Chapter Seven

A Static-Equilibrium Version of the Model

7.1 INTRODUCTION

The methodology of this study has *not* been to develop a static-equilibrium model first and then to construct a dynamic version of it, but rather to specify from the very beginning a dynamic model. It is the author's view that static-equilibrium models are not of much use in providing insights into how an economy actually functions and that too much attention has been devoted in macroeconomic theory to analyzing static-equilibrium models. The static-equilibrium IS-LM model and its various extensions, for example, have come to dominate much of the teaching of macroeconomic theory. Although, as mentioned in Chapter One, it has recently been debated whether this model is an adequate representation of what Keynes actually had in mind, the model continues to be widely used. This model will be called the "textbook" model in the following discussion.

There are two reasons why a static-equilibrium version of the present dynamic model has been developed in this study. One reason is to show explicitly how much is lost in going from a dynamic model to a static-equilibrium model. It will be seen that many of the important characteristics of the dynamic model are lost when the model is converted into a static-equilibrium model. The other reason is to provide a model that is directly comparable to the textbook model. It is easier to compare two static-equilibrium models than it is to compare a dynamic model and a static-equilibrium model. The comparison between the present dynamic model and the textbook model is thus indirect. It will first be seen how the dynamic model compares to its static-equilibrium version, and it will then be seen how the static-equilibrium version compares to the textbook model. Because of the popularity of the textbook model, it was felt that some kind of a comparison between the dynamic model and the textbook model might aid in understanding the characteristics of the dynamic model. The static model is also useful in helping to point out an error in one of Christ's models [7].

7.2 THE STATIC-EQUILIBRIUM VERSION

An "equilibrium state" of a model is defined to be a state in which none of the variables in the model changes over time. The self-repeating run that was concocted for the dynamic model in Chapter Six is a run in which the dynamic model is in equilibrium. A static model is defined to be a model in which there are no time subscripts.

Some of the main differences between the basic dynamic model in this study and a static-equilibrium version of it are the following. First, in equilibrium no constraints can be binding, and so no distinction needs to be made in the static-equilibrium version between unconstrained and constrained quantities. Second, there can be no net savings or dissavings in equilibrium, for otherwise assets would be changing. This means that the net investment of the firm sector must be zero (gross investment equal to depreciation), savings of the households must be zero, and savings of the government must be zero (a balanced budget).

Third, there can be no excess labor and capital in equilibrium, for otherwise the firm sector would, among other things, be changing its price. Fourth, production must equal sales in equilibrium, for otherwise inventories would be changing. Fifth, there can be no capital gains and losses in equilibrium and no excess supply of bills and bonds. Sixth, the actual level of bank reserves must equal the desired level in equilibrium, for otherwise the bank sector would be changing its decisions. Seventh and finally, prices, wage rates, and interest rates must be determined in equilibrium in a way that clears the goods, labor, and financial markets. This condition usually means that prices, wage rates, and interest rates are determined *implicitly* in a static-equilibrium model. The values of these variables are usually determined by equating the quantities demanded to the quantities supplied.

It should be clear already that in the present case many of the important characteristics of the dynamic model will not be present in the static-equilibrium version. The price level and the wage rate cannot be set by the firm sector, but must be determined implicitly so as to clear the goods and labor markets. The loan rate cannot be set by the bank sector and the bill rate cannot be set by the bond dealer, but must be determined implicitly so as to clear the financial markets. No constraints can ever be binding, and no errors of expectations can ever be made. In the present case, in other words, the static-equilibrium version is more than just the dropping of time subscripts from

Table 7-1. Notation for the Static-Equilibrium Model in Alphabetic Order

BR	= actual bank reserves
BR^*	= required bank reserves
d_1	= profit tax rate
d_3	= personal tax rate
DDB	= demand deposits of the bank sector
DDF	= demand deposits of the firm sector
DDH	= demand deposits of the household sector
DEP	= depreciation of the firm sector
DIV	= total dividends paid and received in the economy
$DIVB$	= dividends paid by the bank sector
$DIVF$	= dividends paid by the firm sector
$FUNDS$	= loanable funds of the bank sector
g_1	= reserve requirement ratio
$\overline{H}$	= maximum number of hours that each machine can be used each period
HPF	= number of worker hours paid for by the firm sector
HPG	= number of worker hours paid for by the government
HPH	= number of hours that the household sector is paid for
I	= number of machines purchased by the firm sector in a period
INV	= number of goods purchased by the firm sector for investment purposes
K	= total number of machines on hand in the firm sector
L	= total value of loans of the bank sector
LF	= value of loans taken out by the firm sector
LH	= value of loans taken out by the household sector
m	= length of life of one machine
P	= price level
r	= bill rate and loan rate and bond rate
SD	= savings deposits of the household sector (and of the bank sector)
TAX	= total taxes paid
$TAXB$	= taxes paid by the bank sector
$TAXF$	= taxes paid by the firm sector
$TAXH$	= taxes paid by the household sector
VBB	= value of bills and bonds held by the bank sector
VBG	= value of bills and bonds issued by the government
W	= wage rate
X	= total number of goods sold
XG	= number of goods purchased by the government
XH	= number of goods purchased by the household sector
Y	= total number of goods produced
YG	= minimum guaranteed level of income
YH	= before-tax income of the household sector
δ	= number of goods that it takes to create one machine
λ	= amount of output produced per worker hour
μ_1	= amount of output produced per machine hour
ΠB	= before-tax profits of the bank sector
ΠF	= before-tax profits of the firm sector

Table 7–2. The Equations of the Static-Equilibrium Model

(1) $VBG = VBB$,

(2) $FUNDS = (1-g_1)DDB + SD$,

(3) $HPF = \frac{Y}{\lambda}$,

(4) $K = \frac{Y}{\mu_1 \bar{H}}$,

(5) $I = \frac{1}{m} K$,

(6) $INV = \delta I$,

(7) $\lambda = 1.3212 \delta^{0.3212}$

$$\overline{\Pi F} = (1-d_1)\,[P \cdot Y - W \cdot HPF - (r + \frac{1}{m}) \cdot P \cdot \delta \cdot K]$$

$$= (1-d_1)\,[P \cdot Y - W \cdot \frac{Y}{\lambda} - (r + \frac{1}{m}) \cdot P \cdot \delta \cdot \frac{Y}{\mu_1 \bar{H}}] ,$$

(8) $\frac{\partial \overline{\Pi F}}{\partial \delta} = 0 \Rightarrow \frac{0.3212 \cdot W}{1.3212 \cdot \delta^{1.3212}} - \frac{(r + \frac{1}{m}) \cdot P}{\mu_1 \bar{H}} = 0$,

(9) $\frac{\partial \overline{\Pi F}}{\partial Y} = 0 \Rightarrow P - \frac{W}{\lambda} - \frac{(r + \frac{1}{m}) \cdot P \cdot \delta}{\mu_1 \bar{H}} = 0$,

(10) $Y = X$

(11) $HPH = e^{8.350} P^{-0.40} W^{0.40} r^{0.77} d_3^{-0.30} DIV^{-0.01} (SD-LH)^a - 0.78\,YG$,

(12) $XH = e^{4.398} P^{-1.24} W^{1.24} r^{-0.54} d_3^{-0.18} DIV^{0.08} (SD-LH)^b + 0.36\,YG$,

(13) $X = XH + INV + XG$,

(14) $L = LF + LH$,

(15) $HPH = HPF + HPG$,

(16) $DEP = P \cdot INV$,

(17) $\Pi F = P \cdot Y - W \cdot HPF - DEP - r \cdot LF$,

(18) $TAXF = d_1 \Pi F$,

(19) $DIVF = \Pi F - TAXF$,

(20) $DDF = \beta_{14} W \cdot HPF$,

(21) $DDH = \gamma_1 \cdot P \cdot XH$,

Table 7–2. (continued)

(22) $DDB = DDF + DDH$,

(23) $YH = W \cdot HPH + DIV + r \cdot SD$,

(24) $TAXH = d_3(YH - r \cdot LH) - YG$,

(25) $YH - TAXH - P \cdot XH = 0$,

(26) $\Pi B = r(L + VBB - SD)$,

(27) $TAXB = d_1 \, \Pi B$,

(28) $DIVB = \Pi B - TAXB$,

(29) $DIV = DIVB + DIVF$,

(30) $TAX = TAXB + TAXF + TAXH$,

(31) $BR = DDB + SD - L - VBB$,

(32) $BR^* = g_1 DDB$,

(33) $BR = BR^* \; [\Rightarrow FUNDS = L + VBB]$,

(34) $P \cdot XG + W \cdot HPG + r \cdot VBG - TAX = 0$.

Given values of $\mu_1, \bar{H}, m, \beta_{14}$, and γ_1, the above set of equations consists of 34 equations in 42 unknowns. The unknowns are:

1. BR
2. BR^*
*3. d_1
*4. d_3
5. DDB
6. DDF
7. DDH
8. DEP
9. DIV
10. $DIVB$
11. $DIVF$
12. $FUNDS$
*13. g_1
14. HPF
*15. HPG
16. HPH
17. I
18. INV
19. K
20. L
21. LF
22. LH
23. P
24. r
25. SD
26. TAX
27. $TAXB$
28. $TAXF$
29. $TAXH$
30. VBB
*31. VBG
32. W
33. X
*34. XG
35. XH
36. Y
*37. YG
38. YH
39. δ
40. λ
41. ΠB
42. ΠF

*Variable of the government

One of Equations (25) and (34) is redundant, which means that there are 33 independent equations in 42 unknowns. Given values of *LF*, *LH*, and *SD* and given values of 6 of the 7 government variables, the system of equations consists of the same number of independent equations as unknowns.

Note: Since *SD* and *LH* are exogenous, it does not matter what the values of *a* and *b* are in Equations (11) and (12).

the variables of the dynamic model. The determination of some of the key variables and the interactions among the behavioral units are substantially changed when going from the dynamic model to the static-equilibrium version.

The static-equilibrium version consists of 34 equations and is presented in Table 7-2. The notation for the model is presented in Table 7-1. The variables in Table 7-2 are roughly in the order in which they appear in Table 6-2. Time subscripts have been dropped from all the variables in this chapter. The bond dealer serves no useful purpose in the model, since the bill rate is implicitly determined, and so the bond dealer has been dropped from the model. There is also no reason to have more than one interest rate in the model, and so the loan rate and the bond rate have been dropped. The only interest rate in the model is the bill rate, r, and so this is the rate not only on government debt, but also on private loans and savings deposits.

Equation (1) in Table 7-2 is a market clearing equation, equating the supply of bills and bonds from the government (VBG) to the demand for bills and bonds by the bank sector (VBB). Since there are no capital gains and losses in equilibrium and since the bond rate is always equal to the bill rate, there is no need to distinguish between bills and bonds. The interest payment of the government on VBG, for example, is simply $r \cdot VBG$. Since there is no bond dealer in the static model, the desired value of bills and bonds of the bond dealer in the dynamic model, VBD^*, does not appear in Equation (1). Equation (2) in Table 7-2 defines the level of loanable funds and is the same as Equation (1) in Table 2-4 without the time subscripts and without the $EMAXDD$ and $EMAXSD$ terms. Since there is no uncertainty in the static model, $EMAXDD$ and $EMAXSD$ serve no useful purpose and can be dropped.

Equations (3) through (10) determine the production of the firm sector and its demand for investment goods and employment. Since the price level and the wage rate are not decision variables of the firm sector in the static-equilibrium model, a much different and simpler behavioral model of the firm sector must be considered. No longer can a firm's decisions be assumed to be based on the solution of an optimal control problem in which the price level and the wage rate are among the decision variables. The simpler model of firm behavior in Equations (3)–(10) is as follows.

Since there can be no excess labor and capital in equilibrium, Equations (3) and (4) must hold. Equation (3) states that the number of worker hours that the firm sector pays for must be equal to the number required to produce the output. Equation (4) states that the number of machines on hand must be equal to the minimum number required to produce the output. Since net investment must be zero in equilibrium, Equation (5) must hold. In equilibrium the number of machines wearing out in a period must be $\frac{1}{m}K$, where m is the length of life of a machine. Equation (5) states that the number of machines purchased in a period (I) must be equal to the number wearing out. In Equation (6) the number of machines purchased is translated into the equivalent number of goods purchased.

Equation (7) determines λ as a function of δ. The three important parameters regarding the technology in the firm sector are λ, output per worker hour, μ_1, output per machine hour, and δ, the number of goods required to create one machine. In the non-condensed model two types of machines were postulated, so that λ, μ, and δ each took on two possible values (λ_1 and λ_2, μ_1 and μ_2, δ_1 and δ_2). μ_1 and μ_2 were, however, assumed to be equal, so that the two types of machines differed only in their λ and δ coefficients.

In the condensed model only one type of machine was postulated, so that λ, μ, and δ each took on only one value. In the condensed model investment was still a function of the loan rate because the firm sector's price decision was a function of the loan rate. The price decision had an effect on the investment decision through its effect on expected sales and planned production. In the static-equilibrium model λ and δ are assumed to be continuous variables, and so there are in effect assumed to be an infinite number of different types of machines. The parameter μ_1 is still assumed to be the same for all of the different types of machines. In Equation (7) λ is a positive function of δ: the more expensive a machine is in terms of the number of goods it takes to produce it, the greater is the output per worker hour on the machine. The ratio μ_1/λ is the worker-machine ratio, and with μ_1 fixed, Equation (7) merely states that machines with lower worker-machine ratios cost more.

The choice of the coefficients in Equation (7) is discussed in the next section. The specification of Equation (7) is a way of keeping the putty-clay nature of the technology for the static-equilibrium model. The worker-machine ratio is fixed ex post, but ex ante the firm sector has a choice of which technology to use.

The next equation in Table 7-2 defines after-tax profits of the firm sector. The total revenue is $P \cdot Y$, the total cost of labor is $W \cdot HPF$, and the total cost of capital is $(r+\frac{1}{m}) \cdot P \cdot \delta \cdot K$. Since each machine has a life of m periods and since one machine costs $P \cdot \delta$ to purchase, the cost of capital for one machine is $(r+\frac{1}{m})P \cdot \delta$. Multiply this by K, the total number of machines on hand, and one has the total cost of capital. The second expression for after-tax profits in Table 7-2 replaces HPF and K by their definitions in Equations (3) and (4).

The two decision variables of the firm sector are the choice of the technology, represented by δ, and the level of output Y. The firm sector is assumed to maximize after-tax profits, and so Equations (8) and (9) must hold. The derivation in Equation (8) uses the fact, from Equation (7), that λ is a function of δ. Equation (9) merely states that in equilibrium the price of a unit of output must equal the cost of producing it. The last equation in the production block for the firm sector, Equation (10), states that output must equal sales in equilibrium, for otherwise inventories would be changing.

Equations (11) and (12) determine the two main decision variables of the household sector, the number of hours to work (*HPH*) and the number of goods to purchase (*XH*). The existence of two different households serves no useful purpose in the static-equilibrium model, and so the households have been aggregated into one. Equation (11) is similar to Equations (2) and (1)′ in Table 4-6, and Equation (12) is similar to Equations (3) and (2)′ in Table 4-6. In Equations (2) and (3) in Table 4-6, the level of savings deposits and the stock price were added together, but in Equations (11) and (12) in Table 7-2 the two have been separated. Since the stock price is DIV/r and since r is already included in the equations, the separation of the level of savings deposits and the stock price merely means that *DIV* is included as a separate variable in Equations (11) and (12) in Table 7-2.

Equation (11) states that *HPH* is a positive function of the wage rate and the interest rate, and a negative function of the price level, the proportional tax rate, the level of dividends, and the minimum guaranteed level of income. Equation (12) states that *XH* is a positive function of the wage rate, the level of dividends, and the minimum guaranteed level of income, and a negative function of the price level, the interest rate, and the proportional tax rate. The choice of the coefficients in the two equations is discussed in the next section. The coefficients are based on the coefficients in Equations (2), (3), (1)′, and (2)′ in Table 4-6.

Equations (13)–(31) in Table 7-2 are very similar to the relevant equations in Table 6-2, appropriately simplified. Equation (13) determines total sales and is similar to Equation (16) in Table 6-2. The equation in the present context is the market clearing equation for goods. Equation (14) determines the total value of loans, and Equation (15) is the market clearing equation for labor. Equations (16)–(20) determine the financial variables of the firm sector: depreciation, before-tax profits, taxes paid, dividends paid, and demand deposits. Depreciation in Equation (16) is merely the value of investment in equilibrium. Equation (17), determining before-tax profits, does not include an inventory valuation term because the term is zero in equilibrium. If *LF* is zero in Equation (17), as it is taken to be for the results in the next section, then the level of before-tax profits as defined in Equation (17) is merely $r \cdot K$ because of Equation (9). If *LF* were set equal to *K*, then profits in Equation (17) would, of course, be zero. The choice for the value of *LF* is discussed in the next section.

Since there is no uncertainty in the static model, the DDF_2 term in the dynamic model serves no useful purpose in the static model, and so it has not been included in Equation (20) in Table 7-2. Equation (21) determines the demand deposits of the household sector, and Equation (22) determines total demand deposits. Equations (23) and (24) determine the income and taxes of the household sector. Equation (25) is an equilibrium condition and states that the savings of the household sector must be zero in equilibrium. Equations (26)-(28) determine the before-tax profits, taxes, and dividends of the bank sector, and Equations (29) and (30) determine total dividends and total taxes. Equation (31) determines actual bank reserves, and Equation (32) determines required bank reserves. Equation (33) is an equilibrium condition and states that actual reserves must equal required reserves in equilibrium. Equation (34) is also an equilibrium condition and states that the savings of the government must be zero in equilibrium.

Aside from the specific coefficients in Equations (7), (11), and (12), there are five parameters in the static model: μ_1, $\bar{H}$, m, β_{14}, and γ_1. Not counting these parameters, the static model consists of 34 equations in 42 unknowns. The unknowns are listed at the end of Table 7-2. Although it may not be immediately obvious from the model, one of Equations (25) and (34) is redundant. Given all the other equations in the model and one of the two equations, the other is automatically satisfied. Consider, for example, Equation (34), which says that government savings are zero. This equation must be redundant, given the rest of the equations in the model, for the following reason. The firm and bank sectors retain no earnings and so are neither net savers nor net dissavers. Equation (25) states that the household sector is neither a net saver nor a net dissaver. Therefore, since all flows of funds are accounted for in the system, zero net savings in the private sector of the economy must imply that the net level of savings of the government is zero, which is Equation (34). If the government were a net saver or a net dissaver, this would show up somewhere in the savings of the private sector. Equation (34) is thus redundant, given the other equations of the model.

The static model thus consists of 33 independent equations in 42 unknowns, so that there are nine more unknowns than equations. There are seven variables of the government: three tax parameters, d_1, d_3, and *YG;* the reserve requirement ratio, g_1; the value of bills and bonds issued, *VBG;* the number of goods purchased, *XG;* and the number of worker hours paid for, *HPG.* There are also three stock variables in the model for which there are no explicit equations: the value of savings deposits of the household sector, *SD;* the value of loans of the household sector, *LH;* and the value of loans of the firm sector, *LF.* If these three variables are treated as exogenous, then there are six more unknowns than equations, and so the government can choose six of its seven values. In this case, because of the requirement that the government budget be balanced in equilibrium, given six of the seven government values, the other value is automatically determined.

It seems reasonable in the present context to treat *SD, LH,* and *LF* as exogenous. Consider, for example, *SD-LH-LF,* and denote this variable as A, which is the stock of assets of the private, nonbank sector (not counting common stocks and demand deposits). Let A_{-1} denote the stock of assets of the previous period. Then A is determined as A_{-1} plus the level of savings. Therefore, given

Table 7-3. Equations of the Static-Equilibrium Model by Blocks (*LF, LH, SD,* and all government values except *VBG* are assumed to be exogenous)

Block 1: 12 equations, 14 endogenous variables: *DIV, HPF, HPH, I, INV, K, P, r, W, X, XH, Y,* δ, λ

Equation Can Be Used to Compute:	Equation No. in Table 7-2	
Y	(3)	$HPF = \frac{Y}{\lambda}$,
K	(4)	$K = \frac{Y}{\mu_1 \bar{H}}$,
I	(5)	$I = \frac{1}{m} K$,
INV	(6)	$INV = \delta I$,
λ	(7)	$\lambda = 1.3212 \cdot \delta^{0.3212}$,
δ	(8)	$\delta = \left[\frac{\mu_1 \bar{H} \cdot 0.3212}{(r + \frac{1}{m}) \cdot 1.3212} \frac{W}{P} \right]^{\frac{1}{1.3212}}$,
$W, P, W/P,$ or r	(9)	$\frac{W}{P} = \lambda \left[1 - (r + \frac{1}{m}) \frac{\delta}{\mu_1 \bar{H}} \right]$,
X	(10)	$Y = X$,
HPH	(11)	$HPH = e^{8.350} P^{-0.40} W^{0.40} r^{0.77} d_3^{-0.30} DIV^{-0.01} (SD-LH)^{a} - 0.78YG$,
$W, P, W/P,$ or r	(12)	$XH = e^{4.398} P^{-1.24} W^{1.24} r^{-0.54} d_3^{-0.18} DIV^{0.08} (SD-LH)^{b} + 0.36YG$,
XH	(13)	$X = XH + INV + XG$,
HPF	(15)	$HPH = HPF + HPG$.
Block 2:		
L	(14)	$L = LH + LF$,
DDF	(20)	$DDF = \beta_{14} \cdot W \cdot HPF$,
DDH	(21)	$DDH = \gamma_1 \cdot P \cdot XH$,
DDB	(22)	$DDB = DDF + DDH$,

Table 7–3. (continued)

FUNDS	(2)	$FUNDS = (1 - g_1)DDB + SD,$
VBB	(33)	$FUNDS = L + VBB,$
VBG	(1)	$VBG = VBB,$
DEP	(16)	$DEP = P \cdot INV,$
ΠF	(17)	$\Pi F = P \cdot Y - W \cdot HPF - DEP - r \cdot LF,$
TAXF	(18)	$TAXF = d_1\, \Pi F,$
DIVF	(19)	$DIVF = \Pi F - TAXF,$
ΠB	(26)	$\Pi B = r(L + VBB - SD),$
TAXB	(27)	$TAXB = d_1\, \Pi B,$
DIVB	(28)	$DIVB = \Pi B - TAXB,$
DIV	(29)	$DIV = DIVB + DIVF,$
YH	(23)	$YH = W \cdot HPH + DIV + r \cdot SD,$
TAXH	(24)	$TAXH = d_3(YH - r \cdot LH) - YG,$
TAX	(30)	$TAX = TAXB + TAXF + TAXH,$
W, P, or r	(34)	$P \cdot XG + W \cdot HPG + r \cdot VBG - TAX = 0.$

A_{-1}, A must be equal to it in the static model because savings must be zero in equilibrium. Since the model has no way of determining A_{-1} endogenously, it likewise has no way of determining A. Therefore, it is reasonable to treat A as exogenous. If, say, *SD* were not treated as exogenous (but *LH* and *LF* were), there would be seven more unknowns than equations, and so the government could choose all seven of its values. The requirement of a balanced budget for the government in equilibrium would not lead in this case to one of the government values being automatically determined, given the other six. This would, however, only be because of the unreasonable treatment of *SD* as endogenous.[a]

This completes the specification of the static-equilibrium model. The solution of the model is discussed in the next section, and some results are presented of solving the model for alternative values of the government variables. As was the case for the dynamic model, the results in the next section are meant only to aid in understanding the properties of the static model and are not meant to be a "test" of the model in any sense. The static model is compared to the textbook model in Section 7.4 below.

7.3 THE SOLUTION OF THE STATIC MODEL

Given values of *SD*, *LH*, and *LF* and given six of the seven government values, the model consists of the same number of independent equations as endogenous variables. The model is nonlinear in the variables and so must be solved by some iterative technique. For the results in this section the model was solved using the Gauss-Seidel technique. Before this technique was applied, however, the model was broken up into two blocks, and it will be useful to consider this breakdown. The two blocks are presented in Table 7-3. The first block corresponds to the real sector of the model, and the second block corresponds to the financial sector. The equations in Table 7-3 are in the same form as they appear in Table 7-2 except for Equations (8) and (9), which have been rearranged. The zero-savings equation of the government (Equation (34)) has been included in Table 7-3, and so the zero-savings equation of the household sector (Equation (25)) has not been included.

If *VBG* is taken to be the one endogenous government variable, then the real block consists of 12 equations in 14 endogenous variables. The model was solved in the endogenous *VBG* case in the following way. Given values of two of the 14 endogenous variables in block 1, block 1 was solved for the other 12 variables using Gauss-Seidel. Block 2 was then solved for the other variables in the model, including the two variables taken as given for the solution of block 1. Block 1 was then resolved using the new values of the two variables and block 2 was resolved again. This process was repeated until overall convergence was reached. There are clearly other ways that the model could be solved using Gauss-Seidel, but the way just described converged fairly quickly and so no further experimentation with ways of solving the model was carried out. In addition to its computational convenience, breaking the model up into the two blocks has the advantage of indicating clearly the links between the real and financial sectors.

It was decided for purposes of the static model to make *HPH* and *XH* a function of the real wage, W/P. The coefficients for W and P in Equation (11) were taken to be of opposite sign and equal in absolute value, as were the coefficients for W and P in Equation (12). The 0.40 coefficient for W/P in Equation (11) is the average of the absolute values of the coefficients for W and P in Equations (2) and (1)′ in Table 4-6 $[(0.41 + 0.71 + 0.25 + 0.22)/4 = 0.40]$. Likewise, the 1.24 coefficient for W/P in Equation (12) is the average of the absolute values of the coefficients for W and P in Equations (3) and (2)′ in Table 4-6. The other coefficients in Equations (11) and (12) are similarly averages of the relevant coefficients in Equations (2), (3), (1)′, and (2)′ in Table 4-6. It makes no difference what coefficients are used for *SD-LH* in Equations (11) and (12) because *SD* and *LH* are both treated as exogenous and thus never change. When SD_{t-1} and PS_t are split up in Equations (2) and (3), the coefficients for PS_t change from −0.38 and 0.14 to −0.22 and 0.08 because of the change of the base from $SD_{t-1} + PS_t$ to SD_{t-1} and PS_t separately. The one change that was made in going from Equation (2) in Table 4-6 to Equation (11) in Table 7-2 was to make the coefficient for *DIV* smaller in absolute value, from −0.22 to −0.01. (Remember that *DIV/r* is merely *PS* in the static model.) This was done to make the solution values in the real block somewhat less sensitive to the values determined in the financial block.

The parameter values, government values, and values of *LF, LH,* and *SD* that were used for the basic solution of the model are presented in Table 7-4. These values and the values for the constant terms in Equations (11) and (12) were chosen to make the basic set of solution values come out to be roughly the same as the base run values for the dynamic model in Chapter Six. The values for *LF* and *LH* were, however, taken to be zero, and the value for *SD* was taken to be 203.2. The value 203.2 is the difference between the base run value for SD_t in Chapter Six (1013.4) and the sum of the base run values for LF_t and LH_t (*328.1 + 482.1*). The firm sector was also assumed to hold no demand deposits, so that β_{14} was taken to be zero. These changes have very little effect on the final properties of the model.

Table 7–4. Parameter Values, Government Values, and Values of *LF, LH,* and *SD* for the Base Run in Table 7–5

Parameter Values	*Government Values*	*Values of LF, LH, SD*
$\bar{H} = 1.0$	$d_1 = 0.5$	$LF = 0.0$
$m = 10$	$d_3 = 0.2391$	$LH = 0.0$
$\beta_{14} = 0.0$	$g_1 = 0.1667$	$SD = 203.2$
$\gamma_1 = 0.32044$	$HPG = 124.7$	
$\mu_1 = 0.6787$	$XG = 93.3$	
	$YG = 0.0$	

Note: The two constant terms were chosen in Equations (11) and (12) under the assumption that the values of *a* and *b* were zero.

The two coefficients in Equation (7) (1.3212 and 0.3212) were chosen as follows. The solution value for λ for the basic run was first chosen to be 1.3212, the same as the value of λ_1 in the condensed, dynamic model; and the solution value for δ was taken to be 1.0, also the same as in the condensed, dynamic model. This meant that the first coefficient in Equation (7) had to be 1.3212. Given values for λ, δ, *W, P, r, m,* and $\bar{H}$, Equation (9) was then solved for μ_1. Finally, given values for *W, P, r, m,* $\bar{H}$, μ_1, and the 1.3212 coefficient already determined, Equation (8) was solved for the remaining coefficient, which turned out to be 0.3212.

Regarding the solution of the model using Gauss-Seidel, it is somewhat arbitrary as to which two of the 14 variables in block 1 are taken as given for purposes of solving the block. The main choice in the overall model is which equations to use to compute *W*, *P*, and *r*. Note that if the coefficients for *W* and *P* in Equations (11) and (12) are of opposite sign and equal in absolute value, as they are specified here to be, then *W* and *P* always enter as *W/P* in block 1. It is not important for purposes of solving the model, however, whether *W* and *P* enter separately in Equations (11) and (12) or only as *W/P*. It should also be noted that it is not important for purposes of solving the model whether *r* is included in the demand deposit equations, (20) and (21). There is, in other words, nothing in the model that requires that demand deposits be a function of the rate of interest in order to solve the model.

The results of solving the model are presented in Table 7-5. For all the runs in Table 7-5, *VBG* was taken to be the endogenous government variable. The first set of results in the table is based on the values in Table 7-4. For each of the other runs in the table, one of the six exogenous government values was changed. For all of these results the model was solved by using Equation (9) to compute *r*, Equation (12) to compute *W/P*, and Equation (34) to compute *P*. *W* was computed as *W/P* times *P*. This meant that the two variables taken as given for purposes of solving block 1 were *DIV* and the breakdown of *W/P* into *W* and *P*. The advantage of solving the model in this way is that block 2 becomes linear in the unknown variables in the block and so can be solved without having to use the Gauss-Seidel technique. Only values for the most important variables in the model are presented in Table 7-5. Real *GNP* in the table, *GNPR*, is the sum of *Y* and *HPG*.

For the first experiment in Table 7-5, the number of goods purchased by the government (*XG*), was increased by 2.5. This caused output, *Y*, to rise by 4.77, from 842.03 to 846.80. The price level, the wage rate, and the interest rate were all higher, and the real wage was slightly lower. The values for δ and λ decreased, which meant that the firm sector switched to a cheaper type of machine with a higher worker-machine ratio. Both a higher interest rate and a lower real wage induce the firm sector to switch to a more labor intensive type of machine. The price level and the wage rate each rose by about 12 percent corresponding to the increase in *XG* of about 2.7 percent. The higher price level corresponded to larger values for the financial variables. Demand deposits increased by about 12 percent, from 200.35 to 225.25, and *VBG* increased from 370.16 to 390.91. The aggregate level of dividends increased from 45.79 to 51.91, and the aggregate level of taxes increased from 242.26 to 272.80.

Because the model is fully simultaneous, it is not possible to talk about one endogenous variable *causing* another endogenous variable to behave in a certain way. Nevertheless, it is possible to speak loosely about the relationship between one endogenous variable and another. Consider, for example, why the

Table 7–5. Results of Solving the Static-Equilibrium Model for the Endogenous *VBG* Case

	Base Run	1 XG: +2.5	2 XG: −2.5	3 d_3: −0.00304	4 d_3: +0.00304	5 YG: +2.5	6 YG: −2.5	7 HPG: +2.5	8 HPG: −2.5	9 g_1: −0.10	10 g_1: +0.10
GNPR	966.73	971.50	962.12	972.49	961.20	969.60	963.01	971.06	962.58	967.70	965.77
P	1.0009	1.1216	0.8980	1.1131	0.9047	1.1371	0.8551	1.1325	0.8902	1.0221	0.9807
W	1.0009	1.1202	0.8990	1.1124	0.9052	1.1355	0.8565	1.1310	0.8913	1.0218	0.9809
W/P	1.0000	0.9988	1.0011	0.9994	1.0006	0.9986	1.0016	0.9987	1.0012	0.9997	1.0002
r	0.06500	0.06564	0.06439	0.06533	0.06471	0.06573	0.06416	0.06569	0.06435	0.06515	0.06486
Y	842.03	846.80	837.42	847.79	836.50	844.90	838.31	843.86	840.38	843.00	841.07
λ	1.3213	1.3196	1.3238	1.3204	1.3220	1.3193	1.3234	1.3194	1.3229	1.3208	1.3216
δ	1.0000	0.9961	1.0037	0.9980	1.0018	0.9956	1.0051	0.9959	1.0040	0.9991	1.0008
I	124.06	124.77	123.39	124.91	123.25	124.49	123.52	124.33	123.82	124.21	123.92
INV	124.06	124.28	123.84	124.67	123.47	123.94	124.15	123.82	124.31	124.10	124.03
HPF	637.32	641.73	633.08	642.09	632.77	640.40	633.47	639.56	635.26	638.23	636.42
HPH	762.02	766.43	757.78	766.79	757.47	765.10	758.17	766.76	757.46	762.93	761.12
XH	624.67	626.72	622.78	629.83	619.73	627.66	620.86	626.74	622.77	625.60	623.74
DDB	200.35	225.25	179.20	224.65	179.67	228.70	170.13	227.45	177.65	204.90	196.01
VBG	370.16	390.91	352.53	390.41	352.92	393.78	344.97	392.74	351.24	394.44	346.94
DIV	45.79	51.91	40.61	51.44	40.98	52.58	38.61	52.28	40.37	47.55	44.11
TAX	242.26	272.80	216.34	268.07	220.13	273.56	208.72	275.32	214.58	248.48	236.32

price level is higher in experiment 1 than it is in the base run. When the government increases XG without increasing tax rates, some way must be found for satisfying the zero savings equation of the government, Equation (34). Now, a rising price level increases both the money expenditures of the government and taxes, but the relationships in the model are such that taxes rise more than money expenditures as the price level increases. Therefore, speaking loosely, Equation (34) can be met by having the price level rise. The government is, in other words, financing the increase in XG by an increase in the price level.

Regarding the increase in VBG in experiment 1, consider how VBG is determined. From Equations (1), (2), and (33), VBG equals $(1-g_1)DDB + SD-L$. Since SD and L are exogenous, the only endogenous variable on the right-hand side of this equation is the level of demand deposits, DDB. Since DDB is proportional to the price level, VBG increases as the price level increases. Another way of looking at this is as follows. From Equations (2) and (33) the demand for bills and bonds by the bank sector, VBB, is $(1-g_1)DDB + SD - L$. Since SD and L are exogenous, VBB increases as DDB increases. From Equation (1) VBG must equal VBB, so that VBG must increase as DDB increases to meet the increased demand for bills and bonds from the bank sector.

Consider finally the behavior of the household sector. In order to increase the level of output, the household sector has to be induced to work more. The savings of the household sector must be zero, so that if the sector works more, it must also consume more. One way of inducing the sector to work more is for the interest rate to increase, and for the results in Table 7-5 the interest rate is an important factor in inducing the sector to work more. The higher interest rate in experiment 1 also had, however, a negative effect on the number of goods purchased by the household sector, but this was more than offset by the higher level of dividends. The zero savings constraint of the household sector was also met in part by the fact that the price level increased slightly more than did the wage rate. Holding HPH, XH, r, and DIV constant, an increase in P, holding W constant, has a negative effect on the savings of the household sector, and an increase in W, holding P constant, has a positive effect. An increase in P relative to W thus has a negative effect on savings. It is also the case, however, given the coefficients used in Equations (11) and (12), that a decrease in W/P decreases HPH less than XH, so that on this score a decrease in W/P has a positive effect on savings. Overall, of course, the solution values are such that the zero savings constraint is satisfied, and all that can be done here is to give a rough indication of how this comes about.

For the second experiment, the value of XG was decreased by 2.5. The results in Table 7-5 are almost exactly opposite, even quantitatively, to those for the first experiment, and so require no further discussion. Even though the model is nonlinear, the response of the model is quite symmetrical for the size of the changes considered here.

For the third experiment, the personal income tax parameter, d_3, was decreased by 0.00304. With no other changes, this corresponds to an aggregate tax decrease of 2.5. This change had similar effects to the increase in *XG* in experiment 1. The interest rate rose, although it rose less than it did in experiment 1. A decrease in d_3 has a positive effect on the work effort of the household sector, so that, again speaking loosely, the interest rate needed to rise less in experiment 3 than it did in experiment 1 in order to have the household sector work more. The number of goods purchased by the household sector was greater in experiment 3 than in experiment 1 because the induced increase in output in experiment 3 did not correspond to any increase in the number of goods purchased by the government. The total level of output was also somewhat greater in experiment 3 than in experiment 1. It is interesting to note that even though d_3 was decreased in experiment 3, the aggregate level of tax collections in money terms (*TAX*) rose substantially because of the increase in the price level. The results for the fourth experiment, an increase in d_3 of 0.00304, are again almost exactly opposite to those for the third experiment.

For the fifth experiment, the minimum guaranteed level of income, *YG,* was increased by 2.5. This change had similar effects to the increase in *XG* in experiment 1 and to the decrease in d_3 in experiment 3. In this case, however, the interest rate was higher than it was in experiment 1, and the total level of output was somewhat lower. In contrast to the case in experiment 3, where a decrease in d_3 has a positive effect on the work effort of the household sector, an increase in *YG* has a negative effect on work effort. Therefore, the increase in output was somewhat less in experiment 5 than in experiment 3, and the interest rate was somewhat greater in order to induce the household sector to work more. In other words, decreasing taxes by decreasing the proportional tax rate has more of an effect on output than does decreasing taxes by increasing the minimum guaranteed level of income because of the work response of the household sector. The results for the sixth experiment, a decrease in *YG* of 2.5, are opposite to those for the fifth experiment.

For the seventh experiment, the number of worker hours paid for by the government (*HPG*) was increased by 2.5. Real GNP was about the same in this case as in experiment 1, although in this case 2.5 of the increase in real GNP was due to the increase in *HPG.* The number of goods produced, *Y,* was less in experiment 7 than in experiment 1. Overall, however, the results for experiments 1 and 7 are quite similar. The results for the eighth experiment, a decrease in *HPG* of 2.5, are opposite to those for the seventh experiment.

For the ninth experiment, the reserve requirement ratio, g_1, was decreased from 0.1667 to 0.0667. This change had a stimulative effect on the economy and led, for example, to an increase in output, the price level, the wage rate, the interest rate, and the level of employment. The reason the decrease in g_1 had a positive effect on the economy is roughly as follows. Since *SD* and *L* are exogenous, it can be seen from Equations (2) and (33) that a decrease in g_1 leads, other things being equal, to an increase in *VBB*. In other words, more funds are now available for the bank sector to buy bills and bonds. From Equation (1), *VBG* must then increase to meet the increase in the demand for bills and bonds.

An increase in *VBG* means that the level of interest payments from the government to the bank sector is increased, which in turn means that the level of dividends is increased. A higher aggregate dividend level then has a positive effect on the number of goods purchased by the household sector. An expansion in the economy thus takes place when g_1 is decreased because government spending is increased. Government spending is increased because of the increase in interest payments. It may seem puzzling at first glance as to why the interest rate would increase when g_1 is decreased, since a decrease in g_1 frees up more funds, but one of the reasons this happens is because a higher interest rate is needed to induce the household sector to work more. In a loose sense one might say that the interest rate is tied more to the equations in the real block than it is to the equations in the financial block. The results for the tenth experiment, an increase in g_1 to 0.2667, are opposite to those for the ninth experiment.

The results in Table 7-5 are all based on the treatment of *VBG* as the one endogenous variable of the government. *VBG* can be made exogenous if one of the other seven government variables is made endogenous. For the results in Table 7-6, *VBG* was treated as exogenous and d_3 was taken to be the endogenous variable of the government. When d_3 is endogenous, the equations in Table 7-3 can be solved as follows.

The solution in block 1 can remain the same. In block 2, Equation (1) can be used to solve for *VBB*, given the now exogenous value for *VBG*. Equation (33) can be used to solve for *FUNDS*, and Equation (2) can be used to solve for *DDB*. When *VBG* is exogenous, *DDB* is in effect also exogenous. Given *DDB* and *DDF* (which is actually zero since β_{14} is zero), *DDH* is *DDB* − *DDF* from Equation (22). Given *DDH*, *P* can then be determined from Equation (21). *W* is then *W*/*P* times *P*, where *W*/*P* is available from block 1. Given *P* and *W*, *TAX* can be computed from Equation (34), the zero savings equation of the government. *TAXF* and *TAXB* can be computed in the usual way, and then given these two values and given *TAX*, *TAXH* can be computed from Equation (30). Given *TAXH*, d_3 can then be computed from Equation (24).

The value chosen for *VBG* for the results in Table 7-6 is the solution value of *VBG* for the base run in Table 7-5. All the other exogenous values for the base run in Table 7-6 were taken to be the same as the values used for the base run in Table 7-5. The base run in Table 7-6 is thus exactly the same as the base run in Table 7-5. For the first experiment in Table 7-6, *XG* was increased by 2.5. This had a positive effect on the price level, the wage rate, and the interest rate, but a negative effect on the level of output. In the endogenous d_3

Table 7-6. Results of Solving the Static-Equilibrium Model for the Endogenous d_3 Case

		Experiment			
	Base Run	*1 XG: +2.5*	*2 XG: −2.5*	*3 g_1: −0.10*	*4 g_1: +0.10*
GNPR	966.73	965.67	967.83	960.83	973.30
P	1.0009	1.0060	0.9958	0.9012	1.1269
W	1.0009	1.0054	0.9964	0.9016	1.1263
W/P	1.0000	0.9994	1.0006	1.0004	0.9995
r	0.06500	0.06531	0.06470	0.06477	0.06528
Y	842.03	840.97	843.13	836.13	848.60
λ	1.3212	1.3204	1.3220	1.3217	1.3205
δ	1.0000	0.9981	1.0018	1.0014	0.9983
I	124.06	123.91	124.23	123.20	125.03
INV	124.06	123.68	124.46	123.37	124.82
HPF	637.32	636.90	637.78	637.58	642.64
HPH	762.02	761.60	762.48	757.28	767.34
XH	624.67	621.50	627.88	619.47	630.48
DDB	200.35	200.35	200.35	178.89	227.67
DIV	45.79	46.08	45.49	41.41	51.36
d_3	0.2391	0.2422	0.2360	0.2429	0.2351
TAX	242.26	245.93	238.61	220.48	269.75

Note: Value used for *VBG* was 370.16 for all of the runs in this table.

case the increase in *XG* is financed by an increase in d_3, and an increase in d_3 has a negative effect on the work effort of the household sector. This effect was such in experiment 1 as to lead to a lower value of hours worked by the household sector and a lower value of output. The value for d_3 increased from 0.2391 to 0.2422. The price level and wage rate rose much less in experiment 1 in Table 7-6 than they did in experiment 1 in Table 7-5, since in Table 7-6 the increase in *XG* was in effect financed by an increase in d_3 rather than an increase in the price level. The results for the second experiment in Table 7-6, a decrease in *XG* of 2.5, are again the opposite to those for the first experiment.

For the third experiment in Table 7-6, g_1 was decreased to 0.0667. This change had a significant contractionary effect on the economy. The reason for the large contractionary effect can be seen roughly as follows. Given *VBG, L,* and *SD,* a decrease in g_1 means from Equations (1), (2), and (33) that *DDB* must decrease. Given *DDF* from Equation (20), this means from Equation (22) that *DDH* must decrease. Given the decrease in *DDH, P* must then decrease from Equation (21). *W*, being determined as *W/P* times *P*, must then decrease. The decrease in *P* leads, among other things, to a lower level of dividends and turns out to have a contractionary effect on the economy. The results for the fourth experiment in Table 7-6, an increase in g_1 to 0.26667, are opposite to those for the third experiment.

This concludes the presentation of results for the static model. Although the results of treating other government variables as endogenous could be presented, enough evidence has been presented to give a good indication of the properties of the model.

A few general remarks about the model will be made to conclude this section. First, it should be obvious that it makes an important difference regarding the response of the model to a change in an exogenous variable as to which government variable is made endogenous. When, for example, *VBG* is endogenous, an increase in *XG* leads to an increase in output and a much higher price level, whereas when d_3 is endogenous, an increase in *XG* leads to a slight decrease in output but only a slightly higher price level. It also should be obvious that when *VBG* is endogenous, the multiplier effect of an increase in *XG* on output is not one over the marginal tax rate. In Christ's model [8] the multiplier is over the marginal tax rate, but his model is much simpler than the present model. Christ's model, for example, does not have a labor sector and does not endogenously determine the price level. When a more complicated model than Christ's is considered, there is no reason to expect that his result regarding the multiplier will generalize, and in the present case it clearly does not.

It was mentioned above that it makes no difference from the point of view of solving the model whether the level of demand deposits is a function of the rate of interest or not. It also turns out to make little quantitative difference as to whether this is true or not. The experiments in Tables 7-5 and 7-6 were carried out under the assumption that *DDH* is a function of *r*:

$$DDH = e^{-2.733}\,\gamma_1 P \cdot XH \cdot r^{1.00} \,. \qquad (21)'$$

The constant term in Equation (21)′ is such as to make the base run value of *DDH* unchanged. The results of replacing Equation (21) with Equation (21)′ were little changed from the results in Tables 7-5 and 7-6. For the first experiment in Table 7-5, for example, the new solution value of *r* was 0.06563 compared to 0.06564. The new level of output was 846.72 compared to 846.80. For the third experiment in Table 7-6, the new solution value of *r* was the same to four significant digits, and the new level of output was 836.04 compared to 836.13. For none of the experiments were the results in the two cases noticeably different. As mentioned above, the interest rate is, in a loose sense, more influenced by the equations in the real block than by the equations in the financial block, and so making *DDH* a function of *r* has very little effect on the quantitative properties of the model.

Two of the equations that are quite important in influencing the properties of the model are the two main equations of the household sector, Equations (11) and (12). Equation (11) in particular is quite important because, holding the technology constant, the level of output in the economy is constrained by the work effort of the household sector. The coefficients in Equations (11) and (12) were chosen to be consistent with the coefficients in the condensed model, which were in turn chosen to be consistent with the results obtained by solving the optimal control problems of the households in Chapter Four. Although it might be of interest to examine the properties of the static model under different choices for the coefficients in Equations (11) and (12), this will not be done here.

7.4 A COMPARISON OF THE STATIC MODEL TO THE TEXTBOOK MODEL

A version of the standard macroeconomic textbook model is presented in Table 7-7. This version is taken from a textbook by Branson [6], one of the more advanced textbooks in the field. The notation for the most part is Branson's, and the model is what Branson calls "the extended model."[b]

The model in Table 7-7 consists of (1) a consumption function in disposable income and assets, (2) an investment function in the rate of interest and income, (3) an income identity, (4) a real money demand function in the rate of interest and income, (5) a money supply function in the rate of interest, (6) an equilibrium condition equating money supply to money demand, (7) a production function in employment (with the capital stock held fixed), (8) a demand for labor equation equating the marginal product of labor to the real wage rate, (9) a labor supply function in either the money wage or the real wage,[c] and (10) an equilibrium condition equating the supply of labor to the demand for labor. Taking A and $\bar{K}$ to be exogenous and taking the government variables (g, $\bar{M}$, and the parameters in the tax function, $t(y)$) to be exogenous, the model consists of ten equations in ten unknowns (c, i, y, M^D, M^S, N^D, N^S, P, W, and r). The following are some of the differences between the textbook model and the static model in this chapter.

Consumption in the textbook model is a function of after-tax income and the real value of assets, and the supply of labor is a function of the wage rate and perhaps the price level. In the present model both consumption and the supply of labor are functions of the same variables, since they are both decision variables of the household sector and are thus jointly determined by the maximization processes of the households. The explanatory variables in the equations are the price level, the wage rate, the interest rate, the level of savings deposits and loans, the level of dividends, and the two tax parameters, d_3 and YG.

In the textbook model investment is a function of the rate of interest and income, and the demand for labor is a function of the real wage and the shape of the production function. The price level and the wage rate are

Table 7-7. The Equations of the Textbook Model

(1)	$c = c(y - t(y), \frac{A}{P})$,	[consumption function]
(2)	$i = i(r,y)$,	[investment function]
(3)	$y = c + i + g$,	[income identity: equilibrium condition for the goods market]
(4)	$\frac{M^D}{P} = l(r) + k(y)$,	[demand for money function]
(5)	$M^S = \overline{M}$ *or* $M^S = M(r)$,	[supply of money function]
(6)	$M^S = M^D$,	[equilibrium condition for the money market]
(7)	$y = f(N^D, \overline{K})$,	[production function]
(8)	$f'(N^D) = \frac{W}{P}$,	[demand for labor function]
(9)	$N^S = h(W \text{ or } \frac{W}{P})$,	[supply of labor function]
(10)	$N^S = N^D$.	[equilibrium condition for the labor market]

Given values for A, $\overline{K}$, g, and $\overline{M}$, the model consists of 10 equations in 10 unknowns: c, i, y, M^D, M^S, N^D, N^S, W, P, r.

Notation in alphabetic order:

A = value of assets in money terms

c = value of consumption in real terms

g = value of government purchases of goods in real terms

i = value of investment in real terms

$\overline{K}$ = value of the capital stock

M^D = quantity of money demanded

M^S = quantity of money supplied

$\overline{M}$ = quantity of money supplied by the government

N^D = quantity of labor demanded

N^S = quantity of labor supplied

P = price level

r = interest rate

W = wage rate

y = value of output in real terms

implicitly determined in the textbook model, being determined essentially by the market clearing equations for goods and labor (Equations (3) and (10)). In the present model the firm sector chooses the technology and the level of output so as to maximize after-tax profits. The net result of this is that both the demand for investment and the demand for labor are a function of the real wage rate and the interest rate. The price level and the wage rate are also implicitly determined in the present static model.

In the textbook model there are no government bills or bonds in existence, and no zero savings constraint is postulated.[d] It is thus somewhat difficult to compare the financial sector of the textbook model to the financial sector of the present model. In both models the interest rate is determined implicitly. In the textbook model this comes about by equating the demand for money to the supply of money. In the present model this comes about by equating the demand for bills and bonds (*VBB*) to the supply (*VBG*), and by equating actual bank reserves (*BR*) to required reserves (BR^*). In the present model, unlike in the textbook model, the interest rate has a direct effect on the demand for labor, the supply of labor, and the consumption demand of the household sector. The interest rate actually affects the supply of labor and consumption in two ways, one directly and one through its effect on the aggregate stock price (DIV/r). The interest rate is thus in some sense a more integral part of the present model than it is of the textbook model.

It is well known that the demand for money equation in the textbook model is an important equation in influencing the properties of the model, and much empirical work has to be done on estimating the interest rate sensitivity of the demand for money. In the present model, as was seen above, it is not very important whether the level of demand deposits is or is not a function of the rate of interest. The interest rate is more influenced by the equations in the real block. This appears to be a significant difference between the present model and the textbook model, and puts the importance of empirical studies of the demand for money in a somewhat different light.

The main differences between the present model and the textbook model can be summarized as follows. In the present model the demand for investment and the demand for labor are joint decision variables of the firm sector and are determined jointly through a maximization process. Likewise, the supply of labor and the demand for consumption are joint decision variables of the household sector and can be considered to be determined jointly through a maximization process. Neither of these characteristics is true of the textbook model. The present model also accounts explicitly for all flows of funds in the model and for the zero-savings constraints, which the textbook model does not.

While these are important differences and while the present model does appear to be an improvement over the textbook model, it is still the author's opinion that the most significant weakness of both models is their static-equilibrium nature. What is hoped this chapter has demonstrated is how many important characteristics of the dynamic model are lost when the model is converted into a static-equilibrium model.

NOTES

[a]Christ's model [7] is actually in error in this regard. His model consists of 11 equations (counting the zero-savings equation of the government) in 14 unknowns. Four of the unknowns are government values, and one of the unknowns is real private wealth (w in his notation). w is similar to the variable A in the above discussion. Christ treats w as endogenous and argues that the government can choose only three of its four values. If w were treated as exogenous, as it is argued here it should be, then Christ's model would seem to imply that the government could choose all four of its values. The error in Christ's model, however, is the treatment of two interest rates (r, the yield on bonds, and r', the yield on physical capital) as endogenous. In equilibrium these two rates should be equal, and yet Christ does not impose any restrictions on the two rates. If one of the two rates were dropped, or an equation was added equating the two rates, w could be treated as exogenous and the government would still be able to choose only three of its four values.

[b]See in particular Chapter 14 in Branson [6].

[c]Usually in textbooks the "classical" model is the version in which the supply of labor is a function of the real wage, while the "Keynesian" model is the version in which the supply of labor is a function of the money wage.

[d]This latter point has been emphasized by Christ [8], among others. As mentioned above, both of Christ's models, [7] and [8], incorporate a zero savings constraint, but neither model has a labor sector, and in both models the price level is exogenous.