after taxes and dividends, $\overline{CF}^e_{it}$, is negative, and conversely if $\overline{CF}^e_{it}$ is positive. The change in the value of loans for the firm is thus

$$LF_{it} - LF_{it-1} = (DDF_{1it} + DDF_{2i} - DDF_{it-1}) - \overline{CF}^e_{it}. \tag{3.46}$$

At the end of the period, after all transactions have taken place, actual demand deposits, DDF_{it}, will be equal to $DDF_{1it} + DDF_{2i}$ only in the case in which the firm's expectation of $\overline{CF}_{it}$ is completely accurate. DDF_{it} will be less than $DDF_{1it} + DDF_{2i}$ if the firm underestimates $\overline{CF}_{it}$ and has to use some of DDF_{2i} to meet the unexpected decrease. From the definition of DDF_{2i}, the firm is assured that DDF_{it} will never be less than DDF_{1it}. DDF_{it} will be greater than $DDF_{1it} + DDF_{2i}$ if the firm overestimates $\overline{CF}_{it}$ and takes out more loans than it really needed. The actual change in demand deposits of the firm for period t is a residual and is defined by Equation (3.21). The determination of the value of loans for periods $t+1$ and beyond is a straightforward extension of the above analysis for period t.

3.4 THE SOLUTION OF THE CONTROL PROBLEM

It was seen in the last section that given the paths of the decision variables, the corresponding value of the objective function can be computed. In order to solve the control problem of the firm, algorithms were written to search over various sets of paths for the optimum. The main algorithm searched over different price paths. The base price path, from which other paths were tried, was taken to be the path in which the price in each period was the same and equal to PF^e_{jt} in (3.23). PF^e_{jt} is the price that firm i expects firm j to set for period t. From (3.24) it can be seen that this price path corresponds to firm i expecting that firm j's price path will be the same as firm i's price path, which from (3.28) and (3.29) corresponds to firm i expecting that its market share will remain the same in periods t and beyond as it was in period $t-1$.

For each price path chosen by the algorithm, a submaximization problem was solved to determine the optimal production, investment, and employment paths corresponding to the given price path. This submaximization problem was solved by scanning over the various possible paths. First, given the expected sales path corresponding to the price path, various production paths were tried. The production paths are constrained, given the sales path, by the fact that inventories cannot be negative and by the terminal condition on inventories. For each production path, various investment paths were tried. The investment paths are constrained by the fact that there must be enough machines on hand to produce the amount of output required from the production path and by the terminal conditions. For each production and investment path, various employment paths were tried. The employment paths are constrained by the fact worker hours paid for each period must be at least as great as worker hour requirements.

Two extreme production paths that were tried were a path in which production changed as little as possible from period to period, and a path in which inventories changed as little as possible from period to period. Other paths were then tried as weighted averages of these two paths. There is a tradeoff between costs of production fluctuations (due to costs of investment and employment fluctuations) and costs of inventory fluctuations, and so trying various weighted averages of the two extreme paths should lead to a computed optimum path that is close to the true optimum path.

Given the level of production for a particular period and given the past history of investment, one can compute the number of machines of type 1 or of type 2 that need to be purchased in the period to produce the output of the period, assuming that all machines are utilized to full capacity ($\bar{H}$ hours per period). Two investment paths that were tried were a path in which only machines of type 1 were purchased, and a path in which only machines of type 2 were purchased. Both of these paths were taken to be characterized by full capacity utilization all the time, unless full capacity utilization required negative gross investment, which was not allowed. Other paths were tried in which investment fluctuations were lessened by not having the firm be at full capacity utilization all the time. Paths in which some of type 1 machines and some of type 2 machines were purchased were not tried since it was costly to do so and it did not seem likely that the computed optimum values for period t would be sensitive to this omission.

Given the level of production and the number of the two types of machines on hand for a particular period, given the expected deviation of inventories from β_1 times sales for the period, given the expected change in sales for the period, given the change in worker hours paid for of the previous period, and given the value of net investment for the period, worker hour requirements can be computed from Equations (3.1) and (3.8) – (3.11). Two extreme employment paths that were tried were a path in which worker hours paid for were always kept equal to worker hour requirements, and a path in which fluctuations in worker hours paid for were kept small. Other paths were then tried as weighted averages of these two paths. As was the case for the production paths, trying various weighted averages of the two extreme paths should lead to a computed optimum path that is close to the true optimum path. All paths except the path in which worker hours paid for were equal to worker hour requirements were characterized by the firm paying for more hours than required during some periods.

Given a price path for firm i and its path of worker hours paid for, and given firm i's expectation of the price path of firm j and the path of the average price level in the economy, one can compute from Equations (3.30) – (3.37) the wage path that firm i expects is necessary to yield the path of worker hours paid for that it has set. In other words, once the firm has chosen its price path and its path of worker hours paid for, the wage path is automatically determined.

The loan constraint was handled by throwing out as infeasible those paths that implied a loan value greater than the constraint.

3.5 SOME EXAMPLES OF SOLVING THE CONTROL PROBLEM OF FIRM *i* PARAMETER VALUES AND INITIAL CONDITIONS

The parameter values and initial conditions that were used for the first example are presented in Table 3-2. The most important parameters are β_6, the measure of the extent to which firm i expects firm j to respond to firm i's price setting behavior; β_9, the measure of the extent to which firm i loses or gains market share as its price deviates from firm j's price; β_{10}, the measure of the extent to which firm i expects firm j to respond to firm i's wage setting behavior; β_{13}, the measure of the extent to which firm i loses or gains its market share of labor as its wage deviates from firm j's wage; β_7, the measure of the extent to which firm i expects firm j to change its price in period t as a result of firm j's inventory situation in period $t{-}1$; and the four parameters reflecting inventory, sales adjustment, hours adjustment, and capital adjustment costs, β_2, β_3, β_4, and β_5.

The parameter values and initial conditions were chosen, after some experimentation, so that the optimum values of each control variable for periods t through $t{+}T$ would be essentially the same as the initial value for period $t{-}1$. This was done to make it easier to analyze the effects on the behavior of the firm of changing various initial conditions. As can be seen from Table 3-2, the initial conditions correspond to firm i's having half the sales in period $t{-}1$ and half the labor employed. The firm holds no excess labor and excess capital in period $t{-}1$. The two firms' prices and wage rates in period $t{-}1$ are the same. All the machines held by firm i are type 1 machines. The length of the decision

Table 3-2 Parameter Values and Initial Conditions for the Control Problem of Firm *i*

Parameter	*Value*	*Variable*	*Value*
$T+1$	30	K^a_{1it-1}	250.0
d_1	0.5	K^a_{2it-1}	0.0
m	10	$I_{1it-1}, \ldots, I_{1it-m}$	25.0, . . . , 25.0
λ_1	1.3212	$I_{2it-1}, \ldots, I_{2it-m}$	0.0, . . . , 0.0
λ_2	1.3000	V_{it-1}	52.625
μ_1	1.684	HPF_{it-1}	318.65
μ_2	1.684	HPF_{it-2}	318.65
$\overline{H}$	1.0	$PFF_{it-1}, \ldots, PFF_{it-m+1}$	1.0, . . . , 1.0
δ_1	1.0	LF_{it-1}	164.05
δ_2	0.9	DDF_{it-1}	25.15
β_1	0.125	XF_{it-1}	421.0
β_2	0.075	X_{t-1}	842.0
β_3	0.125		
β_4	0.050	PF_{it-1}	1.0
β_5	0.250	PF_{jt-1}	1.0
β_6	0.5	HP_{t-1}	637.3
β_7	−0.03	$HPUN_{t-1}$	637.3
β_8	−0.30	WF_{it-1}	1.0
β_9	−8.0	WF_{jt-1}	1.0
β_{10}	0.5	XF_{jt-1}	421.0
β_{11}	1.0	V_{jt-1}	52.625
β_{12}	−1.0	DDF_{2i}	2.5
β_{13}	2.0	RF_{it}	0.0750
β_{14}	0.07108	Y_{it-1}	421.0[$=\lambda_1 HPF_{it-1}$ $=\mu_1 K^a_{1it-1}$]
$\overline{K}$	250.0		
$EMAXHP_i$ $+EMAXMH_i$	12.7		

horizon is 30 periods, and the length of life, m, of a machine is 10 periods. The values in Table 3-2 correspond to the firm having profitable investment opportunities in the sense that, ignoring adjustment costs, the present discounted value of the revenue stream generated by an extra unit of investment is greater than the initial cost.

THE RESULTS

The results of solving the control problem of the firm for the parameter values and initial conditions in Table 3-2 are presented in the first row of Table 3-3. Only a small subset of the results are presented in Table 3-3, as it is not feasible to present all 30 values for each variable. Values of the price variable are given for periods t, $t+1$, and $t+2$, and then values for period t are given for the

Table 3-3. Results of Solving the Control Problem of Firm *i*

Initial Conditions from Table 3-2 except:	PF_{it}	PF_{it+1}	PF_{it+2}	PF^e_{jt}	XF^e_{it}	Y^p_{it}	INV_{it}	HPF^e_{it} $(HPFMAX_{it})$	WF_{it}	LF_{it}	Planned excess labor for period t	Planned excess capital for period t
1. No exceptions	1.0000	1.0000	1.0000	1.0000	421.0	421.0	25.0	318.65	1.0000	164.05	0.0	0.0
2. (demand increase, firms *i* and *j*)[a]	1.0090	1.0095	1.0080	1.0075	425.4	427.0	28.6	337.78	1.0269	171.46	0.0	0.0
3. (demand decrease, firms *i* and *j*)[b]	0.9923	0.9933	0.9938	0.9938	416.3	408.3	24.4	314.08	0.9915	154.36	0.0	7.0
4. (demand increase, firm *i* only)[c]	1.0020	1.0025	1.0010	1.0000	424.6	424.5	24.4*	338.12	1.0244	165.56	0.0	0.0
5. (demand decrease, firm *i* only)[d]	0.9985	0.9995	1.0000	1.0000	415.5	409.0	24.4	314.46	0.9944	156.41	0.0	6.5
6. $HPUN_{t-1}$ = 653.2 (+2.5%)	1.0000	1.0000	1.0000	1.0000	421.0	421.0	25.0	318.65	0.9902	163.83	0.0	0.0
7. (excess labor)[e]	0.9995	1.0000	1.0000	1.0000	422.7	422.5	25.9	320.35	0.9922	163.94	0.0	0.0
8. (excess capital)[f]	1.0000	0.9995	1.0000	1.0000	421.0	421.1	25.0	319.08	1.0005	163.03	0.0	11.2
9. PF_{jt-1} = 1.0050 (+0.5%)	1.0025	1.0025	1.0025	1.0025	421.0	421.0	25.0	318.65	1.0000	164.22	0.0	0.0
10. PF_{jt-1} = 0.9950 (−0.5%)	0.9975	0.9975	0.9975	0.9975	421.0	421.0	25.0	318.65	1.0000	163.84	0.0	0.0
11. WF_{jt-1} = 1.0100 (+1.0%)	1.0000	1.0005	1.0005	1.0000	421.0	420.2	24.5	318.15	1.0034	162.86	0.0	0.0
12. WF_{jt-1} = 0.9900 (−1.0%)	1.0000	1.0000	1.0000	1.0000	421.0	421.0	25.0	318.65	0.9960	163.96	0.0	0.0
13. RF_{it} = 0.0788 (+5.0%)	1.0000	1.0005	1.0005	1.0000	421.0	420.2	24.5	318.15	0.9994	162.77	0.0	0.0

Table 3-3. (continued)

Initial Conditions from Table 3-2 except:	PF_{it}	PF_{it+1}	PF_{it+2}	PF^e_{jt}	XF^e_{it}	Y^p_{it}	INV_{it}	HPF^e_{it} ($HPFMAX_{it}$)	WF_{it}	LF_{it}	Planned excess labor for period t	Planned excess capital for period t
14. $RF_{it} = 0.0825$ (+10.0%)	1.0000	1.0005	1.0005	1.0000	421.0	419.5	22.1*	318.66	1.0000	161.61	0.0	0.0
15. $RF_{it} = 0.0338$ (−55.0%)	0.9995	0.9995	0.9995	1.0000	422.7	422.9	26.1	320.81	1.0026	165.48	0.0	0.0
16. $LFMAX_{it} = 159.13$ (−3.0%)	1.0005	1.0005	1.0005	1.0000	419.3	418.0	20.9*	318.11	0.9994	159.10	0.0	0.0

[a] $X_{t-1} = 863.0, XF_{it-1} = 431.5, V_{it-1} = 42.1, XF_{jt-1} = 431.5, V_{jt-1} = 42.1$
[b] $X_{t-1} = 821.0, XF_{it-1} = 410.5, V_{it-1} = 63.1, XF_{jt-1} = 410.5, V_{jt-1} = 63.1$
[c] $X_{t-1} = 852.5, XF_{it-1} = 431.5, V_{it-1} = 42.1$
[d] $X_{t-1} = 831.5, XF_{it-1} = 410.5, V_{it-1} = 63.1$
[e] $HPF_{t-1} = 326.62, HPF_{it-2} = 326.62, HPUN_{t-1} = 653.2, HP_{t-1} = 653.2, DDF_{it-1} = 25.71$
[f] $K^a_{1it-1} = 262.5; I_{1it-1}, \ldots, I_{1it-m} = 26.25$
*Firm switched to machines of type 2.

expected price of firm j, the expected level of sales, the planned level of production, the expected supply of labor, the wage rate, the value of loans, planned excess labor (HPF_{it}^e - MH_{it}^e), and planned excess capital in units of machines (K_{1it}^a - $KMIN_{1it}$ + K_{2it}^a - $KMIN_{2it}$). The value in the first row of Table 3-3 for each variable for period t is the same as the corresponding initial value in Table 3-2, which reflects the way the parameter values and initial conditions were chosen.

One of the most important reactions of a firm is how the firm responds to an increase or decrease in sales. For the results in row 2 of Table 3-3, sales in period $t-1$ were increased by 2.5 percent. Production for period $t-1$ was not changed, and so inventories for period $t-1$ were assumed to fall. Both firm i and firm j were assumed to have the same rise in sales and thus the same drop in inventories. The drop in inventories of firm j led firm i to expect firm j's price for period t to rise to 1.0075. Firm i raised its price a little above this level, which caused its market share to decrease somewhat. Firm i ended up with expected sales of 425.4 for period t, compared to the level of 431.5 that it would have expected had it kept its price equal to the expected price of firm j.

Planned production, investment, employment,[h] and loans were all higher as a result of the sales increase. The wage rate was also higher since firm i needed to attract more workers to meet the increased employment requirements. Also, since firm i expected the average price in the economy to be higher in period t, this had a negative effect on firm i's expectation of the aggregate supply of labor, which caused the firm to have to raise its wage rate more than it otherwise would have to attract the same amount of labor. Although not shown in the table, the higher expected average price also had a negative effect on firm i's expectation of the aggregate demand for goods.

For the results in row 3 of Table 3-3, sales in period $t-1$ were decreased by 2.5 percent. The results are essentially the opposite to those in row 2. Firm i lowered its price slightly from what it expected firm j's price to be, which had the effect of increasing expected sales somewhat from what would have been the case had firm i kept its price the same as the expected price of firm j. Planned production, investment, employment, loans, and the wage rate were all lower as a result of the sales decrease. The firm also planned to hold excess capital in period t, which means that the firm did not plan to lower investment as much as it could have and still produce the planned output.

For the results in row 4, only the level of sales of firm i in period $t-1$ was increased. This change had essentially the same effects as did the demand increase in row 2, except that the rise in price of firm i was less. The price rise was less in row 4 because in this case firm i did not expect firm j to increase its price in period t. The firm switched to the cheaper type 2 machines in row 4. Although employment was slightly greater in row 4 than in row 2, the wage rate in row 4 was less. The wage rate was less because the expected average price level was less. For the results in row 5, the level of firm i's sales in period $t-1$ was decreased. The results are essentially the opposite to those in row 4. In this case, as was the case in row 3, the firm planned to hold some excess capital in period *t*.

For the results in row 6, $HPUN_{t-1}$, the aggregate unconstrained supply of labor in period *t–1*, was increased. The only major effect this had on firm *i* was for firm *i* to lower its wage rate. Because of the larger expected aggregate supply, firm *i* needed a lower wage rate to attract the same amount of labor.

For the results in row 7, the employment of firm *i* for period *t–1* was increased, with no corresponding increase in production. This meant that firm *i* held excess labor in period *t–1*. This change caused firm *i* to decrease its employment in period *t* from the level existing in period *t–1*, to lower its price slightly in period *t*, and to increase its expected sales, planned production, and investment. Excess labor in period *t–1* thus caused the firm to lower its price and expand slightly in period *t*. Employment was decreased in period *t*, but not all the way back to the level in row 1. The wage rate was lower in this case, which was caused by the fact that the aggregate supply of labor in period *t–1* was also increased for this run.

For the results in row 8, the number of machines held by firm *i* in period *t–1* was increased, with no corresponding increase in production. This meant that firm *i* held excess capital in period *t–1*. This change caused the firm to lower its price slightly for period *t+1*. Investment dropped by 1.25 machines—from past gross investments of 26.25 to a gross investment of 25.0 in period t. The firm chose to hold excess capital in period *t* of 11.2 machines. Employment rose because of the investment adjustment costs.

For the results in row 9, the price of firm *j* in period *t–1* was increased by 0.5 percent to 1.0050. This caused firm *i* to expect firm *j*'s price to be 1.0025 in period *t*. Firm *i* raised its price to this amount, keeping its expected market share the same. Planned production, investment, and employment were unchanged. For the results in row 10, the price of firm *j* in period *t–1* was decreased by 0.5 percent, which had the opposite effect from the price increase in row 9.

For the results in row 11, the wage rate of firm *j* in period *t–1* was increased by 1.0 percent to 1.0100. This caused firm *i* to expect firm *j*'s wage rate for period *t* to be higher than firm *i* expected it to be in row 1. (Although not shown in the table, firm *i* expected firm *j*'s wage rate in period *t* to be 1.0000 for the results in row 1 and 1.0050 for the results in row 11.) The higher expected wage rate of firm *j* for period *t* caused firm *i* to raise its wage rate for period *t*. Firm *i* also raised its price slightly for periods *t+1* and beyond and cut back its production, investment, and employment slightly. For the results in row 12, the wage rate of firm *j* in period *t–1* was decreased by 1.0 percent, which caused firm *i* to lower its wage rate in period *t*. In this case firm *i* was not led to lower its price as a result of the lower expected wage rate of firm *j*.

For the results in row 13, the loan rate was increased by 5.0 percent. This caused the firm to raise its price for periods $t+1$ and beyond and to produce, invest, and borrow less in period t. For the results in row 14, the loan rate was increased by 10.0 percent, which caused the firm to switch to type 2 machines. Investment in terms of the number of goods purchased was thus lower in row 14 than in row 13 because of the switch to the cheaper machines. Employment was higher in row 14 than in row 13 because of the greater employment requirements on type 2 machines. For the results in row 15, the loan rate was decreased by 55.0 percent. This caused the firm to lower its price for periods t and beyond, which caused expected sales to increase and the firm to produce, invest, and borrow more. In this case it is, of course, not possible for the firm to switch to more expensive machines, since only two types of machines were postulated and the firm was already using the more expensive type. It was necessary to decrease the loan rate by slightly over 50.0 percent to get the firm to react in any significant way to the change.

For the results in row 16, the loan constraint was assumed to be binding on the firm. $LFMAX_{it}$ was set to 159.13 compared with the unconstrained choice of the firm of 164.05. This constraint caused the firm to raise its price and to produce and invest less in period t. The firm switched to type 2 machines, which allowed the firm to spend less for investment than it otherwise would have had to, given the level of production, and thus to lower the amount of money it needed to borrow.

For none of the runs in Table 3-3, given the parameter values used, did the firm plan to hold excess labor in period t. The adjustment-cost parameter for employment, β_4, was too low relative to the other cost parameters for it to be profitable for the firm to hold excess labor. In general, however, one would expect firms to adjust to falling demand situations by holding some excess labor in the current period.

Some of the main properties of the model that can be gleaned from the results in Table 3-3 are the following.

1. When demand increases and inventories decrease, the firm raises its price and increases its production, investment, employment, wage rate, and loans. The firm raises its price for two reasons. One is because it expects other firms to raise their prices, and the other is a desire to lower its market share somewhat to avoid having as large an increase in investment and employment as would be required if it kept its market share the same. If only the demand for firm i's goods increases, then firm i raises its price less than otherwise because it does not expect other firms to raise their prices in the current period.
2. The opposite effects from 1 take place when demand decreases and inventories increase.

3. The existence of excess labor in a period causes the firm to decrease employment in the next period and to lower its price and expand production slightly.
4. The existence of excess capital in a period causes the firm to decrease investment in the next period and perhaps to lower its price path slightly.
5. The main effect of an increase in the aggregate unconstrained supply of labor is for the firm to decrease its wage rate.
6. The main effect of a change in other firms' prices or wage rates is for firm *i* to change its price or wage rate in the same direction.
7. The firm responds to an interest rate increase by raising its price, at least for periods *t+1* and beyond, and by lowering its production, investment, and loans. Employment may respond in either direction depending on whether or not the firm moves into cheaper types of machines with higher employment requirements.
8. Essentially the opposite effects from 5 take place for an interest rate decrease.
9. The firm responds to a constraint on its borrowing behavior in a similar way that it responds to an interest rate increase, by raising its price and lowering its production and investment. Lower investment in this case may also take the form of purchasing cheaper machines.

It is also evident from the results in Table 3-3 that the behavior of the firm is not necessarily symmetrical for increases and decreases in a particular variable. For the results in rows 2 and 3 of Table 3-3, for example, the firm chose to increase production by only 6.0 units corresponding to a 10.5 increase in sales of the previous period, whereas it chose to decrease its production by 12.7 units corresponding to a 10.5 decrease in sales.

A second example of an asymmetrical reaction is reflected in rows 11 and 12. An increase in the wage rate of firm *j* led firm *i* to increase its price for period *t+1* and beyond, whereas a decrease in the wage rate of firm *j* did not induce firm *i* to lower its price. A third example of an asymmetrical reaction is reflected in rows 13 and 15. An interest rate increase of only 5.0 percent led to a price increase in periods *t+1* and beyond, but it took an interest rate decrease of about 55.0 percent to lead to a price decrease. The firm's reaction to the loan constraint is, of course, another asymmetrical reaction in the sense that the firm is forced to respond to the constraint, but is not forced in the opposite direction when there is no constraint.

One important reason for the firm's asymmetrical behavior regarding increases and decreases in demand is the ability of the firm to hold excess labor and capital during contractions, but having no corresponding ability during expansions when already at full capacity. A decrease in demand means that the firm has the opportunity to hold excess labor and capital to help smooth out adjustment costs, whereas an increase in demand from a situation in which no excess labor and capital is being held means that the firm must either increase investment and employment immediately or must decrease its inventories. It was quite evident from examining various runs for the firm, using in many cases different sets of parameter values, that the firm was more inclined to choose to raise its price and lower its expected sales and production paths than to lower its price and raise its expected sales and production paths.

Another important factor influencing the firm's proclivity to raise or lower its price is the size of the parameters β_6 and β_9 in Equations (3.23)-(3.24) and (3.28)-(3.29). The larger is β_6, the more does firm i expect firm j to follow its price setting behavior, and the larger is β_9 in absolute value, the more does firm i expect its market share to change as its price deviates from firm j's price. Large values of β_6, other things being equal, increase the proclivity of firm i to raise its price, because in this case firm j is expected to follow quickly along. If firm j follows quickly along, firm i will not lose much of its market share as a result of its higher price. Large absolute values of β_9, other things being equal, decrease the proclivity of firm i to raise its price, because in this case it expects to lose a lot of its market share as a result of the higher price. For the particular values of β_6 and β_9 tried in this study, the proclivity of firm i was definitely toward raising its price.

Regarding the effect of the loan rate on the behavior of the firm, it should be noted that the loan rate can affect the investment of the firm in ways that have nothing to do with capital-labor substitution in the sense of the firm purchasing different types of machines. In row 13 in Table 3-3, for example, an increase in the loan rate caused the firm to produce and invest less, and yet it was still optimal for the firm to purchase the more expensive type 1 machines. The higher loan rate caused the firm to raise its price for periods $t+1$ and beyond, which caused expected sales to be less for periods $t+1$ and beyond, which in turn caused production to be less for periods t and beyond. Production was less in period t, even though the expected level of sales was not changed for period t, because of production smoothing considerations. Because of adjustment costs, it was optimal for the firm to begin lowering production in period t. Because of the lower level of production in period t, investment was also less in period t. The loan rate has in this case affected the level of investment without causing any capital-labor substitution to take place in the sense of the firm switching to the cheaper type of machines.

3.6 THE CONDENSED MODEL FOR FIRMS

The firm behavioral equations for the condensed model are presented in Table 3-4. The superscripts p and pp refer to planned values, which may get modified during the course of the decision process. As was the case for banks, all i subscripts have been dropped, since for the condensed model there is only a firm

Table 3–4. Firm Equations for the Condensed Model

(1) $P_t^p = 0.661(P_{t-1})^{0.50}\left(\dfrac{V_{t-1}}{\beta_1 X_{t-1}}\right)^{-0.03}(X_{t-1})^{0.10}(RL_t)^{0.10}\left(\dfrac{HPF_{t-1}}{MH_{t-1}}\right)^{-0.02}\left(\dfrac{K_{t-1}^a}{KMIN_{t-1}}\right)^{-0.01},$

(2) $X_t^{ep} = X_{t-1}\left(\dfrac{P_t^p}{P_{t-1}}\right)^{-0.30}$

The values of P_t, Y_t^p, INV_t, $HPFMAX_t$, W_t, LF_t, $XFMAX_t$, $INVUN_t$, and $LFUN_t$ are determined by the following algorithm:

[1] $Y_t^{pp} = X_t^{ep} + \beta_1 X_t^{ep} - V_{t-1}$,

[2] $K_t^{pp} = \dfrac{Y_t^{pp}}{\mu_1 \overline{H}}$,

[3] If $K_t^{pp} \geqslant K_{t-1}^a$, then $K_t^p = K_{t-1}^a + 0.5(K_t^{pp} - K_{t-1}^a)$ and $Y_t^p = \mu_1 K_t^p \overline{H}$,

[4] If $K_t^{pp} < K_{t-1}^a$, then $K_t^p = K_{t-1}^a + 0.2(K_t^{pp} - K_{t-1}^a)$ and
$Y_t^p = \mu_1(K_{t-1}^a + 0.5(K_t^{pp} - K_{t-1}^a))\overline{H}$,

[5] $V_t^p = V_{t-1} + Y_t^p - X_t^{ep}$,

[6] $MH_{1t}^p = \dfrac{Y_t^p}{\lambda_1}$,

[7] $MH_{3t}^p = \beta_2(V_t^p - \beta_1 X_t^{ep})^2$,

[8] $MH_{4t}^p = \beta_3(X_t^{ep} - X_{t-1})^2$,

[9] $MH_{5t}^p = \beta_4(HPF_{t-1} - HPF_{t-2})^2$,

[10] $MH_{6t}^p = \beta_5(K_t^p - K_{t-1}^a)^2$,

[11] $MH_t^p = MH_{1t}^p + MH_{3t}^p + MH_{4t}^p + MH_{5t}^p + MH_{6t}^p$,

[12] If $MH_t^p = HPF_{t-1}$, then $HPF_t^p = MH_t^p$,

[13] If $MH_t^p < HPF_{t-1}$, then $HPF_t^p = HPF_{t-1} + 0.2(MH_t^p - HPF_{t-1})$,

[14] If $MH_t^p > HPF_{t-1}$, then $HPF_t^p = HPF_{t-1} + 0.5(MH_t^p - HPF_{t-1})$;
Y_t^p = maximum amount that can be produced given K_t^p , X_t^{ep}, and $MH_t^p = HPF_t^p$

$K_t^p = \dfrac{Y_t^p}{\mu_1 \overline{H}}$;

$MH_t^p = HPF_t^p + \beta_5(K_t^p - K_{t-1}^a)^2 - MH_{6t}^p$;

Table 3-4. (continued)

$HPF_t^p = MH_t^p$;

$V_t^p = V_{t-1} + Y_t^p - X_t^{ep}$,

[15] $W_t^p = (W_{t-1})^{0.50} (P_t^p)^{0.40} \left(\frac{HPF_t^p}{HPF_{t-1}}\right)^{0.40} \left(\frac{HPUN_{t-1}}{HP_{t-1}}\right)^{-0.40}$,

[16] $INV_t^p = K_t^p - K_{t-1}^a + INV_{t-m}$,

[17] $DDF_{1t}^p = \beta_{14} W_t^p HPF_t^p$,

[18] $DEP_t^p = \frac{1}{m} [P_t^p INV_t^p + P_{t-1} INV_{t-1} + \ldots + P_{t-m+1} INV_{t-m+1}]$,

[19] $\overline{CF}_t^p = DEP_t^p - P_t^p INV_t^p + P_{t-1} V_{t-1} - P_t^p V_t^p$,

[20] $LF_t^p = LF_{t-1} + DDF_{1t}^p + DDF_2 - DDF_{t-1} - \overline{CF}_t^p$,

[21] $INVUN_t = INV_t^p, LFUN_t = LF_t^p, PUN_t = P_t^p, Y^pUN_t = Y_t^p, WUN_t = W_t^p,$

$HPFMAXUN_t = HPF_t^p$,

[22] If $LF_t^p \leqslant LFMAX_t$, go to statement [42],

[23] $P_t = P_t^p + 0.05\, P_t^p \left(\frac{LFUN_t - LFMAX_t}{LFUN_t}\right)$,

[24] $X_t^e = X_{t-1} \left(\frac{P_t}{P_{t-1}}\right)^{-0.30}$,

[25] $DEP_t^p = \frac{1}{m} [P_t INV_t^p + P_{t-1} INV_{t-1} + \ldots + P_{t-m+1} INV_{t-m+1}]$,

[26] $V_t^p = V_{t-1} + Y_t^p - X_t^e$,

[27] $\overline{CF}_t^p = DEP_t^p - P_t INV_t^p + P_{t-1} V_{t-1} - P_t V_t^p$,

[28] $LF_t^p = LF_{t-1} + DDF_{1t}^p + DDF_2 - DDF_{t-1} - \overline{CF}_t^p$,

[29] $Y_t^p = old\ Y_t^p - \frac{LF_t^p - LFMAX_t}{(\frac{1}{\mu_1 \overline{H}}) (\frac{m-1}{m}) P_t + P_t}$,

[30] $INV_t = INV_t^p - \left(\frac{old\ Y_t^p - new\ Y_t^p}{\mu_1 \overline{H}}\right)$,

[31] $K_t^a = K_t^p - (INV_t^p - INV_t)$,

[32] $HPF_t^p = old\ HPF_t^p - \left(\frac{old\ Y_t^p - new\ Y_t^p}{\lambda_1}\right)$,

Table 3-4. (continued)

[33] $V_t^p = V_{t-1} + Y_t^p - X_t^e$,

[34] MH_t^p = computed as in statements [6] – [11] with X_t^e replacing X_t^{ep}, K_t^a replacing K_t^p, and new Y_t^p and V_t^p being used,

[35] If $MH_t^p > HPF_t^p$, then $HPF_t^p = MH_t^p$,

[36] $W_t = (W_{t-1})^{0.50} (P_t)^{0.40} \left(\frac{HPF_t^p}{HPF_{t-1}}\right)^{0.40} \left(\frac{HPUN_{t-1}}{HP_{t-1}}\right)^{-0.40}$,

[37] $DEP_t = \frac{1}{m} [P_t INV_t + P_{t-1} INV_{t-1} + \ldots + P_{t-m+1} INV_{t-m+1}]$,

[38] $\overline{CF}_t^p = DEP_t - P_t INV_t + P_{t-1} V_{t-1} - P_t V_t^p$,

[39] $DDF_{1t}^p = \beta_{14} W_t HPF_t^p$,

[40] $LF_t = LF_{t-1} + DDF_{1t}^p + DDF_2 - DDF_{t-1} - \overline{CF}_t^p$,

[41] Go to statement [48] ,

[42] $P_t = P_t^p$,

[43] $X_t^e = X_t^{ep}$,

[44] $INV_t = INV_t^p$,

[45] $W_t = W_t^p$,

[46] $LF_t = LF_t^p$,

[47] $K_t^a = K_t^p$,

[48] $HPFMAX_t = HPF_t^p$,

[49] $XFMAX_t = V_{t-1} + min \left\{ Y_t^p, \ [Y_t^p - \lambda_1 (EMAXHP + EMAXMH) + \lambda_1 (HPF_t^p - MH_t^p)] \right\}$,

[50] $KMIN_t^p = \frac{Y_t^p}{\mu_1 \overline{H}}$,

[51] $MH_{4t}^p = \beta_3 (X_t^e - X_{t-1})^2$.

sector rather than individual firms. There is also assumed for the condensed model to be only one type of machine in existence, and so the subscripts referring to the types of machines have been dropped. The existence of only one type of machine rules out the possibility of capital-labor substitution, but, as just discussed, the loan rate can still have an effect on the investment decision of the firm. Because of this, it did not seem necessary in specifying the condensed model to consider more than one type of machine. The price for period t is now denoted P_t rather than PF_{it}, the wage rate is denoted W_t rather than WF_{it}, and the loan rate is denoted RL_t rather than RF_{it}.

Equation (1) in Table 3-4 determines the first planned price of the firm sector. The planned price is a positive function of last period's price, of last period's sales, and of the current loan rate, and is a negative function of last period's ratio of the level of inventories to β_1 times sales, of last period's ratio of worker hours paid for to worker hour requirements (excess labor), and of last period's ratio of the number of machines on hand to the minimum number required (excess capital).

Equation (1) is based on the results in Table 3-3. The size of the various coefficients are for the most part consistent with the size of the responses in Table 3-3, although the coefficient for the loan rate was made somewhat larger than the responses in Table 3-3 would indicate it should be. The equation is, of course, also symmetric and does not capture any of the asymmetries in Table 3-3. The choice of the constant term in Equation (1) is explained in Chapter Six. Equation (2), determining the expected demand for goods, is the same as equation (3.26) for the non-condensed model.

The algorithm described in Table 3-4 determines all the values of the decision variables of the firm. The algorithm is written like a FORTRAN program would be written, and so the logic of the algorithm should be fairly clear to readers with a knowledge of the FORTRAN language. The following is a brief verbal description of the algorithms.

Y_t^{pp} in statement [1] is the output that is necessary for the firm sector to produce in period t, given the expected level of sales for period t, in order for it to end up with the level of inventories at the end of period t being equal to β_1 times sales. K_t^{pp} in statement [2] is the minimum number of machines needed to produce this amount. If the number of machines needed is greater than the actual number on hand in period $t-1$, so that positive net investment is necessary to produce Y_t^{pp}, then, as in statement [3], planned net investment $(K_t^p - K_{t-1}^a)$ is taken to be 50.0 percent of that originally planned $(K_t^{pp} - K_{t-1}^a)$. Planned production is then decreased in statement [3] accordingly. If the originally planned net investment is negative, then, as in statement [4], planned net investment is taken to be 20.0 percent of that originally planned. Planned production is then increased in statement [4]. In statement [3] planned production is decreased by the amount necessary for the firm to be able to produce the output, given the fewer number of machines on hand than originally planned. The firm sector plans to hold no excess capital in this case. In statement [4], however, the firm sector plans to hold some excess capital. Planned production is increased, but not enough to correspond to full utilization of the number of machines on hand. Planned production is rather increased to correspond to what would have been full utilization had the new planned investment been 50.0 percent (rather than 20.0 percent) of that originally planned. Statements [3] and [4] are meant to capture the effects of investment-adjustment costs on the behavior of the firm sector.

Statement [5] defines the planned level of inventories, and statements [6]–[11] determine worker hour requirements. If the computed level of worker hour requirements is equal to last period's level of worker hours paid for, then, as in statement [12], the firm sector plans not to change the number of worker hours paid for in the current period. If the computed level of worker hour requirements is less than last period's level of worker hours paid for, then, as in statement [13], the firm sector plans to decrease the level of worker hours paid for in the current period by 20.0 percent of the difference between the computed level of worker hour requirements and last period's level of worker hours paid for. In this case the firm sector plans to hold excess labor in the current period. If the computed level of worker hour requirements is greater than last period's level of worker hours paid for, then, as in statement [14], the firm sector plans to increase the level of worker hours paid for in the current period by 50.0 percent of the difference between the computed level of worker hour requirements and last period's level of worker hours paid for.

Planned production is then decreased in statement [14] by the amount necessary for the firm sector to be able to produce the output. Planned production must be decreased in this case because the new planned level of worker hours paid for is now less than is necessary for the firm sector to be able to produce the originally planned output. Because worker hour requirements are a function of the current level of inventories (statement [7]), computing the level of production in this case requires solving a quadratic equation in output.[i] The planned number of machines on hand is then decreased in statement [14] to the number necessary to produce the new planned output. This change has an effect on worker-hour requirements (MH^p_{6t} in statement [10]), and so worker hour requirements are recomputed. The planned level of worker hours paid for is then set equal in statement [14] to this recomputed amount. The planned level of inventories is also recomputed using the new planned output. The effect of the new planned level of inventories on worker hour requirements has already been taken into account in the solving of the quadratic equation to compute the new planned level of output. Statements [13] and [14] are meant to capture the effects of employment adjustment costs on the behavior of the firm sector.

Statement [15] determines the first planned wage rate, which is a positive function of last period's wage rate, of the current period's planned price, and of the ratio of the current period's planned level of worker hours paid for to last period's actual level of worker hours paid for, and a negative function of last period's ratio of the unconstrained to the constrained aggregate supply of labor. This equation is again based on the results in Table 3-3.

Statements [16] - [19] determine the variables necessary to compute the planned level of loans of the firm sector. The planned level of loans is then computed in statement [20]. The equations in these statements are all comparable to the equations for the non-condensed model. In statement [21] various unconstrained quantities are defined. These are the values that the firm sector would choose if it were not subject to a loan constraint. The reason that separate notation is established for the unconstrained quantities in statement [21] is because of the presentation of these values in Table 6-6 in Chapter Six. By comparing the values in statement [21] with the final values chosen by the firm sector, one can see directly how the loan constraint has affected the decisions of the firm sector.

If the planned level of loans in statement [20] is less than the maximum value allowed, then the loan constraint is not binding on the firm sector. In this case the actual values are set equal to the planned values, as in statements [42] - [48]. If the planned level of loans is greater than the maximum value allowed, then the firm must modify its original plans. Statement [23] - [40] describe the modifications. In statement [23] the price level is raised from the originally planned level. The size of the increase is a positive function of the percentage difference between the unconstrained and maximum value of loans. The expected demand for goods is then recomputed in statement [24], being based now on a higher price level. The new planned level of loans, based on the higher price, is determined by statements [25] - [28]. A higher price, with the same level of production and investment, has an overall positive effect on the demand for loans, and so LF_t^p in statement [28] is greater than the originally planned level of loans. The new planned level of loans is, of course, not feasible, and it is recomputed in statements [25] - [28] only so it can be used in statement [29]. In statement [29] planned production is decreased. The size of the decrease is a positive function of the difference between the new planned level of loans and the maximum value. The statement is based on the following analysis. Using statements [25] - [27], statement [28] can be written

$$
\begin{aligned}
LF_t^p = {} & LF_{t-1} + DDF_{1t}^p + DDF_2 - DDF_{t-1} \\
& - \frac{1}{m}[P_{t-1}INV_{t-1} + \ldots + P_{t-m+1}INV_{t-m+1}] + \frac{m-1}{m} P_t INV_t^p - P_{t-1}V_{t-1} \\
& + P_t V_{t-1} + P_t Y_t^p - P_t X_t^e. \qquad (3.47)
\end{aligned}
$$

Now, for each unit decrease in Y_t^p, LF_t^p decreases by P_t units, and for each unit decrease in INV_t^p, LF_t^p decreases by $\frac{m-1}{m}$ P_t units. Also, for each unit decrease in Y_t^p, INV_t^p can decrease by $\frac{1}{\mu_1 \bar{H}}$ units. Therefore, a unit decrease in Y_t^p, with the appropriate decrease in INV_t^p, corresponds to a decrease in LF_t^p of $P_t + \left(\frac{1}{\mu_1 \bar{H}}\right) \left(\frac{m-1}{m}\right) P_t$ units. Since the firm must cut back its loans by LF_t^p - $LFMAX_t$ units, the planned level of output must be cut back by $(LF_t^p - LFMAX_t) / (P_t + \left(\frac{1}{\mu_1 \bar{H}}\right) \left(\frac{m-1}{m}\right) P_t)$ units, which is the expression in statement [29]. In statement [30] investment is cut back by the appropriate amount. Statement [31] then defines the resulting new value for the number of machines on hand.

In statement [32] the planned number of hours paid for per worker is decreased corresponding to the decrease in planned production. The new planned level of inventories is computed in statement [33], and the new level of worker hour requirements is computed as described in statement [34]. If the new level of worker hour requirements is greater than the planned level of worker hours paid for, then, as in statement [35], the latter is set equal to the former. In statement [36] the wage rate is changed as a result of the change in the price level and the planned number of worker hours paid for. Statements [37]–[40] determine the new level of loans corresponding to the various changes.

It should be noted that the new level of loans will not be exactly equal to $LFMAX_t$ because the wage rate and the planned number of worker hours paid for, both of which have an effect on DDF_{1t}^p in statement [39], are decreased from their original values. This decrease was not taken into account when planned production was changed in statement [29], and so LF_t as computed in statement [40] will be slightly less than $LFMAX_t$. This is a very small effect, however, which is the reason it was ignored in computing the change in planned production in statement [29].

Statements [48] and [49], determining the maximum number of worker hours that the firm will pay for and the maximum number of goods that the firm will sell, are the same as in the non-condensed models. Statement [50] defines the minimum number of machines needed to produce the planned output ($KMIN_t^p$), and statement [51] defines the number of worker hours required to meet the expected change in sales (MH_{4t}^p). The value of $KMIN_t^p$ is needed for the results in Table 6-6, where the ratio of K_t^a to $KMIN_t^p$ is presented. The ratio is a measure of the planned excess capital of the firm sector. The value of MH_{4t}^p is also presented in Table 6-6. Since the actual level of sales will generally not be equal to the expected level, MH_{4t}^p will generally not be equal to the number of worker hours required to meet the actual change in sales. The value of MH_{4t}^p is presented in Table 6-6 because it is of some interest to compare this number to the number of worker hours actually required to meet the change in sales.

In summary, while the details of the algorithm in Table 3-4 are somewhat tedious, the overall design is fairly clear. The price level and expected sales are determined first in equations (1) and (2). Statements [1] - [14] are then concerned with computing the levels of production, investment, and employment. The decisions involved in statements [3], [4], [13], and [14] reflect the fact that there are costs involved in changing net investment and employment. These statements are designed to approximate the actual production smoothing decisions of the firms in the non-condensed model. Given the planned level of employment and the price level, the wage rate is computed in statement [15]. Statements [16] - [20] then determine the planned value of loans. If the planned value is less than the maximum value, then the algorithm is essentially finished. Otherwise, the firm sector modifies its decisions in a fairly straightforward way in statements [23] - [40].

The optimal control problem of the firm is clearly the most complicated of the control problems in the model, and the algorithm in Table 3-4 can certainly only be considered to be an approximation to it. One of the main differences between the actual control problem and the approximation in Table 3-4 is that the latter is recursive while the former is not. In the actual control problem the decisions on price, production, investment, employment, the wage rate, and loans are made simultaneously (all coming out of the solution of the maximization problem), whereas in Table 3-4 the decisions are made more or less recursively.

NOTES

[a]It should be obvious in what follows that the number of different types of machines can be generalized to any number.

[b]Since all expectations are made by firm *i*, no *i* subscript or superscript has been added to the relevant symbols to denote the fact that it is firm *i* making the expectation.

[c]In the programming for the non-condensed model, firm *i* was assumed to estimate the parameter β_8 in Equations (3.26) and (3.27) on the basis of its past observations of the correlation between changes in the aggregate demand for goods and changes in the average price level. Similarly, the parameters β_{11} and β_{12} in Equations (3.33) and (3.34) were assumed to be estimated by firm *i*. The exact procedure by which these parameters were assumed to be estimated is described in the Appendix.

[d]There is an asymmetry in the specification of Equations (3.26)–(3.27) and (3.33)–(3.34), aside from the fact that the former are in terms of the *constrained* demand for goods and the latter are in terms of the *unconstrained* supply of labor. In (3.26)–(3.27) firm *i*'s expectation of the aggregate demand for goods is only a function of prices and not wages, whereas in (3.33)–(3.34) its expectation of the aggregate supply of labor is a function of both prices and wages. In general, a firm's expectation of the aggregate demand for goods may also be a function of wages, but for reasons of computational convenience this possibility was not allowed for here.

[e]As was the case for banks, although Equations (3.1)–(3.21) are written only for period *t*, they are also meant to hold for periods $t{+}1, \ldots, t{+}T$ as well. In addition, an *e* or a *p* superscript should be added to a variable when firm *i* only has an expectation or a planned value of that variable. For example, Equation (3.7) should be written

$$V^e_{it} = V_{it-1} + Y^p_{it} - XF^e_{it}, \tag{3.7)'$$

$$V^e_{it+k} = V^e_{it+k-1} + Y^p_{it+k} - XF^e_{it+k}, k=1,2,\ldots,T. \tag{3.7)''$$

To conserve space, Equations (3.1)–(3.21) will not be written out in this expanded way, but the expansion in each case is straightforward.

[f]A firm's expectation of the aggregate (constrained) supply of labor depends, of course, on its expectation of the aggregate unconstrained supply of labor from Equation (3.33), which in turn depends on its price and wage expectations from Equations (3.23), (3.25), and (3.32).

[g]Since actual worker hour requirements for period t are dependent on the actual level of sales in period t and since a firm only has at the beginning of period t an expectation of sales for the period, it likewise only has at the beginning of period t an expectation of worker hour requirements for the period.

[h]By employment in this case and in what follows is meant the expected supply of labor, HPF^e_{it}.

[i]The quadratic equation is $Y^p_t/\lambda_1 + \beta_2(V_{t-1} + Y^p_t - X^{ep}_t - \beta_1 X^{ep}_t)^2 + MH^p_{4t} + MH^p_{5t} + MH^p_{6t} = HPF^p_t$. This equation is obtained by substituting [5] in [7], adding [6] through [10], and setting this sum equal to HPF^p_t, the planned number of worker hours.