A Model of Macroeconomic Activity

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Volume II:
The Empirical Model

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To My Parents

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Preface

The work described in this volume is a continuation of my effort to try to improve the specification of macroeconomic models. The model presented in this volume is an empirical version of the theoretical model developed in Volume I. Three important features of the theoretical model that distinguish it from earlier models are that it is based on solid microeconomic foundations, it accounts explicitly for disequilibrium effects, and it accounts for all flows of funds in the system. These three features have been carried over to the empirical model.

The methodology of this study is unusual enough to require some explanation. There is, first of all, no unique way to specify an empirical version of the theoretical model; the model is simply too abstract for this to be possible. Thus, although I have been guided closely by the theoretical model in my empirical specification, it will be clear in what follows that my particular specification is not the only one that could be said to be consistent with the theoretical model.

If there is no unique empirical version of the theoretical model, the question immediately arises as to how the theoretical model is to be judged. My answer to this question occurs on page 16 of Volume I:

> The author looks on a theoretical model of the sort developed in this study as not so much true or false as useful or not useful. The model is useful if it aids in the specification of empirical relationships that one would not already have thought of from a simpler model and that are in turn confirmed by the data. It is not useful if it either does

> not aid in the specification of empirical relationships that one would not have thought of from a simpler model or aids in the specification of empirical relationships that are in turn refuted by the data.

I argue in Chapter Eight that the present empirical model is confirmed by the data in the sense of its being more accurate than other models. It is also the case that I do not think that I would have been led to the present empirical specifications had I not had the theoretical model as a guide. Consequently, my conclusion is that as of this writing the theoretical model is useful. Whether this conclusion holds up as new models are developed, new tests performed, and new data collected is, of course, unknown. One can never rule out the possibility that a more accurate empirical model will be developed that is based on a different theoretical model.

One of the key assumptions of the theoretical model is that economic agents engage in maximizing behavior. In particular, each of the main behavioral units in the model makes its decisions on the basis of the solution to an optimal control problem. This is what is meant by the statement that the model is based on solid microeconomic foundations. It is true, of course, that economic agents do not actually solve optimal control problems explicitly in making their decisions. The assumption that they do so is used here as it is used in most of microeconomics: as a possibly useful approximation. As just mentioned, my way of testing whether assumptions such as this are useful for macroeconomic model building is to specify a theoretical model based on them, use the theoretical model as a guide to the specification of an empirical model, and then test the empirical model in standard ways. The results that I have obtained so far suggest that the maximizing assumption is useful for the specification of macroeconomic models. Additional tests are needed, however, before one can place too much confidence on this conclusion.

Another basic feature of the theoretical model is that expectations play an important role in influencing people's decisions (i.e., in influencing the solutions to the optimal control problems). For the simulation work in Volume I, most of these expectations were assumed to be formed in simple ways on the basis of past data. This is also true for the work in this volume, in the sense that lagged endogenous and lagged exogenous variables are used as explanatory variables in the stochastic equations to try to capture expectational effects.

It would have been possible for the simulation work in Volume I to use more sophisticated mechanisms of expectation formation. It could have been assumed, for example, that each behavioral unit estimates its own relevant econometric model each period, and uses this model to forecast the future values of the variables that it needs to know in order to solve its control problem. Assumptions similar to this were in fact made for some of the expectations formed by the banks, the firms, and the bond dealer. (See in particular the discussion on pages 205, 208, and 209 in Volume I.) Banks, firms, and the bond dealer were assumed to estimate from past data some of the key parameters that influence their expectations. Although I have not carried out such experiments, I doubt that the properties of the theoretical model would be changed very much if more of these types of assumptions were made. What is a crucial characteristic of the model, however, is the assumption that behavioral units do not have perfect foresight. This is one of the four characteristics listed on page 3 of Volume I that the model was deliberately designed to have.

Even if more sophisticated mechanisms of expectation formation had been postulated in Volume I, expectations would still have been based on past data. Consequently, I would probably still have been led to use lagged values in the empirical model to try to capture expectational effects. It is true, however, that if expectations of behavioral units are fairly accurate, one might expect actual future values to be better proxies for these expectations than are current and lagged values. I did in fact do some experimentation in the initial development of the empirical model to see if future values of some of the key explanatory variables (e.g., prices, wage rates, and interest rates) explained the current values of the decision variables better than did the current and lagged values of the explanatory variables. This empirical work did not support the use of future values, however, and in the end no future values were used as explanatory variables in any of the equations of the empirical model. The treatment of expectations in the empirical model is discussed in section 1.2 of Chapter One of this volume.

In a model building effort of this sort there are a number of detailed decisions that have to be made about how certain variables are to be treated and about what kinds of data are to be used. Realizing that not everyone is as interested in these details as I am, I have tried to write this volume so that the discussion of these details can be easily skipped or skimmed. In particular, I have relegated most of this discussion to section 1.3 of Chapter One and to Chapter Two. Most of the discussion of econometric issues is contained in Chapter Three, and this material can also be easily skipped or skimmed by readers who are not particularly interested in such things.

The first section of Chapter One, section 1.1, contains a summary of the central features and properties of the empirical model and of the major conclusions reached in this study. For those who are primarily interested in getting a general ideal of the properties of the model and of how it differs from other models, reading this section should be enough. Section 1.2 contains a discussion of some of the basic principles that guided the empirical specification, and section 1.3 contains a discussion of the linking of the national income accounts with the flow-of-funds accounts by sector. Although the details in section 1.3 can be skipped without much loss of continuity, it should be stressed that the linking of the two accounts is an important part of the present empirical work and has an important effect on the properties of the model.

The complete model is presented in Chapter Two, except for the discussion of the individual stochastic equations. The stochastic equations are explained in Chapters Four, Five, Six, and Seven. These latter four chapters are important in understanding the model, and by considering most of the data and econometric issues in Chapters Two and Three, I have tried to keep Chapters Four through Seven relatively free from discussion other than that directly related to the specification of the stochastic equations. The complete model is presented in tabular form in Tables 2-1, 2-2, and 2-3 in Chapter Two, and these tables are used for reference purposes throughout the rest of the text.

The predictive accuracy of the model is examined in Chapter Eight, and the properties of the model are examined in detail in Chapters Nine and Ten. The properties of the model are examined in Chapter Ten via the computation of optimal controls. Chapter Eleven contains some brief concluding remarks.

This volume can be read without a detailed knowledge of Volume I. One should, however, have some understanding of the theoretical model before reading this volume. At a minimum, Chapters One and Eight (Introduction and Conclusion) in Volume I should be read to get a general idea of the theoretical model.

I would like to stress that the empirical model presented here is not in any direct sense an expanded or revised version of my earlier forecasting model [14]. My interest in developing the forecasting model was to see if an econometric model could be developed that produced reasonably accurate forecasts when used in as mechanical a way as possible. My interest in Volume I, on the other hand, was theoretical and was to develop a general, dynamic macroeconomic model that was based on solid microeconomic foundations and that was not based on the restrictive assumptions of perfect information and the existence of tâtonnement processes that clear markets every period. My interest in this volume, although empirical, is more of an extension of my interest in Volume I than of my interest in the forecasting model. (The forecasting model does, however, provide a good basis of comparison for other models in terms of prediction accuracy, and it has been used for this purpose in Chapter Eight.)

Although my earlier work with the forecasting model has not had a direct effect on the specification of the present model, some of my work with monthly three-digit industry data has. This work is described in references [23], [21], and [15]. The results in these three studies have had an influence on my specification of both the theoretical and empirical models. Some of the links between my work with the monthly three-digit industry data and my work here are discussed in Chapter Five.

I am indebted to a number of people who have commented on my model building effort during the past few years. I would particularly like to thank Gregory Chow, Robert Hall, Donald Hester, Sharon Oster, and James Tobin for comments that led to important additions to this volume. I reluctantly assume responsibility for any errors. Most of this study was financed by grants from the National Science Foundation and the Ford Foundation to the Cowles Foundation for Research in Economics at Yale University.

Ray C. Fair
October 1975

A Model of Macroeconomic Activity

Chapter One

Introduction

1.1 A BRIEF SUMMARY

This section contains a brief discussion of the central features and properties of the empirical model and a summary of the major conclusions reached in this study. Some of the main differences between the present model and other econometric models are also indicated. Before proceeding to this discussion, some of the main features of the theoretical model that have motivated the specification of the empirical model will be reviewed.

The theoretical model is general, dynamic, based on microeconomic foundations, and not based on the assumptions of perfect information and the existence of tâtonnement processes that clear markets every period. It accounts for wealth effects, capital gains effects, all flow-of-funds constraints, and the government budget constraint. The decisions of the main behavioral units in the model (banks, firms, and households) result from the solutions of multiperiod optimal control problems. Expectations play an important role in the model in that the behavioral units must form expectations of future values before solving their control problems. The main decision variables of a bank are its loan rate and the maximum amount of money that it will lend in the period. The main decision variables of a firm are its price, production, investment, wage rate, and the maximum amount of labor that it will hire in the period. The main decision variables of a household are the number of goods to purchase and the number of hours to work. There is also a "bond dealer" in the model, representing the stock and bond markets.

An important distinction is made in the theoretical model between the *unconstrained* and *constrained* decisions of firms and households. A firm or household in a period may be constrained in how much money it can borrow at the current loan rate, and a household may also be constrained in how many hours it can work at the current wage rate. An unconstrained decision of a firm is defined to be a decision that results from the solution of the firm's optimal control problem when the loan constraint is not imposed, and a constrained decision is defined to be a decision that results when the loan constraint is imposed. There are obviously other constraints facing a firm, but for present purposes it is sufficient to distinguish only between the cases of a binding and nonbinding loan constraint. The words "constrained" and "unconstrained" thus refer only to whether the loan constraint is imposed or not. Similarly, an unconstrained decision of a household is defined to be a decision that results from the solution of the household's optimal control problem when neither the loan constraint nor the hours constraint is imposed, and a constrained decision is defined to be a decision that results when one or both constraints are imposed. The actual quantities traded in a period in the theoretical model are the quantities determined from the constrained optimization problems. Comparisons between these actual constrained solutions and the hypothetical unconstrained solutions are used to determine such things as the amount of (involuntary) unemployment in a period.

There are different "regimes" in the theoretical model, corresponding to the different cases of binding and nonbinding constraints. The four main regimes are (1) the regime in which none of the constraints are binding, (2) the regime in which only the loan constraints are binding, (3) the regime in which only the hours constraints are binding, and (4) the regime in which both the loan and hours constraints are binding. Because of the different possible regimes that can exist, there are many examples of asymmetrical reactions in the model. The responsiveness of the economy to various government actions, for example, depends in important ways on which regime is in effect at the time that the policy change is made.

The main determinants of a household's decision variables in the theoretical model, other than the loan and hours constraints when they are binding, are the price of goods, the wage rate, the interest rates, the tax rates, and the value of assets or liabilities at the beginning of the period. These are all variables that one expects on microeconomic grounds to affect a household's decisions. These variables, in conjunction with variables designed to measure the loan and hours constraints, are also used to explain the consumption and work effort variables of the household sector in the empirical model.

Consumption of the household sector is disaggregated into four components in the empirical model: consumption of services (other than services from durable goods and housing), consumption of nondurable goods, consumption of services from durable goods, and consumption of services from housing. Three work effort variables of the household sector are also considered: the labor force participation of men 25–54, the labor force participation of persons 16 and over except men 25–54, and a variable measuring the number of moonlighters.

The equations explaining the consumption of the household sector in the empirical model differ from standard consumption functions in at least two important ways. First, only the variables (other than the constraint variables) that one expects on microeconomic grounds to affect households' decisions (prices, wage rates, interest rates, tax rates, nonlabor income, and the value of assets or liabilities at the beginning of the period) are included on the right-hand side of the equations. Disposable personal income, for example, is *not* included as an explanatory variable in any of the equations because it is in part a consequence of the households' work effort decisions.

The consumption equations in the empirical model are further distinguished from standard consumption functions by their explicit treatment of the loan and hours constraints. It seems likely in practice that these constraints are sometimes binding and sometimes not, and variables have been constructed here that are designed to try to capture this inherent asymmetry of the constraints. When the hours constraint is binding, a household no longer controls its work effort decision, and its optimization problem degenerates into a simple optimal consumption decision. Under these conditions, since work effort is no longer a decision variable of the household, a reasonable specification of a consumption function may involve the inclusion of something like disposable personal income as an explanatory variable.

The consumption equations here do have the property that when the hours constraint is binding on the household sector, the specification is similar to having income as an explanatory variable in the equations. When the hours constraint is not binding, however, the only explanatory variables in the equations are those that one would expect on microeconomic grounds to affect the households' unconstrained decisions.

The treatment of the loan constraint on the household sector is similar to the treatment of the hours constraint. In particular, the equation explaining housing consumption differs depending on whether or not the loan constraint is binding.

The treatment of the consumption and work decisions of the household sector as being jointly determined also distinguishes the model from most other macroeconomic models. The same set of variables affects both types of decisions in the present model; in most other models the link between the two types of decisions is not made very explicit.

Work effort decisions clearly differ depending on whether or not the hours constraint is binding. In particular, in the model, the labor force participation rate of persons 16 and over except men 25–54 is less when the hours constraint is binding than when it is not. This effect can be interpreted as being similar to what are sometimes referred to in the literature as "discouraged worker" effects. The main difference here is, again, that the hours constraint affects both the consumption and work effort decisions simultaneously; thus there are both "discouraged consumption" and "discouraged worker" effects in the model.

The underlying technology of a firm in the theoretical model is of a "putty-clay" type, where at any one time different types of machines with differing worker-machine ratios can be purchased. The worker-machine ratio is fixed for each type of machine. Given this technology, given the past history of investment of a firm, and given an assumption about the maximum number of hours that each machine can be used each period, it is possible to calculate the minimum number of machines required to produce any given level of output. The difference between the actual number of machines on hand and the minimum number required to produce the actual level of output of the period is referred to as the amount of "excess capital" on hand.

It is likewise possible to compute the minimum number of worker hours required to produce any given level of output, and the difference between the actual number of worker hours paid for in a period and the minimum number required to produce the actual level of output of the period is referred to as the amount of "excess labor" on hand. Because of adjustment costs, it may sometimes be optimal for a firm to plan to hold either excess capital or excess labor or both during certain periods.

Market share considerations play an important role in the theoretical model in determining a firm's price and wage behavior. A firm has a certain amount of monopoly power in the short run in the sense that raising its price above prices charged by other firms will not result in an immediate loss of all its customers and lowering its price below prices charged by other firms will not result in an immediate gain of everyone else's customers. There is, however, a tendency for high price firms to lose customers over time and for low price firms to gain customers. A firm also expects that the future prices of other firms are in part a function of its own past prices. Similar considerations apply to a firm's wage decisions and its ability to gain or lose workers. Because of this market share nature of the model, some of the most important factors affecting a firm's decisions are its expectations of other firms' price and wage decisions.

A firm's price, production, investment, employment, and wage rate decisions are determined simultaneously in the theoretical model through the solution of the firm's optimal control problem. There are two constraints that may be binding on a firm. One is the loan constraint. The other, a labor constraint, results from the fact that a firm lacks perfect foresight and thus may at times set a wage rate that is too low to attract sufficient labor. In this case, actual output may fall short of planned output unless there is enough excess labor on hand to take up the slack.

The main determinants of a firm's decision variables in the theoretical model, other than the loan and labor constraints when they are binding, are the loan rate, the amounts of excess labor and capital on hand, the stock of inventories on hand, and variables affecting the firm's expectations of other firms' price and wage decisions. These variables, in conjunction with variables designed to measure the loan and labor constraints, are also used to explain the main decision variables of the firm sector in the empirical model. Lagged variables are generally used in the empirical model to try to capture expectational effects.

There are a number of differences between the explanation of the five main decision variables of the firm sector in the empirical model and their explanation in most other econometric models. First, the five variables are treated as being jointly determined. In most other models the variables are determined in a piecemeal fashion, with little thought given to the fact that they may be for each firm the result of the solution of a single optimizing process. Second, inventory investment is not treated in the empirical model as a direct decision variable of the firm sector. Instead, production is treated as a direct decision variable, and inventory investment is determined residually as the difference between production and sales. In most other macroeconomic models inventory investment is explained directly by a stochastic equation.

A third characteristic that distinguishes the present empirical model of the firm sector from other models is the explicit treatment of excess labor and excess capital. By postulating that firms may hold as an optimizing strategy excess labor and/or excess capital during certain periods, an explanation is provided for the commonly observed cyclical swings in "productivity." Most other models contain no explicit treatment of excess labor and excess capital and cannot reconcile productivity swings with optimizing behavior.

Finally, loan and labor constraints are considered explicitly in the empirical model, something that is generally not done in other models. The loan constraint is designed to try to capture some of the effects of the financial sector on the firm sector when credit is tight. Effects of this sort are sometimes called "credit rationing" effects. The labor constraint reflects the fact that firms lack perfect foresight, and it tries to capture some of the effects of the household sector on the firm sector when wage rates are set too low and labor markets are tight. In tight labor markets the labor constraint is binding on the firm sector in the model, while in loose labor markets the hours constraint is binding on the household sector. Thus in tight labor markets the level of employment is determined by the household sector, whereas in loose labor markets it is determined by the firm sector.

An important characteristic of the empirical model regarding the financial sector is the accounting for all flows of funds in the system. The data from the national income accounts have been linked by sector to the data from the flow-of-funds accounts. Accounting for all flows of funds means that one can consider explicitly in the model the direct purchase and sale of securities by the government. This is not true of models that have not accounted for all flows of funds, where it has to be assumed that the government has direct control over nonborrowed reserves or some similar type of variable. Accounting for all flows of funds also means that the government budget constraint is satisfied, so that any nonzero level of saving of the government must result in the change in at least one financial variable in the model.

Accounting for all flows of funds produces one equation out of 83 independent equations in the model for which there is not an obvious left-hand side endogenous variable. This "extra" equation allows the bill rate to be implicitly determined. There is thus no stochastic equation explaining the bill rate; the bill rate is rather determined implicitly through the solution of the 83 equations. The solution value for the bill rate each period should be thought of as being the rate that is necessary to equate the aggregate *constrained* supply of funds to the aggregate *constrained* demand for funds. The constraints can still be binding on the firms and households in the model even though the bill rate clears the financial markets each period.

The determination of the bill rate in the empirical model is somewhat different from its determination in the theoretical model. In the theoretical model the bond dealer sets the bill and bond rates for the next period with the aim of equating the supply of bills and bonds to the demand for bills and bonds in that period. There is thus an explicit equation for the bill rate in the theoretical model that is absent in the empirical model. This difference is due to the different treatment of bank reserves in the two models. For the theoretical model the level of bank reserves is a residual, whereas in the empirical model the level of bank reserves is linked directly to the level of demand deposits. The length of a period in the empirical model is a quarter, and on a quarterly basis it seems likely that banks have close control over their reserves. It thus does not seem reasonable in the empirical model to treat the level of bank reserves as a residual. The length of a period in the theoretical model is most realistically taken to be much less than a quarter, so that it does not seem unreasonable to assume in the theoretical model that the level of bank reserves is residually determined.

The main equation in the foreign sector explains the real value of imports. The real value of imports is, among other things, a negative function of the import price deflator and a positive function of the price deflator for domestically produced goods. Accounting for all flows of funds in the model means that all flows of funds between the domestic and foreign sectors are kept track of. The two most important exogenous variables in the foreign sector are the price of imports (the import price deflator) and the real value of exports.

Most of the variables in the government sector are exogenous. The exogenous variables include a profit tax rate, two personal income tax rates, an indirect business tax rate, employer and employee social security tax rates, the investment tax credit, the number of goods purchased, the number of worker hours purchased (civilian and military), some transfer payments, the reserve requirement ratio, the discount rate, the value of government securities outstanding, the value of currency outstanding, and the value of gold and foreign exchange of the government. There are two stochastic equations in the government sector—one explaining unemployment insurance benefits and one explaining the interest paid by the government. The two main differences between the treatment of the government sector here and its treatment in other models are the explicit treatment of the government budget constraint and the fact that the value of government securities outstanding can be taken to be a direct policy variable of the government.

The complete model consists of 83 independent equations, 26 of which are stochastic. There are 83 endogenous variables and, counting strike dummies, 78 exogenous variables plus the constant term. The model is simultaneous, nonlinear in variables, and includes lagged endogenous variables as explanatory variables. The error terms in some of the equations show evidence of first order serial correlation, and, after some experimentation, the serial correlation assumption was retained for 12 of the 26 stochastic equations. There are 166 unknown coefficients to estimate in the 26 stochastic equations, counting the serial correlation coefficients but not counting the variances and covariances of the error terms.

Data were collected for the 1952I–1975I period in this study. The basic period of estimation was taken to be 1954I–1974II (82 observations), which leaves three outside sample observations at the end of the period to analyze. All the unknown coefficients were estimated by two stage least squares (TSLS), and some of the coefficients were also estimated by full information maximum likelihood (FIML). It is not yet computationally feasible to obtain FIML estimates for an entire model of the present size, but the procedure described in Chapter Three allows one to proceed at least part way towards the attainment of true FIML estimates of the model.

The predictive accuracy of the empirical model is examined in Chapter Eight. The results in Chapter Eight indicate that the empirical model is more accurate than my earlier forecasting model. Previous results, which are also discussed in Chapter Eight, indicate that my forecasting model is at least as accurate (on an ex post basis) as other models, and probably more so. Consequently, this indirect comparison of the empirical model with other models indicates that the empirical model is more accurate. This conclusion is, of course (as mentioned in the Preface), clearly tentative.

The properties of the model are examined in Chapters Nine and Ten. It is difficult to summarize some of these properties without presenting the model in more detail than has been done so far. The following discussion concentrates on the properties of the model that relate to five important issues in macroeconomics.

The first issue concerns the relationship between the unemployment rate and the rate of inflation. There is no reason to expect this relationship to be at all stable in the model. The unemployment rate and the rate of inflation are both endogenous variables and are influenced by a number of diverse factors. The price set by the firm sector is affected by the bond rate, the labor constraint variable (when it is binding), and three variables designed to pick up expectational effects: the price level lagged one quarter, the wage rate lagged one quarter, and the price of imports. The unemployment rate is residually determined as one minus the ratio of employment to the labor force. The two labor force variables in the model are two of the work effort variables of the household sector. The main factors affecting the labor force are the wage rate, the price level, an interest rate, the marginal personal income tax rate, the value of assets of the previous period, nonlabor income (including the level of transfer payments from the government), and the hours constraint variable (when it is binding). The factors that affect employment are at times the factors that affect the work effort of the household sector (when the labor constraint is binding on the firm sector) and at times the factors that affect the employment demand of the firm sector (when the hours constraint is binding on the household sector).

Given the large number of diverse factors that influence the price level, the labor force, and the level of employment, it would be surprising if the net result of all of these effects were a stable relationship between the unemployment rate and the rate of inflation. There is in fact nothing in the model that indicates that this relationship should be stable, and so the model suggests that one is unlikely to observe a stable Phillips curve in practice.

Before proceeding to the second issue, it is of interest to examine the work effort variables of the household sector in a little more detail. The real wage rate in the model has a positive effect on work effort. A rise in the real wage rate, for example, increases the size of the labor force, other things being equal, which in turn increases the unemployment rate. The real wage rate thus has a direct positive effect on the unemployment rate. Interest rates also have a direct positive effect on the unemployment rate in the model because they have a positive effect on work effort.

The marginal personal income tax rate has a negative effect on both work effort and the unemployment rate, whereas the level of transfer payments from the government to households has a positive effect. Therefore, decreasing net taxes paid by decreasing the marginal tax rate has a direct positive effect on the unemployment rate, whereas decreasing net taxes paid by increasing the level of transfer payments has a direct negative effect. Finally, the value of assets in a period has a negative effect on work effort in the next period. Assets in this case are inclusive of corporate stocks, so that this effect means that an increase in stock prices in a period has a direct negative effect on the unemployment rate in the next period.

The second issue concerns the relationship between aggregate demand and the rate of inflation. There is also no reason to expect this relationship to be stable in the model. On the one hand, the price level has a direct negative effect on the consumption demand of the household sector. The main way that the firm sector contracts in the model is in fact to increase its price, which lowers sales, and then to decrease its production, investment, and demand for employment. On these grounds one would thus expect to observe a negative relationship between aggregate demand and the rate of inflation. On the other hand, tight labor markets, which exist during periods of high aggregate demand, have a positive effect on the price level. The price that the firm sector sets is directly affected by tight labor markets through the labor constraint variable and indirectly affected through the lagged wage rate, the wage rate being directly affected by the conditions of the labor market. On these grounds one would thus expect to observe a positive relationship between aggregate demand and the rate of inflation.

There are also many other factors at work on both the price level and the level of aggregate demand. The bill rate, for example, has a positive effect on the price level (through its effect on the bond rate) and a negative effect on consumption. The wage rate has a positive effect on consumption and, as just mentioned, a positive effect on the price level. Again, because of the many diverse factors that influence the price level and the level of aggregate demand, it would be surprising if the net result of all these effects were a stable relationship between aggregate demand and the rate of inflation. There is nothing in the model that suggests that this relationship should be stable.

The third issue involves the relationship between real output and the unemployment rate. There is once again no reason to expect this relationship to be stable in the model. Although the relationship between real output and employment is likely to be fairly stable (especially in the long run), a stable relationship between real output and the unemployment rate is unlikely because of the large number of factors that influence the labor force. The conclusion here is thus to be wary of Okun's law.

The fourth issue concerns the relationship between aggregate demand and the money supply. The model does predict a close relationship between aggregate demand and the money supply in the long run. The two main factors affecting demand deposits and currency of the household sector are the bill rate and the taxable income of the household sector. The two main factors affecting demand deposits and currency of the firm sector are the bill rate and the sales (in current dollars) of the firm sector. In the long run the sales of the firm sector and the taxable income of the household sector are closely tied to current dollar GNP, so that one would expect the aggregate value of demand deposits and currency to be closely tied to current dollar GNP in the long run. In the short run, however, changes in the bill rate can cause the relationship between the value of demand deposits and currency and current dollar GNP to be far from stable.

The final issue concerns the effectiveness of monetary policy and fiscal policy. Let *XG* denote the real value of goods purchased by the government, and let *VBG* denote the value of government securities outstanding.[a] *XG* is a fiscal policy variable under the control of the Administration and the

Congress in the United States. *VBG* is a monetary policy variable controlled by the Federal Reserve. If monetary policy is defined as a change in *VBG* with no change in any other exogenous variable, then the results in Chapter Nine indicate that monetary policy is effective: a change in *VBG* has, other things being equal, important effects on the economy. Similarly, if fiscal policy is defined as a change in *XG* with no change in any other exogenous variable, the results in Chapter Nine indicate that fiscal policy is also effective. Fiscal policy defined in this way is a policy that offsets any change in the saving of the government caused by the change in *XG* by changes in bank reserves and bank borrowing.

A decrease in *XG*, other things being equal, has a contractionary effect on the economy. So also does an increase in *VBG*. The net result of a decrease in both *XG* and *VBG* depends on the size of the two decreases. An equal initial decrease in both variables is contractionary in the model. A decrease in *XG* matched by a sufficient decrease in *VBG* to keep the money supply unchanged is, on the other hand, contractionary only for the first few quarters after the change. In fact, given any change in *XG*, it is possible to change *VBG* enough so that some important endogenous variable such as, say, real output is unchanged even in the current quarter. The model thus shows clearly the power of the Federal Reserve to influence the economy and to offset efforts of the fiscal branch of the government.

Many of the experiments in Chapter Nine are designed to explore possible asymmetrical properties of the model. The results show that the quantitative impact of a government policy action is different depending on the state of the economy at the time that the action is taken. The absolute size of the impact is also different depending on whether the policy action is contractionary or expansionary. The experimental results in Chapter Nine also show the different effects that result from changing different government policy variables. The effects of changing thirteen government variables are analyzed in Chapter Nine. These results will not be summarized here; they are summarized in Table 9-6 in Chapter Nine.

The final property of the model that will be discussed in this section relates to the optimal control results in Chapter Ten. When loss functions that target a given level of output and a given rate of inflation each quarter are minimized, the optima tend to correspond much more closely to the output targets being achieved than they do to the inflation targets being achieved. This is true even when the output target is weighted much less than the inflation target in the loss function. The model turns out to have the property that during most periods the level of output can be increased to a level of high activity without having too serious an effect on the rate of inflation. (The inflation rate may, of course, already be high. All this property implies is that it will not be much higher if output is increased.) It is generally not possible, however, to lower the inflation rate without having serious effects on the level of output. Consequently, when loss functions in the level of output and the rate of inflation are minimized, the optima tend to correspond to a more closely met output target. If this characteristic is also true of the real world, it has, of course, important policy implications.

This completes the summary of the model and its properties. Some general remakrs about the specification of the model are presented in the next section, and then the linking of the national income accounts with the flow-of-funds accounts by sector is explained in section 1.3.

1.2 FOUR GENERAL REMARKS ABOUT THE SPECIFICATION OF THE EMPIRICAL MODEL

It will be useful for the discussion in the rest of the text to make four general remarks now about the principles or tenets that have guided the specification of the empirical model. The first remark concerns the question of aggregation. The theory in Volume I is concerned with the behavior of individual units, whereas the data that are used to estimate the empirical model are aggregated into only five sectors: financial, firm, household, foreign, and government.

One of the key premises of this study is the assumption that one can use the behavior of the individual firms and households in the theoretical model to guide the specification of the equations relating to the behavior of the entire firm and household sectors in the empirical model. The main defense of this procedure is one of feasibility. Even if all the necessary data were available, which is not the case, it is clearly not feasible in a study of this sort to develop a highly disaggregated model. Consequently, little more will be said about the aggregation question except to admit that this study depends heavily on the premise just stated. (It is also the case that the various types of securities that exist in practice are aggregated in the empirical model into only five different types. This issue is discussed in the next section.)

The second remark concerns the question of unobserved variables. Expectations, which play an important role in the theoretical model, are generally not observed. Likewise, *unconstrained* decision values are generally not observed. Unconstrained decision values are observed in the theoretical model if none of the constraints are binding on the behavioral unit in question, but otherwise only the constrained decision values are assumed to be observed.

One generally tries to account for expectational effects in empirical work through the use of lagged values, and in this study lagged endogenous variables have been used freely as explanatory variables to try to account for these effects. It is generally not possible, of course, to separate expectational effects from lagged response effects, and no attempt has been made here to do so. Each stochastic equation of the model, however, has been estimated under the assumption of first order serial correlation of the error terms to make sure that the lagged endogenous variables are not erroneously picking up serial correlation effects. When the estimate of the serial correlation coefficient was significant for a particular equation, the serial correlation assumption was retained for the equation.

The problem of not generally observing unconstrained decision values is perhaps even more difficult to deal with than the problem of not observing expectations. Much of the discussion in Chapters Four and Five is concerned with explaining how this problem was handled in this study. As will be seen in these two chapters, there are other ways that one might try to deal with this problem than the way chosen here, and an important area for further research is the consideration of alternative procedures.

The third remark, which is related to the aggregation question, concerns the use of quarterly data. Although quarterly data have been used to estimate the empirical model, the time period postulated in the theoretical model is probably most realistically taken to be shorter than quarterly. Many of the interactions among the behavioral units that take place over more than one period in the theoretical model are likely to take place within a quarter in practice. This situation requires some differences of specification between the theoretical and empirical models, especially relating to the firm and financial sectors. One of the most important of these differences is that the empirical model is simultaneous, whereas the theoretical model is recursive.

The final remark, which is related to the question of expectations and lags, concerns the question of how many a priori constraints to impose on the data before estimation. This question is particularly important with regard to the effects of changes in tax laws. There is, unfortunately, much uncertainty regarding both the short run and long run response of the economy to various tax law changes. The data do not appear to be very good at discriminating among different lag structures and among alternative assumptions about how tax law changes affect the economic decisions of the private sector.

The imposition of a priori constraints in this study can be considered, in a loose sense, to be on two different levels. On the first level, the theoretical model is used as a guide to the specification of the empirical model. This procedure imposes very important constraints on the data. On the second level, further constraints on the parameters and equations of the model may be imposed that are not a direct consequence of anything in the theoretical model. The imposition of constraints on the second level works within the basic framework of the model that has been established on the first level.

The various constraints that have been imposed in this study are discussed in the following chapters. In some cases important constraints have been imposed regarding the effects of tax law changes, and in some cases not. An important constraint has, for example, been imposed concerning the effects of indirect business taxes. Households are assumed to respond to a change in indirect business taxes in the same way that they respond to any other type of change in the price level. On the other hand, severe constraints have not been imposed regarding the effects of changes in the personal income tax structure and the investment tax credit.

Constraints are imposed on the shapes of the lag distributions in the model by the use of lagged endogenous variables to try to capture expectational and lag effects. Some experimentation was done in estimating alternative lag structures, and in a few cases from the results of this work further constraints were imposed on the shapes of the lag distributions.

1.3 LINKING THE NATIONAL INCOME ACCOUNTS WITH THE FLOW-OF-FUNDS ACCOUNTS BY SECTOR

The most important issue regarding data in this study is the linking of the national income accounts (NIA) with the flow-of-funds accounts (FFA) by sector. Since this linking plays such an important role in the model, it is necessary to consider it in some detail before proceeding to a general discussion of the model. The rest of this chapter is concerned with explaining the linking.

As mentioned above, there are five sectors in the model: household, firm, financial, foreign, and government. The household sector is an aggregate of three sectors in the FFA: the households, personal trusts, and nonprofit organizations sector; the farm business sector; and the nonfarm noncorporate business sector. The government sector is an aggregate of four sectors: the state and local governments sector; the U.S. government sector; the federally sponsored credit agencies sector; and the monetary authorities sector. And the financial sector is an aggregate of two sectors: the commercial banking sector, and the private nonbank financial institutions sector. The commercial banking sector in the FFA is in turn an aggregate of four subsectors, and the private nonbank financial institutions sector is an aggregate of eleven subsectors. The relationship between the sectors in this study and the sectors in the FFA is summarized in Table 1-1.

Let y_{ijt} denote the payments from sector i to sector j during period t, and let N be the total number of sectors. The total amount paid by sector i during period t is

$$\sum_{j=1}^{N} y_{ijt},$$

and the total amount received by sector i during period t is

$$\sum_{j=1}^{N} y_{jit}.$$

Table 1–1. The Five Sectors of the Model

Sector in the Model (Abbreviation)	*Corresponding Sector(s) in the Flow-of-Funds Accounts*
1. Household (H)	1a. Household, Personal Trusts, and Nonprofit Organizations 1b. Farm Business 1c. Nonfarm Noncorporate Business
2. Firm (F)	2. Nonfinancial Corporate Business
3. Financial (B)	3a. Commercial Banking: (1) Commercial Banks (2) Domestic Affiliates of Commercial Banks (3) Edge Act Corporations and Agencies of Foreign Banks (4) Banks in U.S. Possessions 3b. Private Nonbank Financial Institutions: (1) Savings and Loan Associations (2) Mutual Savings Banks (3) Credit Unions (4) Life Insurance Companies (5) Private Pension Funds (6) State and Local Government Employee Retirement Funds (7) Other Insurance Companies (8) Finance Companies (9) Real Estate Investment Trusts (10) Open-End Investment Companies (11) Security Brokers and Dealers
4. Foreign (R)	4. Rest of the World
5. Government (G)	5a. State and Local Governments 5b. U.S. Government 5c. Federally Sponsored Credit Agencies 5d. Monetary Authorities

The difference between the total amount received and the total amount paid is the amount saved (or dissaved) by the sector during the period. Let s_{it} denote the amount saved by sector i during period t:

$$s_{it} = \sum_{j=1}^{N} y_{jit} - \sum_{j=1}^{N} y_{ijt}.$$

Dissaving corresponds to negative values of s_{it}. By definition, the savings of all sectors must sum to zero:

$$\sum_{i=1}^{N} s_{it} = 0.$$

Let TA_{it} denote the total net worth of sector i at the end of period t. If TA_{it} is negative, then sector i is a net debtor. Ignoring capital gains and losses, the change in net worth of sector i during period t is equal to its saving: $TA_{it} - TA_{it-1} = s_{it}$. TA_{it} is the sum of many different kinds of securities. Let A_{kit} denote the value of security k held by sector i at the end of period t, and let K be the total number of different kinds of securities in existence. Liabilities correspond to negative values of A_{kit}. By definition,

$$TA_{it} = \sum_{k=1}^{K} A_{kit}.$$

For this study, data must be collected on y_{ijt} ($i, j = 1, \ldots, N$) and on A_{kit} ($k = 1, \ldots, K$; $i = 1, \ldots, N$) for each time period. With five sectors ($N = 5$), this means that there are 25 values of y_{ijt} to collect for each period, although a few of these values are always zero. For many ij pairs, data on components of y_{ijt} are also available, and in most of these cases data on at least some of the components are needed.

Although data on y_{ijt} are NIA data, the best source for the data are the flow-of-funds publications. Some of the breakdown on the NIA data by sector is not published in the *Survey of Current Business*, but the breakdown can be obtained from the flow-of-funds publications. The data that were collected on y_{ijt} and its various components for each of the 25 pairs of values of i and j are presented in Table 1-2. Because of the somewhat tedious nature of this data collection, enough detail is presented in Table 1-2 so that in any future work with these data, one should be able to duplicate the collection fairly easily.

The numbers in parentheses in the table are the actual values for 1971, actual as of July 1975.[b] The numbers are at an annual rate in billions of current dollars. The first number in brackets for each variable is the code number of the variable on the flow-of-funds tape. The second number is the page number in the flow-of-funds publication (see reference [3]) where the variable can be found. For those variables in Table 1-2 that are not available on the flow-of-funds tape, the table numbers in the *Survey of Current Business* where the variables can be found are presented in brackets. The table numbers are taken from the July 1974 issue of the *Survey of Current Business*. It should be noted that the actual values in parentheses in the table are values that appear in either the flow-of-funds publication [3] or the *Survey of Current Business without any change of sign.* If a minus sign precedes the description of a variable, the number in parentheses does *not* include this minus sign.

The following is an explanation of the construction of Table 1-2. The first letter of a variable name in Table 1-2 denotes the sector making the payment, and the second letter denotes the sector receiving the payment. For example, $FH\text{—}_t$ is a payment by the firm sector to the household sector for period t, while $HF\text{—}_t$ is a payment by the household sector to the firm sector for period t. In I.1 in the table, $HHINT_t$ is the value of interest paid by the

Table 1–2. The Data from the National Income Accounts by Sector

I. Receipts to the Household Sector from:

1. The Household Sector (y_{HHt}):			
$HHINT_t$	= Consumer Interest	(17.746)	[156901103, p. 3]
$HHDIV_t$	= Dividends, Farms	(0.054)	[136120003, p. 8]
2. The Firm Sector (y_{FHt}):			
$FHWAG_t$	= Wages and Salaries, Private	(449.469)	[SCB, 1.10]
$-FHWLD_t$	= −Wage Accruals Less Disbursements, Private	(0.373)	[836700003, p. 6]
$FHOTH_t$	= Other Labor Income	(36.386)	[SCB, 1.10]
	−General Government (Compensation of Employees of Fed. Gov. and S & L Gov.)	(124.646)	[SCB, 1.7[a]]
	+Wages and Salaries, Government Civilian	(104.702)	[SCB, 1.10]
	+Wages and Salaries, Military	(19.419)	[SCB, 1.10]
$FHSUB_t$	= −Subsidies Less Current Surplus of Fed. Gov. Enterprises	(5.181)	[316402001, p. 4]
	−Subsidies Less Current Surplus of S & L Gov. Enterprises	(−4.058)	[206402003, p. 5]
$FHPRI_t$	= Proprietors' Income	(69.179)	[166111105, p. 3]
$FHRNT_t$	= Rental Income	(25.168)	[116112103, p. 3]
$FHINT_t$	= Net Interest	(41.589)	[86130003, p. 3]
$FHTRP_t$	= Transfer Payments, From Business	(4.274)	[146401003, p. 3]
$FHDIV_t$	= Dividends, Nonfinancial Corporations	(20.171)	[106120005, p. 8]
	+Dividends, Net Foreign	(2.869)	[266120001, p. 8]
$FHPFA_t$	= Profits, Farms	(0.101)	[136060003, p. 8]
$FHCCA_t$	= Capital Consumption, Owner-Occupied Homes	(9.304)	[156300203, p. 8]
	+Capital Consumption, Nonprofit Institutions	(1.853)	[156300103, p. 8]
	+Capital Consumption, Farm Noncorporate	(6.476)	[136300203, p. 8]
	+Capital Consumption, Nonfarm Noncorporate Business	(15.682)	[116300005, p. 8]
	+Capital Consumption, Corporate Farms	(0.515)	[136300103, p. 8]
$FHCSI_t$	= Employer Social Insurance Contributions	(33.080)	[146601005, p. 1]
3. The Financial Sector (y_{BHt}):			
$BHDIV_t$	= Dividends, Financial Corporations	(1.897)	[796120001, p. 8]
$BHCGD_t$	= Capital Gains Dividends	(0.776)	[656120000, p. 16]
4. The Foreign Sector: None			
5. The Government Sector (y_{GHt}):			
$GHCIV_t$	= Wages and Salaries, Government Civilian	(104.702)	[SCB, 1.10]
$GHMIL_t$	= Wages and Salaries, Military	(19.419)	[SCB, 1.10]
$-GHWLD_t$	= −Wage Accruals Less Disbursements, Fed. Gov.	(0.039)	[316700003, p. 4]
	−Wage Accruals Less Disbursements, S & L Gov.	(0.170)	[206700003, p. 5]

Table 1–2. (continued)

$GHOTH_t$	= General Government (Compensation of Employees of Fed. Gov. and S & L Gov.)	(124.646)	[SCB, 1.7[a]]
	−Wages and Salaries, Government Civilian	(104.702)	[SCB, 1.10]
	−Wages and Salaries, Military	(19.419)	[SCB, 1.10]
$GHTRP_t$	= Transfer Payments, To Persons, Fed. Gov.	(72.311)	[156401005, p. 4]
	+Transfer Payments, S & L Gov.	(16.687)	[206401003, p. 5]
$GHINS_t$	= Insurance Credits to Households, Fed. Gov.	(2.914)	[313154005, p. 25]
$GHRET_t$	= Retirement Credit to Households, S & L Gov.	(6.285)	[224090005, p. 24]
$GHINT_t$	= Net Interest, Fed. Gov.	(13.642)	[316132001, p. 25]
	+Net Interest, S & L Gov.	(−0.224)	[SCB, 3.4[b]]
$GHSUB_t$	= Subsidies Less Current Surplus of Fed. Gov. Enterprises	(5.181)	[316402001, p. 4]
	+Subsidies Less Current Surplus of S & L Gov. Enterprises	(−4.058)	[206402003, p. 5]

II. Receipts to the Firm Sector from:

1. The Household Sector (y_{HFt}):

$HFCON_t$	= Personal Consumption Expenditures, Services	(284.799)	[SCB, 1.1]
	+Personal Consumption Expenditures, Nondurable Goods	(278.408)	[SCB, 1.1]
	+Personal Consumption Expenditures, Durable Goods	(103.918)	[155011001, p. 1]
	−Indirect Business Taxes, Fed. Gov.	(20.448)	[316240001, p. 4]
	−Indirect Business Taxes, S & L Gov.	(82.238)	[206240001, p. 5]
	−Imports	(65.620)	[266903001, p. 1]
	−Profits, Financial Corporations	(15.555)	[796060001, p. 8]
	−Capital Consumption, Financial Business	(2.238)	[796300003, p. 8]
$HFRES_t$	= Residential Construction, 1–4 Family, Household Purchases	(26.906)	[155012001, p. 7]
	+Residential Construction, 1–4 Family, Farm	(0.557)	[135012001, p. 7]
	+Residential Construction, 1–4 Family, Change in Work in Process on Nonfarm Noncorporate	(1.202)	[115012405, p. 7]
	+Residential Construction, Multifamily, Noncorporate Business	(9.051)	[115012200, p. 7]
$HFPAE_t$	= Nonresidential Plant and Equipment Investment, Nonprofit Institutions	(5.574)	[155013001, p. 7]
	+Nonresidential Plant and Equipment Investment, Farm	(6.425)	[135013001, p. 7]
	+Nonresidential Plant and Equipment Investment, Nonfarm Noncorporate Business	(11.479)	[115013001, p. 7]

Table 1–2. (continued)

$HFIVT_t$ = Inventory Investment, Farm (1.394) [135020003, p. 7]
+Inventory Investment, Nonfarm Noncorporate (−0.143) [115020000, p. 7]

2. The Firm Sector (y_{FFt}):
$FFRES_t$ = Residential Construction, 1–4 Family, Change in Work in Process on Nonfarm Corporate (1.201) [105012405, p. 7]
+Residential Construction, Multifamily, Corporate Business (3.793) [105012205, p. 7]
$FFPAE_t$ = Nonresidential Plant and Equipment Investment, Nonfinancial Corporation (77.107) [105013005, p. 7]
$FFIVT_t$ = Inventory Investment, Nonfarm Corporate (5.061) [105020005, p. 7]

3. The Financial Sector (y_{BFt}):
$BFRES_t$ = Residential Construction, Multifamily, REITS (0.134) [645012205, p. 7]
$BFPAE_t$ = Nonresidential Plant and Equipment Investment, Financial Corporations (3.977) [795013005, p. 7]

4. The Foreign Sector (y_{RFt}):
$RFEXP_t$ = Exports (65.450) [266902001, p. 1]

5. The Government Sector (y_{GFt}):
$GFPGO_t$ = Purchases of Goods and Services, Fed. Gov. (97.642) [316901001, p. 4]
+Purchases of Goods and Services, S&L Gov (136.600) [206901001, p. 5]
−General Government (Compensation of Employees of Fed. Gov. and S & L Gov.) (124.646) [SCB, 1.7[a]]

III. Receipts to the Financial Sector from:

1. The Household Sector (y_{HBt}):
$HBPRO_t$ = Profits, Financial Corporations (15.555) [796060001, p. 8]
$HBCCA_t$ = Capital Consumption, Financial Business (2.238) [796300003, p. 8]
2. The Firm Sector: None
3. The Financial Sector: None
4. The Foreign Sector: None
5. The Government Sector: None

IV. Receipts to the Foreign Sector from:

1. The Household Sector (y_{HRt}):
$HRIMP_t$ = Imports (65.620) [266903001, p. 1]
$HRTRP_t$ = Personal Transfer Payments to Foreigners (1.062) [156901203, p. 3]
2. The Firm Sector: None
3. The Financial Sector: None
4. The Foreign Sector: None
5. The Government Sector (y_{GRt}):
$GRTRP_t$ = Transfer Payments to Foreigners, Fed. Gov. (2.585) [266401005, p. 4]

Table 1–2. (continued)

V. Receipts to the Government Sector from:

1. The Household Sector (y_{HGt}):		
$HGIBT_t$	= Indirect Business Taxes, Fed. Gov.	(20.448) [316240001, p. 4]
	+ Indirect Business Taxes, S & L Gov.	(82.238) [206240001, p. 5]
$HGPTX_t$	= Personal Taxes, Fed. Gov.	(89.926) [316210001, p. 4]
	+ Personal Taxes, S & L Gov.	(27.681) [206210001, p. 5]
$HGFRM_t$	= Tax Accruals, Farms	(0.095) [136231003, p. 8]
$HGSI1_t$	= Employer Social Insurance Contributions [$=FHCSI_t$]	(33.080) [146601005, p. 1]
$HGSI2_t$	= Personal Contributions to Social Insurance	(30.719) [156601003, p. 3]
2. The Firm Sector (y_{FGt}):		
$FGTAX_t$	= Profits Tax Accruals, Nonfinancial Corporate Business	(29.685) [106231005, p. 8]
3. The Financial Sector (y_{BGt}):		
$BGTAX_t$	= Profits Tax Accruals, Financial Corporations	(7.769) [796231001, p. 8]
$BGSUR_t$	= Current Surplus, Federally Sponsored Credit Agencies	(0.084) [406006003, p. 26]
	+ Current Surplus, Monetary Authorities	(−0.055) [716006001, p. 27]
4. The Foreign Sector: None		
5. The Government Sector: None		

VI. The Saving of Each Sector:

$$
\begin{aligned}
SAVH_t &= (y_{HHt} + y_{FHt} + y_{BHt} + y_{RHt} + y_{GHt}) - (y_{HHt} + y_{HFt} + y_{HBt} + y_{HRt} + y_{HGt})\\
SAVF_t &= (y_{HFt} + y_{FFt} + y_{BFt} + y_{RFt} + y_{GFt}) - (y_{FHt} + y_{FFt} + y_{FRt} + y_{FGt})\\
SAVB_t &= (y_{HBt}) - (y_{BHt} + y_{BFt} + y_{BGt})\\
SAVR_t &= (y_{HRt} + y_{GRt}) - (y_{RFt})\\
SAVG_t &= (y_{HGt} + y_{FGt} + y_{BGt}) - (y_{GHt} + y_{GFt} + y_{GRt})
\end{aligned}
$$

Note that the savings of all sectors sum to zero:

$$SAVH_t + SAVF_t + SAVB_t + SAVR_t + SAVG_t = 0$$

Notes: [a]Quarterly numbers from SCB 1.7; annual numbers from SCB 3.1 and SCB 3.3.

[b]Quarterly numbers from SCB 3.4; annual numbers from SCB 3.3.

The numbers in parentheses are actual values of the variables for 1971 at an annual rate in billions of current dollars.

See the text for an explanation of the numbers in brackets.

household sector to itself. $HHDIV_t$ is the value of dividends paid by farms. Since farms are part of the household sector, the value of dividends paid by farms is a payment by the household sector to itself.

Payments by the firm sector to the household sector are listed in I.2. $FHOTH_t$ includes other labor income as defined in the NIA plus three other items. The value of these items is the difference between compensation of the government (both state and local and federal) and wages and salaries of the government. The value of this difference is the value of other labor income of the government, which must be subtracted from the other labor income item in the NIA to obtain the other labor income component of the firm sector.

The variable $FHSUB_t$ is composed of two items. The first is minus the value of net subsidies of federal government enterprises. The value of net subsidies is a payment from the government sector to the household sector. In the NIA this value is distributed among various income terms listed in I.2, and so it must be subtracted from the other terms in I.2 in order to measure correctly the income received by the household sector from the firm sector. The second item making up $FHSUB_t$ is minus the value of the net subsidies of state and local government enterprises. It is treated in the same way as the first item. Its value in parentheses is negative, which means that the state and local government enterprises actually run a net surplus.

The five capital consumption items in I.2 represent money received by the household sector, but money that is not included as income in any of the other terms in I.2. Consequently, they are included separately in I.2, as money received by the household sector from the firm sector. Employer contributions for social insurance, $FHCSI_t$, are also counted as money received by the household sector from the firm sector. The second variable is I.2, $FHWLD_t$, is wage accruals less disbursements of the firm sector, and this variable must be subtracted from the income received by the household sector from the firm sector in order to retain the consistency of the accounts.

The payments by the financial sector to the household sector in I.3 are small and consist of two dividend variables. There are no payments by the foreign sector to the household sector. The payments by the government sector to the household sector consist of wages and salaries, other labor income, transfer payments, insurance and retirement credits, interest, and the net subsidies of the government enterprises. Subtracted from these variables is the value of wage accruals less disbursements of the government sector. $GHSUB_t$ in I.5 is the negative of $FHSUB_t$ in I.2, and $GHOTH_t$ in I.5 is the amount subtracted in I.2 from the NIA value of other labor income to get $FHOTH_t$.

The payments by the household sector to the firm sector in II.1 consist of items relating to personal consumption, residential construction, nonresidential plant and equipment investment, and inventory investment. Subtracted from the personal consumption items are indirect business taxes, imports, and profits and capital consumption of financial corporations. These latter terms, which are included in the personal consumption items, are not payments by the household sector to the firm sector, but are instead payments by the household sector to the government sector, the foreign sector, and the financial sector, respectively.

The payments by the firm sector to itself in II.2 consist of investment in residential construction, nonresidential plant and equipment, and inventories. The payments by the financial sector to the firm sector in II.3 consist of investment in residential construction and nonresidential plant and equipment. The payments by the foreign sector to the firm sector in II.4 consist of exports. The payments by the government sector to the firm sector in II.5 are obtained by subtracting from government purchases of goods and services the compensation of employees of the government sector.

The only payments to the financial sector in section III in Table 1-2 consist of payments by the household sector in the form of profits and capital consumption. These are two of the terms that were subtracted from the personal consumption items in II.1. The payments by the household sector to the foreign sector in IV.1 consist of imports and personal transfer payments to foreigners. The payments by the government sector in IV.5 consist of federal government transfer payments to foreigners.

The payments by the household sector to the government sector in V.1 consist of indirect business taxes, personal income taxes, tax accruals of farms, and contributions to social insurance, both employer and personal. The payments by the firm sector in V.2 are merely profits tax accruals. The payments by the financial sector in V.3 consist of profits tax accruals and two small items measuring the current surpluses of the federally sponsored credit agencies and the monetary authorities. No terms are included as payments by the government sector to itself, although a term such as federal government grants in aid to state and local governments could have been. It makes no difference in the following analysis whether terms like this are included or not, and so for simplicity they were not included.[c]

The saving of each sector is defined in section VI in Table 1-2. As mentioned above, the savings of all sectors sum to zero by definition. These savings are net of capital gains and losses, net of increases in the world's gold stock, and net of the creation of SDRs and the like.

Before considering the variables in Table 1-2 any further, it will be useful to consider the collection of the flow-of-funds data. In the FFA there are 24 major kinds of securities. For purposes here, these have been aggregated into five kinds: demand deposits and currency, bank reserves, borrowing at federal reserve banks, gold and foreign exchange, and all other. The all other category includes insurance and pension fund reserves, time deposits and savings accounts, government securities, corporate and foreign bonds, corporate equities, all types of mortgages, consumer credit, bank loans, other loans, security credit, trade credit, profit taxes payable, proprietors' equities, and some miscellaneous financial claims.

The all other category is obviously quite heterogeneous, but it is beyond the scope of this study to consider the detailed portfolio behavior of each sector. Contrary to the thrust of the Brainard-Tobin work [4], the present study ignores any possible effects on the economy of substitution among different types of securities. Considerable effort was expended here, however, in making sure that all aggregate flows of funds are accounted for, since the results in Volume I indicate that it is quite important to do so in a macroeconomic model.

Typo

With five kinds of securities and five sectors, this means, using the notation introduced at the beginning of this section, that there are 25 values of A_{kit} for each t. Some of the values of A_{kit} are, however, always zero. The FFA data that have been collected are presented in Table 1-3. The basic data that have been collected are flow data, not stock data. Although quarterly data on stocks are available from the flow-of-funds tape, it is generally advisable to construct stock data from the flow data, using the stock data only for benchmark purposes for one particular quarter. Because of changes in benchmarks and the like, the change in the stock of a particular variable on the flow-of-funds tape does not always equal the flow. This is true even for securities that are not subject to capital gains and losses.

All the data in Table 1-3 exclude capital gains and losses, increases in the world's gold stock, and the creation of SDRs and the like. Capital gains and losses will be considered later. The fourth quarter of 1971 was used for benchmark purposes, and the benchmark values that were used to create the stock data from the flow data are presented in brackets in Table 1-3. The numbers in parentheses in the table are the values of the flows for 1971. The flow data are at annual rates. Both the stock and flow data are in billions of current dollars. The second set of brackets in the table contains the code numbers of the variables on the flow-of-funds tape and the page numbers in [3] where the variables can be found. As in Table 1-2, the values in brackets and parentheses in Table 1-3 are values that appear in the flow-of-funds publication [3] without any change of sign. The items in the table are all *net* items. An increase in net liabilities, for example, is a negative item.

The construction of Table 1-3 is fairly self-explanatory. The data on the change in the value of all securities by sector are presented first in the table. This change is called "net financial investment" (*NFI*) in the FFA. The change in all securities of each sector in the table is an aggregate of the *NFI* of the corresponding sectors in the FFA. For the financial sector and for two of the four FFA sectors that make up the government sector, data on *NFI* are not available directly in the FFA. In these cases the data on *NFI* must be collected as the difference between the net increase in assets and the net increase in liabilities.

Data on the change in demand deposits and currency by sector are presented next in Table 1-3, followed by the change in bank reserves, the change in borrowing at federal reserve banks, and the change in gold and foreign exchange. The household, firm, and foreign sectors hold no bank reserves and do not borrow from the federal reserve banks. The household, firm, and financial sectors hold no gold and foreign exchange. In section VI of Table 1-3 the change in the value of all other securities for each sector is

Table 1–3. The Data from the Flow-of-Funds Accounts by Sector

Abbreviations Used for the Securities:

TOT = All Securities
DDC = Demand Deposits and Currency
RES = Bank Reserves
BOR = Borrowing at Federal Reserve Banks
GFX = Gold and Foreign Exchange
SEC = All Other Securities (*TOT* less *DDC*, *RES*, *BOR*, and *GFX*)
NFI denotes Net Financial Investment
DIS denotes Discrepancy

I. The Change in TOT by Sector (NFI by Sector):

1. The Household Sector:				
$TOTH_t - TOTH_{t-1}$	= *NFI* of Households, Personal Trusts, and Nonprofit Organizations	[1611.645]	(49.684)	[155000005, p. 16]
	+ *NFI* of Farm Business	[−51.401]	(−1.432)	[135000005, p. 20]
	+ *NFI* of Nonfarm Noncorporate Business	[−53.227]	(−5.903)	[115000005, p. 20]
2. The Firm Sector:				
$TOTF_t - TOTF_{t-1}$	= *NFI* of Nonfinancial Corporate Business	[−204.570]	(29.392)	[105000005, p. 22]
3. The Financial Sector:				
$TOTB_t - TOTB_{t-1}$	= Net Acq. of Fin. Assets of Commercial Banking	[576.712]	(58.492)	[764090005, p. 27]
	− Net Increase in Liabilities of Commercial Banking	[543.175]	(56.840)	[764190005, p. 28]
	+ Net Acq. of Fin. Assets of Private Nonbank Fin. Institutions	[928.577]	(84.887)	[694090005, p. 32]
	− Net Increase in Liabilities of Private Nonbank Fin. Institutions	[865.418]	(82.228)	[694190005, p. 32]
4. The Foreign Sector:				
$TOTR_t - TOTR_{t-1}$	= *NFI* of the Rest of the World	[−2.393]	(13.593)	[265000005, p. 39]
5. The Government Sector:				
$TOTG_t - TOTG_{t-1}$	= *NFI* of S & L Gov.	[−95.865]	(−12.050)	[205000005, p. 24]
	+ *NFI* of U.S. Gov.	[−280.185]	(−24.883)	[315000005, p. 25]
	+ Net Increase in Assets of Federally Sponsored Credit Agencies	[50.339]	(3.410)	[404090005, p. 26]
	− Net Increase in Liabilities of Federally Sponsored Credit Agencies	[49.286]	(3.315)	[404190005, p. 26]
	+ Net Acq. of Fin. Assets of Monetary Authorities	[93.547]	(8.298)	[714090005, p. 27]
	− Net Increase in Liabilities of Monetary Authorities	[93.977]	(8.353)	[714190005, p. 27]

Table 1–3. (continued)

II. The Change in DDC by Sector:				
1. The Household Sector:				
$DDCH_t - DDCH_{t-1}$	= Change in *DDC* of Households, Personal Trusts, and Nonprofit Organizations	[145.484]	(10.964)	[153020001, p. 16]
	+ Change in *DDC* of Farm Business	[6.638]	(0.123)	[133020003, p. 20]
	+ Change in *DDC* of Nonfarm Noncorporate Business	[12.515]	(0.000)	[113020003, p. 20]
2. The Firm Sector:				
$DDCF_t - DDCF_{t-1}$	= Change in *DDC* of Nonfinancial Corporate Business	[36.312]	(0.524)	[103020001, p. 22]
3. The Financial Sector:				
$DDCB_t - DDCB_{t-1}$	= −Net Increase in Net Demand Deposit Liabilities of Commercial Banking	[204.589]	(12.995)	[763120005, p. 28]
	+ Net Acq. of Demand Deposit and Currency Assets of Commercial Banking	[0.514]	(0.127)	[743020003, p. 27]
	+ Change in *DDC* of Private Nonbank Financial Institutions	[14.545]	(1.079)	[693020005, p. 32]
4. The Foreign Sector:				
$DDCR_t - DDCR_{t-1}$	= Change in U.S. *DDC* of the Rest of the World	[6.453]	(0.284)	[263020005, p. 39]
5. The Government Sector:				
$DDCG_t - DDCG_{t-1}$	= Change in *DDC* of S & L Gov.	[13.494]	(1.022)	[213020005, p. 24]
	+ Change in *DDC* of U.S. Gov.	[13.482]	(3.301)	[313020001, p. 25]
	+ Change in *DDC* of Federally Sponsored Credit Agencies	[0.247]	(0.054)	[403020000, p. 26]
	− Net Increase in *DDC* Liabilities due to U.S. Gov. of the Monetrary Authorities	[2.484]	(0.897)	[713123101, p. 27]
	− Net Increase in *DDC* Liabilities due to Rest of the World of the Monetary Authorities	[0.465]	(0.119)	[713122605, p. 27]
	− Net Increase in Liabilities in the form of Currency Outside Banks of the Monetary Authorities	[53.438]	(3.392)	[713125001, p. 27]

Table 1–3. (continued)

III. The Change in RES by Sector:

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1. The Household Sector: None
2. The Firm Sector: None
3. The Financial Sector:

$RESB_t - RESB_{t-1}$ = Change in Vault Cash and Member Bank Reserves of Commercial Banking [35.329] (4.132)[723020005, p. 28]

4. The Foreign Sector: None
5. The Government Sector:

$RESG_t - RESG_{t-1}$ = −Net Increase in Labilities in the form of Member Bank Reserves of the Monetary Authorities [27.788] (3.638)[713113001, p. 27]

−Net Increase in Liabilities in the form of Vault Cash of Commercial Banks of the Monetary Authorities [7.541] (0.494)[723025001, p. 27]

IV. The Change in BOR by Sector:

---

1. The Household Sector: None
2. The Firm Sector: None
3. The Financial Sector:

$BORB_t - BORB_{t-1}$ = −Change in Borrowing at Federal Reserve Banks of Commercial Banking [0.039] (−0.296)[713068001, p. 28]

4. The Foreign Sector: None
5. The Government Sector:

$BORG_t - BORG_{t-1}$ = Change in Federal Reserve Loans to Domestic Banks of the Monetary Authorities [0.039] (−0.296)[713068001, p. 27]

V. The Change in GFX by Sector:

---

1. The Household Sector: None
2. The Firm Sector: None
3. The Financial Sector: None
4. The Foreign Sector:

$GFXR_t - GFXR_{t-1}$ = Change in Gold and *SDR*s of the Rest of the World [36.778] (1.334)[263011005, p. 39]

−Change in U.S. Foreign Exchange Position [0.861] (−1.731)[263110005, p. 39]

Table 1–3. (continued)

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5. The Government Sector:
   $GFXG_t - GFXG_{t-1}$ = Change in Gold and Official Foreign Exchange of U.S. Gov. [2.094] (−2.233)[313011005, p. 25]
   − Change in Gold and Foreign Exchange of the Monetary Authorities [10.073] (−0.832)[713011005, p. 27]

VI. The Change in SEC by Sector:

---

1. The Household Sector:
   $SECH_t - SECH_{t-1} = (TOTH_t - TOTH_{t-1}) - (DDCH_t - DDCH_{t-1})$
2. The Firm Sector:
   $SECF_t - SECF_{t-1} = (TOTF_t - TOTF_{t-1}) - (DDCF_t - DDCF_{t-1})$
3. The Financial Sector:
   $SECB_t - SECB_{t-1} = (TOTB_t - TOTB_{t-1}) - (DDCB_t - DDCB_{t-1}) - (RESB_t - RESB_{t-1}) - (BORB_t - BORB_{t-1})$
4. The Foreign Sector:
   $SECR_t - SECR_{t-1} = (TOTR_t - TOTR_{t-1}) - (DDCR_t - DDCR_{t-1}) - (GFXR_t - GFXR_{t-1})$
5. The Government Sector:
   $SECG_t - SECG_{t-1} = (TOTG_t - TOTG_{t-1}) - (DDCG_t - DDCG_{t-1}) - (RESG_t - RESG_{t-1}) - (BORG_t - BORG_{t-1}) - (GFXG_t - GFXG_{t-1})$

VII. Discrepancy (DIS) by Sector:

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1. The Household Sector:
   $DISH_t$ = *DIS* of Households, Personal Trusts, and Nonprofit Organizations (−0.534)[157005005, p. 17]
   + Capital Consumption of Nonfarm Noncorporate Business (15.682)[116300005, p. 20]
   − Current Surplus of Nonfarm Noncorporate Business (15.686)[116000105, p. 20]
   + Farm Discrepancy (−0.001)[137010005, p. 70]
2. The Firm Sector:
   $DISF_t$ = *DIS* of Nonfinancial Corporate Business (10.190)[107005005, p. 23]
3. The Financial Sector:
   $DISB_t$ = *DIS* of Commercial Banking (−1.051)[727005005, p. 28]
   + *DIS* of Private Nonbank Financial Institutions (−0.049)[697005005, p. 32]
4. The Foreign Sector:
   $DISR_t$ = *DIS* of the Rest of the World (−9.776)[267005005, p. 40]
5. The Government Sector:
   $DISG_t$ = *DIS* of S & L Gov. (9.124)[207005005, p. 24]
   + *DIS* of U.S. Gov. (0.094)[317005005, p. 25]
   + *DIS* of Federally Sponsored Credit Agencies (−0.011)[407005005, p. 26]

Notes: The numbers in the first set of brackets are benchmark values for the fourth quarter of 1971 in billions of current dollars.
The numbers in parentheses are actual values of the (flow) variables for 1971 at an annual rate in billion of current dollars.
See the text for an explanation of the numbers in the second set of brackets.

computed as a residual category, the difference between the value of all securities and the sum of the values of the other four. Finally, the data on the discrepancy for each sector are presented in section VII of Table 1-3.

It is now possible to consider the relationships among the variables in Tables 1-2 and 1-3. For each sector except the firm sector, the saving of the sector as defined in Table 1-2 is equal to the change in all securities (net financial investment) of the sector plus the discrepancy of the sector:

$$SAVH_t = TOTH_t - TOTH_{t-1} + DISH_t, \tag{1.1}$$

$$SAVB_t = TOTB_t - TOTB_{t-1} + DISB_t, \tag{1.2}$$

$$SAVR_t = TOTR_t - TOTR_{t-1} + DISR_t, \tag{1.3}$$

$$SAVG_t = TOTG_t - TOTG_{t-1} + DISG_t. \tag{1.4}$$

For the firm sector, saving equals net financial investment plus the discrepancy of the firm sector in Table 1-3 *plus* wage accruals less disbursements of the firm sector and *plus* the statistical discrepancy of the NIA:

$$SAVF_t = TOTF_t - TOTF_{t-1} + DISF_t + FHWLD_t + STATDIS_t, \tag{1.5}$$

where $STATDIS_t$ denotes the statistical discrepancy of the NIA. The value of $STATDIS_t$ in 1971 was -2.323, its code number is 87005005, and it is found on page 2 in [3].

The fact that Equations (1.1)–(1.5) must hold provides an important consistency check on the data. If in Table 1-2 the saving of any sector has been defined incorrectly, this error will show up when the checks in Equations (1.1)–(1.5) are made. Equations (1.1)–(1.5) provide the key links between the NIA data in Table 1-2 and the FFA data in Table 1-3.

Two other consistency checks are also available for the data in Table 1-3. First, the sum of the change in bank reserves across sectors must equal zero, the sum of the change in borrowing from federal reserve banks across sectors must equal zero, and the sum of the change in gold and foreign exchange across sectors must equal zero:

$$(RESB_t - RESB_{t-1}) + (RESG_t - RESG_{t-1}) = 0, \tag{1.6}$$

$$(BORB_t - BORB_{t-1}) + (BORG_t - BORG_{t-1}) = 0, \tag{1.7}$$

$$(GFXG_t - GFXG_{t-1}) + (GFXR_t - GFXR_{t-1}) = 0. \tag{1.8}$$

Second, the sum of the change in demand deposits and currency across sectors *plus* the change in demand deposit mail floats must equal zero:

$$\begin{aligned}&(DDCH_t - DDCH_{t-1}) + (DDCF_t - DDCF_{t-1}) + (DDCB_t - DDCB_{t-1})\\&+ (DDCR_t + DDCR_{t-1}) + (DDCG_t - DDCG_{t-1}) + MAILFLT_t = 0,\end{aligned} \tag{1.9}$$

where $MAILFLT_t$ denotes the demand deposit mail floats. $MAILFLT_t$ consists of two items: a U.S. government item and an all other item. The values of these two items in 1971 were -0.173 and 0.098; the code numbers are 903023105 and 903029205, respectively; and the items are found on page 70 in [3].

It is also the case, because of Equations (1.1)–(1.5) and the fact that the savings of all sectors sum to zero, that the sum of the change in all securities across sectors, plus the sum of the discrepancies across sectors, plus $FHWLD_t$, and plus $STATDIS_t$ equal zero:

$$\begin{aligned}&(TOTH_t - TOTH_{t-1}) + (TOTF_t - TOTF_{t-1}) + (TOTB_t - TOTB_{t-1})\\&+ (TOTR_t - TOTR_{t-1}) + (TOTG_t - TOTG_{t-1}) + DISH_t + DISF_t\\&+ DISB_t + DISR_t + DISG_t + FHWLD_t + STATDIS_t = 0.\end{aligned} \tag{1.10}$$

This, of course, is not an independent check on the data to the extent that Equations (1.1)–(1.5) have already been checked.

Equations (1.6)–(1.10) and the definition of the change in all other securities for each sector in section VI in Table 1-3 imply that:

$$\begin{aligned}&(SECH_t - SECH_{t-1}) + (SECF_t - SECF_{t-1}) + (SECB_t - SECB_{t-1})\\&+ (SECR_t - SECR_{t-1}) + (SECG_t - SECG_{t-1}) = -(DISH_t + DISF_t\\&+ DISB_t + DISR_t + DISG_t) - FHWLD_t - STATDIS_t + MAILFLT_t.\end{aligned} \tag{1.11}$$

In other words, the sum of the change in all other securities across sectors is equal to the negative of the sum of the discrepancies across sectors, less wage accruals less disbursements of the firm sector, less the statistical discrepancy of the NIA, and plus the demand deposit mail floats.

Aside from the adjustments for the various discrepancies, all that Equations (1.6)–(1.11) state is that each security that is an asset to one sector is a *corresponding liability to some other sector. Since liabilities correspond to* negative values of A_{kit}, the sum of A_{kit} across sectors for a given k and t must be zero, except for the various discrepancies.

This completes the discussion of the linking of the NIA and FFA data by sector. What remains to be done in this section is to discuss the treatment of capital gains and losses on stocks held by the household sector. There is a variable on the flow-of-funds tape that measures households' holdings of corporate equities. Its code number is 153064005, and it is found on page 50 in [3]. The level data on this variable measure the market value of the stock. The flow data, on the other hand, measure the value of the change in the stock *excluding* capital gains and losses. Therefore, the value of capital gains or losses for a period, denoted in the model as CG_t, can be computed as the difference between the change in the value of the stock (using the level data) and the value of the flow (using the flow data). Seasonally unadjusted flow data were used for this purpose because the stock data are seasonally unadjusted. (All the other flow data used in this study are seasonally adjusted.) CG_t measures a few other items aside from capital gains and losses (mostly adjustments to the level data), but these items are quite small compared to the capital gains and losses component.

Data on capital gains and losses for the other sectors in the model were not collected because they were not used anywhere in the model. Data on increases in the world's gold stock were not collected for the same reason. There are no data in the FFA for capital gains and losses on bonds.

NOTES

[a] *VBG* is exclusive of capital gains and losses, so that a change in the price of government securities outstanding caused by a change in interest rates does not affect *VBG*.

[b] Although the model is quarterly, the actual values presented in parentheses in this chapter are annual. The annual data are less rounded than the quarterly data, and for purposes of making the various consistency checks discussed in this chapter, it is better to use less rounded data.

[c] It also makes no difference whether the household sector's payments to itself in I.1 are included or not. These payments were included here merely to avoid any possible confusion that might arise as to how the two items in I.1 are to be treated.

Chapter Two

The Complete Model

2.1 INTRODUCTION

Aside from the specification of the stochastic equations, the complete model is presented and discussed in this chapter. Most of the remaining data questions are also considered. Presenting the complete model now has the advantage of showing very early its closed nature (with respect to the flows of funds) and of establishing all the notation that is needed in later chapters. A model building effort of this sort requires a number of detailed decisions about how certain variables are to be treated and about what kinds of data are to be used, and it seems best to get most of these details out of the way now in order to put the discussion of the stochastic equations in a better perspective.

The complete list of variables in the model is presented in Table 2-1 in alphabetic order, and the complete list of equations in the model is presented in Table 2-2. For reference purposes, the estimates of the stochastic equations are presented in Table 2-3, although this table is not discussed in this chapter. The notation used in this volume corresponds as closely as possible to the notation used for the theoretical model in Volume I. Enough detail has been presented in Table 2-1 so that one should be able to duplicate the collection of the data fairly easily, using also the information in Tables 1-2 and 1-3. The notation for most of the variables has been changed in going from Tables 1-2 and 1-3 to Table 2-1. The notation in Tables 1-2 and 1-3 is designed to try to make clear the relationships among the NIA and FFA data, whereas, as just mentioned, the notation in Table 2-1 is designed to be consistent with the notation in Volume I. The next two sections are a discussion of Tables 2-1 and 2-2. Table 2-2 will be discussed first, and then the data questions that pertain to Table 2-1 will be discussed.

Table 2–1. The Complete List of Variables in the Model in Alphabetic Order

Subscript t denotes variable for quarter t. All flow variables are at quarterly rates. Variables are seasonally adjusted where appropriate. *BCURT* denotes that the unit of the variable is billions of current dollars, and *B*1958 denotes that the unit of the variable is billions of 1958 dollars. A † denotes an exogenous variable.

Value of Variable in 1971IV	*Equation Number in Model*		
1342.4	61	A_t	= value of nondemand deposit securities of the household sector, *BCURT*. [$= SECH_t - \sum_{i=t+1}^{80} CG_i$ for $t < 80$; $= SECH_t + \sum_{i=81}^{t} CG_i$ for $t > 80$; $= SECH_t$ for $t = 80$. $t = 80$ in 1971IV. For $SECH_t$, see Table 1–3. See also the discussion in section 2.3.]
0.0	20	$BORR_t$	= commercial bank borrowing at federal reserve banks, *BCURT*. [$= -BORB_t$ in Table 1–3.]
35.3	45	BR_t	= bank reserves, *BCURT*. [$= RESB_t$ in Table 1–3.]
24.1	46	CD_t	= personal consumption expenditures on durable goods, *B*1958. [SCB, 1.2.]
7.9	53	CF_t	= cash flow before taxes and dividends of the firm sector, *BCURT*. [Defined in Table 2–2.]
−5.2	54	$\overline{CF}_t$	= cash flow net of taxes and dividends of the firm sector, *BCURT*. [$= SAVF_t$ in Table 1–2. Also defined in Table 2–2.]
28.4	23	CG_t	= capital gains (+) or losses (−) during quarter t on corporate stocks held by the household sector, *BCURT*. [See discussion section 1.2.]
53.1	2	CN_t	= personal consumption expenditures on nondurable goods, *B*1958. [SCB, 1.2.]
6.5		$^{\dagger}COM_t$	= farm output, *B*1958. [SCB, 1.8.]
48.8	1	CS_t	= personal consumption expenditures on services, *B*1958. [SCB, 1.2.]
29.2		$^{\dagger}CURR_t$	= value of currency outstanding less the value of demand deposits of the government sector, *BCURT*. [$= -DDCG_t$ in Table 1–3.]
0.412		$^{\dagger}d_{1t}$	= profit tax rate. [$= TAXF_t/\pi F_t$.]
0.084		$^{\dagger}d_{3t}$	= one of the two personal income tax rates. [$= (PTAXH_t/YH_t) - \tau \cdot YH_t$.]
0.223	84	d_{3t}^{M}	= marginal personal income tax rate. [Defined in Table 2–2.]
0.183		$^{\dagger}d_{4t}$	= indirect business tax rate. [$= IBTH_t/(PCD_tCD_t + PCN_tCN_t + PCS_tCS_t - IBTH_t)$.]

Table 2–1. (continued)

Value of Variable in 1971IV	*Equation Number in Model*		
0.059		$^{\dagger}d_{5t}$	= employer social security tax rate. $[=FHCSI_t/(WFF_t(HPFN_t + 1.5HPFO_t)JOBF_t).]$
0.055		$^{\dagger}d_{6t}$	= employee social security tax rate. $[=HGSI2_t/(WFF_t(HPFN_t + 1.5HPFO_t)JOBF_t).]$
0.0		$^{\dagger}D593_t$	= dummy variable that takes on a value of one in 1959III and zero otherwise.
0.0		$^{\dagger}D594_t$	= dummy variable that takes on a value of one in 1959IV and zero otherwise.
0.0		$^{\dagger}D601_t$	= dummy variable that takes on a value of one in 1960I and zero otherwise.
0.0		$^{\dagger}D644_t$	= dummy variable that takes on a value of one in 1964IV and zero otherwise.
0.0		$^{\dagger}\mathrm{D}651_t$	= dummy variable that takes on a value of one in 1965I and zero otherwise.
0.0		$^{\dagger}D652_t$	= dummy variable that takes on a value of one in 1965II and zero otherwise.
0.0		$^{\dagger}D691_t$	= dummy variable that takes on a value of one in 1969I and zero otherwise.
0.0		$^{\dagger}D692_t$	= dummy variable that takes on a value of one in 1969II and zero otherwise.
0.0		$^{\dagger}D693_t$	= dummy variable that takes on a value of one in 1969III and zero otherwise.
0.0		$^{\dagger}D704_t$	= dummy variable that takes on a value of one in 1970IV and zero otherwise.
0.0		$^{\dagger}\mathrm{D}711_t$	= dummy variable that takes on a value of one in 1971I and zero otherwise.
1.0		$^{\dagger}D714_t$	= dummy variable that takes on a value of one in 1971IV and zero otherwise.
0.0		$^{\dagger}D721_t$	= dummy variable that takes on a value of one in 1972I and zero otherwise.
1.0		$^{\dagger}DD661_t$	= dummy variable that takes on a value of zero before 1966I and a value of one from 1966I on.
189.5	62	DDB_t	= value of demand deposits and currency of the financial sector, *BCURT*. [$= -DDCB_t$ in Table 1–3.]
36.3	16	DDF_t	= value of demand deposits and currency of the firm sector, *BCURT*. [$= DDCF_t$ in Table 1–3.]
164.6	8	DDH_t	= value of demand deposits and currency of the household sector, *BCURT*. [$= DDCH_t$ in Table 1–3.]
6.5		$^{\dagger}DDR_t$	= value of demand deposits and currency of the foreign sector, *BCURT*. [$= DDCR_t$ in Table 1–3.]

Table 2–1. (continued)

Value of Variable in 1971IV	*Equation Number in Model*		
14.8		$^{\dagger}DEP_t$	= depreciation of the firm sector, *BCURT*. [F/F, Capital Consumption Allowances of Nonfinancial Corporate Business, 106300005, p. 22.]
0.2		$^{\dagger}DISB_t$	= discrepancies of the financial, firm, government, household, and foreign sectors, respectively, *BCURT*. [Same as in Table 1–3.]
2.1		$^{\dagger}DISF_t$	
1.3		$^{\dagger}DISG_t$	
−1.3		$^{\dagger}DISH_t$	
−2.1		$^{\dagger}DISR_t$	
0.6		$^{\dagger}DIVB_t$	= dividends paid by the financial sector, *BCURT*. [$=BHDIV_t + BHCGD_t$ in Table 1–2.]
5.8	17	$DIVF_t$	= dividends paid by the firm sector, *BCURT*. [$=FHDIV_t$ in Table 1–2.]
6.4	56	$DIVH_t$	= dividends received by the household sector except those dividends paid to itself, *BCURT*. [Defined in Table 2–2.]
1.0		$^{\dagger}DTAXCR_t$	= investment tax credit variable. [=0.5 in 1962III–1963IV and 1971III; 1.0 in 1964I–1966III, 1967II–1969I, and 1971IV–1975I; and 0.0 otherwise.]
83019	81	$EMPL_t$	= total number of people employed, civilian and military, thousands of persons. [Sum of civilian employment and $JOBGM_t$. Data on the former were obtained from EE, *A*-31. Average of monthly data. See discussion in section 2.3 for adjustments.]
12.3		$^{\dagger}EX_t$	= exports, *B*1958. [SCB, 1.2.]
8.8		$^{\dagger}FHCCA_t$	= capital consumption of the household sector, *BCURT*. [Same as in Table 1–2.]
8.4	43	$FHCSI_t$	= employer social security contributions, *BCURT*. [Same as in Table 1–2.]
0.0		$^{\dagger}FHPFA_t$	= profits of farms (household sector), *BCURT*. [Same as in Table 1–2.]
6.4		$^{\dagger}FHRNT_t$	= rental income of the household sector, *BCURT*. [Same as in Table 1–2.]
1.1		$^{\dagger}FHTRP_t$	= transfer payments from the firm sector to the household sector, *BCURT*. [Same as in Table 1–2.]
0.2		$^{\dagger}FHWLD_t$	= wage accruals less disbursements of the firm sector, *BCURT*. [Same as in Table 1–2.]
0.186		$^{\dagger}g_{1t}$	= reserve requirement ratio. [$=BR_t/DDB_t$.]
12.2		$^{\dagger}GFXG_t$	= value of gold and foreign exchange of the government sector, *BCURT*. [Same as in Table 1–3.]
0.3		$^{\dagger}GHSUB_t$	= net subsidies of government enterprises, *BCURT*. [Same as in Table 1–2.]
0.1		$^{\dagger}GHWLD_t$	= wage accruals less disbursements of the government sector, *BCURT*. [Same as in Table 1–2.]

Table 2–1. (continued)

Value of Variable in 1971IV	*Equation Number in Model*		
270.8		GNP_t	= gross national product, *BCURT*. [Defined in Table 2–2. See also F/F, 86901005, p. 1.]
0.7		$^\dagger GRTRP_t$	= transfer payments from the government sector to the foreign sector, *BCURT*. [Same as in Table 1–2.]
7.8	44	$HGSI2_t$	= employee social security contributions, *BCURT*. [Same as in Table 1–2.]
491.3	13	HPF_t	= average number of hours paid per job per quarter by the firm sector. [Unpublished data from BLS.]
451.9	50	$HPFN_t$	= average number of nonovertime hours paid per job per quarter by the firm sector. [$=HPF_t - HPFO_t$.]
39.4	14	$HPFO_t$	= average number of overtime hours paid per job per quarter by the firm sector. [EE, *C*-7. (For manufacturing.) Average of monthly data. Data multiplied by 13 to put on a quarterly basis.]
505.4		$^\dagger HPGC_t$	= average number of hours paid per civilian job per quarter by the government sector. [EE, *B*-5 and *C*-9. Ratio of "man hours" variable for the government in *C*-9 to $JOBGC_t$ in *B*-5. Average of monthly data. Data multiplied by 13 to put on a quarterly basis.]
520.0		$^\dagger HPGM_t$	= average number of hours paid per military job per quarter by the government sector. [Assumed to be 520 hours for all t.]
0.3		$^\dagger HRTRP_t$	= transfer payments from the household sector to the foreign sector, *BCURT*. [Same as in Table 1–2.]
26.5	40	$IBTH_t$	= indirect business taxes, *BCURT*. [$=HGIBT_t$ in Table 1–2.]
7.1	47	IH_t	= residential investment of the household sector, *B*1958. [$=HFRES_t/PIH_t$. For $HFRES_t$, see Table 1–2.]
12.9	24	IM_t	= imports, *B*1958. [SCB, 1.2.]
10.8	18	$INTF_t$	= interest paid by the firm sector, *BCURT*. [$=FHINT_t$ in Table 1–2.]
3.2	26	$INTG_t$	= interest paid by the government sector, *BCURT*. [$=GHINT_t$ in Table 1–2.]
14.0	57	$INTH_t$	= interest received by the household sector, *BCURT*. [Defined in Table 2–2.]
14.5	11	INV_t	= nonresidential plant and equipment investment of the firm sector, *B*1958. [$=FFPAE_t/PFF_t$. For $FFPAE_t$, see Table 1–2.]
−1.1	19	IVA_t	= inventory valuation adjustment, *BCURT*. [F/F, Inventory Valuation Adjustment, 105020601, p. 1.]

Table 2–1. (continued)

Value of Variable in 1971IV	Equation Number in Model		
299.3	75	J_t	= ratio of total worker hours paid for to the total population 16 and over. [Defined in Table 2–2.]
317.4	76	J_t^*	= J_t detrended. [Defined in Table 2–2.]
71667.	12	$JOBF_t$	= number of jobs in the firm sector, thousands of jobs. [Unpublished data from BLS.]
13027.		†$JOBGC_t$	= number of civilian jobs in the government sector, thousands of jobs. [EE, *B*-5. Average of monthly data.]
2690.		†$JOBGM_t$	= number of military jobs in the government sector, thousands of jobs. [EE, *A*-31. Average of monthly data. Difference between total labor force and civilian labor force.]
404.4	72	K_t^a	= actual capital stock of the firm sector, *B*1958. [See discussion in section 5.2.]
356.1	3	KCD_t	= stock of consumer durables, *B*1958. [See discussion in section 2.3.]
518.7	4	KIH_t	= stock of residential structures of the household sector, *B*1958. [See discussion in section 2.3.]
380.9	73	$KMIN_t$	= minimum amount of capital required to produce Y_t, *B*1958. [Defined in Table 2–2.]
250.9	64	$LBVBB_t$	= value of loans of the financial sector, *BCURT*. [$=SECB_t$ in Table 1–3.]
240.9	55	LF_t	= value of loans taken out by the firm sector, *BCURT*. [$=-SECF_t$ in Table 1–3.]
−2.7		†$MAILFLT_t$	= demand deposit mail float, *BCURT*. [See discussion in section 1.3.]
$3.43 \cdot 10^7$	74	$M_tH_t^M$	= number of worker hours required to produce Y_t, thousands of worker hours. [Defined in Table 2–2.]
4366.	7	$MOON_t$	= difference between the total number of jobs in the economy (establishment data) and the total number of people employed (household survey data), thousands of persons. This difference is called "the number of moonlighters." [$=JOBF_t + JOBGC_t + JOBGM_t - EMPL_t$.]
1.116	33	PCD_t	= implicit price deflator for CD_t, 1958 = 1.0. [SCB, 8.1, Deflator for Durable Goods.]
1.333	32	PCN_t	= implicit price deflator for CN_t, 1958 = 1.0. [SCB, 8.1, Deflator for Nondurable Goods.]
1.238		†$PCOM_t$	= implicit price deflator for COM_t, 1958 = 1.0. [SCB, 8.1, Deflator for Farm Output.]
1.501	31	PCS_t	= implicit price deflator for CS_t, 1958 = 1.0. [SCB, 8.1, Deflator for Services.]

Table 2–1. (continued)

Value of Variable in 1971IV	*Equation Number in Model*		
1.218	29	PD_t	= implicit price deflator for $X_t - EX_t + IM_t$ (domestic sales), 1958 = 1.0. [Defined in Table 2–2.]
1.260	28	PEX_t	= implicit price deflator for EX_t, 1958 = 1.0. [SCB, 8.1, Deflator for Exports.]
1.216	9	PF_t	= implicit price deflator for $X_t - COM_t$, 1958 = 1.0. [$=(XX_t - PCOM_t\, COM_t)/(X_t - COM_t)$.]
1.371	35	PFF_t	= implicit price deflator for INV_t, 1958 = 1.0. [SCB, 8.1, Deflator for Nonresidential Fixed Investment.]
1.400	36	PG_t	= implicit price deflator for XG_t, 1958 = 1.0. [$=GFPGO_t/XG_t$. For $GFPGO_t$, see Table 1–2.]
1.374	30	PH_t	= implicit price deflator for domestic sales inclusive of indirect business taxes, 1958 = 1.0. [Defined in Table 2–2.]
1.504	34	PIH_t	= implicit price deflator for IH_t, 1958 = 1.0. [SCB, 8.1, Deflator for Residential Structures.]
1.268		$^{\dagger}PIM_t$	= implicit price deflator for IM_t, 1958 = 1.0. [SCB, 8.1, Deflator for Imports.]
144315.		$^{\dagger}POP_t$	= noninstitutional population 16 and over, thousands of persons. [EE, *A*-1. Average of monthly data. See discussion in section 2.3 for adjustments.]
35181.		$^{\dagger}POP_{1t}$	= noninstitutional population of men 25–54, thousands of persons. [EE, *A*-3. Sum of total labor force and not in labor force of men 25–54. Average of monthly data. See discussion in section 2.3 for adjustments.]
109135.		$^{\dagger}POP_{2t}$	= noninstitutional population of all persons 16 and over except men 25–54, thousands of persons. [$=POP_t - POP_{1t}$.]
31.0	41	$PTAXH_t$	= personal income taxes of the household sector plus tax accruals of farms, *BCURT*. [$=HGPTX_t + HGFRM_t$ in Table 1–2.]
1.217	27	PX_t	= implicit price deflator for X_t, 1958 = 1.0. [$=(PCS_tCS_t + PCN_tCN_t + PCD_tCD_t + PIH_tIH_t + PFF_tINV_t + PEX_tEX_t - PIM_tIM_t + PG_tXG_t - PFF_t(XPAEH_t + XPAEB_t) + PIH_t(XRESF_t + XRESB_t) - IBTH_t)/(CS_t + CN_t + CD_t + IH_t + INV_t + EX_t - IM_t + XG_t + XPAEH_t + XPAEB_t + XRESF_t + XRESB_t)$. See discussion in section 2.3, which demonstrates that $PX_t = XX_t/X_t$.]
7.30	21	$RAAA_t$	= *Aaa* corporate bond rate, percentage points [FRB, *A*30. Average of monthly data.]

Table 2–1. (continued)

Value of Variable in 1971IV	*Equation Number in Model*		
4.23	70	$RBILL_t$	= three-month treasury bill rate, percentage points. FRB, *A*29. Average of monthly data.]
0.94	79	$RBILL_t^*$	= $RBILL_t$ detrended up to 1970IV, percentage points. [Defined in Table 2–2.]
4.84		†RD_t	= *the discount rate, percentage points.* [FRB, *A*8, Rate at F. R. Bank of N.Y. Quarterly average.]
7.74	22	$RMORT_t$	= mortgage rate, percentage points. [FRB, *A*45. Yield in private secondary market on FHA-~~issued~~ insured loans. Average of monthly data. See discussion in section 2.3.]
0.9	63	$SAVB_t$	= saving of the financial sector, *BCURT*. [Same as in Table 1–2. Also defined in Table 2–2.]
−6.6	68	$SAVG_t$	= saving of the government sector, *BCURT*. [Same as in Table 1–2. Also defined in Table 2–2.]
9.0	60	$SAVH_t$	= saving of the household sector, net of capital gains or losses, *BCURT*. [Same as in Table 1–2. Also defined in Table 2–2.]
1.8	65	$SAVR_t$	= saving of the foreign sector, *BCURT*. [Same as in Table 1–2. Also defined in Table 2–2.]
−44.8	66	$SECR_t$	= value of securities of the foreign sector not including demand deposits and currency and gold and foreign exchange, *BCURT*. [Same as in Table 1–3.]
−1.1		†$STATDIS_t$	= statistical discrepancy of the national income accounts, *BCURT*. [See discussion in section 1.3.]
80		†t	= linear time trend, $t = 1$ in 1952I.
58.0	67	TAX_t	= total net taxes paid to the government sector, *BCURT*. [Defined in Table 2–2.]
2.0		†$TAXB_t$	= taxes paid by the financial sector, *BCURT*. [$= BGTAX_t + BGSUR_t$ in Table 1–2.]
7.2	42	$TAXF_t$	= taxes paid by the firm sector, *BCURT*. [$= FGTAX_t$ in Table 1–2.]
48.8	59	$TAXH_t$	= total net taxes paid by the household sector, *BCURT*. [Defined in Table 2–2.]
33619.	5	TLF_{1t}	= total labor force of men 25–54, thousands of persons. [Sum of civilian labor force (seasonally adjusted) and armed forces (not seasonally adjusted) of men 25–54. Data on the former were obtained from the BLS. Data on the latter were obtained from EE, *A*-3, as the difference between the total labor force and the civilian labor force (both not seasonally adjusted) of men 25–54. Average of monthly data. See discussion in section 2.3 for adjustments.]

Table 2–1. (continued)

Value of Variable in 1971IV	*Equation Number in Model*		
54504.	6	TLF_{2t}	= total labor force of all persons 16 and over except men 25–54, thousands of persons. [Difference between total labor force 16 and over (seasonally adjusted) and TLF_{1t}. Data on the former were obtained from EE, *A*-31. Average of monthly data. See discussion in section 2.3 for adjustments.]
1.5	25	TPU_t	= transfer payments in the form of unemployment insurance benefits, *BCURT*. [SCB, 2.1, State Unemployment Insurance Benefits.]
5104.	82	U_t	= number of people unemployed, thousands of persons. [Defined in Table 2–2.]
0.0597	83	UR_t	= civilian unemployment rate. [Defined in Table 2–2.]
205.9	51	V_t	= stock of inventories of the firm sector, *B*1958. [See discussion in section 2.3.]
323.1		$^{\dagger}VBG_t$	= value of government securities, *BCURT*. [$= -SECG_t$ in Table 1–3.]
132.1	15	WF_t	= average hourly earnings, private nonfarm economy, production and nonsupervisory workers, adjusted for overtime (in manufacturing only) and interindustry employment shifts, index of current dollars, 1967 = 100. [EE, *C*-17. See discussion in section 2.3.]
$3.88 \cdot 10^{-6}$	37	WFF_t	= average hourly earnings, excluding overtime, of workers in the firm sector, millions of current dollars per hour per job. [$=(FHWAG_t - FHWLD_t + FHOTH_t + FHPRI_t)/((HPFN_t + 1.5HPFO_t)JOBF_t)$. For the first four variables, see Table 1–2.]
$4.08 \cdot 10^{-6}$	38	WGC_t	= average hourly earnings of government civilian workers, millions of current dollars per hour per job. [$=(GHCIV_t - GHWLD_t + GHOTH_t)/(HPGC_t JOBGC_t)$. For the first three variables, see Table 1–2.]
$3.50 \cdot 10^{-6}$	39	WGM_t	= average hourly earnings of government military workers, millions of current dollars per hour per job. [$=GHMIL_t/(HPGM_t JOBGM_t)$. For $GHMIL_t$, see Table 1–2.]
169.4	48	X_t	= total sales of the firm sector, *B*1958. [Defined in Table 2–2.]
0.5		$^{\dagger}XCCAB_t$	= capital consumption of the financial sector, *B*1958. [$=HBCCA_t/PX_t$. For $HBCCA_t$, see Table 1–2.]
20.4		$^{\dagger}XG_t$	= purchases of goods of the government sector, *B*1958. [Difference between government purchases of goods and services in constant dollars (SCB, 1.2) and general government in constant dollars (SCB, 1.8.)]

Table 2–1. (continued)

Value of Variable in 1971IV	*Equation Number in Model*		
−0.1		$^{\dagger}XIVTH_t$	= inventory investment of the household sector, *B*1958. [$=HFIVT_t/PX_t$. For $HFIVT_t$, see Table 1–2.]
0.9		$^{\dagger}XPAEB_t$	= nonresidential plant and equipment investment of the financial sector, *B*1958. [$=BFPAE_t/PFF_t$. For $BFPAE_t$, see Table 1–2.]
4.3		$^{\dagger}XPAEH_t$	= nonresidential plant and equipment investment of the household sector, *B*1958. [$=HFPAE_t/PFF_t$. For $HFPAE_t$, see Table 1–2.]
3.4		$^{\dagger}XPROB_t$	= profits of the financial sector, *B*1958. [$=HBPRO_t/PX_t$. For $HBPRO_t$, see Table 1–2.]
0.0		$^{\dagger}XRESB_t$	= residential investment of the financial sector, *B*1958. [$=BFRES_t/PIH_t$. For $BFRES_t$, see Table 1–2.]
0.8		$^{\dagger}XRESF_t$	= residential investment of the firm sector, *B*1958. [$=FFRES_t/PIH_t$. For $FFRES_t$, see Table 1–2.]
206.2	49	XX_t	= total sales of the firm sector, *BCURT*. [Defined in Table 2–2.]
170.6	10	Y_t	= production of the firm sector, *B*1958. [$=X_t+V_t-V_{t-1}$.]
23.4		$^{\dagger}YG_t$	= transfer payments from the government sector to the household sector, not counting TPU_t, *BCURT*. [$=GHTRP_t+GHINS_t+GHRET_t-TPU_t$. For the first three variables, see Table 1–2.]
201.8	58	YH_t	= taxable income of the household sector, *BCURT*. [Defined in Table 2–2.]
45.0	71	$YNLH_t$	= nonlabor income of the household sector, *BCURT*. [Defined in Table 2–2.]
0.9664	77	ZJ_t	= hours constraint variable for the household sector. [Defined in Table 2–2.]
0.5962	78	ZJ_t'	= labor constraint variable for the firm sector. [Defined in Table 2–2.]
0.9999	80	ZR_t	= loan constraint variable. [Defined in Table 2.2.]
0.0525		$^{\dagger}\delta_D$	= physical depreciation rate of the stock of durable goods, rate per quarter. [See discussion in section 2.3.]
0.00575		$^{\dagger}\delta_H$	= physical depreciation rate of the stock of residential structures of the household sector, rate per quarter. [See discussion in section 2.3.]
0.0285		$^{\dagger}\delta_K$	= physical depreciation rate of the stock of capital of the firm sector, rate per quarter. [See discussion in section 5.2.]

Table 2–1. (continued)

Value of Variable in 1971IV	*Equation Number in Model*		
$4.975 \cdot 10^{-6}$		${}^{\dagger}\lambda_t$	= amount of output capable of being produced per worker hour, output (*B*1958) per thousand worker hours. [Constructed from peak-to-peak interpolations. See discussion in section 5.2.]
0.4480		${}^{\dagger}(\mu_t \bar{H})$	= maximum amount of output capable of being produced per quarter per unit of the capital stock, output (*B*1958) per unit of capital stock (*B*1958). [Constructed from peak-to-peak interpolations. See discussion in section 5.2.]
17.5	52	πF_t	= before-tax profits of the firm sector, *BCURT*. [Defined in Table 2–2. See also F/F, Profits of Corporate Business, 106060205, p. 8, plus Foreign Profits, 266060001, p. 8.]
1.035		${}^{\dagger}\psi_{1t}$	$= PEX_t/PX_t$
1.042		${}^{\dagger}\psi_{2t}$	$= PCS_t((1 + d_{4t})PD_t)$
0.925		${}^{\dagger}\psi_{3t}$	$= PCN_t/((1 + d_{4t})PD_t)$
0.774		${}^{\dagger}\psi_{4t}$	$= PCD_t/((1 + d_{4t})PD_t)$
1.235		${}^{\dagger}\psi_{5t}$	$= PIH_t/PD_t$
1.126		${}^{\dagger}\psi_{6t}$	$= PFF_t/PD_t$
1.150		${}^{\dagger}\psi_{7t}$	$= PG_t/PD_t$
$2.938 \cdot 10^{-8}$		${}^{\dagger}\psi_{8t}$	$= WFF_t/WF_t$
$3.092 \cdot 10^{-8}$		${}^{\dagger}\psi_{9t}$	$= WGC_t/WF_t$
$2.651 \cdot 10^{-8}$		${}^{\dagger}\psi_{10t}$	$= WGM_t/WF_t$
0.000343		${}^{\dagger}\tau$	= progressivity tax parameter in personal income tax equation. [See discussion in section 2.2.]

Note: The table includes 83 endogenous variables (not counting GNP_t) and 78 exogenous variables (not counting δ_D, δ_H, δ_K, and τ).

Table 2–2. The List of Equations in the Model

Variables Explained by Stochastic Equations

The Household Sector:

1. CS_t	[consumption expenditures on services]
2. CN_t	[consumption expenditures on nondurable goods]
3. KCD_t	[stock of consumer durables]
4. KIH_t	[stock of residential structures of the household sector]
5. TLF_{1t}	[total labor force of males 25–54]
6. TLF_{2t}	[total labor force of all others 16 and over]
7. $MOON_t$	[the number of moonlighters]
8. DDH_t	[value of demand deposits and currency of the household sector]

The Firm Sector:

9. PF_t	[implicit price deflator for $X_t - COM_t$ (total firm sales less farm output)]
10. Y_t	[production of the firm sector]
11. INV_t	[nonresidential plant and equipment investment of the firm sector]
12. $JOBF_t$	[number of jobs in the firm sector]
13. HPF_t	[average number of hours paid per job by the firm sector]
14. $HPFO_t$	[average number of overtime hours paid per job by the firm sector]
15. WF_t	[average earnings adjusted for overtime and interindustry employment shifts]
16. DDF_t	[value of demand deposits and currency of the firm sector]
17. $DIVF_t$	[dividends paid by the firm sector]
18. $INTF_t$	[interest paid by the firm sector]
19. IVA_t	[inventory valuation adjustment]

The Financial Sector:

20. $BORR_t$	[commercial bank borrowing at federal reserve banks]
21. $RAAA_t$	[the bond rate]
22. $RMORT_t$	[the mortgage rate]
23. CG_t	[capital gains (+) or losses (−) on stocks held by the household sector]

The Foreign Sector:

24. IM_t	[imports]

The Government Sector:

25. TPU_t	[unemployment insurance benefits]
26. $INTG_t$	[interest paid by the government sector]

Price Deflators Explained as a Function of PF_t

27. $PX_t = \dfrac{PF_t(X_t - COM_t) + PCOM_t COM_t}{X_t}$	[price deflator for total firm sales]
28. $PEX_t = \psi_{1t} PX_t$	[price deflator for exports]
29. $PD_t = \dfrac{PX_t X_t - PEX_t EX_t + PIM_t IM_t}{X_t - EX + IM_t}$	[price deflator for domestic sales (total firm sales, less exports, plus imports)]
30. $PH_t = PD_t + \dfrac{IBTH_t}{X_t - EX_t + IM_t}$	[price deflator for domestic sales inclusive of indirect business taxes]
31. $PCS_t = \psi_{2t}(1 + d_{4t})PD_t$	[price deflator for expenditures on services]
32. $PCN_t = \psi_{3t}(1 + d_{4t})PD_t$	[price deflator for expenditures on nondurable goods]
33. $PCD_t = \psi_{4t}(1 + d_{4t})PD_t$	[price deflator for expenditures on durable goods]

Table 2–2. (continued)

34. $PIH_t = \psi_{5t} PD_t$ [price deflator for expenditures on residential structures]

35. $PFF_t = \psi_{6t} PD_t$ [price deflator for expenditures on nonresidential plant and equipment investment]

36. $PG_t = \psi_{7t} PD_t$ [price deflator for expenditures on goods by the government sector]

Compensation Rates Explained as a Function of WF_t

37. $WFF_t = \psi_{8t} WF_t$ [average hourly earnings, excluding overtime, of workers in the firm sector]

38. $WGC_t = \psi_{9t} WF_t$ [average hourly earnings of government civilian workers]

39. $WGM_t = \psi_{10t} WF_t$ [average hourly earnings of government military workers]

Taxes Explained as a Function of Tax Rates

40. $IBTH_t = \dfrac{d_{4t}}{1 + d_{4t}} (PCD_t CD_t + PCN_t CN_t - PCS_t CS_t)$ [indirect business taxes]

41. $PTAXH_t = (d_{3t} + \tau . YH_t) YH_t$ [personal income taxes]

42. $TAXF_t = d_{1t} \pi F_t$ [profit taxes of the firm sector]

43. $FHCSI_t = d_{5t}(WFF_t(HPFN_t + 1.5HPFO_t)JOBF_t)$ [employer social security taxes]

44. $HGSI2_t = d_{6t}(WFF_t(HPFN_t + 1.5HPFO_t)JOBF_t)$ [employee social security taxes]

Bank Reserves Explained as a Function of Demand Deposits and the Reserve Requirement Ratio

45. $BR_t = g_{1t} DDB_t$ [bank reserves]

Variables Explained by Definitions That Are Needed to Close the Model

46. $CD_t = KCD_t - (1 - \delta_D) KCD_{t-1}$ [expenditures on durable goods]

47. $IH_t = KIH_t - (1 - \delta_H) KIH_{t-1}$ [expenditures on residential structures by the household sector]

48. $X_t = CS_t + CN_t + CD_t - IH_t + INV_t + EX_t - IM_t + XG_t + XPAEH_t - XPAEB_t + XRESF_t + XRESB_t + XIVTH_t - XPROB_t - XCCAB_t$ [total sales of the firm sector (constant dollars)]

49. $XX_t = PCS_t CS_t + PCN_t CN_t + PCD_t CD_t + PIH_t IH_t + PFF_t INV_t + PEX_t EX_t - PIM_t IM_t - PG_t XG_t - PFF_t(XPAEH_t + XPAEB_t) + PIH_t(XRESF_t + XRESB_t) + PX_t(XIVTH_t - XPROB_t - XCCAB_t) - IBTH_t$ [value of total sales of the firm sector (current dollars)]

50. $HPFN_t = HPF_t - HPFO_t$ [average number of nonovertime hours paid per job by the firm sector]

51. $V_t = V_{t-1} + Y_t - X_t$ [stock of inventories at the end of period t]

Table 2–2. (continued)

52. πF_t	$= XX_t + PX_t(V_t - V_{t-1}) - WFF_t(1+d_{5t})(HPFN_t + 1.5HPFO_t)JOBF_t - FHRNT_t - FHTRP_t - FHPFA_t - FHCCA_t + GHSUB_t - INTF_t - DEP_t - IVA_t - FHWLD_t - STATDIS_t$	[before-tax profits of the firm sector]
53. CF_t	$= XX_t - WFF_t(1 + d_{5t})(HPFN_t + 1.5HPFO_t) \times JOBF_t - FHRNT_t - FHTRP_t - FHPFA_t - FHCCA_t + GHSUB_t - INTF_t - PFF_tINV_t - PIH_tXRESF_t$	[cash flow of the firm sector]
54. $\overline{CF}_t$	$= CF_t - TAXF_t - DIVF_t$	[cash flow net of taxes and dividends of the firm sector]
55. LF_t	$= LF_{t-1} + DDF_t - DDF_{t-1} - \overline{CF}_t + DISF_t + FHWLD_t + STATDIS_t$	[value of loans taken out by the firm sector]
56. $DIVH_t$	$= DIVF_t + DIVB_t$	[dividends received by the household sector]
57. $INTH_t$	$= INTF_t + INTG_t$	[interest received by the household sector]
58. YH_t	$= WFF_t(HPFN_t + 1.5HPFO_t)JOBF_t + WGC_tHPGC_tJOBGC_t + WGM_tHPGM_t \times JOBGM_t + DIVH_t + INTH_t + FHRNT_t + FHTRP_t + FHPFA_t$	[taxable income of the household sector]
59. $TAXH_t$	$= PTAXH_t + IBTH_t + FHCSI_t + HGSI2_t - YG_t - TPU_t$	[total net taxes paid by the household sector]
60. $SAVH_t$	$= YH_t + FHCCA_t + FHCSI_t - PCS_tCS_t - PCN_tCN_t - PCD_tCD_t - PIH_tIH_t - PFF_tXPAEH_t - PX_t XIVTH_t - HRTRP_t - (TAXH_t - IBTH_t)$	[saving of the household sector]
61. A_t	$= A_{t-1} - DDH_t + DDH_{t-1} + SAVH_t + CG_t - DISH_t$	[value of nondemand deposit securities of the household sector]
62. DDB_t	$= DDB_{t-1} + DDH_t - DDH_{t-1} + DDF_t - DDF_{t-1} - DDR_t - DDR_{t-1} - CURR_t + CURR_{t-1} + MAILFLT_t$	[value of demand deposits and currency of the financial sector]
63. $SAVB_t$	$= PX_t(XPROB_t + XCCAB_t) - PFF_tXPAEB_t - PIH_tXRESB_t - DIVB_t - TAXB_t$	[saving of the financial sector]
64. $LBVBB_t$	$= LBVBB_{t-1} + BORR_t - BORR_{t-1} - BR_t + BR_{t-1} + DDB_t - DDB_{t-1} + SAVB_t - DISB_t$	[value of loans of the financial sector]
65. $SAVR_t$	$= PIM_tIM_t + HRTRP_t + GRTRP_t - PEX_tEX_t$	[saving of the foreign sector]
66. $SECR_t$	$= SECR_{t-1} - DDR_t + DDR_{t-1} + GFXG_t - GFXG_{t-1} + SAVR_t - DISR_t$	[value of securities of the foreign sector not including demand deposits and currency and gold and foreign exchange]
67. TAX_t	$= TAXH_t + TAXF_t + TAXB_t$	[total net taxes paid to the government sector]
68. $SAVG_t$	$= TAX_t - PG_tXG_t - WGC_tHPGC_tJOBGC_t - WGM_tHPGM_tJOBGM_t - INTG_t - GRTRP_t - GHSUB_t$	[saving of the government sector]
69. 0	$= VBG_t - VBG_{t-1} - BORR_t + BORR_{t-1} + CURR_t - CURR_{t-1} + BR_t - BR_{t-1} + SAVG_t - GFXG_t + GFXG_{t-1} - DISG_t$	[government budget constraint]

Table 2–2. (continued)

70. 0	$= -LBVBB_t + LBVBB_{t-1} - A_t + A_{t-1} + CG_t + LF_t - LF_{t-1} + VBG_t - VBG_{t-1} - SECR_t + SECR_{t-1} - (DISH_t - DISF_t - DISB_t + DISR_t - DISG_t) - FHWLD_t - STATDIS_t + MAILFLT_t$	[the change in the sum of all other securities (SEC) across sectors must be zero after adjusting for discrepancies]
71. $YNLH_t$	$= DIVH_t + INTH_t + FHRNT_t + FHTRP_t + FHPFA_t + YG_t + TPU_t - HGSI2_t$	[nonlabor income of the household sector]
72. K_t^a	$= (1 - \delta_K)K_{t-1}^a + INV_t$	[actual capital stock of the firm sector]
73. $KMIN_t$	$= \dfrac{Y_t}{(\mu_t \overline{H})}$	[minimum amount of capital required to produce Y_t]
74. $M_t H_t^M$	$= \dfrac{Y_t}{\lambda_t}$	[number of worker hours required to produced Y_t]
75. J_t	$= \dfrac{JOBF_t HPF_t + JOBGC_t HPGC_t + JOBGM_t HPGM_t}{POP_t}$	[ratio of total worker hours paid for to the total population 16 and over]
76. J_t^*	$= \dfrac{J_t}{e^{-0.00073513 \cdot t}}$	[J_t detrended]
77. ZJ_t	$= e^{-1/10000(J_t^* - 335.9)^2}$	[hours constraint variable for the household sector]
78. ZJ_t'	$= 4.454062 + \dfrac{1}{1 - UR_t - 1.199514}$	[labor constraint variable for the firm sector]
79. $RBILL_t^*$	$= \begin{cases} RBILL_t/e^{0.019757 \cdot t} \text{ if } t \leq 76 \\ RBILL_t/e^{0.019757 \cdot 76} \text{ if } t > 76 \end{cases}$	[$RBILL_t$ detrended up to $t = 76$ (1970IV)]
80. ZR_t	$= e^{-1/1000(RBILL_t^* - 0.608)^2}$	[loan constraint variable]
81. $EMPL_t$	$= JOBF_t + JOBGC_t + JOBGM_t - MOON_t$	[total number of people employed]
82. U_t	$= TLF_{1t} + TLF_{2t} - EMPL_t$	[total number of people unemployed]
83. UR_t	$= \dfrac{U_t}{TLF_{1t} + TLF_{2t} - JOBGM_t}$	[civilian unemployment rate]
84. d_{3t}^M	$= d_{3t} + 2\tau \cdot YH_t$	[marginal personal income tax rate]

Note:

GNP_t	$= XX_t + PX_t(V_t - V_{t-1}) - IBTH_t + WGC_t HPGC_t JOBGC_t + WGM_t HPGM_t JOBGM_t + GHWLD_t + PX_t(XPROB_t - XCCAB_t)$	[GNP in current dollars]

Table 2–3. TSLS and FIML Estimates of the 26 Stochastic Equations

Notes:

1. The FIML estimates appear above the TSLS estimates.
2. The sample period is 1954I–1974II (82 observations).
3. The numbers in parentheses are absolute values of the t-statistics of the TSLS estimates.
4. DW = Durbin-Watson statistic for the TSLS estimates.
5. R^2 = coefficient of determination for the TSLS estimates.
6. $\hat{\rho}$ = estimate of the first order serial correlation coefficient for the equation. "0" means the coefficient was constrained to be zero.
7. When $\hat{\rho} \neq 0$, DW and R^2 are computed using the estimates of the transformed residuals.
8. logs are natural logs.
9. $\bar{d}^M_{3t} = (d^M_{3t} + d^M_{3t-1} + d^M_{3t-2} + d^M_{3t-3})/4$.
10. "a" means that the coefficient was not estimated by FIML.

The Household Sector

$$1.\ \log \frac{CS_t}{POP_t} = \underset{(0.50)}{\overset{0.260}{-0.259}} + \underset{(25.43)}{\overset{0.976}{0.976}} \log \frac{CS_{t-1}}{POP_{t-1}} + \underset{(1.06)}{\overset{0.00877}{0.00877}} \log \frac{A_{t-1}}{PH_{t-1}POP_{t-1}}$$

$$- \underset{(1.15)}{\overset{-0.117}{0.117}} \log PCS_t + \underset{(0.97)}{\overset{0.0787}{0.0787}} \log WF_t + \underset{(0.87)}{\overset{0.0223}{0.0223}} \log \frac{YNLH_{t-1}}{PH_{t-1}POP_{t-1}}$$

$$- \underset{(1.54)}{\overset{-0.00984}{0.00984}} \log RMORT_t - \underset{(2.73)}{\overset{-0.00658}{0.00658}} \log RBILL_t + \underset{(1.49)}{\overset{0.0364}{0.0364}} \log ZJ_t$$

$\hat{\rho}$	DW	R^2
0	2.32	0.9995

$$2.\ \log\frac{CN_t}{POP_t} = \overset{-2.74}{\underset{(3.75)}{-2.74}} + \overset{0.508}{\underset{(6.31)}{0.508}}\log\frac{CN_{t-1}}{POP_{t-1}} + \overset{0.0167}{\underset{(1.05)}{0.0168}}\log\frac{A_{t-1}}{PH_{t-1}POP_{t-1}} \qquad \circ \qquad 2.14 \qquad 0.996$$

$$- \overset{-0.131}{\underset{(2.13)}{0.131}}\log PCN_t + \overset{0.102}{\underset{(1.63)}{0.102}}\log WF_t + \overset{0.0219}{\underset{(0.48)}{0.0219}}\log\frac{YNLH_t}{PH_tPOP_t}$$

$$+ \overset{0.150}{\underset{(3.18)}{0.150}}\log\frac{YNLH_{t-1}}{PH_{t-1}POP_{t-1}} + \overset{0.261}{\underset{(2.31)}{0.256}}\log(1.0 - \bar{d}^{M}_{3t-1})$$

$$+ \overset{0.256}{\underset{(3.75)}{0.256}}\log ZJ_t$$

$$3.\ \log\frac{KCD_t}{POP_t} = \overset{-1.22}{\underset{(2.70)}{-1.22}} + \overset{0.873}{\underset{(19.92)}{0.873}}\log\frac{KCD_{t-1}}{POP_{t-1}} - \overset{-0.128}{\underset{(3.69)}{0.128}}\log PCD_t + \overset{0.137}{\underset{(3.49)}{0.137}}\log WF_t \qquad \overset{0.789}{\underset{(6.99)}{0.611}} \qquad 1.94 \qquad 0.9999$$

$$+ \overset{0.0183}{\underset{(1.45)}{0.0183}}\log\frac{YNLH_t}{PH_tPOP_t} - \overset{-0.0152}{\underset{(2.04)}{0.0152}}\log RMORT_t + \overset{0.201}{\underset{(4.11)}{0.215}}\log ZJ_t$$

$$- \overset{a}{\underset{(1.31)}{0.00249}}D644_t + \overset{a}{\underset{(1.38)}{0.00259}}D651_t - \overset{a}{\underset{(1.27)}{0.00247}}D704_t$$

$$+ \overset{a}{\underset{(1.58)}{0.00299}}D711_t$$

Table 2–3. (continued)

	$\hat{\rho}$	DW	R^2
$4.\ \log\frac{KIH_t}{POP_t} = \underset{(4.89)}{\overset{-0.504}{-0.504}} + \underset{(74.66)}{\overset{0.929}{0.929}}\log\frac{KIH_{t-1}}{POP_{t-1}} - \underset{(1.24)}{\overset{-0.0114}{0.0122}}\log PIH_{t-1}$ $+ \underset{(2.99)}{\overset{0.0296}{0.0296}}\log WF_{t-1} - \underset{(2.34)}{\overset{-0.00725}{0.00728}}\log RMORT_{t-1}$ $- \underset{(2.22)}{\overset{-0.00677}{0.00678}}\log RMORT_{t-2} + \underset{(1.10)}{\overset{0.00338}{0.0110}}\log ZJ_{t-1} + \underset{(1.25)}{\overset{0.127}{0.328}}\log ZR_{t-1}$	0.776 0.770 (10.93)	2.26	0.9999
$5.\ \log\frac{TLF_{1t}}{POP_{1t}} = \underset{(0.82)}{\overset{a}{-0.230}} + \underset{(7.70)}{\overset{a}{0.675}}\log\frac{TLF_{1t-1}}{POP_{1t-1}} + \underset{(0.78)}{\overset{a}{0.0268}}\log\frac{WF_t}{PH_t}$ $\underset{(0.87)}{\overset{a}{0.0130}}\cdot\frac{1}{4}\sum_{i=1}^{4}\log\frac{YNLH_{t-i}}{PH_{t-i}POP_{t-i}} + \underset{(2.71)}{\overset{a}{0.0622}}\log(1.0 - \bar{d}^M_{3t-1})$	0	2.03	0.947
$6.\ \log\frac{TLF_{2t}}{POP_{2t}} = \underset{(4.14)}{\overset{a}{0.698}} + \underset{(10.79)}{\overset{a}{0.705}}\log\frac{TLF_{2t-1}}{POP_{2t-1}} - \underset{(2.23)}{\overset{a}{0.0166}}\log\frac{A_{t-1}}{PH_{t-1}POP_{t-1}}$ $+ \underset{(3.74)}{\overset{a}{0.0852}}\log\frac{WF_{t-1}}{PH_{t-1}} + \underset{(1.42)}{\overset{a}{0.0121}}\log RMORT_t + \underset{(4.79)}{\overset{a}{0.173}}\log ZJ_t$	0	1.86	0.983

$$7.\ \log\frac{MOON_t}{POP_t} = \underset{(3.31)}{\overset{a}{-2.70}} + \underset{(4.35)}{\overset{a}{0.528}}\log\frac{MOON_{t-1}}{POP_{t-1}} + \underset{(1.28)}{\overset{a}{0.262}}\log\frac{WF_t}{PH_t}$$

$$+ \underset{(0.27)}{\overset{a}{0.176}}\log(1.0 - \bar{d}^M_{3t-1}) + \underset{(3.33)}{\overset{a}{2.29}}\log ZJ_{t-1}$$

$$8.\ \log\frac{DDH_t}{POP_t} = \underset{(2.44)}{\overset{1.03}{1.03}} + \underset{(6.68)}{\overset{0.695}{0.695}}\log\frac{DDH_{t-1}}{POP_{t-1}} - \underset{(3.05)}{\overset{-0.0255}{0.0255}}\log RBILL_t$$

$$+ \underset{(2.84)}{\overset{0.432}{0.432}}\log\frac{YH_t}{POP_t} - \underset{(2.46)}{\overset{0.00270}{0.00271}}t$$

The Firm Sector

$$9.\ \log PF_t = \underset{(3.01)}{\overset{-0.401}{-0.401}} + \underset{(10.71)}{\overset{0.732}{0.739}}\log PF_{t-1} + \underset{(8.17)}{\overset{0.0845}{0.0795}}\log PIM_t + \underset{(2.71)}{\overset{0.0763}{0.0763}}\log WF_{t-1}$$

$$- \underset{(3.69)}{\overset{0.0332}{0.0332}}\log RAAA_t - \underset{(0.95)}{\overset{0.00108}{0.00122}}DTAXCR_t - \underset{(1.83)}{\overset{-0.000940}{0.00228}}\log ZJ_t^f$$

$$10.\ \log Y_t = \underset{(3.20)}{\overset{0.153}{0.153}} + \underset{(2.40)}{\overset{0.189}{0.189}}\log Y_{t-1} + \underset{(11.27)}{\overset{0.981}{0.981}}\log X_t - \underset{(4.67)}{\overset{-0.191}{0.191}}\log V_{t-1}$$

$$- \underset{(2.91)}{\overset{a}{0.0150}}D593_t + \underset{(0.57)}{\overset{a}{0.00310}}D594_t + \underset{(1.78)}{\overset{a}{0.00878}}D601_t$$

	$\hat{\rho}$	DW	R^2
7.	0	1.90	0.831
8.	0	2.39	0.997
9.	0	1.54	0.9996
10.	0.675 0.596 (6.71)	2.01	0.9996

Table 2–3. (continued)

11.
$$\begin{aligned} INV_t - INV_{t-1} = {} & \underset{(0.80)}{\overset{a}{-0.00256}}(K^a_{t-1} - KMIN_{t-1}) + \underset{(0.78)}{\overset{a}{0.0272}}(Y_t - Y_{t-1}) \\ & + \underset{(3.11)}{\overset{a}{0.0782}}(Y_{t-1} - Y_{t-2}) + \underset{(1.09)}{\overset{a}{0.0241}}(Y_{t-2} - Y_{t-3}) \\ & + \underset{(2.52)}{\overset{a}{0.0558}}(Y_{t-3} - Y_{t-4}) - \underset{(0.82)}{\overset{a}{0.0155}}(INV_{t-1} - \delta_K K^a_{t-1}) \\ & - \underset{(3.74)}{\overset{a}{1.04}} D704_t + \underset{(1.75)}{\overset{a}{0.509}} D711_t \end{aligned}$$

$\hat{\rho}$	DW	R^2
0	1.89	0.579

12.
$$\begin{aligned} \log JOBF_t - \log JOBF_{t-1} = {} & \underset{(2.86)}{\overset{-0.488}{-0.489}} - \underset{(2.85)}{\overset{-0.0780}{0.0780}}(\log JOBF_{t-1} - \log M_{t-1}H^M_{t-1}) \\ & + \underset{(2.97)}{\overset{0.0000966}{0.0000971}}t + \underset{(3.67)}{\overset{0.168}{0.215}}(\log Y_t - \log Y_{t-1}) \\ & + \underset{(3.84)}{\overset{0.163}{0.172}}(\log Y_{t-1} - \log Y_{t-2}) \\ & + \underset{(1.79)}{\overset{0.00316}{0.0725}}(\log Y_{t-2} - \log Y_{t-3}) \\ & - \underset{(2.22)}{\overset{a}{0.00945}} D593_t + \underset{(0.49)}{\overset{a}{0.00196}} D594_t \end{aligned}$$

$\hat{\rho}$	DW	R^2
0.364 0.307 (2.92)	1.96	0.737

13. $\log HPF_t - \log HPF_{t-1} \quad \underset{(4.15)}{\overset{1.42}{1.42}} - \underset{(4.15)}{\overset{-0.269}{0.269}} \log HPF_{t-1}$	$\underset{(2.06)}{\overset{-0.277}{-0.221}}$	1.96	0.345
$- \underset{(2.70)}{\overset{-0.0438}{0.0438}}(\log JOBF_{t-1} \quad \log M_{t-1}H^M_{t-1}) - \underset{(4.20)}{\overset{-0.000250}{0.000253}}t$			
$+ \underset{(5.21)}{\overset{0.162}{0.162}}(\log Y_t - \log Y_{t-1})$			
14. $\log HPFO_t = \underset{(14.00)}{\overset{a}{-18.9}} + \underset{(16.62)}{\overset{a}{0.0420}}(HPF_t + 0.5482t) + \underset{(13.41)}{\overset{a}{0.209}} DD661_t$	0	1.34	0.885
[sample period began in 1956I]			
15. $\log WF_t = \underset{(1.93)}{\overset{-0.386}{-0.386}} + \underset{(29.46)}{\overset{0.972}{0.972}} \log WF_{t-1} + \underset{(0.35)}{\overset{0.0000602}{0.0000577}}t + \underset{(1.74)}{\overset{0.0590}{0.0590}} \log PX_t$	0	1.65	0.9999
$+ \underset{(5.79)}{\overset{0.0904}{0.0904}} \log J^*_t$			
16. $\log DDF_t = \underset{(0.99)}{\overset{0.100}{0.100}} + \underset{(21.53)}{\overset{0.919}{0.919}} \log DDF_{t-1} + \underset{(2.08)}{\overset{0.0404}{0.0404}} \log XX_t$	0	2.04	0.962
$- \underset{(1.35)}{\overset{-0.0143}{0.0143}} \log RBILL_t$			

Table 2–3. (continued)

	$\hat{\rho}$	DW	R^2
17. $\log DIVF_t = \underset{(1.56)}{\overset{a}{-0.0196}} + \underset{(69.35)}{\overset{a}{0.941}} \log DIVF_{t-1} + \underset{(4.42)}{\overset{a}{0.0592}} \log(\pi F_t - TAXF_t)$	$\underset{(2.42)}{\overset{a}{-0.258}}$	1.84	0.997
$+ \underset{(2.41)}{\overset{a}{7.88}} \log ZR_t$			
18. $INTF_t = \underset{(0.37)}{\overset{a}{-0.107}} + \underset{(6.52)}{\overset{a}{0.574}} INTF_{t-1} + \underset{(4.41)}{\overset{a}{0.0159}} LF_t + \underset{(2.88)}{\overset{a}{0.144}} RAAA_t$	$\underset{(24.95)}{\overset{a}{0.940}}$	1.99	0.9998
19. $IVA_t = \underset{(4.00)}{\overset{a}{3.53}} - \underset{(10.59)}{\overset{a}{171.9}} PX_t + \underset{(9.40)}{\overset{a}{162.4}} PX_{t-1} + \underset{(4.90)}{\overset{a}{0.0399}} V_{t-1}$	0	1.75	0.861
The Financial Sector			
20. $\frac{BORR_t}{BR_t} = \underset{(3.18)}{\overset{a}{0.0121}} + \underset{(0.95)}{\overset{a}{0.0106}}(RBILL_t - RD_t)$	$\underset{(5.75)}{\overset{a}{0.536}}$	2.18	0.368
21. $\log RAAA_t = \underset{(3.52)}{\overset{0.0613}{0.0642}} + \underset{(43.03)}{\overset{0.922}{0.922}} \log RAAA_{t-1} + \underset{(3.08)}{\overset{0.166}{0.166}} \log RBILL_t$	0	2.05	0.994
$\underset{(2.67)}{\overset{-0.177}{-0.180}} \log RBILL_{t-1} + \underset{(1.99)}{\overset{0.0640}{0.0601}} \log RBILL_{t-2}$			
$+ \underset{(2.33)}{\overset{1.25}{1.87}} \cdot \frac{1}{6}[3(\log PX_{t-1} - \log PX_{t-2})$			
$+ 2(\log PX_{t-2} - \log PX_{t-3}) + (\log PX_{t-3} - \log PX_{t-4})]$			

$$22.\ \log RMORT_t = \underset{(3.73)}{\overset{0.188}{0.186}} + \underset{(24.72)}{\overset{0.859}{0.859}} \log RMORT_{t-1} + \underset{(0.92)}{\overset{0.0355}{0.0355}} \log RBILL_t \qquad \underset{(2.58)}{\overset{0.240}{0.274}} \qquad 2.00 \qquad 0.988$$

$$+ \underset{(1.32)}{\overset{0.0690}{0.0642}} \log RBILL_{t-1} - \underset{(2.79)}{\overset{-0.0997}{0.0939}} \log RBILL_{t-2}$$

$$+ \underset{(2.19)}{\overset{0.0436}{0.0453}} \log RBILL_{t-3} + \underset{(2.39)}{\overset{2.51}{2.36}} \cdot \tfrac{1}{6}[3(\log PX_{t-1} - \log PX_{t-2})$$

$$+ 2(\log PX_{t-2} - \log PX_{t-3}) + (\log PX_{t-3} - \log PX_{t-4})]$$

$$23.\ CG_t = \underset{(2.57)}{\overset{a}{13.1}} \quad \underset{(1.88)}{\overset{a}{77.1}}(RAAA_t - RAAA_{t-1}) + \underset{(1.70)}{\overset{a}{19.7}} \cdot \tfrac{1}{6}\{3[(CF_t - TAXF_t) \qquad 0 \qquad 2.28 \qquad 0.167$$

$$- (CF_{t-1} - TAXF_{t-1})] + 2[(CF_{t-1} - TAXF_{t-1}) - (CF_{t-2} - TAXF_{t-2})]$$

$$+ [(CF_{t-2} - TAXF_{t-2}) - (CF_{t-3} - TAXF_{t-3})]\}$$

The Foreign Sector

$\hat{\rho}$ DW R^2

$$24.\ \log \frac{IM_t}{POP_t} = \underset{(0.99)}{\overset{a}{-1.60}} - \underset{(2.26)}{\overset{a}{0.426}} \log PIM_{t-2} + \underset{(4.79)}{\overset{a}{1.62}} \log PX_{t-1} \qquad \underset{(12.21)}{\overset{a}{0.803}} \qquad 2.03 \qquad 0.996$$

$$+ \underset{(4.96)}{\overset{a}{1.17}} \log \frac{X_t}{POP_t} - \underset{(3.27)}{\overset{a}{0.0712}} D651_t + \underset{(1.49)}{\overset{a}{0.0325}} D652_t$$

$$- \underset{(4.11)}{\overset{a}{0.0947}} D691_t + \underset{(2.43)}{\overset{a}{0.0640}} D692_t + \underset{(1.14)}{\overset{a}{0.0261}} D693_t - \underset{(2.82)}{\overset{a}{0.0614}} D714_t$$

Table 2–3. (continued)

	$\hat{\rho}$	DW	R^2
$+\underset{(3.05)}{\overset{a}{0.0662}}D721_t$			
The Government Sector			
25. $\log TPU_t = \underset{(20.53)}{\overset{a}{-14.4}} + \underset{(19.99)}{\overset{a}{1.71}}\log U_t + \underset{(9.61)}{\overset{a}{1.13}}\log PX_{t-1}$	$\underset{(4.57)}{\overset{a}{0.451}}$	1.85	0.971
26. $\log INTG_t = \underset{(3.61)}{\overset{a}{-1.21}} + \underset{(15.51)}{\overset{a}{0.786}}\log INTG_{t-1} + \underset{(3.45)}{\overset{a}{0.223}}\log VBG_t$	$\underset{(3.05)}{\overset{a}{0.319}}$	2.11	0.997
$+\underset{(3.92)}{\overset{a}{0.0501}}\log RBILL_t + \underset{(1.43)}{\overset{a}{0.0643}}\log RAAA_t$			

2.2 A DISCUSSION OF TABLE 2–2

Consider the stochastic equations in Table 2-2 first. There are eight stochastic equations for the household sector, explaining: (1) consumption of services in real terms, CS_t, (2) consumption of nondurables in real terms, CN_t, (3) the stock of consumer durables in real terms, KCD_t, (4) the stock of houses in real terms, KIH_t, (5) the total labor force of men 25–54, TLF_{1t}, (6) the total labor force of all persons 16 and over except men 25–54, TLF_{2t}, (7) the number of moonlighters, $MOON_t$, and (8) demand deposits of the household sector, DDH_t.

There are eleven stochastic equations for the firm sector, explaining: (9) the price variable that the firm sector is assumed to set, PF_t, (10) production in real terms, Y_t, (11) investment in real terms, INV_t, (12) the number of jobs in the firm sector, $JOBF_t$, (13) the average number of hours paid per job, HPF_t, (14) the average number of overtime hours paid per job, $HPFO_t$, (15) the wage rate that the firm sector is assumed to set, WF_t, (16) demand deposits of the firm sector, DDF_t, (17) dividends paid, $DIVF_t$, (18) interest paid, $INTF_t$, and (19) the inventory valuation adjustment, IVA_t.

There are four stochastic equations for the financial sector, explaining: (20) commercial bank borrowing at the federal reserve banks, $BORR_t$, (21) the bond rate, $RAAA_t$, (22) the mortgage rate, $RMORT_t$, and (23) capital gains on stocks held by the household sector, CG_t. There is one stochastic equation for the foreign sector, explaining: (24) the value of imports in real terms, IM_t. There are, finally, two stochastic equations for the government sector, explaining: (25) transfer payments in the form of unemployment insurance benefits, TPU_t, and (26) interest paid, $INTG_t$. Putting capital gains in the financial sector and imports in the foreign sector, rather than both in the household sector, is somewhat arbitrary, but for expository purposes this seemed like the best procedure.

The next set of equations in Table 2-2 concerns the treatment of the various price deflators in the model. PX_t in Equation 27 is the implicit price deflator for total firm sales, X_t. PF_t, on the other hand, which is the price the firm sector is assumed to set according to Equation 9, is the implicit price deflator for total firm sales *less* farm output, $X_t - COM_t$. Farm output in real terms is denoted as COM_t and will be referred to, somewhat loosely, as "commodity sales." The implicit price deflator for COM_t is denoted as $PCOM_t$ and will be referred to as the "price of commodities." Since PF_t is the price deflator for $X_t - COM_t$, $PCOM_t$ the price deflator for COM_t, and PX_t the price deflator for X_t, the following equation is true by definition: $PX_t X_t = PF_t(X_t - COM_t) + PCOM_t COM_t$, which is Equation 27 in Table 2-2.

Equation 29 defines the price deflator for domestic sales, PD_t, where domestic sales are taken to be total firm sales, less exports, and plus imports. Equation 30 then defines the price deflator for domestic sales *inclusive* of indirect business taxes, PH_t. Given that PD_t is the price deflator for domestic sales net of indirect business taxes and that $IBTH_t$ is the value of indirect business taxes, the following equation is true by definition: $PH_t(X_t - EX_t + IM_t) = PD_t(X_t - EX_t + IM_t) + IBTH_t$, which is Equation 30. PH_t is used as an explanatory variable in some of the stochastic equations of the household sector. Since PH_t is inclusive of indirect business taxes, using it as an explanatory variable means that one is assuming that the prices the households are being influenced by are inclusive of indirect business taxes. This is an example in the model in which an important constraint is put on the specification of the way that taxes affect behavior. Notice also that it is the price of domestic sales that is assumed to affect household behavior, not the price of total firm sales. In other words, the price of imports is assumed to affect household behavior, but the price of exports is not.

The next six deflators in the table are explained as a function of PD_t (Equations 31–36). Consider, for example, Equation 34 explaining PIH_t, the price deflator for housing expenditures. ψ_{5t} in the equation is (from Table 2-1) the actual ratio of PIH_t to PD_t that existed in quarter t. This ratio is taken to be exogenous in the model. PIH_t is then explained as $\psi_{5t}PD_t$. This procedure has the effect of making PIH_t an endogenous variable, since PD_t is an endogenous variable, but making the ratio of PIH_t to PD_t an exogenous variable. It is beyond the scope of this study to consider the determination of relative prices, and the procedure just described is a simple way of allowing there to be more than one endogenous price variable in the model while at the same time allowing relative prices to remain exogenous.

The price deflators PFF_t and PG_t are handled the same way as PIH_t. The price deflators for service, nondurable, and durable consumption expenditures (PCS_t, PCN_t, and PCD_t) are, however, handled slightly differently because of the treatment of indirect business taxes. Indirect business taxes are a part of consumption in current dollar terms, but they are not a part of consumption in real terms. Consequently, the price deflators for the various consumption categories include indirect business tax rates.

Unfortunately, indirect business taxes are not disaggregated by consumption category, and so some assumption has to be made regarding this disaggregation. What is assumed here is that the same indirect business tax rate applies to all three consumption categories. This assumption allows the indirect business tax rate, d_{4t}, to be defined in Table 2-1 as:

$$d_{4t} = \frac{IBTH_t}{PCD_t CD_t + PCN_t CN_t + PCS_t CS_t - IBTH_t}.$$

$IBTH_t$ is subtracted from the other terms in the denominator because indirect business tax rates usually apply to the cost of the item net of indirect business taxes. d_{4t} is taken to be exogenous in the model.

Because of the assumption just made about indirect business taxes, PCS_t, PCN_t, and PCD_t are larger than the actual before-tax prices of the items. If $\overline{PCS}_t$, $\overline{PCN}_t$, and $\overline{PCD}_t$ denote the before-tax prices of the items, then PCS_t equals $(1+d_{4t})\overline{PCS}_t$, PCN_t equals $(1+d_{4t})\overline{PCN}_t$, and PCD_t equals $(1+d_{4t})\overline{PCD}_t$. PD_t does not include indirect business taxes, and so the ratios $\overline{PCS}_t/PD_t$, $\overline{PCN}_t/PD_t$, and $\overline{PCD}_t/PD_t$ are the natural ratios to take as exogenous regarding the consumption categories. These ratios are denoted as ψ_{2t}, ψ_{3t}, and ψ_{4t} in Table 2-1. In Table 2-2, PCS_t is then determined as $\psi_{2t}(1+d_{4t})\,PD_t$, PCN_t is determined as $\psi_{3t}(1+d_{4t})\,PD_t$, and PCD_t is determined as $\psi_{4t}(1+d_{4t})\,PD_t$.

The price deflator for exports, PEX_t, is determined in Equation 28 as a function of PX_t. Since total firm sales include exports and not imports, the natural ratio to take as exogenous regarding the price of exports is PEX_t/PX_t. ψ_{1t} is defined in Table 2-1 to be this ratio, and so PEX_t is determined in Table 2-2 as $\psi_{1t}PX_t$.

Two price deflators are taken to be exogenous in the model, the price of commodities, $PCOM_t$, and the price of imports, PIM_t. The assumption that PIM_t is exogenous is much more important than the assumption that $PCOM_t$ is exogenous. PIM_t enters as an explanatory variable in the equation explaining PF_t, the key price variable in the model, whereas $PCOM_t$ does not. The only place that $PCOM_t$ is used in the model is in Equation 27 in going from PF_t to PX_t.

The treatment of $PCOM_t$ and PIM_t as exogenous reflects the assumption that both variables are determined by world supply and demand conditions for the various items and are beyond the control of the firm sector in the United States. PIM_t is also influenced by changes in the value of the dollar relative to other currencies, and these changes are likewise assumed to be beyond the control of the firm sector. It is obvious that supply and demand conditions in the United States have some effect on prices determined in world markets, but these effects have to be ignored here. It is clearly beyond the scope of this study to build the kind of model that would be necessary to explain the prices of the major commodities in the world. This study is thus subject to at least a small amount of bias from ignoring the fact that $PCOM_t$ and PIM_t are determined in part by some of the endogenous variables in the model. The present approach is similar to the approach taken by Nordhaus and Shoven [36], who divide the economy into a sector in which prices are endogenously determined in the sector and a sector in which prices are exogenously determined by world supply and demand conditions. (Nordhaus and Shoven also take the price of labor to be exogenous, but this is not done here.)

Equations 37–39 in Table 2-2 determine three wage rates in the model as a function of WF_t. WF_t is the wage rate that is assumed to be set by the firm sector according to Equation 15. It is a series (see Table 2-1) on average hourly earnings in the private nonfarm economy of production and nonsupervisory workers, adjusted for overtime (in manufacturing only) and interindustry employment shifts. Since WF_t is adjusted for overtime and interindustry shifts, it is about as good a measure of an aggregate "wage rate" that one can hope to get. It is not the case, however, that WF_t provides a direct link between the employment data used in this study and the NIA data. The wage variables that do provide this link are WFF_t, WGC_t, and WGM_t, which are defined in Table 2-1 and are explained in the next section. Consequently, Equations 37–39 can be considered as providing the link between the employment data and the NIA data.

The ratio of each of the three wage variables to WF_t is assumed to be exogenous in the model. These three ratios are denoted as ψ_{8t}, ψ_{9t}, and ψ_{10t} and are defined in Table 2-1. This treatment of the wage variables is similar to the treatment of the price deflators: it allows the three wage variables to be endogenous while keeping the relative wage rates exogenous. Equations 37–39 are not, however, an important part of the model, since the three wage variables are only needed for some of the income and profit definitions. WF_t is always the wage variable that is used in the specification of the stochastic equations.

Equations 40–44 explain taxes as a function of tax rates. There are six tax rates in the model: the (already defined) indirect business tax rate, d_{4t}; two personal income tax rates, d_{3t} and τ; the corporate profit tax rate, d_{1t}; the employer social security tax rate, d_{5t}; and the employee social security tax rate, d_{6t}. These six rates are assumed to be exogenous. Although the tax rates are assumed to be exogenous, the actual taxes paid are, of course, endogenous because the tax rates multiply endogenous variables.

All the tax rates except τ are defined in Table 2-1. d_{1t}, for example, is the actual ratio of $TAXF_t$ to πF_t that existed in quarter t. $TAXF_t$ is then determined in Table 2-2 as $d_{1t}\pi F_t$. Indirect business taxes and social security taxes are treated in the same way. Personal income taxes, on the other hand, are not. It is not, for example, realistic to take the ratio of $PTAXH_t$ to YH_t as exogenous because of the somewhat progressive structure of the personal income tax system. As YH_t increases, $PTAXH_t$ generally increases more than proportionally. Consequently, some estimate of this progressivity must be made.

The progressivity of the personal income tax system was estimated in the following way. The period 1954I–1975I was first divided into eight subperiods, each subperiod corresponding roughly to a period in which there were no major changes in the tax laws (surtaxes being counted as changes in the tax laws). The eight subperiods are: 1954I–1963IV, 1964I–1965I, 1965II–1968II, 1968III–1969IV, 1970I–1970IV, 1971I–1971IV, 1972I–1972IV, and 1973I–1975I. Two assumptions about the relationship between $PTAXH_t$ and YH_t were then made. The first is that within a subperiod $PTAXH_t$ is equal to $(d_3 + \tau \cdot YH_t)YH_t$ plus a random error term, where d_3 and τ are constants. The second is that changes in the tax laws affect d_3, but not τ. These two assumptions led to the estimation of the following equation:

$$\begin{aligned} PTAXH_t = {} & \underset{(2.65)}{-1.67} + \underset{(11.23)}{0.119}\ YH_t \cdot D1_t + \underset{(10.07)}{0.102}\ YH_t \cdot D2_t + \underset{(9.78)}{0.101}\ YH_t \cdot D3_t \\ & + \underset{(10.47)}{0.115}\ YH_t \cdot D4_t + \underset{(8.97)}{0.102}\ YH_t \cdot D5_t + \underset{(7.72)}{0.090}\ YH_t \cdot D6_t \\ & + \underset{(8.07)}{0.099}\ YH_t \cdot D7_t + \underset{(6.04)}{0.082}\ YH_t \cdot D8_t + \underset{(7.49)}{0.000343}\ YH_t \cdot YH_t, \\ & SE = 0.39,\ R^2 = 0.999,\ DW = 1.58. \qquad (2.1) \end{aligned}$$

$D1_t$ is a dummy variable that takes on a value of one in subperiod 1 and zero otherwise, $D2_t$ is a dummy variable that takes on a value of one in subperiod 2 and zero otherwise, and so on. The equation was estimated over the entire 1954I–1975I period. The coefficient of $YH_t \cdot D1_t$ is the estimate of d_3 for the first subperiod, the coefficient of $YH_t \cdot D2_t$ is the estimate of d_3 for the second subperiod, and so on. The coefficient of $YH_t \cdot YH_t$ is the estimate of τ. Since Equation (2.1) is clearly only a rough approximation to the actual tax system, a constant term was included in the estimated equation even though the two assumptions just mentioned do not call for it. When YH_t is zero, $PTAXH_t$ ought also to be zero, but the zero-zero point is so far removed from any observation in the sample that it seemed unwise from an approximation point of view to constrain the equation to pass through this point.

The assumption that changes in the tax laws do not affect τ is probably not bad as a first approximation, but again it is clearly only an approximation. The estimate of τ in Equation (2.1) is 0.000343, and this is the value of τ that has been used in this study. Given τ, d_{3t} is defined in Table 2-1 to be $PTAXH_t/YH_t - \tau \cdot YH_t$. d_{3t} is taken to be exogenous, and $PTAXH_t$ is then explained as $(d_{3t} + \tau \cdot YH_t)YH_t$ in Equation 41 in Table 2-2. The marginal personal income tax rate for quarter t, denoted as d^M_{3t}, is equal to $d_{3t} + 2\tau \cdot YH_t$, which is Equation 84 in Table 2-2. From Table 2-1 it can be seen that the marginal tax rate (d^M_{3t}) was 0.223 in 1971IV, while the average tax rate ($PTAXH_t/YH_t$) was 0.154 (= 31.0/201.8).

Equation (2.1) could have been used directly as the equation explaining $PTAXH_t$ in the model, rather than Equation 41, but for computational convenience this was not done. The results in the theoretical model indicate that the marginal tax rate ought to be an important explanatory variable in the household sector, and the procedure just outlined provides a convenient way of constructing a marginal tax rate series. If Equation (2.1) were used instead, this task would be more difficult, especially if the equation were estimated jointly with the other equations in the model. The fit of Equation (2.1) is good enough (a standard error of 0.39 billion dollars at a quarterly rate) that treating d_{3t} as exogenous is not likely to introduce any serious biases anywhere. Treating d_{3t} as exogenous effectively converts the $PTAXH_t$ equation into an equation with a perfect fit.

Equation 45 explains bank reserves (BR_t) as a function of the level of demand deposits of the financial sector (DDB_t) and the reserve requirement ratio (g_{1t}). g_{1t} is defined in Table 2-1 as the ratio of BR_t to DDB_t that actually existed in quarter t. g_{1t} is taken to be exogenous, and BR_t is then explained as $g_{1t}\,DDB_t$ in Equation 45 in Table 2-2. The relationship between BR_t and DDB_t is thus assumed to be exogenous, although both variables are themselves endogenous. This assumption is discussed in Chapter Six, but it should be noted now that the assumption says nothing about commercial bank borrowing at federal reserve banks ($BORR_t$). Borrowing can clearly exist even though the ratio of BR_t to DDB_t is taken to be exogenous. $BORR_t$ is in fact explained by Equation 20. As discussed in Chapter Six, the treatment of BR_t in the empirical model is different from its treatment in the theoretical model, where it is treated as a residual. The different treatment in the empirical model is due to the use of quarterly data, rather than data for a shorter period of time.

Equations 46 through 84 in Table 2-2 are definitions that are needed to close the model. Many of the equations are concerned with defining the savings of the sectors and the values of the securities held by the sectors. These types of equations are based on Equations (1.1)–(1.11) in Chapter One and the corresponding definitions in Tables 1-2 and 1-3.

Equation 46 relates the current expenditures on durable goods in real terms, CD_t, to the current and lagged stocks of consumer durables (KCD_t and KCD_{t-1}). δ_D is the depreciation rate on the stock of consumer durables. Its construction is explained in the next section. KCD_t is explained by Equation 3, and Equation 46 is needed to relate current expenditures to KCD_t. Equation 47 is a similar equation for current expenditures on housing of the household sector, IH_t. δ_H is the depreciation rate on the stock of houses, and its construction is also explained in the next section.

Equation 48 defines total firm sales in real terms, X_t, and Equation 49 defines total firm sales in current dollar terms, XX_t. X_t is the sum of the various quantity items, and XX_t is the sum of the various price-times-quantity items. The endogenous variables on the right-hand side of Equation 48 are CS_t, CN_t, CD_t, IH_t, INV_t, and IM_t. The exogenous variables are exports (EX_t), government purchases of goods (XG_t), plant and equipment investment of the household and financial sectors ($XPAEH_t$ and $XPAEB_t$), residential investment of the firm and financial sectors ($XRESF_t$ and $XRESB_t$), inventory investment of the household sector ($XIVTH_t$), and profits and capital consumption in real terms of the financial sector ($XPROB_t$ and $XCCAB_t$). Except for EX_t and XG_t, these exogenous variables are small in value and not very important. The last two variables, $XPROB_t$ and $XCCAB_t$, should be thought of as sales by the financial sector to the household sector, which must be subtracted from the expenditures of the household sector in determining the sales of the firm sector. The only exogenous variable that is in Equation 49 and not in Equation 48 is the price of imports, PIM_t. The value of indirect business taxes ($IBTH_t$) is subtracted from the other variables in Equation 49 because the indirect business tax rates are included in the price deflators. $IBTH_t$ is not a revenue item of the firm sector, and so it must be subtracted from the other variables to net indirect business taxes out of the equation.

In Equation 50 the average number of nonovertime hours paid per job by the firm sector, $HPFN_t$, is defined as the difference between the average number of total hours and the average number of overtime hours. In Equation 51 the current stock of inventories of the firm sector, V_t, is equal to last period's stock plus the difference between production and sales of the current period. $V_t - V_{t-1}$ is inventory investment, and it is not, as in most other macroeconometric models, explained directly by a stochastic equation. Instead, Y_t is explained by a stochastic equation, and inventory investment is residually determined by Equation 51. Y_t is explained directly because it is considered, from the theoretical model, to be a direct decision variable of the firm sector.

Equation 52 defines the before-tax profits of the firm sector, πF_t. The first two items on the right-hand side ($XX_t + PX_t(V_t - V_{t-1})$) equal the value of production. The next item is the wage costs of the firm sector. d_{5t} is the employer social security tax rate, so that $WFF_t(1 + d_{5t})$ is the wage rate paid by the firm sector inclusive of employer social security taxes. The next four items are payments by the firm sector to the household sector that are taken to be exogenous: rental income of the household sector ($FHRNT_t$), transfer payments from the firm sector to the household sector ($FHTRP_t$), profits of farms ($FHPFA_t$), and capital consumption of the household sector ($FHCCA_t$). The next item is the net subsidies of government enterprises ($GHSUB_t$), which is a revenue item of the firm sector. The last six items are interest paid by the firm sector ($INTF_t$), depreciation (DEP_t), inventory valuation adjustment (IVA_t), wage accruals less disbursements of the firm sector ($FHWLD_t$), and the statistical discrepancy of the NIA ($STATDIS_t$). As discussed in the next section, πF_t as defined in Equation 52 is the NIA definition of the profits of the firm sector.

Equation 53 defines the before-tax cash flow of the firm sector, CF_t. Equation 53 differs from Equation 52 by the exclusion of inventory investment, depreciation, the inventory valuation adjustment, wage accruals less disbursements, and the statistical discrepancy, and by the inclusion of current investment expenditures ($PFF_t INV_t$ and $PIH_t XRESF_t$). Equation 54 defines the cash flow of the firm sector *net* of taxes and dividends, $\overline{CF}_t$. $\overline{CF}_t$ is the same as $SAVF_t$ in section VI of Table 1-2.

Equation 55 determines the loans of the firm sector, LF_t. It is the same as Equation (1.5) in Chapter One. The value of loans in the current period is equal to the value last period, plus the change in the value of demand deposits, less the cash flow net of taxes and dividends, plus the discrepancy of the firm sector, plus wage accruals less disbursements of the firm sector, and plus the statistical discrepancy of the NIA. As discussed in Chapter One, this equation provides one of the key links between the FFA and NIA data.

The value of dividends received by the household sector, $DIVH_t$, is defined in Equation 56, and the value of interest received by the household sector, $INTH_t$, is defined in Equation 57. $DIVH_t$ is the sum of the dividends paid by the firm and financial sectors, and $INTH_t$ is the sum of the interest paid by the firm and government sectors.

The taxable income of the household sector, YH_t, is defined in Equation 58. YH_t is the sum of wage, dividend, interest, and rental income, plus two small items: business transfer payments from the firm sector to the household sector ($FHTRP_t$) and farm profits ($FHPFA_t$).

Equation 59 defines the net taxes paid by the household sector, $TAXH_t$, net taxes being defined as taxes paid to the government less transfer payments from the government. YG_t in the equation is defined in Table 2-1 and is equal to transfer payments from the government sector to the household sector (except for unemployment insurance benefits), including insurance and retirement credits. $TAXH_t$ in Equation 59 is equal to the sum of personal income taxes ($PTAXH_t$), indirect business taxes ($IBTH_t$), and social security taxes ($FHCSI_t + HGSI2_t$), less YG_t and less unemployment insurance benefits (TPU_t). TPU_t has not been included in YG_t because it is endogenous, while all the items that make up YG_t are exogenous.

Equation 60 defines the saving of the household sector, $SAVH_t$. This equation is the same as the equation for $SAVH_t$ in Table 1-2, section VI. $SAVH_t$ is equal to household income less household expenditures and net taxes. Household income includes taxable income (YH_t), capital consumption ($FHCCA_t$), and employer social insurance contributions ($FHCSI_t$), the latter being counted as a payment from the firm sector to the household sector. Household expenditures include expenditures on services ($PCS_t CS_t$), nondurable goods ($PCN_t CN_t$), durable goods ($PCD_t CD_t$), housing ($PIH_t IH_t$), plant and equipment ($PFF_t XPAEH_t$), inventories ($PX_t XIVTH$), and transfer payments to the foreign sector ($HRTRP_t$). $IBTH_t$ is subtracted from $TAXH_t$ in the equation because it is already included in the price deflators PCS_t, PCN_t, and PCD_t. It should be noted that since $TAXH_t$ includes both employer and employee social insurance contributions, employer social insurance contributions ($FHCSI_t$) are actually netted out of Equation 60.

Equation 61 determines the value of nondemand deposit securities of the household sector, A_t. It is the same as Equation (1.1) in Chapter One. The value of A_t is equal to its value last period, less the change in the value of demand deposits of the household sector, plus saving and capital gains or losses, and less the discrepancy of the household sector. Equation 61 is similar to Equation 55 for the firm sector and also provides one of the key links between the FFA and NIA data.

Equation 62 determines the value of demand deposits and currency of the financial sector, DDB_t. It is the same as Equation (1.9) in Chapter One. The value of demand deposits and currency of the financial sector in the current period is equal to the value last period, plus the change in the value of demand deposits and currency of the household, firm, and foreign sectors, less the change in $CURR$ (the value of currency outstanding less the value of demand deposits of the government sector), and plus the demand deposit mail floats.

The saving of the financial sector, $SAVB_t$, is defined in Equation 63. This equation is the same as the Equation for $SAVB_t$ in Table 1-2, section VI. $SAVB_t$ is not an important variable in the model, since all of the variables on the right-hand side of Equation 63 are exogenous except for the three price deflators.

Equation 64 determines the value of loans of the financial sector, $LBVBB_t$. It is the same as Equation (1.2) in Chapter One. ($TOTB_t$ in Chapter One is equal to $LBVBB_t + BR_t - BORR_t - DDB_t$ in the notation here.) It is also similar to Equation 61 for the household sector and Equation 55 for the firm sector. The value of $LBVBB_t$ is equal to its value last period, plus the change in borrowing from the federal reserve banks, less the change in bank reserves, plus the change in the value of demand deposits and currency of the financial sector, plus the saving of the financial sector, and less the discrepancy of the financial sector.

The saving of the foreign sector, $SAVR_t$, is defined in Equation 65. This equation is the same as the equation for $SAVR_t$ in Table 1-2, section VI. The two right-hand side endogenous variables in Equation 65 are IM_t and PEX_t.

Equation 66 determines the value of securities of the foreign sector not counting demand deposits and currency and gold and foreign exchange, $SECR_t$. It is the same as Equation (1.3) in Chapter One ($TOTR_t$ in Chapter One is equal to $SECR_t + DDR_t - GFXG_t$ in the notation here). It is also similar to Equations 55, 61, and 64. The value of $SECR_t$ is equal to its value last period, less the change in the value of demand deposits and currency of the foreign sector, plus the change in the value of gold and foreign exchange of the government sector, plus the saving of the foreign sector, and less the discrepancy of the foreign sector.

Equation 67 defines the total net tax collections of the government,

TAX_t, and Equation 68 defines the saving of the government, $SAVG_t$. Equation 68 is the same as the equation for $SAVG_t$ in Table 1-2, section VI. $SAVG_t$ is equal to net tax collections, less expenditures of goods ($PG_t XG_t$), less expenditures on labor ($WGC_t HPGC_t JOBGC_t + WGM_t HPGM_t JOBGM_t$), less interest payments ($INTG_t$), less transfer payments to the foreign sector ($GRTRP_t$), and less the net subsidies of government enterprises ($GHSUB_t$).

Equation 69 is the government budget constraint and is the same as Equation (1.4) in Chapter One. ($TOTG_t$ in Chapter One is equal to $-VBG_t + BORR_t - CURR_t - BR_t + GFXG_t$ in the notation here.) It says that the net saving or dissaving of the government in a period results in the change in at least one of the following items: the value of government securities (VBG_t), the value of borrowing by commercial banks at federal reserve banks ($BORR_t$), the value of currency outstanding less the value of demand deposits of the government sector ($CURR_t$), the value of bank reserves (BR_t), and the value of gold and foreign exchange held by the government sector ($GFXG_t$).

Equation 70 is the same as Equation (1.11) in Chapter One. It says that the sum of the change in all other securities (excluding demand deposits and currency, bank reserves, borrowing at federal reserve banks, and gold and foreign exchange) across sectors must, after adjustment for the various discrepancies, be zero. The notation has, of course, been changed in going from Equation (1.11) to Equation 70, and in order to see clearly that the two equations are the same it is necessary to consult Table 2-1 for the definitions of the variables in Equation 70.

The remaining definitions in Table 2-2 concern variables that are either used as explanatory variables in one or more of the stochastic equations (sometimes only in lagged form) or are needed for the construction of variables that are so used. Equation 71 defines a variable, $YNLH_t$, that is taken to be a measure of the nonlabor income of the household sector. It is equal to dividend, interest, and rental income, plus business transfer payments from the firm sector to the household sector, plus farm profits, plus ($YG_t + TPU_t$), and minus employee contributions for social insurance. $YG_t + TPU_t$ is the value of transfer payments from the government sector to the household sector.

Equation 72 determines the capital stock of the firm sector, K_t^a. δ_K is the depreciation rate of the capital stock; its construction is explained in section 5.2. Equation 73 defines $KMIN_t$, an estimate of the minimum amount of capital required to produce Y_t. The variable ($\mu_t \bar{H}$) in the equation is obtained from peak-to-peak interpolations of the Y_t/K_t^a series. Its construction is also explained in section 5.2. Equation 74 defines $M_t H_t^M$, an estimate of the number of worker hours required to produce Y_t. The variable λ_t in the equation is obtained from peak-to-peak interpolations of a series on output per paid for worker hour. Its construction is explained in section 5.2.

Equation 75 defines a variable J_t, which is the ratio of the total number of worker hours paid for in the economy to the total population 16 and over. J_t has a negative trend, and J_t^* in Equation 76 is J_t detrended. Equation 77 defines a variable, ZJ_t, as a function of J_t^*. ZJ_t is the hours constraint variable. Its construction is explained in section 4.3. Equation 78 defines a variable, ZJ_t', as a function of the unemployment rate, UR_t. ZJ_t' is the labor constraint variable. Its construction is explained in section 5.3.

The bill rate, $RBILL_t$, has a positive trend over part of the sample period, and $RBILL_t^*$ in Equation 79 is $RBILL_t$ detrended up to 1970IV. Equation 80 defines a variable, ZR_t, as a function of $RBILL_t^*$. ZR_t is the loan constraint variable. Its construction is explained in section 4.3.

The total number of people employed, $EMPL_t$, is defined in Equation 81. $EMPL_t$ is equal to the number of jobs in the economy less $MOON_t$, the latter being interpreted as the number of people holding two jobs. The data on jobs are establishment data, and the data on $EMPL_t$ are household survey data. $MOON_t$ is defined in Table 2-1 as the difference between the total number of jobs and $EMPL_t$. Both $MOON_t$ and $JOBF_t$ are explained by stochastic equations, and both $JOBGC_t$ and $JOBGM_t$ are taken to be exogenous. Consequently, $EMPL_t$ is determined residually as the difference between jobs and $MOON_t$ in Equation 81.

The number of people unemployed, U_t, is defined in Equation 82. U_t is equal to the number of people in the labor force less the number of people employed. The two labor force variables in the equation, TLF_{1t} and TLF_{2t}, are determined by stochastic equations. The civilian unemployment rate, UR_t, is defined in Equation 83. It is the ratio of U_t to the civilian labor force, $TLF_{1t} + TLF_{2t} - JOBGM_t$. The marginal personal income tax rate, d_{3t}^M, is defined in Equation 84. d_{3t}^M is the derivative of Equation 41 with respect to YH_t.

The equation at the bottom of Table 2-2 defines GNP in current dollars, GNP_t. GNP_t is useful for reference purposes, but it is not used directly as an explanatory variable in any of the equations in the model. It is equal to the value of production of the firm sector $(XX_t + PX_t(V_t - V_{t-1}))$, plus indirect business taxes, plus the government wage bill, plus wage accruals less disbursements of the government sector, and plus the value of production of the financial sector $(PX_t(XPROB_t + XCCAB_t))$.

This completes the discussion of the equations in Table 2-2. Not counting the equation for GNP_t, the model as presented in Table 2-2 consists of 84 equations. It turns out, however, that one of the equations is redundant. The easiest way to see this is to refer back to Chapter One. Equations (1.1)–(1.5) and the fact that the savings of all sectors sum to zero imply Equation (1.10). Equations (1.6)–(1.10) in turn imply Equation (1.11). Now, Equation

(1.11) is the same as Equation 70 in Table 2-2. The other matchings of equations from Chapter One to Table 2-2 are as follows: (1.1) → 61, (1.2) → 64, (1.3) → 66, (1.4) → 69, (1.5) → 55, and (1.9) → 62.

This takes care of all the equations in Chapter One except Equations (1.6), (1.7), and (1.8). These three equations are, however, implicitly satisfied in Table 2-2 because they have been taken into account in the construction of the table. Consider, for example, Equation (1.6), which says that the sum of the change in bank reserves across the financial and government sectors is zero:

$$(RESB_t - RESB_{t-1}) + (RESG_t - RESG_{t-1}) = 0. \tag{1.6}$$

BR_t is defined in Table 2-1 to be equal to $RESB_t$, and $BR_t - BR_{t-1}$ enters Equation 64 with a minus sign. No variable was defined for $RESG_t$, however, and instead $BR_t - BR_{t-1}$ was merely included in Equation 69 with a plus sign. This means that Equation (1.6) is automatically satisfied in Table 2-2. This same procedure was also followed for Equations (1.7) and (1.8)—$BORR_t$ and $GFXG_t$ being the two variable names used. With these three equations taken into account, the above matching of equations shows that Equations 55, 61, 62, 64, 66, and 69 in Table 2-2 imply Equation 70. One of these equations can thus be dropped, leaving 83 independent equations.

A convenient equation to drop from the model is Equation 69, the government budget constraint. The fact that Equation 69 can be dropped means that the government budget constraint is automatically satisfied once all of the flows of funds have been accounted for. If Equation 69 is dropped, then the only other equation in Table 2-2 for which there is not an obvious left-hand side variable is Equation 70, the equation stating that the change in the sum of all other securities across sectors must be zero after adjusting for the various discrepancies.

There are thus 82 obvious endogenous variables in the model and one not so obvious. The most natural choice for the remaining endogenous variable is the bill rate, $RBILL_t$, and this is the choice made here. It should be noted, however, that any one of a number of government variables could be taken as endogenous instead. If, for example, one felt that the government pegged the bill rate at some particular level each period, then the value of government securities, VBG_t, would be the most natural variable to take as endogenous.

Given that $RBILL_t$ is taken to be endogenous, it is important to note how it is determined in the model. $RBILL_t$ enters as an explanatory variable in a number of the stochastic equations. The overall model is a system of 83 nonlinear equations in 83 unknowns, and this system can be solved numerically. Consequently, $RBILL_t$ is determined through the solution of the 83 equations. There is no one equation for which $RBILL_t$ appears naturally on the left-hand side, and the reason that $RBILL_t$ can be determined in this way is because of the linking of the NIA and FFA data and the accounting for all of the flows of funds. More will be said about this in Chapter Six.

2.3 A DISCUSSION OF TABLE 2–1

All the variables in the model are listed in Table 2-1. Presented in brackets in the table for each variable is either a reference where recent data on the variable can be found or a description of how the variable was constructed from other variables in the model. The comments in brackets rely heavily on the work in Tables 1-2 and 1-3 in Chapter One. For some variables the notation has remained the same in going from Tables 1-2 and 1-3 to Table 2-1, but for the most part the notation has been changed to conform more closely to the notation in Volume I. Also presented in the table is the value of each variable for the fourth quarter of 1971.

The data used in this study were collected for the 1952I–1975I period and are data as of about July 1975. The period prior to 1952I was not considered here because quarterly flow-of-funds data are not available before 1952I. The main sources for the data are the flow-of-funds tape, the *Survey of Current Business, Employment and Earnings*, and the *Federal Reserve Bulletin*. When SCB occurs in brackets in Table 2-1, this means that the data were collected from the *Survey of Current Business* starting with the July 1975 issue and working back. The number following SCB in brackets is the table number in the *Survey* where the variable can be found. Almost all the data are at annual rates in the *Survey*, and for purposes here these data have been divided by 4 to put them at quarterly rates.

When EE occurs in brackets in Table 2-1, this means that the data were collected from *Employment and Earnings* as of the July 1975 issue, and when FRB occurs in brackets, this means that the data were collected from the *Federal Reserve Bulletin* as of the July 1975 issue. The number following EE or FRB in brackets is the table number in the respective publication where the variable can be found. Back data on some of the variables referenced as EE or FRB in Table 2-1 were not obtained by going through past issues of *Employment and Earnings* and the *Federal Reserve Bulletin*, but were obtained from *1973 Business Statistics*. When F/F occurs in brackets in Table 2-1, this means that the data were obtained from the flow-of-funds tape. For these cases the code number of the variable is presented in brackets, as well as the page number in [3] where the variable can be found.

When the phrase "Defined in Table 2-2" appears in brackets, this means that data on the variable do not have to collected because the variable is merely defined in terms of other variables in the model for which data have been collected. In two cases, however (gross national product, GNP_t, and profits of the firm sector, πF_t), alternative sources for the data have been presented. Although these two variables are derived from other variables in the model, it is useful to have a check that the two variables are being defined in the appropriate way.

Much of Table 2-1 is self explanatory. The following discussion concerns only those parts of the table that need further elaboration. The first thing to note is that in going from the notation in Table 1-3 to the notation in Table 2-1, the sign of the variable has sometimes been changed. Liabilities in Table 1-3 are always negative items, but this is not the case in Table 2-1. For example, the value of currency outstanding less the value of demand deposits of the government sector is denoted as $CURR_t$ in Table 2-1, whereas it is denoted as $-DDCG_t$ in Table 1-3.

The variable $LBVBB_t$ is the value of "all other" securities held by the financial sector. In the theoretical model these securities correspond to loans to firms and households and bills and bonds of the government. The former was denoted as LB and the latter as VBB; hence the $LBVBB$ notation used here.

The variable A_t, the value of nondemand deposit securities of the household sector, was constructed by summing the capital gains or losses variable (CG_t) forward and backward from 1971IV ($t = 80$) and then adding the appropriate sum to $SECH_t$ for $t > 80$ and subtracting the appropriate sum from $SECH_t$ for $t < 80$. $SECH_t$ is defined in Table 1-3. It is equal to the difference between the value of all securities held by the household sector ($TOTH_t$) and the value of demand deposits and currency held by the household sector ($DDCH_t$). Its value in 1970IV was 1342.4 billion dollars, which includes the value of corporate stocks held by the household sector. The flow data for $SECH_t$, on the other hand, exclude capital gains or losses, and so constructing $SECH_t$ by summing the flow data (as was done here, using 1971IV as a benchmark) does not produce a series that can be considered to be the value of nondemand deposit securities of the household sector. In order to produce the latter series, cumulative capital gains or losses have to be added to or subtracted from $SECH_t$, as is indicated in Table 2-1. For any period t, the following relationship between A_t and $SECH_t$ holds:

$$A_t - A_{t-1} = SECH_t - SECH_{t-1} + CG_t.$$

The employment variables in Table 2-1 require some explanation. The total number of jobs in the economy is the number of jobs in the government sector plus the number of jobs in the private sector. As in the theoretical model, it is assumed here that there are no jobs in the financial sector, and so all jobs in the private sector have been allocated to the firm sector. In terms of the amount of output produced, the financial sector is quite small, and the assumption that there are no jobs in this sector is not very important. The number of jobs in the government sector is equal to the number of civilian jobs ($JOBGC_t$) plus the number of military jobs ($JOBGM_t$).

Data on the number of jobs in the firm sector ($JOBF_t$) were obtained directly from the BLS. The data are quarterly and pertain to the total private economy, all persons. These data are the data used in the construction of the index of "output per man-hour" for the total private economy in Table C-10 in *Employment and Earnings*. ("Man-hours" in the BLS terminology refers to hours of both men and women. "Worker hours" or "person hours" would be a more appropriate term.)

Data on the average number of hours paid per civilian job per quarter by the government sector ($HPGC_t$) were obtained by taking the ratio of "man-hours" to $JOBGC_t$, as explained in Table 2-1. Data on the same variable for military jobs ($HPGM_t$) could not be obtained in this way because there are no data on "man-hours" for the military. Instead, $HPGM_t$ was just assumed to be 520 hours for all t (40 hours per week). Data on the average number of hours paid per job per quarter by the firm sector (HPF_t) were also obtained as the ratio of "man-hours" to jobs ($JOBF_t$). The data on man-hours were obtained directly from the BLS. The data on man-hours are presented in index number form in Table C-10 in *Employment and Earnings*, but the nonindexed data must be obtained directly from the BLS.

Data on the overtime variable, $HPFO_t$, pertain only to the manufacturing sector, but it has been assumed here that the data in fact pertain to the entire firm sector. In other words, it has been assumed that the (unobserved) amount of overtime per job in the nonmanufacturing part of the firm sector is the same as the (observed) amount in the manufacturing part. As will be discussed shortly, this assumption is not very important because the $HPFO_t$ variable itself is not a very important variable in the model.

The data on jobs and hours are establishment data. The data on population (POP_t, POP_{1t}), labor force (TLF_{1t}, TLF_{2t}), and number of people employed ($EMPL_t$) are household survey data. A few changes had to be made in the household survey data here to account for adjustments to the 1970 Census data. Adjustments to the official data were made by the BLS in January 1972 and March 1973. In terms of the variables used here, the BLS in January 1972 added 787 thousand to POP_t, subtracted 42 thousand from POP_{1t}, subtracted 40 thousand from TLF_{1t}, added 373 thousand to TLF_{2t}, and added 301 thousand to $EMPL_t$. (See the February 1972 issue of *Employment and Earnings*.)

In March 1973 the adjustments were much smaller. The BLS added roughly 8 thousand to POP_t, 3 thousand to POP_{1t}, 26 thousand to TLF_{1t}, 35 thousand to TLF_{2t}, and 58 thousand to $EMPL_t$. This information was obtained directly from the BLS. (See the note to Table A-1 in the April

1973 issue of *Employment and Earnings* for a brief discussion of the March 1973 adjustments.) In order to account for these adjustments here, the data on the various series prior to March 1973 were adjusted by adding or subtracting the amounts necessary to make the series prior to March 1973 comparable to the series from March 1973 on. The data for the first quarter of 1973 were changed by one-third of the March 1973 adjustments. The changes that were made are:

POP_t: +795 for the 1952I–1971IV period; +8 for the 1972I–1972IV period; +3 for 1973I; no change for the 1973II-1975I period,

POP_{1t}: −39 for the 1952I–1971IV period; +3 for the 1972I–1972IV period; +1 for 1973I; no change for the 1973II–1975I period,

TLF_{1t}: −14 for the 1952I–1971IV period; +26 for the 1972I–1972IV period; +9 for 1973I; no change for the 1973II–1975I period,

TLF_{2t}: +408 for the 1952I–1971IV period; +35 for the 1972I–1972IV period; +12 for 1973I; no change for the 1973II–1975I period,

$EMPL_t$: +359 for the 1952I–1971IV period; +58 for the 1972I–1972IV period; +19 for 1973I; no change for the 1973II–1975I period.

These adjustments were made before data on the variables that depend on these five variables were generated.

The variable $MOON_t$ is the difference between the number of jobs in the economy according to the establishment data and the number of people employed according to the household survey data. The main reason that $MOON_t$ is not zero is because of people holding more than one job. If someone holds two jobs, he or she is counted once in the household survey data but twice in the establishment data. Although there are a number of minor discrepancies between the establishment and household survey data that would cause $MOON_t$ to be nonzero even if no one held more than one job, the primary reason that $MOON_t$ is not zero is because of people moonlighting. Consequently, $MOON_t$ will be referred to in this study as the "number of moonlighters." In interpreting $MOON_t$ in this way, one is assuming both that the other discrepancies between the two data bases are negligible and that no one holds more than two jobs.

The next variables in Table 2-1 that need to be explained are the three wage variables: WFF_t, WGC_t, and WGM_t. The numerator of the ratio defining WFF_t in Table 2-1 ($FHWAG_t - FHWLD_t + FHOTH_t + FHPRI_t$) is taken to be the measure of wage payments from the firm sector to the household sector. This measure is the sum of wages and salaries, other labor income, and proprietors income. The denominator of the ratio defining

WFF_t, $(HPFN_t + 1.5HPFO_t)JOBF_t$, is taken to be the measure of the equivalent number of nonovertime hours paid for in the firm sector. Overtime hours are assumed to be paid at time-and-a-half, which is the reason for 1.5 multiplying $HPFO_t$ in the expression. The ratio (WFF_t) is thus the measure of average straight time hourly earnings of workers in the firm sector. The main wage variable in the model is WF_t, and WFF_t is linked to WF_t by taking the ratio of WFF_t to WF_t (defined as ψ_{8t} in Table 2-1) to be exogenous.

WFF_t is needed for three definitions in the model: Equation 52, defining profits of the firm sector; Equation 53, defining cash flow of the firm sector; and Equation 58, defining income of the household sector. Since WF_t is endogenous, the linking of WFF_t and WF_t means that the wage payments of the firm sector are endogenous not only because $HPFN_t$, $HPFO_t$, and $JOBF_t$ are endogenous, but also because WFF_t is. WFF_t was linked to WF_t in the way described, and not itself taken to be the measure of the aggregate wage rate in the model, because WF_t seemed to be a much better measure. The linking of WFF_t to WF_t is not of crucial importance in the model, however, since it only affects the three definitions just mentioned. In the same way, the overtime hours variable, $HPFO_t$, is not of crucial importance in the model because it only affects the same three definitions.

The wage rate WFF_t is net of employer social security taxes. The employer social security tax rate, d_{5t}, is defined in Table 2-1. It is the ratio of employer social security taxes to the wage bill of the firm sector. Consequently, $WFF_t(1 + d_{5t})$ is the wage rate paid by the firm sector inclusive of employer social security taxes. This is the wage rate used in Equations 52 and 53 in Table 2–2, which define the profits and cash flow of the firm sector.

The government wage variables, WGC_t and WGM_t, are treated in the same way as WFF_t, except that no adjustments for overtime are made because no overtime data exist for the government sector. The numerator in the definition of WGC_t is the sum of civilian wages and salaries and the "other labor income" component that pertains to the government sector. The numerator in the definition of WGM_t is merely military wages and salaries. WGC_t and WGM_t are only needed for two definitions in the model: Equation 58, defining income of the household sector; and Equation 68, defining the saving of the government.

Data on WF_t are actually available only from 1964I on. Prior to 1964I, data on a similar type of wage rate are available only for manufacturing, as opposed to the entire private nonfarm economy. The actual series on WF_t used here is a splice of the manufacturing series before 1964I and the private nonfarm series from 1964I on. The ratio of the wage rate for the private nonfarm economy to the wage rate for manufacturing in 1964I was 0.97887, and so the manufacturing series was multiplied by 0.97887 to make it comparable to the private nonfarm series. As indicated in Table 2–1, current data on WF_t are published in *Employment and Earnings*, Table C-17. The past data on both the private nonfarm series and the manufacturing series were obtained directly from the BLS.

The construction of two of the price deflators in Table 2–1 also needs to be explained. The first is PX_t. All the variables in brackets defining PX_t are not themselves defined in terms of PX_t. Call the numerator of the ratio defining PX_t, $E1_t$, and call the denominator $E2_t$. It can be seen from Equation 49 in Table 2–2 that $E1_t$ is equal to $XX_t - PX_t(XIVTH_t - XPROB_t - XCCAB_t)$, and it can be seen from Equation 48 in Table 2–2 that $E2_t$ is equal to $X_t - (XIVTH_t - XPROB_t - XCCAB_t)$. The variable X_t is total firms sales in constant dollars, and the variable XX_t is total firm sales in current dollars. Since $PX_t = E1_t/E2_t$, it is also true, using the expressions just presented for $E1_t$ and $E2_t$, that $PX_t = XX_t/X_t$. Consequently, PX_t can be interpreted as the implicit price deflator for X_t. The reason for this somewhat roundabout process in defining PX_t is that PX_t was taken to be the deflator for the three small exogenous items: $XIVTH_t$, $XPROB_t$, and $XCCAB_t$.

The second deflator whose construction needs explaining is PG_t, the deflator for government purchases of goods, XG_t. Government purchases of goods in current dollars is denoted as $GFPGO_t$ in Table 1–2 in Chapter One. $GFPGO_t$ is government purchases of goods and services *less* government compensation of employees (general government). XG_t is the same thing in constant dollars; therefore, PG_t is defined as the ratio of $GFPGO_t$ to XG_t.

One characteristic that should be noted about the deflators PX_t and PF_t is the difference between the way the deflators are constructed and the way they are determined in the model. In the model in Table 2–2, PF_t is determined by a stochastic equation and PX_t is determined from PF_t. The other deflators are then determined from PX_t. In Table 2–1, however, PX_t is defined in terms of the other deflators. Data on PX_t are used in Table 2–1 to determine, directly or indirectly, the ψ_{it} ratios ($i = 1, \ldots, 7$), which are then taken to be exogenous in the model in Table 2–2.

The treatment of the price deflators in this way means that in any simulation with the model, the predicted value of PX_t will not necessarily equal the predicted value of XX_t divided by the predicted value of X_t. In other words, PX_t equals XX_t/X_t only in the actual data, not in the predicted data. PX_t should thus be interpreted as the implicit price deflator for X_t only in a special sense. There is nothing wrong with treating the deflators in this way; all it changes is the interpretation of PX_t. None of the equations in Table 2–2 require that XX_t be equal to PX_tX_t.

Past data on the mortgage rate series, $RMORT_t$, were obtained directly from the FHA. Prior to May 1960 the yield estimates were based on the assumption of a 30-year maturity. Since May 1960 the assumption of a 25-year maturity has been used. There are a few monthly gaps in this series, and these gaps have been closed here by simple linear interpolation. The series published in the *Federal Reserve Bulletin* is actually lagged one month, and the series was unlagged before the quarterly averages were taken. This particular mortgage rate series is fairly sensitive to recent changes in mortgage market conditions, which is the reason for its present use.

The last three variables in Table 2–1 whose construction needs to be explained are the stock of consumer durables, KCD_t; the stock of residential structures of the household sector, KIH_t; and the stock of inventories of the firm sector, V_t. Consider V_t first. Inventory investment of the firm sector in current dollars is denoted in section II.2 in Table 1–2 as $FFIVT_t$. This series was first divided by PX_t to create a series on inventory investment in real terms. Then a series on the stock of inventories in real terms (V_t) was created by summing the real investment figures forward and backward from a base period value in 1971IV. The base period value that was used is 205.9 billion dollars, which was obtained from the August 1974 issue of the *Survey of Current Business*, p. 51. For a description of the procedure that was used to construct the stock data in the *Survey*, see Loftus [31].

The series on KCD_t was constructed as follows. From Equation 46 in Table 2–2, KCD_t is:

$$KCD_t = (1 - \delta_D)KCD_{t-1} + CD_t. \tag{2.2}$$

Given data on CD_t, a series on KCD_t can be constructed once a base period value and a value for the depreciation rate δ_D are chosen. Using results of a recent study conducted by the U.S. Department of Commerce, Shavell [38] presents estimates of the stock of durable goods for the years 1946, 1950, 1955, 1960, 1965, and 1969. These estimates are end-of-year estimates. The estimate of the net stock for 1955 based on assumptions of straight line depreciation, average life, and L-2 survival patterns is 15[illegible].6 billion dollars (in real terms). This estimate was taken here to be the actual value of KCD_t in 1955IV. From this base period, various values of δ_D were used to generate, from Equation (2.2), different series on the stock of consumer durables. The values from each of these series for 1960IV, 1965IV, and 1969IV were compared to the values published in [38] to see which value of δ_D most closely reproduced the published values. The value finally chosen for δ_D was 0.0525. The use of this value lead to values of KCD_t in the three comparison quarters of 186.7, 242.4, and 320.4, which compare closely to the published values of 186.1, 236.8, and 320.4.

A similar procedure was followed for the construction of the series on KIH_t. From Equation 47 in Table 2–2, KIH_t is:

$$KIH_t = (1 - \delta_H)KIH_{t-1} + IH_t. \tag{2.3}$$

Annual estimates on the stocks of residential structures are presented in the November 1971 issue of the *Survey of Current Business* (Young, Musgrave, and Harkins [39]) for the 1925–1970 period. The estimate of the net stock of residential structures for 1963 for the private nonfarm (1–4 units and 5 or more units) and farm sectors is 434.5 billion dollars (in real terms). This figure is the sum of three figures in Table 1 in [39]. This estimate was taken here to be the actual value of KIH_t in 1963IV. From this base period, the value of δ_H that seemed to reproduce the published series the best was 0.00575, and this was the value chosen to be used in the model. The use of this value led to a value of KIH_t in 1970IV of 504.8, which compares fairly closely to the published value of 510.7. The published series on the stocks could not used directly in this study because the series are not quarterly and because of the necessity of linking the investment series (CD_t and IH_t) to the stock series in some way.

Chapter Three

Econometric Issues

3.1 INTRODUCTION

Most of the econometric issues that pertain to this study are discussed in this chapter. The three main issues that are discussed are the treatment of serial correlation problems, the computation of the two stage least squares (TSLS) estimates, and the computation of the full information maximum likelihood (FIML) estimates. The model is nonlinear in both variables and parameters, and so one cannot rely directly on the standard textbook procedures for estimating linear models in computing the TSLS and FIML estimates of the model.

The following notation will be used for the discussion in this chapter. Let G denote the total number of equations in the model, M the number of stochastic equations, N the total number of predetermined (exogenous and lagged endogenous) variables, and T the number of observations. Write the g^{th} equation of the model as:

$$\phi_g(y_{1t}, \ldots, y_{Gt}, x_{1t}, \ldots, x_{Nt}, \beta_g) = u_{gt}, (g = 1, \ldots, G), (t = 1, \ldots, T), \qquad (3.1)$$

where the y_{it} are the endogenous variables, the x_{it} are the predetermined variables, β_g is the vector of unknown coefficients in equation g, and u_{gt} is the error term corresponding to equation g. For identities, u_{gt} is zero for all t. It will be assumed without loss of generality that the stochastic equations occur first in the model. The first M equations in the model are thus stochastic, with the remaining $G - M$ equations being identities. For the model as presented in the last chapter, M is 26 and G is 83. The basic period of estimation is 1954I–1974II, which gives a value of T of 82.

Counting the strike dummies, there are 78 exogenous variables in the model plus the constant term. There are also a number of lagged endogenous and lagged exogenous variables that appear as explanatory variables in the stochastic equations and in the identities. The value of N for the model is thus some number greater than 78. The error terms in some of the equations show evidence of first order serial correlation, and, as mentioned in section 1.1, the serial correlation assumption was retained for 12 of the 26 stochastic equations. There are 166 unknown coefficients to estimate in the 26 stochastic equations, counting the serial correlation coefficients, but not counting the variances and covariances of the error terms.

It will be useful in the following discussion to consider a particular example of one of the equations in (3.1). Assume that the first equation is:

$$\log \frac{y_{1t}}{x_{1t}} = \beta_{11} + \beta_{12} \log \frac{y_{1t-1}}{x_{1t-1}} + \beta_{13} \log y_{2t} + \beta_{14} \log y_{3t} + \beta_{15} x_{2t} + u_{1t}, \tag{3.2}$$

where

$$u_{1t} = \rho_{11} u_{1t-1} + \varepsilon_{1t}, \qquad (t = 1, \ldots, T). \tag{3.3}$$

The functional form of Equation (3.2) is common to a number of the stochastic equations in the model. Equation (3.2) is nonlinear in variables, but linear in the unknown coefficients. The first order serial correlation assumption in (3.3) is, as just mentioned, common to 12 of the stochastic equations. The error term ε_{1t} in Equation (3.3) is assumed not to be serially correlated.

3.2 THE TREATMENT OF SERIAL CORRELATION PROBLEMS

A convenient way of handling an equation with a first order serially correlated error term is to convert the equation into one that is nonlinear in coefficients, but that has a serially uncorrelated error term. Lagging Equation (3.2) once, multiplying through by ρ_{11}, and subtracting the resulting expression from Equation (3.2) yields, after some rearranging:

$$\begin{aligned}\log \frac{y_{1t}}{x_{1t}} = {} & \rho_{11} \log \frac{y_{1t-1}}{x_{1t-1}} + \beta_{11} - \rho_{11}\beta_{11} + \beta_{12} \log \frac{y_{1t-1}}{x_{1t-1}} - \rho_{11}\beta_{12} \log \frac{y_{1t-2}}{x_{1t-2}} \\ & + \beta_{13} \log y_{2t} - \rho_{11}\beta_{13} \log y_{2t-1} + \beta_{14} \log y_{3t} - \rho_{11}\beta_{14} \log y_{3t-1} \\ & + \beta_{15} x_{2t} - \rho_{11}\beta_{15} x_{2t-1} + \varepsilon_{1t}.\end{aligned} \tag{3.4}$$

Considering ρ_{11} to be just another coefficient to estimate, Equation (3.4) differs from Equation (3.2) by the inclusion of more explanatory variables and by the inclusion of nonlinear restrictions on the coefficients of these variables. The error term in the equation is, however, not serially correlated. The nonlinear restrictions on the coefficients result from the treatment of ρ_{11} as an unknown coefficient.

The treatment of the serial correlation problem in this way means that the u_{gt} error terms in (3.1) can be considered to be serially uncorrelated, where any initial serial correlation of the error terms has been solved out in the manner just described. The interpretation of (3.1) in this way means that the β_g vector should be considered as including the serial correlation coefficient when serial correlation is present in the g^{th} equation. When serial correlation is present in an equation, the number of predetermined variables in the equation should also be considered to be larger than it otherwise would be, and the equation should be considered to be nonlinear in coefficients as well as, possibly, in variables.

If observations on the endogenous and predetermined variables are available for $t = 0, 1, \ldots, T$, then Equation (3.4) must be estimated for $t = 1, \ldots, T$. There are ways of using information on the first observation more efficiently than the approach just described allows, but this added complication was not considered here. Ignoring the extra information on the first observation has no detrimental effect on the large sample properties of the estimators.

The present treatment has also not considered the case where an error term in one equation is directly correlated with the lagged value of an error term in some other equation. This complication would introduce nonlinear restrictions on the coefficients across, as well as within, equations. Since no experimentation with cross-serial correlation effects was carried out in this study, this added complication will not be considered in this chapter. For the linear model case, see Chow and Fair [9] and Fair [17] for a treatment of cross-serial correlation, as well as serial correlation of higher than first order.

3.3 THE COMPUTATION OF THE TWO STAGE LEAST SQUARES ESTIMATES

Since the model is nonlinear, explicit expressions for the reduced form equations cannot be derived. Consequently, consistent estimates of the reduced form coefficients cannot be obtained from any type of first stage regressions. Fortunately, the two stage least squares (TSLS) procedure does not require that consistent estimates of the reduced form coefficients be obtained in order to obtain consistent estimates of the structural coefficients in the second stage.

Consider, for example, the estimation of Equation (3.4) by TSLS. The endogenous terms on the right-hand side of the equation are

$\log y_{2t}$ and $\log y_{3t}$. If ε_{1t} is assumed not to be correlated with any variables on the right-hand side of the equation except $\log y_{2t}$ and $\log y_{3t}$, then consistent estimates of the coefficients of the equation can be obtained by the following two stage procedure. In the first stage, regress $\log y_{2t}$ and $\log y_{3t}$ on a common set of variables. The variables in this set should be variables that one feels, from knowledge of the overall model, have an effect, either directly or indirectly, on $\log y_{2t}$ and $\log y_{3t}$ and are not correlated with ε_{1t}. In other words, these variables should be correlated with $\log y_{2t}$ and $\log y_{3t}$, but not with ε_{1t}. The variables in this set must include the predetermined variables that appear on the right-hand side of Equation (3.4) in the form in which they appear in the equation: the constant, $\log \frac{y_{1t-1}}{x_{1t-1}}$, $\log \frac{y_{1t-2}}{x_{1t-2}}$, $\log y_{2t-1}$, x_{2t}, and x_{2t-1}. Let $\widehat{\log y_{2t}}$ and $\widehat{\log y_{3t}}$ denote the predicted values of $\log y_{2t}$ and $\log y_{3t}$ from the two regressions, and let $\hat{v}_{2t}$ and $\hat{v}_{3t}$ denote the estimated residuals from the two regressions. By definition, $\hat{v}_{2t} = \log y_{2t} - \widehat{\log y_{2t}}$ and $\hat{v}_{3t} = \log y_{3t} - \widehat{\log y_{3t}}$.

Now replace $\log y_{2t}$ and $\log y_{3t}$ in Equation (3.4) with their predicted values:

$$\log \frac{y_{1t}}{x_{1t}} = \rho_{11} \log \frac{y_{1t-1}}{x_{1t-1}} + \beta_{11} - \rho_{11}\beta_{11} + \beta_{12} \log \frac{y_{1t-1}}{x_{1t-1}} - \rho_{11}\beta_{12} \log \frac{y_{1t-2}}{x_{1t-2}}$$
$$+ \beta_{13} \widehat{\log y_{2t}} - \rho_{11}\beta_{13} \log y_{2t-1} + \beta_{14} \widehat{\log y_{3t}} - \rho_{11}\beta_{14} \log y_{3t-1}$$
$$+ \beta_{15} x_{2t} - \rho_{11}\beta_{15} x_{2t-1} + (\varepsilon_{1t} + \beta_{13} \hat{v}_{2t} + \beta_{14} \hat{v}_{3t}). \qquad (3.5)$$

By one of the properties of least squares, all of the variables on the right-hand side of this equation are orthogonal within the sample period to $\hat{v}_{2t}$ and $\hat{v}_{3t}$. This is because a common set of regressors has been used for both first stage regressions and because this set includes all of the predetermined variables on the right-hand side of Equation (3.4) in the form in which they appear in the equation. ε_{1t} is uncorrelated with all of the right-hand side variables in Equation (3.5). It is uncorrelated with the two predicted value variables because these variables are merely linear combinations of variables that are uncorrelated with ε_{1t} by assumption. ε_{1t} is uncorrelated with all of the other variables in the equation by assumption. Consequently, the composite error term in parentheses in Equation (3.5) is uncorrelated with all of the right-hand side variables, and so consistent estimates of this equation can be obtained by minimizing the sum of squared residuals with respect to the six coefficients: β_{11}, β_{12}, β_{13}, β_{14}, β_{15}, and ρ_{11}.

Minimizing the sum of squared residuals in Equation (3.5) is a nonlinear minimization problem because of the presence of ρ_{11}. This problem is not, however, very difficult to solve. One procedure that can be used is the iterative procedure outlined in Fair [22] (p. 509, fn. 3), which is merely the Cochrane-Orcutt [10] procedure adjusted to account for simultaneous equations bias. Since this minimization problem is not very difficult, other procedures could clearly be used. The question of which procedure one uses to minimize the sum of squared residuals in Equation (3.5) is a numerical question, not a statistical one.

The above analysis is also not limited to the particular kind of nonlinearity present in Equation (3.5). One could, for example, have a restriction that says that $\beta_{12} = \beta_{13}\beta_{14}$ and carry out the minimization incorporating this restriction as well. All this would do would be to change possibly the numerical procedure used to carry out the minimization. The Cochrane-Orcutt procedure and its various generalizations, for example, are more or less restricted to nonlinearities caused by the presence of serial correlation of the error terms.

In a very elegant paper, Amemiya [2] discusses the nonlinear two stage least squares estimator. He proves, for the case in which the equation being estimated is only nonlinear in coefficients, that the nonlinear two stage least squares estimator has the same asymptotic distribution as the limited information maximum likelihood estimator, providing that one uses all the predetermined variables in the model as regressors in the first stage regressions. (Amemiya considers only the case in which the predetermined variables are fixed.) For the nonlinear-in-variables case, no such theorem exists. The efficiency of the two stage least squares estimator in this case depends on how closely one has approximated the (unknown) reduced form equations in the first stage regressions.

The TSLS estimates of the model are presented in Table 2–3. The only nonlinearity in coefficients that existed in any of the equations was due to the presence of the serial correlation coefficient, and so the iterative procedure described in [22] was used to minimize the sum of squared errors when nonlinearity existed. A different set of first stage regressors was used for each equation estimated, depending on the predetermined variables and the right-hand side endogenous variables included in the equation. The regressors that were chosen for each equation were, in addition to the ones that were necessary to meet the orthogonality requirement discussed above, ones that seemed likely to have important effects on the included right-hand side endogenous variables.

The "*t*-statistics" that are presented in Table 2–3 are the absolute values of the ratios of the coefficient estimates to the estimates of their asymptotic standard errors. The estimates of the asymptotic standard errors for those equations that were linear in coefficients (no serial correlation) were computed in the usual way for the two stage least squares estimator. The estimates were computed as the square roots of the diagonal elements of

$\hat{\sigma}^2(\hat{Z}'\hat{Z})^{-1}$, where $\hat{\sigma}^2$ is the TSLS estimate of the variance of the error term in the equation being estimated and $\hat{Z}$ is the matrix of observations on the variables used in the second stage regression. A ^ is placed on Z to denote the fact that some of the variables in $\hat{Z}$ are variables of predicted values. $\hat{\sigma}^2$ is the estimate of the variance of the actual error term in the equation, not of the variance of the composite error term that is minimized in the second stage regression.

For those equations that were nonlinear in coefficients because of the serial correlation assumption, the estimates of the asymptotic standard errors (including the estimates of the asymptotic standard errors of the estimates of the serial correlation coefficients) were computed in a manner analogous to that described in [22], p. 514, for the linear model case. Consider, for example, Equation (3.2). Let Z denote the matrix of observations on the right-hand side variables in this equation. Let $\hat{Z}$ denote the matrix that is obtained from Z by replacing $\log y_{2t}$ and $\log y_{3t}$ in Z with $\widehat{\log y_{2t}}$ and $\widehat{\log y_{3t}}$ $(t = 1, \ldots, T)$, the latter two series being obtained in the manner described above.

Define $\hat{Q}$ to be equal to $\hat{Z} - \hat{\rho}_{11}Z_{-1}$, where $\hat{\rho}_{11}$ is the TSLS estimate of ρ_{11} and Z_{-1} is the matrix Z lagged one period. (It is assumed that observations for $t = 0$ are available.) Then the estimates of the asymptotic standard errors of the coefficient estimates other than $\hat{\rho}_{11}$ were computed as the square roots of the diagonal elements of $\hat{\sigma}^2(\hat{Q}'\hat{Q})^{-1}$, where $\hat{\sigma}^2$ is the estimate of the variance of ε_{1t}, the nonserially correlated error term. The estimate of the asymptotic standard error of $\hat{\rho}_{11}$ was computed, as described in [22], as the square root of $(1 - \hat{\rho}_{11}^2)/T$.

The t-statistics and Durbin-Watson statistics presented in Table 2–3 are meant to be interpreted more as just summary measures of the regressions than as precise statistical tests of some hypothesis. Too many assumptions of classical statistical hypothesis testing have been violated in the process of arriving at the estimates in Table 2–3 for any rigorous interpretation of the statistics as test statistics to be warranted. The primary way that the model has been tested in this study is to compare, in the manner described in Chapter Eight, its prediction accuracy with the prediction accuracy of other models.

3.4 THE COMPUTATION OF THE FULL INFORMATION MAXIMUM LIKELIHOOD ESTIMATES

In by now a classic paper, Chow [7] provides an interpretation of the full information maximum likelihood (FIML) estimator of a linear simultaneous equations model as a natural generalization of least squares. The FIML estimates are ones that minimize the generalized variance of the error terms in a model, subject to the restriction that the generalized variance of certain linear combinations of the endogenous variables be equal to a constant. The linear combination aspect of this procedure is the reason why the FIML estimator does not require, as do two and three stage least squares, that there be one natural left-hand side variable per equation.

In the present model there is a natural left-hand side variable for every equation except one, Equation 70 in Table 2–2. Equation 70 is, however, one of the key equations in the model, it being the equation that allows the bill rate to be implicitly determined. Therefore, because of Equation 70 and the implicit determination of the bill rate, the FIML estimator appears to be the natural one to use to estimate the model.

Under the assumption that the error terms for the stochastic equations in (3.1) are jointly normally distributed, the FIML estimates of the unknown coefficients in the model are obtained by maximizing:

$$L = -\tfrac{1}{2}T \log |S| + \sum_{t=1}^{T} \log |J_t| \tag{3.6}$$

with respect to the unknown coefficients,[a] where

$$S = (s_{gh}),\ s_{gh} = \frac{1}{T}\sum_{t=1}^{T} u_{gt}u_{ht}, \qquad (g, h = 1, \ldots, M), \tag{3.7}$$

$$J_t = \left(\frac{\partial \phi_g}{\partial y_{ht}}\right), \qquad (g, h = 1, \ldots, G). \tag{3.8}$$

The matrix S is $M \times M$, and the Jacobian matrix J_t is $G \times G$.

The maximization of L in (3.6) is a computationally difficult problem for a model of even moderate size because of the presence of the Jacobian terms. For every evaluation of L, $T + 1$ determinants have to be computed. T of these determinants are for the J_t matrices, which are generally of much higher dimension than the dimension of S. Since, as just discussed, it seems important to obtain FIML estimates of the model, a considerable effort was put into this study in trying to do so.

It did turn out to be feasible to obtain a set of estimates of the model that may be close to the true set of FIML estimates. This set was obtained as follows. First, 78 of the 166 unknown coefficients were fixed at their TSLS estimates, leaving 88 coefficients to estimate. An attempt was made to choose for the coefficients to estimate by FIML those that appeared to be most important in the model. The coefficients of the strike dummy variables, for example, were never chosen to be estimated. Second, some of the identities in the model were substituted out, decreasing the dimension of J_t in (3.8) to 48 × 48.

Third, J_t is a very sparse matrix, and advantage was taken of this fact in computing its determinant. Although J_t was 48 × 48, there were only 200 nonzero elements in it. There is a considerable literature, apparently largely unknown to economists, on dealing with sparse matrices,[b] and it turned out in the present case that considerable computational time could be saved by taking advantage of the fact that J_t is sparse. A good set of routines for dealing with sparse matrices is available from IBM [29], and when these routines were combined in the appropriate way to take the determinant of J_t, the computational time needed to take the determinant was decreased by a factor of 28 over the time that would otherwise be required. This is an enormous saving, and were it not for this saving, it would clearly not have been feasible to obtain the set of estimates that was in fact obtained.

Fourth, it turned out that a fairly good approximation to $\sum_{t=1}^{T} \log |J_t|$ is $\frac{T}{2}(\log |J_1| + \log |J_T|)$. When $\log |J_t|$ is plotted against t $(t = 1, \ldots, T)$, the points come fairly close to lying on a straight line, so that the average of the first and last points multiplied by T/2 is a fairly close approximation to the sum of the T points. This approximation was used for the work here, which meant that the determinant of J_t only had to be computed twice per evaluation of L rather than 82 times. To give an example of the error introduced by the approximation, the sum of all 82 points using the TSLS estimates was −8056.3, whereas the average of the first and last points multiplied by 41 was −8105.5. This is an error of about 0.6 percent.

The above procedures decreased the computer time needed for one evaluation of L to about 0.4 of a second on the IBM 370-158 computer at Yale. (The 370-158 is not a particularly fast computer for this purpose relative to a number of other computers in existence.) The fifth and final step in the calculation of the estimates was to maximize L using algorithms for maximizing nonlinear functions of coefficients that do not require analytic derivatives. The two algorithms that were considered are the no-derivative algorithm of Powell [37], and a member of the class of gradient algorithms considered by Huang [28]. The gradient algorithm requires first derivatives, and for present purposes the derivatives were obtained numerically. The gradient algorithm that was used is the one that updates the approximation to the inverse of the matrix of second partial derivatives by means of the "rank one correction formula." These two algorithms were used successfully by the author in two other studies, one concerned with solving optimal control problems for econometric models [20] and one concerned with obtaining FIML and robust estimates of econometric models [19]. For the optimal control work, a maximization problem in which there were 239 unknown coefficients to determine was solved using the gradient algorithm.

The TSLS estimates were used as starting points for both algorithms. From the results of some early experimentation, the no-derivative algorithm appeared to be more adept at increasing the value of the likelihood function, and so it was the one used in the final stages of the work. Even the use of the no-derivative algorithm did not, however, result in much of an increase in the value of the likelihood function from the value corresponding to the TSLS estimates. The value of the likelihood function for the TSLS estimates is 907.2. The value of the likelihood function for the "FIML" estimates presented in Table 2–3 is 924.6, which is a gain of only 1.9 percent.

It took the algorithm 24 iterations to achieve this value. The 24 iterations corresponded to 24,449 function evaluations (about 1,000 function evaluations per iteration), which at 0.4 seconds per evaluation took about two hours and 43 minutes of computer time. The value of the likelihood function was only changing in the fourth digit (the first decimal point) at the point that the algorithm was stopped (from having exhausted the computer budget for this project). The coefficient estimates were also changing by only small amounts.

It can be seen in Table 2–3 that the FIML estimates are in most cases quite close to the TSLS estimates. (Generally, only three significant digits are presented in Table 2–3, and in a number of cases the FIML and TSLS estimates are the same to three digits. Almost all the estimates, however, differed in at least the fifth digit.) This can mean either that the TSLS estimates are in fact quite close to the true FLML estimates, or that the algorithm did a poor job in maximizing the likelihood function. Cost considerations prevented any further experimentation to see if the true optimum had in fact been reached. Given the small increase in the value of the likelihood function that occurred, it is clear that more work needs to be done before one can have much confidence that the "FIML" estimates that have been obtained in this study are close to the true FIML estimates.

One final point about the computation of the FIML estimates should be noted. Constraining 78 of the coefficients to be equal to their TSLS estimates resulted in 13 of the 26 stochastic equations not having any coefficients left to be estimated by FIML. These 13 equations were *not*, however, dropped from the model when computing the FIML estimates. The predicted error terms for these equations (based on the TSLS estimates) were used, for example, in the computation of $|S|$ in (3.6). The Jacobian J_t was also not changed. This procedure allows the correlation between the error terms in the 13 unestimated equations and the error terms in the 13 estimated equations to have an effect on the coefficient estimates of the 13 estimated equations.

3.5 THE SOLUTION OF THE MODEL

The model is solved by the use of the Gauss-Seidel technique. For the work in Chapter Eight and for most of the work in Chapter Nine, Equation 8 in Table 2–2, the equation explaining the value of demand deposits and currency of the household sector (DDH_t), was used to solve for the bill rate. Given values of the predetermined variables and given values of $SAVH_t$, CG_t, DDF_t, $BORR_t$, $SAVB_t$, LF_t, and $SECR_t$, Equations 45, 61, 62, 64, and 70 in Table 2–2 form a set of five equations in five unknowns that can be solved analytically. The five unknowns are: BR_t, A_t, DDB_t, $LBVBB_t$, and DDH_t.

These analytic solutions were obtained, and the five equations that resulted from these solutions were used as the equations explaining the five variables. This procedure means that Equation 70 is used in the solution of DDH_t, the DDH_t equation having been "used up" in determining the bill rate. Each of the remaining 76 equations in Table 2–2 was used to solve the variable that appears naturally on the left-hand side of the equation. Equations 77 and 80, which explain the hours and loan constraint variables, were modified slightly in the process of solving the model. These modifications are discussed in Chapters Four and Five.

There are other ways that the model could be solved, but this way was one of the most natural and proved to be quite satisfactory. The number of iterations needed to solve the model each quarter was generally between about 5 and 20, depending on the starting values used. The speed of convergence seemed to be maximized by damping the solution value of the bill rate by about 90 per cent on each iteration. In other words, if $\widehat{RBILL}_t^{(i)}$ denotes the solution value of $RBILL_t$ on the i^{th} iteration and $\widetilde{RBILL}_t^{(i+1)}$ denotes the solution value of $RBILL_t$ that results from solving Equation 8 for $RBILL_t$ on the $(i+1)^{\text{st}}$ iteration, then the value of $\widehat{RBILL}_t^{(i+1)}$ was taken to be $\widehat{RBILL}_t^{(i)} + 0.1(\widetilde{RBILL}_t^{(i+1)} - \widehat{RBILL}_t^{(i)})$. Otherwise, no other damping was used in the solution of the model. One solution of the model for 82 quarters took about ten seconds of computer time on the IBM 370-158 computer at Yale.

The model was solved by setting all the structural error terms equal to zero (their expected value). It is well known that this procedure is incorrect for nonlinear models in the sense that it is not equivalent to setting the reduced form error terms equal to their expected values and then solving the reduced form equations. (See, for example, Howrey and Kelejian [27].) The proper way to solve the model would be by means of stochastic simulation, but this procedure is too costly to use in this study. Consequently, the usual procedure for solving nonlinear models was followed, even though it is not quite right.

For some of the solutions in Chapter Nine, and for all the solutions in Chapter Ten, the bill rate was taken to be exogenous and VBG_t was added as the extra endogenous variable. In this case Equation 70 could be used to solve for VBG_t directly, and each of the other equations could be used to solve for the variable that appears naturally on the left-hand side. Convergence turned out to be somewhat faster in this case than in the endogenous bill rate case.

3.6 A POSSIBLE ESTIMATOR FOR FUTURE USE

The purpose of this section is to describe an estimator that may be of interest to consider in future work. The estimator is not computationally feasible on the IBM 370-158 computer, but it should be feasible on computers about ten to twenty times faster than the 370-158.

To motivate this estimator, consider first the estimation of a linear simultaneous equations model by FIML. Let V denote a $T \times M$ matrix of reduced form error terms, where M is the number of stochastic equations, and let $\hat{V}$ denote a $T \times M$ matrix of predicted reduced form error terms. Given values of the structural coefficients, one can obtain predictions of the reduced form error terms by "simulating" the model over the sample period. This simulation should be thought of for now as being a static simulation. In the linear model case, simulation does not require the use of any iterative procedure to solve the model each period because the reduced form coefficient matrix can be obtained directly from the structural coefficient matrices.

Consider now minimizing $|V'V|$ with respect to the structural coefficients. Since $|\hat{V}'\hat{V}|$ can be computed given a set of values of the structural coefficients, one of the algorithms discussed in section 3.4 could be used to carry out this minimization. If one were successful in this task, the values of the structural coefficients that minimized $|V'V|$ would be the FIML estimates. (See, for example, Malinvaud [33], Ch. 19, p. 677.) The FIML estimates are thus estimates that minimize the generalized variance of the reduced form error terms with respect to the structural coefficients.

The minimization procedure just described could be carried out for a nonlinear model as well, where "simulation" would now require the use of something like the Gauss-Seidel procedure to solve the model each period. The predicted error terms that make up $\hat{V}$ would be the differences between the simulated and actual values of the endogenous variables. The values of the structural coefficients that corresponded to the minimum of $|\hat{V}'\hat{V}|$ would not be FIML estimates in this case because the true reduced form error terms are not additive in nonlinear models. There is, however, at least some analogy between these estimates and the true FIML estimates.

The above minimization procedure can be carried out, in either the linear or nonlinear case, using dynamic simulation rather than static simulation, a dynamic simulation being defined as a simulation that uses generated values of the lagged endogenous variables rather than actual values. The dynamic simulation can either be over the entire sample period or just for a few periods ahead at a time. Let $\hat{V}$ denote the $T \times M$ matrix of predicted error terms obtained from dynamic simulation of the model. The values of the structural coefficients that correspond to the minimum of $|\hat{V}'\hat{V}|$ will be called full information dynamic (FDYN) estimates.

The suggestion here is that it may be of interest in future work to obtain FDYN estimates of the model. It is true, of course, that for a properly specified model the FIML estimates are asymptotically efficient, so that if one knew that the model was properly specified, there would be no reason to be concerned with obtaining FDYN estimates. It is almost never the case, however, that one has complete confidence in the specification of a model, especially regarding the specification of the lag structures. The reason for proposing the FDYN estimator here is the feeling that the estimator may—by taking into account in a somewhat more explicit way than does the FIML estimator the dynamic properties of a model—lessen the effects of misspecification. Whether this is true or not is, of course, unclear, but at least it does seem worthy of some experimentation.

As mentioned in section 3.5, the time taken to solve the model once for 82 quarters is about ten seconds on the IBM 370-158. The time taken does not vary much depending on whether the simulation is static or dynamic. The time that would be required to compute $|\hat{V}'\hat{V}|$ once the model is solved for the 82 quarters is less than one second. Consequently, if the algorithms discussed in section 3.4 were used to minimize $|\hat{V}'\hat{V}|$, the time taken per function evaluation would be about ten seconds. This compares to the time of 0.4 seconds for the evaluation of the likelihood function in (3.6) in the computation of the FIML estimates. The FDYN estimates are thus about 25 times more expensive to compute than the FIML estimates, which means that the problem is really not feasible on the IBM 370-158. It should, however, be feasible to compute the estimates on a computer about ten to twenty times faster.

Klein [30] has suggested that it might be useful to estimate dynamic models by minimizing some function of multiperiod prediction errors. He is not very explicit on what function should be used, although for the linear model case he does suggest in one place (p. 64) that one might use the sum of the variances of the predicted error terms, each variance being normalized by the variance of the endogenous variable to which the error term corresponds. A more natural function to use, however, for both linear and nonlinear models, would appear to be the function $|\hat{V}'\hat{V}|$ suggested above. This function can be interpreted as a generalized variance of the predicted error terms, and it corresponds most naturally to the function that FIML minimizes in the static case.

Another reason for suggesting that some experimentation with the FDYN estimator be done is that fairly good results were obtained in Fair [12] using a single equation DYN estimator. The results in [12], while clearly tentative, do indicate that some gain in prediction accuracy may be attained by the use of DYN estimators. The results in [12] are all within-sample results. If in the future FDYN estimates are obtained, they will clearly have to be judged on grounds of outside-sample prediction accuracy, or at least on some criteria other than within-sample prediction accuracy, since the FDYN estimates are explicitly designed to minimize a generalized variance of within-sample prediction errors.

Another class of estimators that may be worth considering in future work is the class of robust estimators. As discussed in Fair [19], almost any estimator that is based on minimizing some function of the error terms in an equation or a model can be modified to be a robust estimator. Again, some encouraging results were obtained in [19] about the possibility of being able to increase prediction accuracy by the use of robust estimators. These results are also very tentative, but they do at least indicate that further experimentation with robust estimators of econometric models should be undertaken. Primarily because of cost considerations, robust estimators were not considered in this study.

NOTES

[a] See, for example, Chow [8].

[b] See Brayton, Gustavson, and Willoughby [5] for a fairly extensive bibliography on sparse matrices.

Chapter Four

The Household Sector

4.1 INTRODUCTION

The eight stochastic equations that relate to the household sector are explained in this chapter. The eight equations include four consumption equations, three work effort equations, and an equation explaining the value of demand deposits and currency of the household sector. Given the important distinction in the theoretical model between a household's unconstrained and constrained decisions, it will be useful in the following analysis to consider these two types of decisions separately.

In section 4.2 the variables that are assumed to affect the unconstrained decision variables of the household sector are discussed, and then in section 4.3 the treatment of the constraints is discussed. The variables that explain the unconstrained decision variables are those that one expects on microeconomic grounds to affect a household's decisions. The effects of the constraints are handled by adding to the equations determining the unconstrained decision variables certain "constraint" variables (denoted as ZJ_t and ZR_t below).

4.2 THE DETERMINATION OF THE UNCONSTRAINED DECISIONS

In Table 4–1 the decision variables in the theoretical model are matched to the related variables in the empirical model. All the decision variables should be considered for now as being unconstrained. In the theoretical model there is only one type of consumption good, and so there is only one consumption decision variable for each household. In the empirical model, on the other hand, four consumption variables are considered to be decision variables of the household sector. These four variables are expenditures on services, CS_t,

Table 4–1. Matching of Dependent Variables in the Theoretical and Empirical Models for the Household Sector

Decision Variable in the Theoretical Model	*Related Variable(s) in the Empirical Model*
1. XH_{it} (number of goods purchased by household i)	CS_t (expenditures on services) CN_t (expenditures on nondurable goods) CD_t (expenditures on durable goods) KCD_t (stock of consumer durable goods) IH_t (expenditures on housing) KIH_t (stock of houses)
2. HPH_{it} (number of hours that household i is paid for)	$JOBH_t$ (number of jobs in the economy) HPH_t (average number of hours that each job is paid for) $EMPL_t$ (number of people employed in the economy) $MOON_t$ ($=JOBH_t - EMPL_t$, number of moonlighters) TLF_{1t} (labor force of men 25–54) TLF_{2t} (labor force of all persons 16 and over except men 25–54)
3. DDH_{it} (demand deposits of household i)	DDH_t (demand deposits of the household sector)

Note: $JOBH_t = JOBF_t + JOBGC_t + JOBGM_t$

$$HPH_t = \frac{JOBF_t HPF_t + JOBGC_t HPGC_t + JOBGM_t HPGM_t}{JOBH_t}$$

expenditures on nondurable goods, CN_t, the stock of consumer durable goods, KCD_t, and the stock of houses, KIH_t. This separate treatment of the four consumption decision variables can be justified within the context of the theoretical model if it is assumed that the four variables enter the utility function of each household separately. The inclusion of the stocks of consumer durables and houses in the utility function can be justified if it is assumed that the services from durable goods and houses are proportional to the stocks.

The solution of the optimal control problems of the households in Chapter Four in Volume I would proceed in a similar way with four kinds of goods or services rather than one. The main difference that would exist in the four-good case is that the relative prices among the four goods would affect the household's decisions. If services are proportional to stocks and stocks have a life longer than one decision period, then the stocks of durable goods and houses that exist at the beginning of the household's decision period would also, of course, have important effects on the household's decisions. Otherwise, however, the analysis in Chapter Four in Volume I would be little changed. The solution of the control problem would just be slightly more complex.

There is also only one work effort variable for each household in the theoretical model, whereas in the empirical model more than one variable is considered. Six work effort variables are listed in Table 4-1 for the empirical model. The total number of worker hours paid for in the economy is $JOBH_t HPH_t$. If households were never constrained in their work effort, they could be considered as determining this amount. The household sector could also be considered as determining the breakdown of this amount into jobs ($JOBH_t$) and hours per job (HPH_t) and as determining the breakdown of jobs ($JOBH_t$) into the number of different people employed ($EMPL_t$) and the number of moonlighters ($MOON_t$). Finally, the household sector could be considered as determining the total labor force (TLF_{1t} and TLF_{2t}).

In this unconstrained case, "unemployment" (the difference between the number of people in the labor force and the number of people employed) would be completely voluntary. It would be a decision variable of the household sector and would be a function of all the variables that affect the unconstrained decision making processes of the households. (In the theoretical model unemployment was zero in the unconstrained case because search behavior was not considered explicitly.) It will be convenient with respect to the six work effort variables in Table 4–1 to consider in this chapter only the determination of $MOON_t$, TLF_{1t}, and TLF_{2t}. The determination of $JOBH_t$ and HPH_t will be discussed in the next chapter. Once $JOBH_t$ and $MOON_t$ are determined, $EMPL_t$ is simply the difference between the two.

In Table 4–2 the variables that are important in the theoretical model in influencing a household's decisions are listed and are matched to the relevant variables in the empirical model. As can be seen in the table, there are five price deflators in the empirical model that are of relevance to the household sector, instead of only one in the theoretical model. The variable YG in the theoretical model measures the level of transfers payments from the government to each household. The closest variable approximating YG in the empirical model is ($YG_t + TPU_t$), which measures transfer payments from the government sector to the household sector. YG in the theoretical model has a negative effect on work effort and a positive effect on consumption. The variable that was chosen in the empirical model to represent the effects of YG was actually not ($YG_t + TPU_t$), but was instead $YNLH_t$, the measure of nonlabor income of the household sector. $YNLH_t$, which is defined by Equation 71 in Table 2–2, includes ($YG_t + TPU_t$) plus dividend, interest, rental income, and three other items. Two of the three other items are small and not important (transfer payments from the firm sector to the household sector, $FHTRP_t$, and profits of farms, $FHPFA_t$). The other item is employee social security contributions, $HGSI2_t$, which is subtracted from the other variables.

The inclusion of $HGSI2_t$ in the definition of nonlabor income is another example of the imposition of a constraint on the way that taxes affect behavior. Since d_{6t} is the employee social security tax rate, it could have been used directly as an explanatory variable in the stochastic equations of the

Table 4–2. Matching of Explanatory Variables in the Theoretical and Empirical Models for the Household Sector

Explanatory Variable in the Theoretical Model	*Related Variable(s) in the Empirical Model*
1. WH_{it} (wage rate received by household i)	WF_t (average hourly earnings, adjusted for overtime and interindustry employment shifts)
2. PH_{it} (price paid for goods by household i)	PCS_t (price deflator for CS_t) PCN_t (price deflator for CN_t) PCD_t (price deflator for CD_t) PIH_t (price deflator for IH_t) PH_t (price deflator for domestic sales inclusive of indirect business taxes)
3. r_t (bill rate), RH_{it} (loan rate paid by household i)	$RBILL_t$ (bill rate) $RMORT_t$ (mortgage rate)
4. d_3 (personal income tax rate)	d_{3t}^M (marginal personal income tax rate)
5. YG (minimum guaranteed level of income or level of transfer payments to each household)	YG_t (transfer payments from the government sector to the household sector not counting TPU_t) TPU_t (unemployment insurance benefits) $HGSI2_t$ (employee social security taxes) $YNLH_t$ (nonlabor income, $DIVH_t + INTH_t + FHRNT_t + FHTRP_t + FHPFA_t + YG_t + TPU_t - HGSI2_t$)
6. A_{it-1} (value of nondemand deposit assets of the previous period), LH_{it-1} (value of loans taken out of the previous period)	A_{t-1} (value of nondemand deposit securities of the previous period)

Explanatory Variables in the Empirical Model for Which There are no Counterparts in the Theoretical Model

POP_t	(population 16 and over)
POP_{1t}	(population of men 25–54)
POP_{2t}	(population of all persons 16 and over except men 25–54)
KCD_{t-1}	(stock of durable goods of the previous period)
KIH_{t-1}	(stock of houses of the previous period)

household sector. An initial attempt was made to do this, but with little success. It did not appear to be possible to pick up independent effects of d_{6t} in the data, and so $HGSI2_t$ was instead included as a negative item in the definition of nonlabor income. $HGSI2_t$ is, of course, an endogenous variable in the model, but so also are three other variables that are included in the definition of nonlabor income ($DIVH_t$, $INTH_t$, and TPU_t). $YNLH_t$ is thus an endogenous variable in the model.

$HGSI2_t$ is linked to the wage bill of the firm sector, and so it changes when wages change. Therefore, $YNLH_t$ changes as wages change, and so it is not, strictly speaking, a nonlabor income variable. The effect of wages on $YNLH_t$ is, however, fairly small, and for ease of exposition $YNLH_t$ will be referred to simply as nonlabor income.

The lumping together of $YG_t + TPU_t$ and dividend, interest, and rental income in the definition of $YNLH_t$ is yet another example of the imposition of a constraint on the way that the government affects behavior. Transfer payments from the government sector are assumed to be treated by the household sector like any other nonlabor income item. This constraint was again imposed because of the difficulty of estimating separate effects of the two types of income.

In the theoretical model there are both creditor and debtor households. A_{it} in Volume I denotes the value of nondemand deposit assets of creditor households, and LH_{it} denotes the value of loans of debtor households. In the empirical work it is not possible to distinguish between creditor and debtor households, and A_{t-1} in Table 4–2 instead denotes the value of nondemand deposit assets *minus* liabilities of the household sector.

In the estimation of the consumption and work effort equations of the household sector, the explanatory variables for each equation were taken from the variables in Table 4–2. Because of possible multicollinearity problems, only a subset of the variables in the table was tried for any one equation. Some variables that were tried were also dropped if they contributed little to the explanation of the dependent variable. Many of the variables were deflated by population; two of the variables ($YNLH_t$ and A_t) were deflated by the price level; the functional form of all of the equations was taken to be the log form; and some experimentation was done on trying alternative lag structures. The estimated equations are discussed in section 4.4, but before this is done the treatment of the constraints on the household sector must be explained.

4.3 THE TREATMENT OF THE CONSTRAINTS

The hours and loan constraints on the households play an important role in the theoretical model. The existence of constraints poses a very serious problem for empirical work because the unconstrained decision values are observed only if the constraints are not binding. Otherwise, only the constrained decision values are observed. All the discussion in the previous section was concerned with the unconstrained decision variables, and so some modification of the equations that result from this discussion must be made to account for the constraints. There is no one obvious way to account for the constraints, and it should be stressed that the approach that will now be described is only one of a number that might be tried. It would clearly be of interest in future work to consider other possible ways of accounting for the constraints.

Let $CSUN_t$ denote the expenditures on services that the household sector would make if it were not constrained, and let CS_t denote the actual expenditures made. Assume that one has specified, from the previous section, an equation determining $CSUN_t$:

$$CSUN_t = f(\cdots). \tag{4.1}$$

Assume also that all the variables on the right-hand side of this equation are observed. If the household sector is not contrained, the ratio $CS_t/CSUN_t$ is one. If the household sector is constrained, then the ratio is less than one, providing that one assumes—as is done here—that binding constraints cause the household sector to consume less than it would have unconstrained. If one can find a variable, say Z_t, such that:

$$\frac{CS_t}{CSUN_t} = Z_t^{\gamma_1},\ \gamma_1 > 0, \tag{4.2}$$

then one has immediately from Equations (4.1) and (4.2) an equation in observed variables, which can then be estimated:

$$CS_t = Z_t^{\gamma_1} f(\cdots). \tag{4.3}$$

Within this framework, the problem of accounting for the constraints reduces itself to finding a variable Z_t for which the specification in (4.2) seems reasonable.

Consider first the hours constraint on the household sector. What one needs is a variable that takes on a value of one when conditions in the labor market are tight and households are not constrained, and a value of less than one otherwise. When the variable is less than one, it should be proportional to the ratio of the constrained to the unconstrained decision values of the household sector. One obvious measure of labor market tightness is $1 - UR_t$, where UR_t is the civilian unemployment rate. Another measure of labor market tightness is J_t^*, which is defined in Equation 76 of Table 2–2 and which is the detrended ratio of total hours paid for in the economy to the total population 16 and over. The number -0.00073513 used in Equation 76 is the estimate of the coefficient of t in the regression of $\log J_t$ on a constant and t for the 1952I–1974II period.

If, say, J_t^* is used as the measure of labor market tightness, one needs to construct a variable Z_t that is a function of J_t^* and that has the

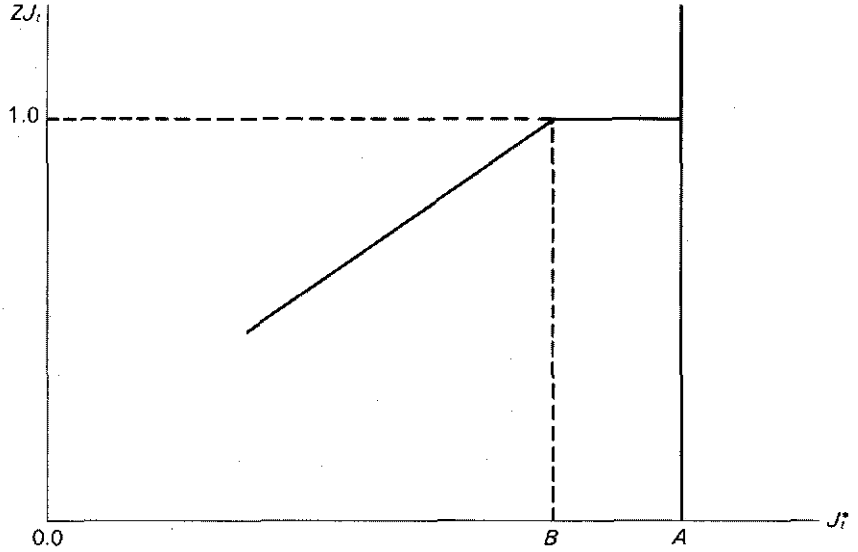

Figure 4-1. Desired Shape of ZJ_t as a Function of J^*

properties just described. The desired shape of Z_t as a function of J_t^* is presented in Figure 4-1. Point A is some value that is larger than the largest value of J_t^* observed in the sample period, and point B is the value of J_t^* above which it seems reasonable to assume that the household sector is not constrained. An approximation to the curve in Figure 4-1 is the left half of the normal density function:

$$Z_t = e^{-\alpha_1(J_t^* - A)^2}. \tag{4.4}$$

For J_t^* equal to A, Z_t is one, and for J_t^* less than A, Z_t is less than one. How good an approximation the normal density is to the curve in Figure 4-1 depends on how close B is to A and how steep the slope of the line to the left of B is. The goodness of the approximation depends also, of course, on the value chosen for α_1, but it turns out, as will be seen shortly, that a value of α_1 does not have to be specified before estimation of the equation. Another possible choice for the Z_t variable would be to replace J_t^* with $1 - UR_t$ and to take for the value of A some value that is larger than the largest value of $1 - UR_t$ in the sample period. J_t^* turned out to give somewhat better results than did $1 - UR_t$, although both sets of results were fairly close. Only the results using J_t^* are reported below.

Consider next the loan constraint. One needs to find a variable that takes on a value of one when conditions in the financial markets are

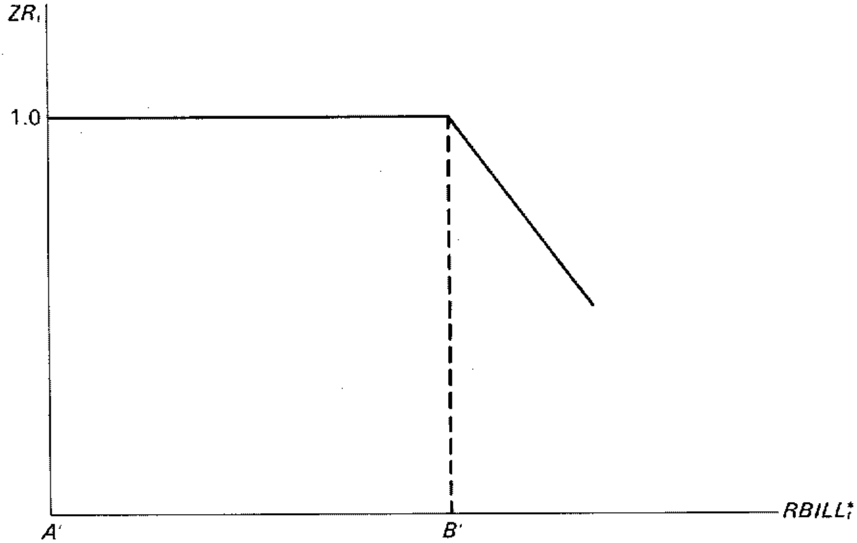

Figure 4-2. Desired Shape of ZR_t as a Function of $RBILL_t^*$

loose and households are not constrained, and a value of less than one otherwise. When the variable is less than one, it should again be proportional to the ratio of the constrained to the unconstrained values of the household sector. One possible measure of the tightness of the financial markets is the bill rate, $RBILL_t$. $RBILL_t$ does, however, have a positive trend during much of the postwar period, and a possibly better measure of the tightness of the financial markets is a partly detrended version of the bill rate. The version used in this study is $RBILL_t^*$, defined by Equation 79 in Table 2-2. $RBILL_t^*$ is $RBILL_t$ detrended up to the 1970IV. The number 0.019757 used in Equation 79 is the estimate of the coefficient of t in the regression of log $RBILL_t$ on a constant and t for the 1952I–1970IV period.

If $RBILL_t^*$ is used as the measure of tightness in the financial markets, one needs to construct a variable, Z_t, that is a function of $RBILL_t^*$ and that has the properties just described. The desired shape of Z_t as a function of $RBILL_t^*$ is presented in Figure 4-2. Point A' is some value that is smaller than the smallest value of $RBILL_t$ observed in the sample period, and point B' is the value of $RBILL_t^*$ below which it seems reasonable to assume that the household sector is not constrained. An approximation to the curve in Figure 4-2 is the right half of the normal density function:

$$Z_t = e^{-\alpha_2(RBILL_t^* - A')^2}. \tag{4.5}$$

For $RBILL_t^*$ equal to A', Z_t is one, and for $RBILL_t^*$ greater than A', Z_t is less than one. It also turns out in this case that a value of α_2 does not have to be specified before estimation.

To distinguish the Z_t for the hours constraint in Equation (4.4) from the Z_t for the loan constraint in Equation (4.5), the former will be denoted as ZJ_t and the latter as ZR_t. This is the notation used in Table 2–2. Both constraints may, of course, be binding at the same time. If both constraints are binding, they are assumed to interact multiplicatively:

$$\frac{CS_t}{CSUN_t} = ZJ_t^{\gamma_1} ZR_t^{\gamma_2},\ \gamma_1 > 0,\ \gamma_2 > 0. \tag{4.6}$$

This equation says that if neither constraint is binding ($ZJ_t = 1$ and $ZR_t = 1$), then CS_t equals $CSUN_t$. Otherwise, CS_t is less than $CSUN_t$.

Consider now the estimation of the equation explaining CS_t. Assume for sake of argument that the equation explaining $CSUN_t$, i.e., $f(\cdots)$ in (4.1), is simply:

$$CSUN_t = e^{\beta_0} Q_t^{\beta_1} e^{\varepsilon_t}, \tag{4.7}$$

where Q_t is the one explanatory variable in the equation and ε_t is an error term. Substituting (4.7) into (4.6) and taking logs yields:

$$\log CS_t = \beta_0 + \beta_1 \log Q_t + \gamma_1 \log ZJ_t + \gamma_2 \log ZR_t + \varepsilon_t. \tag{4.8}$$

Substituting the expressions for ZJ_t and ZR_t in (4.4) and (4.5) into (4.8) then yields:

$$\log CS_t = \beta_0 + \beta_1 \log Q_t - \gamma_1\alpha_1(J_t^* - A)^2 - \gamma_2\alpha_2(RBILL_t^* - A')^2 + \varepsilon_t. \tag{4.9}$$

Given values for A and A', Equation (4.9) can be directly estimated. There are no longer any unobserved variables to be concerned about. The coefficients γ_1 and α_1 cannot be separately estimated, nor can the coefficients γ_2 and α_2, but it is not really important to be able to do so. What is important is that the variables $(J_t^* - A)^2$ and $(RBILL_t^* - A')^2$ pick up the effects of the constraints, not that one be able to separate their coefficient estimates into estimates of the γ_i parameters in Equation (4.6) and estimates of the α_i parameters in the approximating equations (4.4) and (4.5).

Since the choice of either γ_1 or α_1 is arbitrary, α_1 has been chosen for scale purposes to be 1/10000 (see Equation 77 in Table 2–2). Similarly, α_2 has been chosen for scale purposes to be 1/1000 (see Equation 80 in Table 2–2). The value of A was taken to be 335.9, which is slightly larger than the largest value of J_t^* in the sample period, and the value of A' was taken to be 0.608, which is slightly smaller than the smallest value of $RBILL_t^*$ in the sample period.

To summarize, the constraints on the household sector were handled in this study by adding to the equations explaining the decision variables of the household sector the variables $(J_t^* - A)^2$ and $(RBILL_t^* - A')^2$. This converts each equation from one with an unobserved variable on the left-hand side (the unconstrained decision value) to one with an observed variable on the left-hand side (the constrained decision value). It is clear that this treatment of the constraints requires a number of restrictive assumptions. It does have the advantage, however, of allowing one not to have to estimate separately the α_i coefficients in Equations (4.4) and (4.5) and the γ_i coefficients in Equation (4.6). The data are effectively allowed to estimate both sets of coefficients at the same time, thus allowing there to be fewer a priori constraints imposed on the data than might be the case with other specifications. No a priori constraints of a zero-one type, for example, are imposed on the data.

Regarding the loan constraint, considerable thought was given in this study to possible ways of using the flow-of-funds data to help measure the constraint. The problem with the flow-of-funds data, however, is that they all measure the effects of the constrained decisions, and there seemed no obvious way to use the data to get a direct indication of when the loan constraint was binding. In terms of the notation in Volume I, there seemed no obvious way to measure $LBMAX_t$, the maximum value of loans that the bank sector chooses in the period. All that one observes is the net result of what happens after the firm and household sectors have taken $LBMAX_t$ into account in their decision making processes.

The two constraint variables, ZJ_t and ZR_t, are endogenous and are treated as such in the estimation work. ZJ_t is a function of J_t^*, which is a function of J_t, which in turn is a function of $JOBF_t$ and HPF_t. The latter two variables, as will be seen in the next chapter, are two endogenous variables in the model. ZR_t is a function of $RBILL_t^*$, which in turn is a function of $RBILL_t$, $RBILL_t$ being another endogenous variable in the model.

Although ZJ_t and ZR_t can be treated like any other endogenous variables for purposes of estimation, a slight modification of the variables has to be made for purposes of solving the model. Consider Equation 77 in Table 2-2 explaining ZJ_t:

$$77.\ ZJ_t = e^{-1/10000(J_t^* - 335.9)^2}.$$

In the data, J_t^* is always less than 335.9 by construction (see above). In the solution of the model, however, there is nothing that guarantees that the predicted value of J_t^* will always be less than 335.9. If the predicted value of

J_t^* is greater than or equal to 335.9, this indicates a very tight labor market (tighter than ever existed in the data). In tight labor markets, ZJ_t is supposed to take on a value of one (or close to one). Consequently, in the solution program for the model, the predicted value of ZJ_t was set equal to one whenever the predicted value of J_t^* was greater than 335.9. Otherwise, Equation 77 was used to determine the predicted value of ZJ_t. A similar procedure was followed for ZR_t. The predicted value of ZR_t was set equal to one whenever the predicted value of $RBILL_t^*$ was less than 0.608, but otherwise Equation 80 in Table 2–2 was used to determine the predicted value of ZR_t.

One final point about the treatment of the constraints should be made, which has to do with the assumption that all the variables in $f(\cdots)$ in Equation (4.1) are observed. It will be seen in the next section, from examining the results in Table 2-3, that the lagged dependent variable in each equation is an important explanatory variable in the equation. Since only constrained decision values are assumed always to be observed, a lagged dependent variable in the present context is a lagged constrained decision value, not a lagged unconstrained value. Therefore, the assumption here is that lagged constrained values enter functions like $f(\cdots)$ in Equation (4.1). For example, CS_{t-1} is assumed to enter $f(\cdots)$ in (4.1), not $CSUN_{t-1}$. Since lagged dependent variables are used to try to capture expectational effects, there is no compelling reason for making one assumption or the other regarding whether lagged unconstrained or lagged constrained decision values enter functions like $f(\cdots)$. The assumption that lagged constrained decision values enter the functions was made primarily on grounds of convenience.

It is also the case, as will be seen in the next section, that some left-hand side variables have been deflated by population. This, however, poses no added difficulties in interpreting the effects of the constraints. If, for example, $CSUN_t$ in Equation (4.1) is divided by POP_t, the equation can be multiplied through by POP_t, leaving $CSUN_t$ on the left-hand side. Then after adjusting the equation by use of the constraint variables to have CS_t be the left-hand side variable, the equation can be divided back through by POP_t.

4.4 THE ESTIMATES OF THE EQUATIONS FOR THE HOUSEHOLD SECTOR

There are eight stochastic equations for the household sector. The functional forms chosen for these equations, the explanatory variables used in each equation, and the TSLS and FIML coefficient estimates of the equations are presented in Table 2–3 in Chapter Two. These equations will not be repeated here, but instead reference will be made throughout this section to Table 2–3.

The first four equations are consumption equations, explaining

$$\log\frac{CS_t}{POP_t},\ \log\frac{CN_t}{POP_t},\ \log\frac{KCD_t}{POP_t},\quad \text{and}\quad \log\frac{KIH_t}{POP_t},$$

where POP_t is the population of all persons 16 and over. The next three equations are work effort equations, explaining

$$\log \frac{TLF_{1t}}{POP_{1t}}, \log \frac{TLF_{2t}}{POP_{2t}}, \quad \text{and} \quad \log \frac{MOON_t}{POP_t}.$$

TLF_{1t}/POP_{1t} is the labor force participation rate of men 25–54, and TLF_{2t}/POP_{2t} is the participation rate of all persons 16 and over except men 25–54. $MOON_t/POP_t$ is the percent of the population holding two jobs. The eighth equation explains log DDH_t/POP_t, where DDH_t/POP_t is the value of demand deposits and currency of the household sector deflated by population.

Each of the first seven equations in Table 2–3 includes as explanatory variables a subset of the variables listed in Table 4–2. These are again the variables that are important in the theoretical model in influencing a household's decisions. It will be useful to consider all seven equations together regarding the estimated effects of the various explanatory variables. First, the price deflators have a negative effect in all seven equations, and the wage rate has a positive effect in all seven equations. In the consumption equations the price deflator and the wage rate were not constrained to have equal coefficients in absolute value, but in the work effort equations they were.[a] The price deflator used in the work effort equations is PH_t, the price deflator for domestic sales inclusive of indirect business taxes.

One or more interest rate variables are included in three of the consumption equations (Equations 1, 3, and 4), all with negative coefficient estimates; and one interest rate variable is included in one of the work effort equations (Equation 6), with a positive coefficient estimate. The two interest rate variables considered in the estimation work for the household sector are the bill rate and the mortgage rate, the former being taken as a proxy for the short term rates affecting the household sector and the latter being taken as a proxy for the long term rates.

The nonlabor income variable is included in three of the consumption equations (Equations 1, 2, and 3), with positive coefficient estimates; and in one of the work effort equations (Equation 5), with a negative coefficient estimate. The value of assets of the previous period is included in two of the consumption equations (Equations 1 and 2), with positive coefficient estimates; and in one of the work effort equations (Equation 6), with a negative coefficient estimate. The marginal personal income tax rate is included in one of the consumption equations (Equation 2), with a negative estimated effect; and in two of the work effort equations (Equations 5 and 7), with negative estimated effects.

All the results just cited are consistent with the results in the theoretical model. In the theoretical model the price level has a negative effect and the wage rate has a positive effect on consumption and work effort; the interest rate has a negative effect on consumption and a positive effect on work effort; nonlabor income (i.e., YG, the minimum guaranteed level of income) has a positive effect on consumption and a negative effect on work effort; the value of assets of the previous period has a positive effect on consumption and a negative effect on work effort; and the personal income tax rate has a negative effect on consumption and work effort.

The hours constraint variable enters all four consumption equations and two of the three work effort equations, with positive coefficient estimates. From Equation (4.8) it can be seen that the coefficient estimates are expected to be positive, since γ_1 is postulated to be positive. The estimates are not, of course, estimates of γ_1, because α_1 has been arbitrarily set equal to 1/10000. As discussed above, it is not possible to obtain separate estimates of γ_1 and α_1. The loan constraint variable enters only the housing equation (Equation 4), with the expected positive coefficient estimate. Otherwise, the variable did not appear to be important in explaining any of the other decision variables of the household sector.

Some experimentation was done with estimating alternative lag structures, and in the end the following constraints were imposed on the data. First, a four quarter average of the marginal tax rate variable lagged one quarter ($\bar{d}^M_{3t-1}$) was used as the tax rate variable. This seemed like a reasonable procedure in the sense that it may take people a few quarters to perceive a change in their marginal tax rate. Since the equations are in log form, the explanatory variable relating to $\bar{d}^M_{3t-1}$ was taken to be $\log(1 - \bar{d}^M_{3t-1})$. If $\bar{d}^M_{3t-1}$ were zero, then this form says that $\bar{d}^M_{3t-1}$ would have no effect on the decision variables. If instead the variable $\log \bar{d}^M_{3t-1}$ were used, this would imply an effect of plus infinity (assuming the coefficient estimate of $\log \bar{d}^M_{3t-1}$ to be negative) if $\bar{d}^M_{3t-1}$ were zero, which does not seem reasonable.

In one of the two labor force participation equations (Equation 5), a four quarter average of the log of $YNLH_t/(PH_t\, POP_t)$ lagged one quarter was used as the nonlabor income variable. This procedure is equivalent to constraining the coefficients of $\log(YNLH_{t-i}/(PH_{t-i}\, POP_{t-i}))$, $i = 1, 2, 3, 4$, to be the same.

All the explanatory variables in the housing equation are lagged at least one quarter. Since it generally takes longer than a quarter to build a house, the longer lags that seemed to pertain to the housing equation are consistent with what one would expect. The mortgage rate is included twice in the housing equation, with lags of one and two quarters. The data seemed capable of picking up separate effects of the two lagged values. The other equation where it seemed possible to pick up separate effects of the lagged values of the same variable was Equation 2 for nondurable consumption, where the nonlabor income variable was included contemporaneously and with a lag of one quarter.

Lagged dependent variables are included in all seven equations. As mentioned in Chapter One, each of the equations was initially estimated under the assumption of first order serial correlation to make sure that the lagged dependent variables are not erroneously picking up serial correlation effects. Serial correlation turned out to be important in only two equations, Equations 3 and 4, explaining the stocks of durable goods and houses. The serial correlation assumption was retained for these two equations, and the estimates of the serial correlation coefficients for these two equations are presented in Table 2–3 along with the other coefficient estimates. For each of the other equations, the serial correlation coefficient was constrained to be zero.

The final equation estimated for the household sector is Equation 8, explaining the household sector's holdings of demand deposits and currency (DDH_t). In the theoretical model, DDH_t is a function of the household sector's expenditures on goods, but here the best results were obtained by taking DDH_t to be a function of taxable income (YH_t) and the bill rate. A time trend was also included in the equation to pick up any possible trend in the relationship between DDH_t/POP_t and the other explanatory variables in the equation.

Four dummy variables were added to Equation 3 explaining KCD_t to account for the effects of the automobile strikes in 1964 and 1970. Otherwise, the strikes did not appear to have a strong enough effect on any of the other variables in the household sector to warrant the further use of dummy variables.

Regarding the four consumption categories, some experimentation was done in this study to see if it were possible to pick up substitution or complementary effects among the categories. In theory, all the variables listed in Table 4–2 should be included in each consumption equation. The price of services, for example, should have an effect on all four consumption categories, not just on the consumption of services. It did not appear to be possible, however, to pick up these effects in the data, and so no substitution or complementary effects of this kind are included in the model.

Equations 1, 2, 3, 4, and 8 are the equations in the household sector for which FIML estimates were obtained. As can be seen in Table 2–3, the TSLS and FIML estimates of these five equations are quite close. The largest differences occur for the serial correlation coefficient in Equation 3 and for the coefficients of the two constraint variables in Equation 4.

This completes the discussion of the stochastic equations for the household sector in Table 2–3. The explanatory variables that have been used in the equations, other than the constraint variables and the lagged dependent variables, are variables that one would expect on microeconomic grounds to affect households' unconstrained decisions. After adjusting for the effects of the constraints and for expectational and lag effects, the results do seem to indicate that these variables have important effects on the decision variables of the household sector. The question of how the household sector interacts with the other sectors in the model is taken up in Chapter Nine. To conclude this chapter, four further comments about the household sector will be made.

First, it should be noted, as mentioned in section 1.1, that when the hours constraint is binding on the household sector, the specification of the consumption equations is similar to a specification that would include labor income directly as an explanatory variable in the equations. When the hours constraint variable, ZJ_t, is not close to one, it is a function of the number of hours paid for in the economy. ZJ_t is not close to one when the hours constraint is binding on the household sector, so that when the hours constraint is binding, there is a variable on the right-hand side of the consumption equations that is a function of the number of hours paid for. Since the wage rate is also included in the consumption equations, there is something like a labor income variable on the right-hand side of the equations when the constraint is binding. When the constraint is not binding (ZJ_t close to one), then only the wage rate part of labor income is included as an explanatory variable.

The second comment concerns the inclusion of the hours constraint variable in the work effort equations. ZJ_t is an important explanatory variable in the equation explaining the labor force participation of all persons 16 and over except men 25–54 (Equation 6). This result is interpreted within the context of the model as indicating that when the hours constraint is binding on the household sector, the participation rate that results from the solutions of the households' constrained optimal control problems is less than the rate that would result if the households were not constrained. As mentioned in section 1.1, effects of this sort are sometimes referred to in the literature as "discouraged worker" effects. The hours constraint variable in Equation 6 can thus be thought of as picking up discouraged worker effects if one wants to use this terminology. The hours constraint variable also has, of course, important effects in the consumption equations, where the "discouraged" terminology is generally not used.

The third comment concerns the question of real versus nominal interest rates. All of the interest rates considered in this study are nominal interest rates. The concept of a real interest rate is not needed. In the theoretical model a household solves its multiperiod optimal control problem after having formed expectations of future prices, wages rates, and interest rates. Any "real" interest rate effects are captured through these expectations and the other factors that affect the solution of the household's problem. In the empirical model, prices, wage rates, and interest rates are included together in the equations, along with lagged endogenous variables to capture expectational effects, and so any "real" interest rate effects should be picked up, at least in some approximate way, through these variables.

It should be noted in passing that interest rates have an effect on the decision variables of the household sector through the A_{t-1} variable, as well as directly. A_{t-1} is the value of the securities of the household sector at the end of period $t-1$. It has a positive effect on service and nondurable consumption in Table 2–3 and a negative effect on the labor force participation of persons 16 and over except men 25–54. A_{t-1} includes capital gains or losses on corporate stocks. The value of capital gains or losses during period $t-1$ (CG_{t-1}) is, as will be seen in Chapter Six, a negative function of the bond rate in period $t-1$. The bond rate in period $t-1$ is in turn a positive function of the bill rate in period $t-1$. Consequently, the bill rate in period $t-1$ has an effect on consumption and work effort in period t through its effect on A_{t-1}.

The fourth and final comment concerns the treatment of financial disequlibrium effects in the housing market. These effects are assumed to be captured in the model through the inclusion of the loan constraint variable in the equation explaining the stock of houses (Equation 4). This approach differs from the approach that I took in specifying the monthly housing starts sector in my forecasting model [14]. For the forecasting model separate equations explaining the supply of and demand for housing starts were specified and estimated, and the two equations were estimated under the assumption that the housing market is not always in equilibrium. The equations were estimated by one of the techniques described in Fair and Jaffee [24]. One of the key assumptions of this approach is the assumption that the observed quantity of a variable is equal to the minimum of the quantity demanded and the quantity supplied. In the case of housing, the assumption is that one observes the minimum of the demand for housing starts from the household sector and the equivalent supply of housing starts from the financial sector. In periods of disequilibrium, either the household sector is constrained by the financial sector from borrowing the amount of money that it wants to at the current prices and interest rates or the financial sector is prevented from making as many loans to finance housing investment as it wants to at the current interest rates.

Although the approach taken in the present study differs in important ways from my earlier approach, the two approaches are not inconsistent with each other. In the present specification, the loan constraint is at times binding on the household sector and at times not. When it is binding, it causes the household sector to spend less on housing than otherwise. This case corresponds in the earlier approach to the case in which the observed quantity of housing starts is equal to the equivalent supply from the financial sector. When the loan constraint is not binding, it has no effect on the housing expenditures of the household sector. This case corresponds in the earlier approach to the case in which the observed quantity of housing starts is equal to the demand from the household sector. (Both approaches assume that the supply of housing from the construction sector is never a constraint in the market. The questions of how the construction sector may at times be a constraint in the housing market and how one might handle this are discussed in Fair [14], Chapter 8, and Fair [16].) This comparison of the two approaches provides a good example of there being more than one way to specify disequilibrium effects. As mentioned in the previous section, it is clearly of interest in future work to consider alternative approaches.

NOTE

[a] The results in Fair [18] for sixteen age-sex groups indicate that labor force participation rates are responsive to the real wage, which is one of the reasons for imposing in this study the constraint in the work effort equations that the coefficients of the wage rate and price deflator be equal in absolute value.

Chapter Five

The Firm Sector

5.1 INTRODUCTION

The eleven stochastic equations that relate to the firm sector are discussed in this chapter. The equations explain the eleven variables that are listed on the right-hand side of Table 5-1. Table 5-1 contains for the firm sector a matching of the variables in the theoretical model to the related variables in the empirical model. The six most important variables explained in this chapter are: the price level (PF_t), production (Y_t), investment (INV_t), the number of jobs ($JOBF_t$), the average number of hours paid per job (HPF_t), and the wage rate (WF_t).

The treatment of the firm sector in the theoretical model was summarized in Chapter One, section 1.1. A firm's price, production, investment, employment, and wage rate decisions are determined simultaneously in the theoretical model through the solution of the firm's optimal control problem. The underlying technology of a firm is of the "putty-clay" type, and it may at times be optimal for a firm to plan to hold either excess labor or excess capital or both. Market share considerations and expectations play an important role in determining a firm's price and wage behavior. The two possible constraints on a firm are the loan constraint and the labor constraint.

Although a firm's decisions are determined simultaneously in the theoretical model, it is sometimes useful for descriptive purposes to consider the decisions as being made sequentially. This sequence is from the price decision, to the production decision, to the investment and employment decisions, to the wage rate decision. A firm should first be considered as having chosen its optimal price path. This path implies a certain expected sales path, from which the optimal production path can be chosen. Given the optimal production path, the optimal paths of investment and employment can be chosen. Finally, given the optimal employment path, the optimal wage rate path can be chosen. The optimal wage rate path is that path that the firm

Table 5–1. Matching of Dependent Variables in the Theoretical and Empirical Models for the Firm Sector

Decision Variable in the Theoretical Model (Notation for Condensed Model)	*Related Variables in the Empirical Model*
1. P_t (price level)	PF_t (deflator for $X_t - COM_t$)
2. Y_t (number of goods produced)	Y_t (production of the firm sector, B1958)
3. INV_t (number of goods purchased for investment purposes)	INV_t (nonresidential plant and equipment investment of the firm sector, B1958)
4. HPF_t (number of worker hours paid for)	$JOBF_t$ (number of jobs in the firm sector) HPF_t (average number of hours paid per job) $HPFO_t$ (average number of overtime hours paid per job)
5. W_t (wage rate)	WF_t (average hourly earnings, adjusted for overtime and interindustry employment shifts)
6. DDF_t (demand deposits)	DDF_t (demand deposits)
7. $DIVF_t$ (dividends paid)	$DIVF_t$ (dividends paid)
8. RL_tLF_t (interest paid)	$INTF_t$ (interest paid)
9. $(P_t - P_{t-1})V_{t-1}$ (inventory valuation adjustment)	IVA_t (inventory valuation adjustment)

expects is necessary to attract the amount of labor implied by its optimal employment path. It will be useful to keep this sequence in mind for the discussion in section 5.3.

Before discussing the stochastic equations, it is necessary to consider the measures of excess labor and excess capital that have been used. These measures are discussed in the next section. (This section can be skipped if desired without much loss of continuity.) The empirical model of the firm sector is outlined in section 5.3, and the equation estimates are explained in section 5.4. Section 5.5 contains a brief review of the model.

5.2 THE TECHNOLOGY OF THE FIRM SECTOR AND THE MEASUREMENT OF EXCESS LABOR AND EXCESS CAPITAL

Two possible ways of measuring the capital stock of the firm sector have been considered in this study. The first, more conventional way is to assume that the capital stock deteriorates at some rate δ_K each quarter and thus to postulate that:

$$K_t^a - K_{t-1}^a = INV_t - \delta_K K_{t-1}^a, \tag{5.1}$$

where K_t^a is the value of the capital stock (in real terms) in quarter t and INV_t is the value of plant and equipment investment of the firm sector (in real terms) in quarter t. For this measure of the capital stock, the production function of the firm sector is postulated to be:

$$Y_t = \min\{\lambda_t(M_t H_t^M), \mu_t(K_t^a H_t^K)\}, \tag{5.2}$$

where M_t is the number of workers employed, H_t^M is the number of hours worked per worker, H_t^K is the number of hours each unit of K_t^a is utilized, and λ_t and μ_t are coefficients that may change over time due to technical progress.

Equations (5.1) and (5.2) are not consistent with the putty-clay assumptions of the theoretical model. Each machine in the theoretical model wears out after m periods, but its productiveness does not lessen as it gets older. Machines do not change at all until age m, when they just fall apart completely. Consequently, even if there were only one type of machine ever in existence, Equation (5.1) would not be true. Rather, $K_t^a - K_{t-1}^a$ would equal $INV_t - INV_{t-m}$, where INV_{t-m} would be the number of machines that wear out at the end of period $t - 1$. It is also the case that no technical change was postulated in the theoretical model, but even if technical change were postulated, it would not enter in the way specified in Equation (5.2). Technical change would take the form of machines having different λ and μ coefficients according to when they were purchased. One could not write down an equation like (5.2), but would instead have to keep track of when each machine was purchased and what the coefficients were for that machine in order to be able to calculate how much output could be produced with the existing stock of machines. Equations (5.1) and (5.2) are thus at best only approximations to the production technology in the theoretical model.

Since Equations (5.1) and (5.2) are only approximations, a slightly different way of approximating the technology was tried to see if this led to better results. Consider INV_t to be the number of machines purchased in period t, and assume that these machines are all alike. Let μ_t stand for the amount of output that can be produced per machine hour on one of these machines. Assume, finally, that all machines wear out after m periods, but do not deteriorate physically before that time. Then the amount of output that can be produced per hour with all of the machines running is:

$$\frac{Y_t}{H_t^K} = \mu_t INV_t + \mu_{t-1} INV_{t-1} + \cdots + \mu_{t-m+1} INV_{t-m+1}, \tag{5.3}$$

where Y_t/H_t^K is output per hour when all machines are running. Associated with each machine is a λ_t coefficient, which is the amount of output that can be produced per worker hour on machines purchased in period t. Assume that all machines are used H_t^K hours, so that Y_t in Equation (5.3) is the actual amount of output produced. The number of worker hours required to produce Y_t in this case is:

$$M_tH_t^M = \frac{\mu_t INV_t H_t^K}{\lambda_t} + \frac{\mu_{t-1} INV_{t-1} H_t^K}{\lambda_{t-1}} + \cdots + \frac{\mu_{t-m+1} INV_{t-m+1} H_t^K}{\lambda_{t-m+1}}. \tag{5.4}$$

This second technology, which will be considered below, is thus represented by Equations (5.3) and (5.4).

Two variables that are needed for the estimation work in the next section are a variable that measures the amount of excess labor on hand and a variable that measures the amount of excess capital on hand. For the technology represented by Equations (5.1) and (5.2), these two variables were constructed in the following way. For the measurement of excess labor, output per paid for worker hour ($Y_t/(JOBF_tHPF_t)$) was first plotted for the 1952I–1975I period. The peaks of this series were assumed to correspond to cases where the number of worker hours paid for ($JOBF_tHPF_t$) equals the number of worker hours actually worked ($M_tH_t^M$). This assumption implies that values of λ_t in Equation (5.2) are observed at the peaks. The values of λ_t other than those at the peaks were then assumed to lie on straight lines between the peaks. Values of λ_t, in other words, were estimated from a peak-to-peak interpolation of the output per paid for worker hour series.

Given data series on λ_t and Y_t, a series on the number of worker hours required to produce Y_t, $M_tH_t^M$, is then merely Y_t/λ_t from Equation (5.2). This series can then be compared to the observed series on worker hours paid for, $JOBF_tHPF_t$, to determine the amount of excess labor on hand in any period. The quarters that were used as peaks for the interpolation are 1952I, 1953II, 1955II, and 1966I. The line drawn between the 1955II and 1966I peaks was extended beyond 1966I in determining the values of λ_t between 1966I and 1975I.

This procedure of constructing a series on $M_tH_t^M$ is the same as that used in Fair [23] and [14], the first for monthly seasonally unadjusted three-digit industry data and the second for quarterly seasonally adjusted data on the private nonfarm sector of the economy. It was argued in [23] that seasonally adjusted data should not be used to estimate production function parameters and worker hour requirements series because technical relationships are not likely to be subject to much seasonal variation. Unfortunately, however, much of the NIA data are not available on a seasonally unadjusted basis, and it is beyond the scope of this study to try to piece together enough data to be able to estimate the empirical model on a nonseasonally adjusted basis. Consequently, seasonally adjusted data have been used here, as well as in [14], in constructing the worker hour requirements series.

For the measurement of excess capital for the technology represented by Equations (5.1) and (5.2), a capital stock series first had to be constructed. Given data on INV_t, a series on K_t^a can be constructed once a base period value and a value for the depreciation rate δ_K are chosen. In a recent study [40], the Bureau of Economic Analysis (BEA) has estimated on an annual basis the fixed nonresidential business capital in the United States for the 1925–1973 period. The results of the BEA study were used here to estimate a base period value for K_t^a and a value of δ_K. The net stocks series on page 1 in [40] was first multiplied by 0.7 to scale it down to a series that pertains to the firm sector. 0.7 is roughly the ratio of plant and equipment investment in the firm sector to total plant and equipment investment. The net stocks series on page 1 in [40] pertains to all plant and equipment investment (firm, household, and financial). It is based on the assumptions of straight line depreciation and service lives equal to 85.0 percent of Bulletin F.

The base period for K_t^a was taken to be 1952IV, and the base period value was taken to be 197.2 billion (1958) dollars. This latter figure is 0.7 times the value on page 1 in [40] for the end of 1952. From this base period, various values of δ_K were used to generate different capital stock series, using the formula:

$$K_t^a = (1 - \delta_K)K_{t-1}^a + INV_t. \tag{5.5}$$

These series were compared to the "actual" series derived from [40] to see which value of δ_K most closely reproduced the actual series. It was apparent from this exercise that one value of δ_K for the whole period was not adequate to approximate the actual series at all accurately. There appeared to be a shift around 1966 in the value needed for δ_K, a larger value being needed after 1966.

In the end, two values of δ_K were chosen, a value of 0.0255 before 1966I and a value of 0.0285 from 1966I on. The use of the value of 197.2 for K_t^a in 1952IV and the value of 0.0255 for δ_K resulted in a value of 308.9 for K_t^a in 1965IV, which compares quite closely to the actual value of 308.6. The value for K_t^a in 1965IV was taken to be 308.9, and from this base the rest of the K_t^a series was generated using the value of 0.0285 for δ_K. The generated value of K_t^a for 1971IV was 404.4 (see Table 2-1), which compares fairly closely to the actual value of 406.3. The actual series from the BEA could not be used directly here because it is annual and because of the necessity of having a link between the investment series (INV_t) and the capital stock series.

Regarding the measurement of excess capital, there are no data on hours paid for or worked per unit of K_t^a, and so, given a series on K_t^a, one must be content with plotting Y_t/K_t^a. This is, from Equation (5.2), a plot of $\mu_t H_t^K$, where H_t^K is the average number of hours that each machine is utilized. If it is assumed that at each peak of this series H_t^K is equal to the same constant, say $\overline{H}$, then one observes at the peaks $\mu_t\overline{H}$. Interpolation between peaks can then produce a complete series on $\mu_t\overline{H}$. If, finally, $\overline{H}$ is assumed to be the maximum number of hours per period that each unit of K_t^a can be utilized, then $Y_t/(\mu_t\bar{H})$ is the minimum amount of capital required to produce Y_t. This variable is denoted as $KMIN_t$ in Table 2–2.

The observations that were used for the peaks are 1953II, 1966I, and 1973I. The values of $\mu_t\bar{H}$ between 1973II and 1975I were all taken to be equal to the 1973I value. The line drawn between 1953II and 1966I had a positive slope, but the line drawn between 1966II–1973I had a slight negative slope. There seemed to be some evidence of a slight deterioration in output per machine hour after 1966I. It is true, however, that the plot of Y_t/K_t^a over the entire 1952I–1975I period showed little evidence of either a positive or a negative trend. The slopes of both of the interpolation lines were fairly small.

This takes care of the measurement of excess labor and excess capital for the technology represented by Equations (5.1) and (5.2). Consider next the measurement of excess capital for the technology represented by Equations (5.3) and (5.4). The BEA study [40] was first used to get an estimate of m, the length of life of one unit of capital. The BEA study presents estimates of both the gross and net capital stocks, and for purposes of estimating m the gross capital stock series on page 1 in [40] was used. If it is assumed that machines do not physically depreciate until age m, when they fall apart, an estimate of m can be obtained by summing past values of gross investment (also presented in [40]) until the sum is equal to the BEA estimate of the gross capital stock. The number of periods that one uses in this sum is an estimate of m. This procedure can be followed for each yearly estimate of the gross capital stock. One will not, of course, necessarily get the same estimate of m for each year. It was quite evident when carrying out this procedure for the 1952–1972 period that m began to get much smaller in the 1960s, a result that is consistent with having to use a larger value of δ_K beginning around 1966 to approximate the net capital stock series. There is nothing in the following analysis that requires m to be constant over time, and so instead of choosing only one or two values of m, an entire time series for m (denoted as m_t) was constructed from the BEA gross investment and gross capital stock data.

Given a series for m_t, the next step in the construction of an excess capital series was to get estimates of the μ_t series in Equation (5.3). (Equation (5.3) should now be modified by adding a t subscript to m.) To do this, it was first assumed that $\mu_t = \bar{\mu}(1 + \delta)^t$, where $\bar{\mu}$ and δ are parameters to be estimated. If δ is zero, then μ_t is constant over time; otherwise μ_t is changing at rate δ each period. Next, a few quarters were chosen where it seemed plausible to assume that all machines were utilized $\bar{H}$ hours. These quarters, in other words, were assumed to be quarters in which the amount of excess capital on hand was zero. If quarter s is one of these quarters, then it is the case from Equation (5.3), and the assumption just made about μ_t, that:

$$Y_s = \bar{\mu}\bar{H}[(1 + \delta)^s INV_s + (1 + \delta)^{s-1} INV_{s-1} + \cdots + (1 + \delta)^{s-m_s+1} INV_{s-m_s+1}]. \quad (5.6)$$

Given data on investment, output, and m_t and given two quarters for which Equation (5.6) holds, one has two equations in two unknowns, the unknowns being $\bar{\mu}\bar{H}$ and δ. The two equations are nonlinear, but they can easily be solved numerically. If one has more than two quarters for which Equation (5.6) is assumed to hold, then different pairs of equations can be solved to see, among other things, how sensitive the solution values are to alternative pairs.

Values of δ and $\bar{\mu}\bar{H}$ were computed in this way for alternative pairs of equations, and it turned out that a value of zero for δ seemed quite consistent with the data. There did not appear, in other words, to be any evidence of capital's getting either more efficient or less efficient over time in terms of output per unit of capital. This result is consistent with the observation made earlier that the plot of Y_t/K_t^a for the first technology showed little evidence of a trend.

If δ is zero, then one can merely sum up past values of investment to get a measure of the capital stock:

$$K_t^a = INV_t + INV_{t-1} + \cdots + INV_{t-m_t+1}. \tag{5.7}$$

An estimate of the minimum amount of capital required to produce Y_t can in this case be obtained as merely $Y_t/(\bar{\mu}\bar{H})$, where $\bar{\mu}\bar{H}$ is estimated from solving one of the pairs of equations discussed above. It turned out that the estimates of $\bar{\mu}\bar{H}$ were roughly the same for alternative pairs of equations (with estimates of δ of approximately zero), so that it did not matter very much which pair of equations was used to estimate $\bar{\mu}\bar{H}$. The value of $\bar{\mu}\bar{H}$ that was chosen for the work below is 0.2660.

For the measurement of excess labor for this technology, it was first assumed that $\lambda_t = \bar{\lambda}(1 + \delta_\lambda)^t$. A few quarters were then chosen where it seemed plausible to assume that all machines were utilized $\bar{H}$ hours (no excess capital) and that the number of worker hours paid for equals the number of worker hours actually worked (no excess labor). If quarter s is one of these quarters, then it is the case from Equation (5.4) and the assumption just made about λ_t that:

$$JOBF_sHPF_s = \frac{\bar{\mu}\bar{H}}{\bar{\lambda}}\left[\frac{INV_s}{(1+\delta_\lambda)^s} + \frac{INV_{s-1}}{(1+\delta_\lambda)^{s-1}} + \cdots + \frac{INV_{s-m_s+1}}{(1+\delta_\lambda)^{s-m_s+1}}\right]. \tag{5.8}$$

Given data on worker hours paid for, investment, and m_t and given two quarters for which Equation (5.8) holds, one has two equations in two unknowns, $\bar{\mu}\bar{H}/\bar{\lambda}$ and δ_λ. Again, the equations are nonlinear, but they can easily be solved numerically. It turned out that the estimates of $\bar{\mu}\bar{H}/\bar{\lambda}$ and δ_λ were not highly sensitive to the choice of alternative pairs of equations to solve, but in the end two sets of estimates were considered. The two quarters chosen for the first set were 1953II and 1966I, and for these quarters the solution values were 118894.4 for $\bar{\mu}\bar{H}/\bar{\lambda}$ and 0.005204 for δ_λ. The two quarters chosen for the second set were 1953II and 1968II, with solution values of 121927.8 and 0.005602.

From the above information it is now possible to compute a series on worker hour requirements. Since δ_λ is positive, it is always optimal for a firm to utilize the newer machines first. Therefore, given Y_t and the estimate of $\bar{\mu}\bar{H}$, it is possible to compute from Equation (5.6), using also data on investment and the estimate of zero for δ, the age of the oldest machine operating in quarter t in the production of Y_t. This age—call it $\bar{m}_t$—will not be equal to the age of the oldest machine in existence in quarter t (denoted above as m_t) except for those quarters for which there is no excess capital on hand. Now, given values for $\bar{m}_t$, $\bar{\mu}\bar{H}/\bar{\lambda}$, δ_λ, and investment, one can compute from Equation (5.8) the number of worker hours required in period t to produce Y_t. This procedure can be carried out for each quarter, and so a series on worker hour requirements, $M_tH_t^M$, can be constructed.

It turned out that the two different sets of assumptions about the technology of the firm sector led to similar results. Some of these results are presented and discussed in Appendix A to this volume. In the end, the first technology was chosen to be used in the model because of its simpler nature. The fact that the two sets of results were similar means that the aggregate data used in this study do not appear to be capable of discriminating among alternative assumptions about the technology. Both technologies are clearly approximations, and what the data seem to indicate is that both approximations are about as equally good or as equally bad. The purpose of presenting both technologies in this chapter is to show that the results of this study do not appear to be sensitive to the choice of the technology for the model.

In the theoretical model it was possible for a firm to substitute capital for labor (or vice versa) over time through the purchase of different types of machines with differing worker-machine ratios. The type of machine that it was optimal for a firm to purchase in any one period resulted from the solution of its optimal control problem in the period. With the aggregate data used here, it seems highly unlikely that one would be able to pick up substitution effects of this sort, especially considering the fact that the data do not even appear to be capable of discriminating between the two somewhat different technologies considered above.

Using three digit industry data, some evidence was found in Fair [15] for the existence of capital-labor substitution of the kind just outlined, but the aggregate data used in this study do not permit the kind of test that was performed in [15]. Consequently, no attempt was made here to try to estimate the effects of this type of capital-labor substitution. This does not mean, however, that the cost of capital has no effect on the investment of the firm sector in the present model. This issue is discussed in section 5.5.

5.3 AN OUTLINE OF THE EMPIRICAL MODEL OF THE FIRM SECTOR

As was the case for the household sector, it is necessary regarding the firm sector to distinguish between its unconstrained and constrained decisions. The loan constraint on the firm sector can be handled in the same way that it was handled for the household sector, namely by including log ZR_t as an explanatory variable in the estimated equations (the equations being in log form). Under the assumptions made in the last chapter, adding this variable to an equation converts the left-hand side variable from an unconstrained decision variable to a constrained decision variable.

The treatment of the labor constant on the firm sector requires considerably more explanation. The labor constraint relates to the fact that a firm may not get as much labor in a period as it expected that it would at the wage rate that it set and may thus be forced to produce in the period less than it planned to. In other words, although the variable $JOBF_tHPF_t$ is the number of worker hours actually paid for by the firm sector in quarter t (see Table 5–1), it is not necessarily the number the firm sector planned to pay for. $JOBF_tHPF_t$ will be less than the planned number if the labor constriant is binding. When the labor constraint is binding on the firm sector, $JOBF_tHPF_t$ is determined by the household sector. Otherwise, it is determined by the firm sector. There are thus two regimes to consider regarding the determination of $JOBF_tHPF_t$.

In order to consider the two regimes problem in more detail, government employment must first be taken into account. The total number of worker hours paid for by the government sector in quarter t is, using the notation in Table 2–1, $JOBGC_tHPGC_t + JOBGM_tHPGM_t$. It will be useful for the present discussion to denote this variable as $MHPG_t$. The total number of hours that the household sector is paid for is, therefore, $JOBF_tHPF_t + MHPG_t$, which is also denoted as $JOBH_tHPH_t$ in Table 4–1. If the household sector is not constrained in its work effort, then it determines $JOBH_tHPH_t$.

If it is assumed, as is done in the theoretical model, that the government sector always gets the amount of labor that it wants, then in those cases where the household sector is not constrained in its work effort, $JOBF_tHPF_t$ is determined as the difference between $JOBH_tHPH_t$ and $MHPG_t$. This amount of labor may, as just mentioned, be less than the amount of labor that the firm sector planned at the beginning of the period to hire. If, on the other hand, the household sector is constrained in its work effort, then $JOBF_tHPF_t$ is determined by the firm sector, and $JOBH_tHPH_t$ is determined as the sum of $JOBF_tHPF_t$ and $MHPG_t$.

One possible approach to the two regimes problem would be to break up the sample period some way into two regimes and estimate separate equations in the two regimes. In one regime an equation explaining $JOBH_t HPH_t$ would be estimated, with $JOBF_t HPF_t$ being determined as the residual; and in the other regime an equation explaining $JOBF_t HPF_t$ would be estimated, with $JOBH_t HPH_t$ being determined as the residual. The either/or nature of this approach, however, has the disadvantage of making the results more sensitive to the choice of regimes than one might want. Since any procedure of choosing regimes is not error free, one would like to design a model that is not highly sensitive to errors made in choosing regimes.

In order to see how the two regimes problems was handled here, it is necessary to consider first a rough outline of the equations explaining the main decision variables of the firm sector. The following outline is based on the assumption that the loan constraint is not binding on the firm sector and on the assumption that sales expectations for the current period are perfect. A superscript p on a variable denotes the planned value of the variable, the plans being made at the beginning of period t. Consider the following seven equations:

$$PF_t^p = f_1(\cdots), \tag{5.9}$$

$$X_t^p = f_2(PF_t^p, \ldots), \tag{5.10}$$

$$Y_t^p = f_3(X_t^p, \ldots), \tag{5.11}$$

$$INV_t^p = f_4(Y_t^p, \ldots), \tag{5.12}$$

$$JOBF_t^p = f_5(Y_t^p, \ldots), \tag{5.13}$$

$$HPF_t^p = f_6(Y_t^p, \ldots), \tag{5.14}$$

$$WF_t^p = f_7(PF_t^p, JOBF_t^p HPF_t^p, \ldots). \tag{5.15}$$

PF_t^p is the price the firm sector plans to set. The variables that explain PF_t^p will be discussed later. X_t^p is the number of goods the firm sector plans to sell in period t. It is a function of PF_t^p and other variables. Y_t^p is the number of goods the firm sector plans to produce in period t. It is a function of X_t^p and other variables. INV_t^p is the amount of investment the firm sector plans to make in period t. It is a function of Y_t^p and other variables. $JOBF_t^p HPF_t^p$ is the number of worker hours the firm sector plans to pay for in period t. $JOBF_t^p$ and HPF_t^p are explained separately in the model; both are functions of Y_t^p and other variables. Finally, WF_t^p is the wage rate that the firm sector expects it will have to pay to attract the planned amount of labor. It is a function of PF_t^p, $JOBF_t^p HPF_t^p$, and other variables. Equations (5.9)–(5.15) are consistent with the decision sequence discussed at the end of section 5.1.

Let $JOBF_t^s HPF_t^s$ denote the supply of labor to the firm sector from the household sector at the wage rate WF_t^p. If this supply is greater than or equal to $JOBF_t^p HPF_t^p$, then all the plans of the firm sector can be realized. The planned values in (5.9)–(5.15) can be taken to be the observed values. If, on the other hand, the supply is less than $JOBF_t^p HPF_t^p$, the firm sector has to adjust. One must thus decide when the firm sector has to adjust and how it adjusts when it has to. With respect to the question of when the firm sector adjusts, assume for now that one has found a variable ZJ_t' that takes on a value of one when the firm sector does not have to adjust and a value of less than one otherwise. The construction of ZJ_t' will be explained later.

Consider now the question of how the firm sector adjusts when it receives less labor than it expects. The firm sector is assumed to adjust in this case by raising its price, thus cutting sales, and lowering its production, investment, and employment. In particular, it is assumed that:

$$\frac{PF_t}{PF_t^p} = (ZJ_t')^{\gamma_3}, \ \gamma_3 < 0, \tag{5.16}$$

where PF_t is the observed price. The price is assumed to be raised enough in the constrained case ($ZJ_t' < 1$) so as to lead to the new values of $JOBF_t^p$ and HPF_t^p chosen by the firm sector to be equal to the supply from the household sector. The final set of equations for the firm sector is then postulated to be:

$$PF_t = (ZJ_t')^{\gamma_3} PF_t^p = (ZJ_t')^{\gamma_3} f_1(\cdots), \tag*{(5.9)$'$}$$

$$X_t = f_2(PF_t, \ldots), \text{ [this equation stands for a number of equations in the model]} \tag*{(5.10)$'$}$$

$$Y_t = f_3(X_t, \ldots), \tag*{(5.11)$'$}$$

$$INV_t = f_4(Y_t, \ldots), \tag*{(5.12)$'$}$$

$$JOBF_t = f_5(Y_t, \ldots), \tag*{(5.13)$'$}$$

$$HPF_t = f_6(Y_t, \ldots), \tag*{(5.14)$'$}$$

$$WF_t = f_7(PF_t, JOBF_t HPF_t, \ldots), \tag*{(5.15)$'$}$$

where all the variables are now observed variables. The possible labor constraint on the firm sector was thus handled by adding to the price equation, which is in log form, the term $\gamma_3 \log ZJ_t'$.

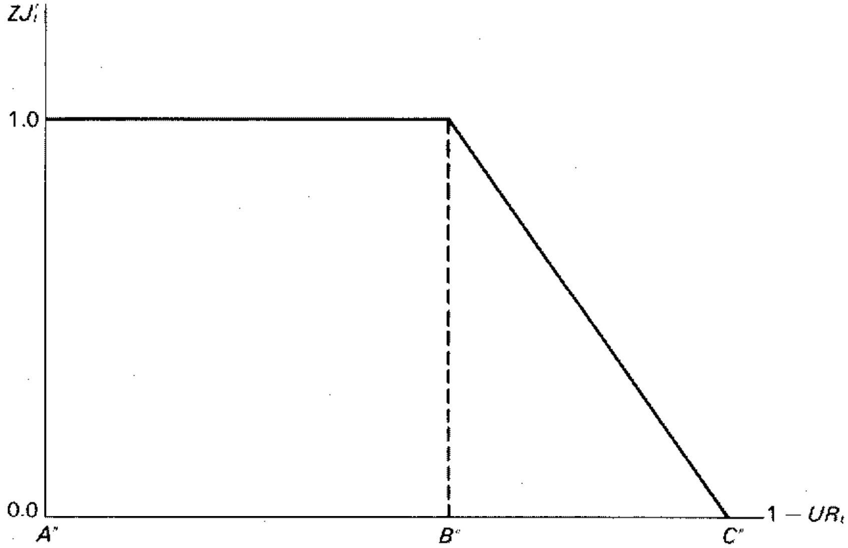

Figure 5-1. Desired Shape of ZJ_t' as a Function of $1 - UR_t$

The construction of ZJ_t' will now be explained. Although J_t^* was used as the measure of labor market tightness in the last chapter in computing ZJ_t, one minus the unemployment rate, $1 - UR_t$, is used as the measure of labor market tightness in this chapter in computing ZJ_t'. The reason for this difference is explained below. The desired shape of ZJ_t' as a function of $1 - UR_t$ is depicted in Figure 5-1. Point A'' is some value that is smaller than the smallest value of $1 - UR_t$ observed in the sample period; point B'' is the value of $1 - UR_t$ below which it seems reasonable to assume that the firm sector always gets as much labor as it expected; and point C'' is some value that is larger than the largest value of $1 - UR_t$ observed in the sample period.

If one wants ZJ_t' to equal 0.0 when $1 - UR_t$ equals C'', which, as explained in the next paragraph, is wanted here, then the right half of the normal density function cannot be used to approximate the curve in Figure 5-1. Instead, the following equation was used for the approximation:

$$ZJ_t' = \alpha_3 + \frac{1}{1 - UR_t - \alpha_4}, \qquad (5.17)$$

where α_3 and α_4 are chosen so that ZJ_t' equals 1.0 when $1 - UR_t$ equals A'' and 0.0 when $1 - UR_t$ equals C''. The value chosen for A'' was 0.910 (a 9.0 percent unemployment rate), and the value chosen for C'' was 0.975 (a 2.5 percent unemployment rate). These values are slightly outside the range of observed values of $1 - UR_t$ in the sample period. For these two values, the values of α_3 and α_4 that lead to the above requirements being met are 4.454062 and 1.199514, respectively.

The procedure just described for constructing ZJ'_t constrains the unemployment rate always to lie above 2.5 percent in the model. When $1 - UR_t$ is equal to 0.975, ZJ'_t is equal to zero, which from Equation (5.9)′ implies (for $\gamma_3 < 0$) a value of PF_t of infinity. It turned out that the single equation fit of the price equation and the fit of the overall model were not very sensitive to the use of alternative values of the minimum unemployment rate. This result is not particularly surprising since during most of the sample period the economy was operating considerably above an unemployment rate of 2.5 percent. As a general rule, one would not expect the fit of a model to be sensitive to the imposition of a constraint on the behavior of the model regarding values that lie outside the range of values observed in the sample period. The constraint was imposed here not for any goodness-of-fit reasons, but to guarantee that the unemployment rate would never be driven below 2.5 percent in the optimal control experiments in Chapter Ten. It does seem unlikely that the unemployment rate in the United States could be driven much below 2.5 percent, and so the lower bound of 2.5 percent was imposed on the model.

The desire to impose this constraint on the model is the reason for the use of $1 - UR_t$ rather than J^*_t as the measure of labor market tightness for the construction of ZJ'_t. For the solution of the model, the predicted value of ZJ'_t was set equal to one whenever the predicted value of UR_t was greater than or equal to 9.0 percent. Otherwise, Equation (5.17) (Equation 78 in Table 2–2) was used to determine the predicted value of ZJ'_t. This procedure is similar to the procedures followed for ZJ_t and ZR_t, which were discussed in the last chapter.

As was the case for the treatment of the hours constraint on the household sector, the treatment of the labor constraint on the firm sector has the advantage of allowing the data some flexibility in estimating the effects of the constraint. No a priori constraints of a zero-one type are imposed on the data. The procedure followed here does have the disadvantage, however, of not necessarily using all the information on the labor market that is available. Only equations for $JOBF^p_t$ and HPF^p_t have been estimated; no attempt has been made to estimate also equations explaining $JOBH^s_t$ and HPH^s_t. When the hours constraint is not binding on the household sector, $JOBH^s_t$ and HPH^s_t are equal to the observed values ($JOBH_t$ and HPH_t). Since it is known from the theoretical model what variables affect $JOBH^s_t$ and HPH^s_t (the variables listed in Table 4–2), one has two potential equations to estimate that have not been estimated.

In order to put the present treatment of the labor constraint on the firm sector in a somewhat better perspective, it will be useful to review briefly three other ways that one could consider dealing with the demand for and supply of labor in the model. The present approach converts the two demand equations (explaining $JOBF_t^p$ and HPF_t^p) into equations with observed left-hand side variables. No explicit equations are postulated for $JOBH_t^s$ and HPH_t^s. Another approach would be to postulate also equations explaining $JOBH_t^s$ and HPH_t^s, convert the equations in some manner into equations with observed left-hand side variables, and then estimate the two resulting equations. One could perhaps convert the equations into equations with observed left-hand side variables by the use of some sort of a Z_t variable, as has been done in this study for other equations. Taking the government employment variables to be exogenous and using the definitions, $JOBF_t = JOBH_t - JOBGC_t - JOBGM_t$ and $JOBF_tHPF_t = JOBH_tHPH_t - JOBGC_t HPGC_t - JOBGM_tHPGM_t$, this approach would result in two equations explaining $JOBF_t$ (one in the firm sector and one in the household sector) and two explaining HPF_t. In solving the model the predicted values of $JOBF_t$ and HPF_t could be taken to be some weighted average of the predictions from the two equations for each variable.

A second alternative approach would be to postulate explicitly that the observed $JOBF_t$ is equal to the minimum of $JOBF_t^p$ and $JOBH_t^s - JOBGC_t - JOBGM_t$ (and similarly for HPF_t) and to use some of the recent econometric techniques that have been developed for estimating markets in disequilibrium to estimate the equations. (For a discussion of these techniques, see Fair and Jaffee [34], Fair and Kelejian [25], Ameniya [1], and Maddala and Nelson [32].)

A third alternative approach, but one that is not consistent with the specification of the theoretical model, would be to assume that the wage rate adjusts each quarter to clear the labor market, drop the equation explaining the wage rate from the model, and estimate separate equations explaining $JOBF_t^pHPF_t^p$ and $JOBH_t^sHPH_t^s$. In this case both $JOBF_t^pHPF_t^p$ and $JOBH_t^sHPH_t - JOBGC_tHPGC_t - JOBGM_tHPGM_t$ would always be equal to the observed value ($JOBF_tHPF_t$).

To summarize, there are clearly a number of other ways of dealing with the labor market than the approach taken here. The present approach has the advantages of flexibility and computational ease, but it does throw away some potentially important information. In future work it would be of interest to consider alternative approaches.

Before considering the other variables that appear in f_1 through f_7 in equations (5.9)′–(5.15)′, it should be noted that the inclusion of ZJ_t' in the price equation has introduced simultaneity into the model where there did not exist any before. PF_t affects X_t, which affects Y_t, which in turn affects $JOBF_t$. $JOBF_t$ affects UR_t, which in turn affects ZJ_t'. Consequently, PF_t has an effect on ZJ_t', as well as vice versa. Since in theory (i.e., not considering the approximation for ZJ'_t that has been used) ZJ'_t only enters the price equation in the case of a binding labor constraint on the firm sector, there is only simultaneity of the kind just described in the constrained case. The simultaneity takes the form in the constrained case of links between the expenditure equations in the household sector and the price equation in the firm sector.

One should think of the simultaneity in the constrained case as reflecting the outcome of a number of interactions between the household and firm sectors within the quarter. It is important to think this way to justify the rather strong assumption made following Equation (5.16) that the price is always raised enough in the constrained case so as to lead to the new values of $JOBF_t^p$ and HPF_t^p being equal to the supply from the household sector. It should also be noted that as the price is raised during this within-quarter adjustment process, the wage rate is also likely to be raised. Increasing the wage rate increases, of course, the supply of labor from the household sector. It is not, however, theoretically unambiguous that the wage rate will rise in this case. The effect on the wage rate is not unambiguous, because while the higher price level has a positive effect on the wage rate, the decrease in employment demand from the contraction of the firm sector has a negative effect.

5.4 THE ESTIMATES OF THE EQUATIONS FOR THE FIRM SECTOR

The eleven stochastic equations for the firm sector are Equations 9–19 in Table 2–3. The most important equations are 9, 10, 11, 12, 13, and 15, explaining, respectively, PF_t, Y_t, INV_t, $JOBF_t$, HPF_t, and WF_t. The following is a discussion of each of the eleven equations.

The PF_t Equation

Equation 9, explaining PF_t, is in log form and includes as explanatory variables the price of imports (PIM_t), the wage rate lagged one period (WF_{t-1}), the bond rate ($RAAA_t$), an investment tax credit variable ($DTAXCR_t$), the labor constraint variable (ZJ'_t), and PF_{t-1}.

The bond rate has a positive effect on PF_t, and the investment tax credit variable has a negative effect. $DTAXCR_t$ is defined in Table 2–1. It takes on a value of 1.0 when the credit of 7 percent is in full force, a value of 0.0 when the credit is not in force, and a value of 0.5 when the credit is estimated to about half in force. The value of 0.5 was used for the 1962III–1963IV, when the Long amendment was in effect, and for 1971III, when the credit was in effect for only about half of the quarter.

The inclusion of $RAAA_t$ and $DTAXCR_t$ in the price equation was guided by the results for the theoretical model. In the theoretical model the interest rate had a positive effect on the price that a firm sets. An increase in the interest rate, for example, caused a firm to contract, and the way that a firms contracts in the theoretical model is to raise its price, thus lowering expected sales, and to decrease its production, investment, and employment. The inclusion of $RAAA_t$ and $DTAXCR_t$ in Equation 9 should thus be considered an attempt to pick up this effect. Both variables are taken to measure part of the cost of capital to the firm sector.

The inclusion of the price of imports, the wage rate lagged one period, and the price level itself lagged one period in the price equation is designed to try to pick up expectational effects. As mentioned in Chapter One, section 1.1, a firm's expectations of other firms' prices plays an important role in the theoretical model in determining the price that the firm sets for the period. After some experimentation, the three variables just mentioned were chosen to represent expectational effects in the empirical model. Any choice of this sort is, of course, only a rough approximation to the actual way that expectations are formed.

Four variables that had some influence in the theoretical model on the price that a firm set were not found to be important in the experimentation that was done here. These four variables are the ratio of the stock of inventories to the level of sales, the level of sales itself, the amount of excess labor on hand, and the amount of excess capital on hand, all of the previous period.

Some experimentation was done, primarily through the use of dummy variables, to see if the effects of price controls should be taken into account in the estimation of the price equation. The only two quarters in which there appeared to be any important effects were 1971IV (the quarter affected by the first price freeze), where the price level seemed to change less than it otherwise would have, and 1972I (the quarter following the lifting of the freeze), where the price level seemed to change more than it otherwise would have. The smaller change in 1971IV seemed to be offset by the larger change in 1972I. When, for example, the dummy variables $D714_t$ and $D721_t$ were added to the PF_t equation, the coefficient estimates for the two variables were -0.00352 and 0.00684, respectively, with t-statistics of -1.14 and 2.28. The other coefficient estimates changed very little from the addition of the two dummy variables to the equation. Because of the small changes in the other coefficient estimates, and because there were no other quarters in which the effects of price controls seemed to be important, it was decided to ignore price controls altogether in the model and lump any effects from the controls into the error term in the equation. Price and wage controls may have some effects on the aggregate variables considered in the model, but the effects seem small enough to be able to be ignored with little harm.

The Y_t Equation

Equation 10 explains the production of the firm sector, Y_t. The equation is in log form and includes as explanatory variables the level of production of the previous period (Y_{t-1}), the current level of sales (X_t), the stock of inventories at the end of the previous period (V_{t-1}), and three dummy variables to account for the effects of the steel strike in 1959. This equation is based on the assumption that the firm sector first sets its price, knows then what its sales for the current period will be, and from this latter information decides on what its production for the current period will be. In practice, information on sales is available and decisions on production are made more than once during a quarter, and so firms have some flexibility within a quarter to adjust their production to unexpected changes in sales.

The assumption just made that sales expectations for the current quarter are perfect implies that firms can adjust their production completely during a quarter to any unexpected change in sales. This does not mean that firms are always assumed to plan to produce what they expect to sell, only that, given their plans and sales expectations at the beginning of the quarter, they can adjust their plans as actual sales deviate from expected sales. Some experimentation was done, using alternative assumptions about the formation of sales expectations, to see if production could be better explained under some other assumption than the assumption that sales expectations within a quarter are perfect, but this did not appear to be the case.

In the theoretical model production is smoothed relative to sales —i.e., the optimal production path of a firm generally has less variance than its expected sales path. This is because of various costs of adjustment that *have been postulated in the model. The two most important adjustment costs* are costs of changing employment and costs of changing the capital stock. There are also costs included in having the stock of inventories deviate from β_1 times sales, where $\beta_1 > 0$. If a firm were only interested in minimizing these latter costs, it would produce in period t according to the following equation (assuming perfect sales expectations for period t):

$$Y_t = X_t + \beta_1 X_t - V_{t-1}. \tag{5.18}$$

Since by definition $V_t - V_{t-1} = Y_t - X_t$, producing according to Equation (5.18) would insure that $V_t = \beta_1 X_t$.

Since there are other adjustment costs, it is generally not optimal for a firm to produce according to Equation (5.18). In the theoretical model there was no need to postulate explicitly how a firm's production plan deviated from Equation (5.18) because its optimal production path just resulted, along with the other optimal paths, from the direct solution of its optimal control problem. In the present case, however, it is necessary to postulate an explicit equation explaining the firm sector's production decision, an equation that can be considered to be an approximation to the way the firm sector actually makes its production decision. The standard assumption to make regarding the effects of adjustment costs on behavior is, in the present context, the following:

$$Y_t - Y_{t-1} = \lambda(Y_t^* - Y_{t-1}), 0 < \lambda \leqslant 1, \tag{5.19}$$

where, say, Y_t^* is the Y_t in Equation (5.18):

$$Y_t^* = X_t + \beta_1 X_t - V_{t-1}. \tag*{(5.18)$'$}$$

Equation (5.19) states that the actual change in production in period t is some proportion of the change that the firm would make if it were only interested in minimizing the costs of having V_t deviate from $\beta_1 X_t$. Substituting Equation (5.18)′ into (5.19) yields:

$$Y_t = (1 - \lambda)Y_{t-1} + \lambda(1 + \beta_1)X_t - \lambda V_{t-1}. \tag{5.20}$$

Equation 10 in Table 2–3 is similar to Equation (5.20) except that it is in log form and has added to it a constant term and three dummy variables. It is also the case that the restrictions on the coefficients in Equation (5.20) have not been imposed in Equation 10. There are three variables on the right-hand side of Equation (5.20), but only two coefficients. Since Equations (5.19) and (5.18)′ are considered to be only a rough approximation to the production decision of the firm sector, the imposition of the restrictions in (5.20) did not seem warranted. The lagged output term in Equation 10 should be considered as picking up only in some rough way the effects of adjustment costs on the current production decision of the firm sector.

The production equation estimated here is consistent with the equation estimated for the lagged adjustment model in Fair [21]. The data used in [21] were monthly, seasonally unadjusted, three digit industry data, and for these data significant effects of future sales expectations were obtained. One would expect, if firms smooth production relative to sales, that the current production decision would depend in part upon expected future sales. This certainly appeared to be the case for the data used in [21], where significant effects of up to six months ahead were obtained. For the aggregate data used in this study, however, it did not appear to be possible to pick up any significant effects of future sales expectations on current production.

The INV_t Equation

Equation 11 explains the investment in plant and equipment of the firm sector, INV_t. The equation is in linear form, with the left-hand side variable being the change in investment. The explanatory variables include the amount of excess capital on hand at the end of the previous period ($K^a_{t-1} - KMIN_{t-1}$), the current change in output, the change in output lagged one, two, and three periods, the difference between gross investment and depreciation of the previous period ($INV_{t-1} - \delta_K K^a_{t-1}$), and two dummy variables to account for the effects of the automobile strike in 1970. The equation is based on the assumption that the firm sector decides on its level of production before deciding on its level of investment.

As was the case for the production decision, it is necessary with respect to the investment decision to postulate in the empirical model an explicit equation explaining this decision. The investment decision of a firm in the theoretical model results from the solution of its optimal control problem, and some approximation to this decision must be made here. Adjustment costs play an important role in the theoretical model in influencing a firm's investment decision, and because of these costs, it is sometimes optimal for a firm to hold excess capital. It is also the case, not surprisingly, that the amount of excess capital on hand at the beginning of a firm's decision period has an important effect on the solution values obtained in that period, especially on the solution values for investment.

Equation 11 is based on the following three equations:

$$
\begin{aligned}
(K^a_t - K^a_{t-1})^* = {} & \alpha_0(K^a_{t-1} - KMIN_{t-1}) + \alpha_1(Y_t - Y_{t-1}) + \alpha_2(Y_{t-1} - Y_{t-2}) \\
& + \alpha_3(Y_{t-2} - Y_{t-3}) + \alpha_4(Y_{t-3} - Y_{t-4}),
\end{aligned}
\tag{5.21}
$$

$$
INV^*_t = (K^a_t - K^a_{t-1})^* + \delta_K K^a_{t-1},
\tag{5.22}
$$

$$
INV_t - INV_{t-1} = \lambda(INV^*_t - INV_{t-1}), 0 < \lambda \leqslant 1.
\tag{5.23}
$$

For sake of the current discussion, call $(K^a_t - K^a_{t-1})^*$ in (5.21) "desired net investment" and call INV^*_t in (5.22) "desired gross investment." Equation (5.21) states that desired net investment is a function of the amount of excess capital on hand and of four change-in-output terms. If output is not changing and has not changed for the past four periods, and if there is no excess capital on hand, then desired net investment is zero. The past change-in-output terms in Equation (5.21) can best be thought of as being proxies for expected future output terms.

Equation (5.22) relates desired gross investment to desired net investment. $\delta_K K^a_{t-1}$ is the physical depreciation of the capital stock during period $t - 1$, δ_K being the estimated depreciation rate of the capital stock. By definition $INV_t = K^a_t - K^a_{t-1} + \delta_K K^a_{t-1}$, and Equation (5.22) is merely this same equation for the desired values. Equation (5.23) is a stock-adjustment equation relating the desired change in gross investment to the actual change. Equation (5.23) is meant to approximate cost-of-adjustment effects.

Combining Equations (5.21)–(5.23) yields:

$$\begin{aligned} INV_t - INV_{t-1} = {} & \lambda\alpha_0(K^a_{t-1} - KMIN_{t-1}) + \lambda\alpha_1(Y_t - Y_{t-1}) \\ & + \lambda\alpha_2(Y_{t-1} - Y_{t-2}) + \lambda\alpha_3(Y_{t-2} - Y_{t-3}) \\ & + \lambda\alpha_4(Y_{t-3} - Y_{t-4}) - \lambda(INV_{t-1} - \delta_K K^a_{t-1}), \end{aligned} \tag{5.24}$$

which is Equation 11 in Table 2–3 except for the dummy variables. The coefficient estimates in Equation 11 are of the expected signs, but the estimate of λ of 0.0155 is unreasonably small. Surely the actual change in gross investment in any one period is greater than 1.55 percent of the desired change. What the results appear to indicate is that the appropriate left-hand side variable in Equation (5.21) is not desired net investment, but rather the change in gross investment.

A number of investment equations were estimated in this study using different functional forms and different measures of excess capital, and invariably results were obtained for the change in gross investment that one would instead have expected to be true for net investment. One difficulty may be that depreciation has not been measured very precisely. The data on net investment depend on the particular measure of depreciation used, whereas the data on gross investment are direct NIA data.

It may be, for example, that a more accurate measure of depreciation would result in a larger coefficient estimate for the last term in Equation (5.24) (i.e., in a larger estimate of λ) and thus in a more reasonable set of results. It is also likely that the past change-in-output terms in the equation are picking up some cost-of-adjustment effects as well as expected future output effects, so that all cost-of-adjustment effects are not necessarily reflected in λ.

Whatever the case, the decision was made to take Equation 11 as the investment equation even though the estimate of λ seems too small. It should be noted that the long run properties of Equation (5.21) are still reasonable even if the change in gross investment is the left-hand side variable. One does expect the change in gross investment to be zero if there is no excess capital on hand and no recent changes in output.

The $JOBF_t$ and HPF_t Equations

Equation 12 explains the number of jobs in the firm sector, $JOBF_t$. The equation is in log form, with the left-hand side variable being $\log JOBF_t - \log JOBF_{t-1}$. The explanatory variables include a variable measuring the amount of excess labor on hand during the previous period ($\log JOBF_{t-1} - \log M_{t-1}H^M_{t-1}$), a time trend, three change-in-output terms, and two dummy variables to account for the effects of the steel strike in 1959. Equation 13 contains one less change-in-output term than does Equation 12, no dummy variables, and one added variable, $\log HPF_{t-1}$. Equations 12 and

13 are based on the assumption that the firm sector decides on its level of production before it decides on the number of jobs and the number of hours paid per job.

Equations 12 and 13 are meant to represent in an approximate sense the employment decisions of firms that result from the solutions of their optimal control problems. As was the case for a firm's investment decision, adjustment costs play an important role in the theoretical model in influencing a firm's employment decision. Because of these costs, it is sometimes optimal for a firm to hold excess labor. The amount of excess labor on hand at the beginning of the period has an important effect on the decisions made in that period, especially on the employment decision.

The excess labor variable in Equations 12 and 13 is explained as follows. $M_{t-1}H^M_{t-1}$ is, from the discussion in section 5.2, the number of worker hours required to produce Y_{t-1}. Let HS_{t-1} denote the average number of hours per job that a firm would like to be worked in period $t-1$ if there were no adjustment costs to contend with. $M_{t-1}H^M_{t-1}/HS_{t-1}$ is then the number of jobs required to produce Y_{t-1} if the average number of hours worked per job were HS_{t-1}. For the sake of the following discussion, this number will be referred to as the "desired" number of jobs for period $t-1$.

A measure of excess labor for period $t-1$ is the ratio of the actual number of jobs in the period to the desired number. The log form of this measure is $\log JOBF_{t-1} - \log(M_{t-1}H^M_{t-1}/HS_{t-1})$, or $\log JOBF_{t-1} - \log M_{t-1}H^M_{t-1} + \log HS_{t-1}$. If it is finally assumed that HS_{t-1} is a smoothly trending variable, namely $\bar{H}e^{\delta t}$, then the measure of excess labor is $\log JOBF_{t-1} - \log M_{t-1}H^M_{t-1} + \log \bar{H} + \delta t$. In Equations 12 and 13 the $\log JOBF_{t-1} - \log M_{t-1}H^M_{t-1}$ terms enters separately, and the equations include a constant term and time trend to pick up the effects of $\log HS_{t-1}$.

In the theoretical model, employment is generally smoothed relative to production because of the adjustment costs. The specification of Equations 12 and 13 and the coefficient estimates reflect this fact. The change in jobs times hours paid per job ($JOBF_tHPF_t$) is less than proportional to the current change in output. The past change-in-output terms in the two equations can be interpreted either as representing the effects of past output behavior on current employment decisions that are not captured in the excess labor terms or as being proxies for expected future output changes (or as both).

The $\log HPF_{t-1}$ term in Equation 13 reflects the fact that, unlike $JOBF_t$, which can move steadily upward or downward over time, HPF_t fluctuates around a relatively constant level of hours (such as 40 hours per week). If HPF_t is not equal to this level, this should, other things being equal, bring forces into play causing it to return to this level. Therefore, a term like $\log HPF_{t-1} - \log HS_{t-1}$ should be added to Equation 13, which, given the assumption made about HS_{t-1} above, is equivalent to adding $\log HPF_{t-1}$, a constant, and a time trend to the equation.

Equations 12 and 13 are similar to the equations estimated in Fair [23]. The data used in [23] are monthly, seasonally unadjusted, three digit industry data. For these data, significant effects (of up to six months ahead) of future output expectations on current employment decisions were obtained. For the aggregate data used in this study it is not possible to obtain such precise effects, although, as mentioned above, the past change-in-output terms in Equations 12 and 13 can be considered to be picking up in part expected future output effects.

Equations 12 and 13 are explained in detail in [23]. See in particular the discussion in Chapter 8 regarding the reasons for estimating separate equations for $JOBF_t$ and HPF_t, rather than just one equation explaining $JOBF_tHPF_t$. Although the study in [23] was completed before the theoretical model in Volume I was developed, the basic equations in [23] are consistent with the theoretical model if they are interpreted as representing approximations to the employment decisions of a firm that result from the solution of its optimal control problem. Equation 12 is also similar to the employment equation estimated in [14] for the private nonfarm sector, the only main difference here being the inclusion of one more change-in-output term.

The WF_t Equation

The last major equation estimated for the firm sector is Equation 15, explaining WF_t. The equation is in log form and includes as explanatory variables the price of firm sales (PX_t), a measure of labor market tightness (J_t^*), a time trend, and the wage rate lagged one period.

In the theoretical model a firm's optimal wage path is that path the firm expects it needs to set to attract the amount of labor implied by its optimal employment path. Two important factors influencing a firm's wage rate decision, in addition to the amount of labor that it wants, are its expectations of other firms' wage rates and its expectations of the labor supply curve facing it. It is thus necessary in empirical work to attempt to account for these expectations in some way.

The condensed model for the firm sector in Volume I is an approximation to the way a firm actually behaves in solving its optimal control problem each period. Equation 15 in Table 2–3 is similar to the equation representing the firm sector's wage rate decision in the condensed model. In the condensed model the current wage rate of the firm sector is a function of the wage rate of the previous period, the current price level, and two terms representing general labor market conditions and the firm sector's demand for labor (see statement [15] on page 66 in Volume I). The WF_{t-1}, PX_t, and J_t^* terms in Equation 15 can be considered to be accounting for the effects of these variables.

Equation 15 can also be considered, at least in a loose sense, as reflecting the outcome of bargaining over time between the firm and household sectors over the real wage rate. In the theoretical model bargaining takes the form of the firm sector adjusting over time, i.e., over more than one period, to changes in the labor supply curve facing it, the labor supply curve being determined each period by the household sector. If an equation like Equation 15 is interpreted in this way, an important question is which wage rate variable and which price variable are relevant for the bargaining process. The choice for the price variable here is PX_t, the price of total firms sales. PX_t excludes import prices and indirect business taxes. Neither an increase in import prices nor an increase in indirect business taxes benefits the firm sector, although both increases do hurt the household sector. The relevant price variable for the household sector is PH_t, which is inclusive of import prices and indirect business taxes. The use of PX_t in Equation 15 thus reflects the assumption that the household sector is aware that some price increases benefit the foreign sector and the government sector rather than the firm sector and considers only the prices that benefit the firm sector in its bargaining process with the firm sector.

The main question regarding which wage rate variable to use in an equation like 15 is whether the wage rate should be inclusive or exclusive of employer social security taxes. WF_t is exclusive of these taxes, whereas $(1 + d_{5t})WF_t$ is inclusive of the taxes. d_{5t} is the employer social security tax rate. If the firm sector effectively pays the taxes, then the appropriate wage rate variable in Equation 15 is WF_t. In this case, an increase in d_{5t} does not affect the bargaining process. If, on the other hand, the household sector effectively pays the taxes, then the appropriate wage rate variable is $(1 + d_{5t})WF_t$.

The procedure followed here regarding this question was to attempt to let the data tell how much of the tax is paid by each sector. Assume that the appropriate wage rate variable is $(1 + d_{5t})^{\gamma} WF_t$, where $0 \leqslant \gamma \leqslant 1$. The log of this variable is $\gamma \log(1 + d_{5t}) + \log WF_t$, so that this specification introduces the term $-\gamma \log(1 + d_{5t})$ on the right-hand side of Equation 15, with $\log WF_t$ being the left-hand side variable. γ is a parameter that can be estimated. The estimates of γ that were obtained in the experimentation with the wage equation were generally close to zero and not significant.

In the end the decision was made to constrain γ to be zero and so to drop the term $-\gamma \log(1 + d_{5t})$ from the equation. The results obtained in this study thus indicate that the firm sector pays the taxes. This conclusion should, however, be interpreted with a certain amount of caution because of the crude nature of the test and the highly aggregative nature of the data. See Brittain [6] for a more detailed study of the incidence of social security taxes.

One final question about the wage equation that was considered in this study is whether any long run constraints should be imposed on the equation. Ignore for now the effects of J_t^*, which can be considered to be short run in nature, and take WF_t/PX_t to be the real wage rate. If productivity is growing at roughly a constant rate (g) over time, then one might want to postulate that WF_t/PX_t also grows on average at rate g:

$$\frac{WF_t}{PX_t} = Ae^{gt}, \tag{5.25}$$

where A is a constant. Equation (5.25) in log form is:

$$\log WF_t = \log PX_t + \log A + gt. \tag{5.26}$$

Imposing this long run constraint on Equation 15 requires constraining the coefficients of $\log WF_{t-1}$ and $\log PX_t$ to sum to one. The TSLS and FIML estimates in Equation 15, which are not constrained, both sum to 1.031. This sum is close enough to one that it would make little difference regarding the properties of the model whether the sum was constrained to be one or not. In the final analysis the decision was made not to impose the constraint, primarily because of the feeling that the equation is too approximate to warrant this kind of refinement.

The Equations Explaining $HPFO_t$, DDF_t, $DIVF_t$, $INTF_t$ and IVA_t

The remaining five stochastic equations for the firm sector are not nearly as important as the others. Equation 14 explains the average number of overtime hours paid per quarter by the firm sector, $HPFO_t$. This variable, as explained in section 2.3, is needed for three definitions in the model. $HPFO_t$ is explained as a function of HPF_t, the total average number of hours paid per quarter by the firm sector. One would expect $HPFO_t$ to be related to HPF_t in roughly the manner indicated in Figure 5-2. Up to some point A (e.g., 40 hours per week), $HPFO_t$ should be zero or some small constant number, and after point A, increases in $HPFO_t$ and HPF_t should be one for one. An approximation to the curve in Figure 5-2 is:

$$HPFO_t = e^{\alpha_1 + \alpha_2 HPF_t}, \tag{5.27}$$

which in log form is:

$$\log HPFO_t = \alpha_1 + \alpha_2 HPF_t. \tag{5.28}$$

Equation 14 in Table 2-3 is the same as Equation (5.28) ,with two exceptions. First, HPF_t has a negative trend, and it was detrended before being used in the equation. The 0.5482 coefficient in Equation 14 was obtained from a regression of HPF_t on a constant and t for the 1952I–1974II period. Second, there appeared to be an important shift in the relationship

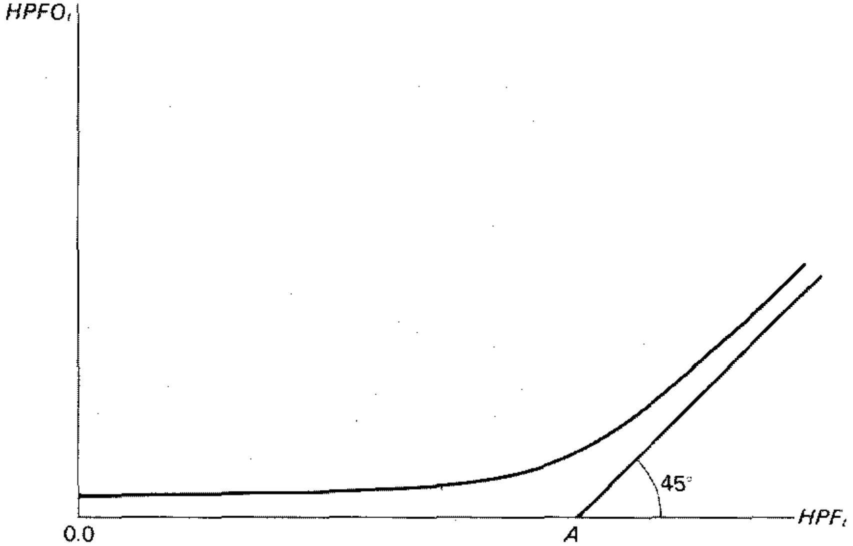

Figure 5-2. Expected Relationship Between $HPFO_t$ and HPF_t

between $HPFO_t$ and HPF_t beginning in 1966I, and so a dummy variable, $DD661_t$, was added to the equation to account for this shift. $DD661_t$ takes on a value of 0.0 before 1966I and a value of 1.0 from 1966I on. The sample period began in 1956I for this equation because data on $HPFO_t$ do not exist before this time.

Equation 16 explains demand deposits and currency of the firm sector, DDF_t. The equation is in log form, and the explanatory variables include the value of sales of the firm sector in current dollars, XX_t, the bill rate, and DDF_{t-1}. In the theoretical model there is a difference between aimed-for demand deposits and actual demand deposits. The former is a function of the firm's wage bill. The latter is determined residually and may differ from the former if the firm's expectations of its cash flow do not turn out to be correct. Actual demand deposits act in part as a buffer to absorb in a period any difference between actual and expected cash flow.

In the empirical model, DDF_t does not act as a buffer, but is rather assumed to be a direct decision variable of the firm sector and determined according to Equation 16. What is residually determined in the empirical model is the value of the loans of the firm sector, LF_t (see Equation 55 in Table 2-2). In the theoretical model, LF_t is not residually determined, but is itself a direct decision variable of a firm. It was useful in the theoretical model to take LF_t to be a direct decision variable because of the way that the loan constraint was treated. In the empirical model, however, it is useful *to take DDF_t to be a direct decision variable because of the different treatment* of the loan constraint. Although the firm's aimed-for demand deposits were tied to the firm's wage bill in the theoretical model, slightly better results were obtained here by the use of the value of sales of the firm sector in Equation 16.

Equation 17 explains the value of dividends paid by the firm sector, $DIVF_t$. The equation is in log form and includes as explanatory variables profits after taxes ($\pi F_t - TAXF_t$), the loan constraint variable (ZR_t), and $DIVF_{t-1}$. This is a typical kind of dividend equation that is estimated in the literature, except for the loan constraint variable. What the results indicate is that the firm sector pays out less of its after tax profits in dividends when the loan constraint is binding than otherwise. In the theoretical model a firm each period pays out all its after-tax profits in dividends, so that there is nothing in the model that can be used to guide the specification here.

Equation 18 explains the value of interest paid by the firm sector, $INTF_t$. The equation is in linear form and includes as explanatory variables the value of loans of the firm sector (LF_t), the bond rate ($RAAA_t$), and $INTF_{t-1}$. In the theoretical model $INTF_t$ equals RL_tLF_t, where RL_t is the loan rate, and Equation 18 is an attempt to approximate this.

The final stochastic equation for the firm sector, Equation 19, explains the inventory valuation adjustment, IVA_t. The equation is linear and includes as explanatory variables the current price of firm sales (PX_t), the price of firm sales lagged one period (PX_{t-1}), and the stock of inventories at the end of period $t - 1$ (V_{t-1}). In the theoretical model IVA_t equals $-(P_t - P_{t-1})V_{t-1}$, and Equation 19 is an attempt to approximate this equation. The coefficient estimates for PX_t and PX_{t-1} in Equation 19 are almost exactly equal in absolute value, which is as expected.

A Brief Comparison of the TSLS and FIML Estimates

FIML estimates were obtained for Equations 9, 10, 12, 13, 15, and 16 in the firm sector, which explain, respectively, PF_t, Y_t, $JOBF_t$, HPF_t, WF_t, and DDF_t. The TSLS and FIML estimates of these six equations are generally quite close. The largest differences between the estimates occur for the coefficient of the labor constraint variable in the PF_t equation and for the coefficient of $\log Y_t - \log Y_{t-1}$ in the $JOBF_t$ equation. Otherwise, the differences are very small.

5.5 A REVIEW OF THE MODEL OF FIRM SECTOR

Before concluding this chapter, it will be useful to review briefly some of the important properties of the empirical model of the firm sector. One good way of reviewing the model is to consider the effects that the bond rate have on the firm sector. The bond rate has a positive effect on the price that the firm sector sets (Equation 9). The household sector responds negatively to higher prices, so that a higher price leads to lower consumption by the household sector and thus a lower level of sales of the firm sector. A lower level of sales has a negative effect on the production of the firm sector (Equation 10).

A lower level of production in turn has a negative effect on the investment of the firm sector (Equation 11) and on the number of jobs and the average number of hours paid per job in the firm sector (Equations 12 and 13). The bond rate thus has a negative effect on investment and employment in the firm sector. The bond rate does not appear directly as an explanatory variable in the investment and employment equations, but instead affects investment and employment through its effect on the price that the firm sector sets. In a similar fashion, the investment tax credit affects investment and employment through its effect on the firm sector's price. A higher credit leads to a lower price, which then leads to more investment and employment.

In the theoretical model a binding loan constraint has a positive effect on the price set by a firm and thus a negative effect on its production, investment, and employment. In the empirical work here, however, no important effect of the loan constraint variable on PF_t could be found. The only place where the loan constraint variable did appear to have an effect on the firm sector was in the dividend equation (Equation 17), where a more restrictive loan constraint implies fewer dividends paid by the firm sector than otherwise.

The amount of excess capital on hand has a direct negative effect on investment (Equation 11), and the amount of excess labor on hand has a direct negative effect on employment (Equations 12 and 13). In the theoretical model the amounts of excess capital and labor on hand had negative effects on the price set by a firm, but no important effects of this sort could be found in the empirical work here. Likewise, no important effects of the ratio of inventories to sales of the previous period (V_{t-1}/X_{t-1}) could be found on PF_t, even though this ratio had a negative effect on the price set by a firm in the theoretical model.

The measure of labor market tightness J_t^* has a contemporaneous positive effect on the wage rate set by the firm sector (Equation 15). The wage rate in turn has an effect on the price level with a lag of one period. The inclusion of the lagged wage rate in the price equation is designed to pick up expectational effects. The other variables in the price equation that are assumed to be picking up expectational effects are the lagged price (PF_{t-1}) and the price of imports (PIM_t).

An important variable in the price equation is the labor constraint variable, ZJ'_t. In theory, this variable affects PF_t only when the labor constraint is binding on the firm sector, although in practice it actually has at least a small effect all the time because of the approximation for ZJ'_t that has been used. ZJ'_t is a nonlinear function of $1 - UR_t$. The use of ZJ'_t in the price equation is designed to try to pick up the effect of the labor constraint on the firm sector. When the firm sector receives less labor than it expected it would at the wage rate that it set, it is assumed (within the quarter) to raise its price and contract.

This completes for now the discussion of the equations for the firm sector. The relationship between the firm sector and the other sectors in the model is examined in more detail in Chapter Nine.

Chapter Six

The Financial Sector

6.1 INTRODUCTION

As mentioned in section 1.1, an important characteristic of the empirical model regarding the financial sector is the accounting for all flows of funds in the system. This allows the bill rate ($RBILL_t$) to be implicitly determined in the model through the solution of the 83 independent equations. There is no stochastic equation in the model in which the bill rate appears naturally as the left-hand side variable—i.e., naturally as the variable explained by the equation.

There are four stochastic equations in the financial sector, Equations 20–23 in Table 2–3, explaining, respectively: commercial bank borrowing at federal reserve banks ($BORR_t$), the corporate bond rate ($RAAA_t$), the mortgage rate ($RMORT_t$), and capital gains or losses on corporate stocks held by the household sector (CG_t). There is also an important nonstochastic equation explaining bank reserves (BR_t), Equation 45 in Table 2–2:

45. $BR_t = g_{1t} DDB_t$.

This equation and the four stochastic equations are explained in the next section. The treatment of the loan constraints in the model is then discussed in section 6.3.

6.2 EQUATION 45 AND THE FOUR STOCHASTIC EQUATIONS IN THE FINANCIAL SECTOR

Equation 45 in Table 2–2 links the level of bank reserves to the level of demand deposits and currency of the financial sector (DDB_t). g_{1t} in the equation is defined in Table 2–1 and is the actual ratio of BR_t to DDB_t observed in quarter t. The relationship between BR_t and DDB_t is thus reflected in g_{1t} in the model, and this relationship is taken to be exogenous.

Some experimentation was done with alternative specifications of the relationship between BR_t and DDB_t before deciding to take the relationship to be exogenous. It is possible, for example, to obtain data on actual reserve requirement rates on demand deposits from past issues of the *Federal Reserve Bulletin*. (See, for example, page A9 of the July 1974 issue.) These data were used to construct a variable, denoted as $\bar{g}_{1t}$, that was the quarterly average of the actual reserve requirement rates on demand deposits for reserve city banks. Given data on $\bar{g}_{1t}$ and g_{1t}, it is then possible to compare the two series to see how closely they correspond.

There are a number of reasons why the two series are not likely to correspond exactly. One reason is that there are different reserve requirement rates for different types of banks. $\bar{g}_{1t}$ pertains only to reserve city banks. Another reason is that DDB_t is not exactly the correct base to use to calculate required reserves. DDB_t, for example, excludes time deposits in commercial banks, for which there is also a reserve requirement rate, and it has netted out of it demand deposits held by nonbank financial intermediaries in commercial banks. (Remember that the financial sector in the empirical model is an aggregate of nonbank financial intermediaries and commercial banks.) A third reason g_{1t} and $\bar{g}_{1t}$ do not coincide exactly is that banks may at times hold excess reserves in the aggregate. BR_t includes all reserves, not necessarily just required reserves.

To summarize, then, the factors that cause g_{1t} to change (i.e., cause the ratio of BR_t to DDB_t to change) include not just changes in the actual reserve requirement rates set by the government, which $\bar{g}_{1t}$ does capture, but also changes in the proportion of excess reserves held by the bank sector, shifts of demand deposits among different types of banks, and shifts of funds between time deposits and other nondemand deposit securities, all of which $\bar{g}_{1t}$ does not capture.

Even though g_{1t} and $\bar{g}_{1t}$ are not expected to coincide exactly, a plot of g_{1t} and $\bar{g}_{1t}$ over time reveals a fairly close agreement between the two series. It did not seem unreasonable from observing this plot to take g_{1t} as exogenous in the model. Nevertheless, some experimentation was done to see if $BR_t - \bar{g}_{1t}DDB_t$, which one might interpret as a measure of excess reserves, could be explained as a function of the bill rate or other interest rates. One might expect there to be fewer excess reserves held when interest rates are high than otherwise. $BR_t - \bar{g}_{1t}DDB_t$ did not appear, however, to be sensitive to the level of the bill rate or any other interest rate, and so in the end the decision was made to take g_{1t} to be exogenous.

The treatment of BR_t here is in contrast to its treatment in the theoretical model, where it is treated as a residual. This difference is due to the view that on a quarterly basis banks are likely to have fairly close control over their reserves and thus that it is not reasonable to treat the level of bank reserves as a residual when quarterly data are used. It is interesting to note that if BR_t were treated as a residual in the empirical model, in the sense that no equation for it was specified, but yet it still was taken to be endogenous, then one would need an explicit equation determining the bill rate. The bill rate could no longer be taken to be implicitly determined in the model. In the theoretical model there is effectively an equation for the bill rate, since the bond dealer sets the bill rate.

It should finally be noted that the treatment of g_{1t} as exogenous implies nothing about the behavior of bank borrowing, $BORR_t$. As will be discussed next, $BORR_t$ is determined by Equation 20 and responds to the difference between the bill rate and the discount rate. The level of nonborrowed reserves is by definition $BR_t - BORR_t$, and since BR_t and $BORR_t$ are both endogenous variables in the model, the level of nonborrowed reserves is also endogenous.

The first stochastic equation in the financial sector to be discussed is Equation 20 in Table 2–3, explaining $BORR_t$. The equation is quite simple. The ratio of $BORR_t$ to bank reserves is taken to be a function of the difference between the bill rate and the discount rate (RD_t). The positive estimate of the constant term in the equation implies that there is still some borrowing even if the bill rate and the discount rate are the same.

Consider next Equations 21 and 22, explaining $RAAA_t$ and $RMORT_t$. In the theoretical model the bond rate was determined according to the expectations theory, i.e., as a function of the current bill rate and of expected future bill rates. $RAAA_t$ and $RMORT_t$ are likewise assumed here to be determined according to the expectations theory. Both are taken to be a function of the current bill rate, of past values of the bill rate, and of past values of the rate of inflation. The past values of the bill rate and the rate of inflation are used as proxies for the (unobserved) expected future bill rates.

Both Equations 21 and 22 are in log form. The same rate of inflation variable is used in both equations, namely a weighted average of the rates of inflation in the past three quarters, with weights of 3, 2, and 1. This weighted average was chosen after some experimentation with alternative weighting schemes. Each of the two equations includes as explanatory variables both the lagged dependent variable and lagged values of the bill rate, which implies a fairly complicated lag structure of the bill rate on both of the long term rates.

The last stochastic equation to be considered in the financial sector is Equation 23, determining CG_t. Not counting new issues and retirements, CG_t is the *change* in the market value of stocks held by the household sector. In the theoretical model the aggregate value of stocks is determined as the present discounted value of expected future dividend levels, the discount rates being the current and expected future bill rates. Consequently, the theoretical model implies that CG_t ought to be a function of the *changes* in expected future dividend levels and of the *changes* in the current and expected future bill rates.

The two explanatory variables in Equation 23 are the change in the bond rate and a weighted average of the change in the after-tax cash flow of the firm sector. The change in the bond rate is taken to be a proxy for the (unobserved) changes in expected future bill rates, and the weighted average of the change in after-tax cash flow is taken to be a proxy for the (unobserved) changes in expected future dividend levels. The weights for the cash flow variable are 3, 2, and 1, which were also chosen after some experimentation with alternative weighting schemes.

The coefficient estimates in Equation 23 are of the expected signs, but the fit of the equation is not particularly good. Only 16.7 percent of the variance of CG_t has been explained. For present purposes the equation does provide some link between other variables in the model and CG_t, but it is not likely to be an equation that one can use to make money in the stock market. Some attempt was made here to try to improve upon Equation 23, but to no avail.

6.3 THE TREATMENT OF THE LOAN CONSTRAINTS

The final issue to discuss regarding the financial sector is the treatment of the loan constraints. In Chapters Four and Five the loan constraints were handled by adding log ZR_t to the various equations (the equations being in log form). Log ZR_t is equal to $-(1/1000)(RBILL_t^* - 0.608)^2$, where $RBILL_t^*$ is $RBILL_t$ partly detrended. Some of the equations in Chapters Four and Five also, of course, include log $RBILL_t$ directly as an explanatory variable. The variables log $RAAA_t$ and log $RMORT_t$ are also explanatory variables in some of the equations, and both of these variables are in turn influenced directly by log $RBILL_t$.

In the estimation and solution of the model, log ZR_t is treated as an endogenous variable, since it is a function of $RBILL_t$. Consequently, adding log ZR_t to some of the equations of the model can be looked upon as merely allowing $RBILL_t$ to enter the model in a more nonlinear way than otherwise would be the case. The reason for this added nonlinearity is justified by the discussion in Chapter Four, where it is argued that adding log ZR_t (and log ZJ_t) to an equation converts the equation from one with an unobserved left-hand side variable (the unconstrained decision value) to one with an observed left-hand side variable (the constrained decision value).

The procedure of determining $RBILL_t$ by solving the complete model is equivalent to assuming that $RBILL_t$ is determined by equating each period the aggregate supply of and demand for funds in the economy. Because of the addition of log ZR_t, log ZJ_t, and log ZJ'_t to the model, however, this procedure is *not* equivalent to equating the *unconstrained* supply of and demand for funds. What enter on the left-hand side of the equations for the household and firm sectors are the constrained decision values, and these are the values that are used in solving the model. The "constrained" aggregate demand for funds is equated to the "constrained" aggregate supply.

Near the end of Chapter Four a brief comparison was made between the treatment of the housing market in [14] and its treatment here. It was pointed out that the two treatments are not inconsistent with each other, although it is true that the model in [14] is incomplete because the mortgage rate and deposit flows into Savings and Loan Associations and Mutual Savings Banks are treated as exogenous. It is now possible within the context of the present model to consider more explicitly what happens when there is disequilibrium in the housing market.

If the loan constraint is binding on the household sector, housing investment is less than otherwise. This means that the demand for funds on the part of the household sector is less than otherwise. This lower demand (the "constrained" demand) is what is in theory used in the solution of the model (and thus of the bill rate) within the period. Since, however, the loan constraint variable is itself a function of the bill rate, the effect of the loan constraint on the household sector is assumed to be captured by means of the bill rate entering the model in the constrained case in a more nonlinear way than othewise.

If the loan constraint is not binding on the household and firm sectors (i.e., ZR_t is almost equal to one), then this added nonlinearity does not exist. The hours constraint may, however, still be binding on the household sector, or the labor constraint may still be binding on the firm sector, so that it may still be the case that it is the "constrained" aggregate demand for funds that is equated to the "constrained" aggregate supply in the solution of the model. The supplies of and demands for funds are affected by all of the constraints, not just by the loan constraint.

The periods in which the loan constraint is not binding on the household and firm sectors can be referred to loosely as periods of "easy money." It is important to note, however, that periods of easy money do *not* correspond to periods in which the financial sector holds excess reserves. The financial sector never holds excess reserves in the model, since BR_t is always equal to $g_{1t}DDB_t$. Periods of easy money just mean that the bill rate is low, a low bill rate implying that the loan constraint is not binding (i.e., that ZR_t is almost equal to one). In the theoretical model, a period of easy money might correspond to the banks holding excess reserves because of expectation errors, but, as discussed above, the financial sector is assumed in the empirical model not to hold excess reserves on a quarterly basis.

Chapter Seven

The Foreign and Government Sectors

7.1 THE FOREIGN SECTOR

There is one stochastic equation in the foreign sector, an equation determining the value of imports in real terms, IM_t. There is no foreign sector in the theoretical model, so that one cannot use anything in Volume I to guide the specification of the equation determining IM_t. The equation determining IM_t is Equation 24 in Table 2–3. The equation is the log form, and the left-hand side variable is the real per capita value of imports. The explanatory variables in the equation include: the price deflator for imports lagged two quarters (PIM_{t-2}), the price deflator for the sales of the firm sector lagged one quarter (PX_{t-1}), the real per capita value of sales of the firm sector (X_t/POP_t), and seven dummy variables to account for the effects of three dock strikes.

The price of imports relative to the price of domestically produced goods ought to be important in determining the demand for imports, as well as the size of the domestic sector itself, and this is what Equation 24 is designed to try to pick up. The coefficient estimates for Equation 24 indicate that the level of imports is more responsive to the price of domestically produced goods than it is to the price of imports. The estimate of the coefficient of $\log PX_{t-1}$ is about 3.8 times larger in absolute value than the estimate of the coefficient of $\log PIM_{t-2}$. The coefficient estimate for $\log(X_t/POP_t)$ is slightly greater than one.

The three other variables in the foreign sector that are endogenous, aside from IM_t, are the price of exports, PEX_t, the saving of the foreign sector, $SAVR_t$, and the value of securities held by the foreign sector, $SECR_t$. The value of $SECR_t$ is actually negative, as can be seen for 1971IV in Table 2–1, which means that the foreign sector is a net debtor with respect to the "all other" securities category. $SECR_t$ is determined in Equation 66 in Table 2–2. It is endogenous because $SAVR_t$ is endogenous. $SAVR_t$ is determined in Equation 65 in Table 2-2, and it is endogenous because IM_t and PEX_t are endogenous. The negative of $SAVR_t$ is the U.S. balance of payments on current account. PEX_t is determined in Equation 28 in Table 2–2 as a function of PX_t.

The two most important variables that are taken to be exogenous in the foreign sector are the value of exports in real terms, EX_t, and the price deflator for imports, PIM_t. The other variables in the foreign sector that are exogenous are transfer payments from the household and government sectors to the foreign sector, $HRTRP_t$ and $GRTRP_t$, the value of demand deposits of the foreign sector, DDR_t, the gold and foreign exchange holdings of the government sector, $GFXG_t$, and the discrepancy of the foreign sector, $DISR_t$. Since DDR_t and $GFXG_t$ are exogenous, any net saving or dissaving on the part of the foreign sector takes the form in the model of an increase or decrease in the value of $SECR_t$.

Although the present treatment of the foreign sector is fairly simple, it does take into account relative price effects on the demand for imports and keeps track of the flows of funds between the domestic and foreign sectors. It is clearly beyond the scope of this study, however, to endogenize either the import price deflator or the real value of exports.

Some thought was given to the question of whether it is reasonable to take the gold and foreign exchange holdings of the government sector ($GFXG_t$) as exogenous. If capital flows into and out of the United States are responsive to interest rates in the United States, then $GFXG_t$ should not be taken to be exogenous. An alternative approach would be to take some foreign interest rate (such as the Eurodollar rate) as exogenous, endogenize $GFXG_t$, and take $GFXG_t$ to be responsive to the spread between the U.S. bill rate and the foreign interest rate. It appears to be the case, however, that foreign interest rates are quite responsive to the U.S. interest rates (see Cooper [11] for a good discussion of this issue), and so it seemed more reasonable to take $GFXG_t$ as exogenous (from the point of view of U.S. domestic activities) than to take something like the Eurodollar rate as exogenous.

7.2 THE GOVERNMENT SECTOR

Accounting for all the flows of funds in the system implies (as was seen in Chapter Two) that the government budget constraint is automatically satisfied. It also means that one can consider explicitly in the model the direct purchase and sale of government securities. In other words, the value of government securities outstanding, VBG_t, can be taken to be a direct policy variable of the government. The fact that the government budget constraint is satisfied means, from Equation 69 in Table 2–2, that any nonzero level of saving of the government must result in the change in at least one of the following five items: VBG_t, $BORR_t$, $CURR_t$, BR_t, and $GFXG_t$.

Since $CURR_t$ and $GXFG_t$ are taken to be exogenous, this means that any nonzero level of saving of the government must result in the change in either the value of government securities outstanding, commerical bank borrowing, or bank reserves.

There are two stochastic equations in the government sector, one determining the value of unemployment insurance benefits, TPU_t, and one determining the interest payments of the government, $INTG_t$. Equation 25 in Table 2–3 explains TPU_t. It is in log form and includes as explanatory variables the number of people unemployed (U_t) and the price deflator for firm sales lagged one quarter (PX_{t-1}). The inclusion of the price deflator in the equation reflects the assumption that the government changes the current dollar value of unemployment insurance benefits as the general price level changes.

Equation 26 in Table 2–3 explains $INTG_t$. It is in log form and includes as explanatory variables the value of government securities outstanding, the bill rate, the bond rate, and $INTG_{t-1}$. In the theoretical model the interest paid by the government is equal to $r_t VBILLG_t + BONDG_t$, where r_t is the bill rate, $VBILLG_t$ is the value of bills outstanding, and $BONDG_t$ is the number of bonds (consols) outstanding. In the empirical model VBG_t includes both the value of bills and the value of bonds, and so it seems reasonable to include both the bill rate and the bond rate in Equation 26 to try to pick up the effects of both interest rates on the interest paid by the government. It appeared to be possible in this case to pick up separate effects of the two rates. This is in contrast to the case for the firm sector, where separate effects could not be obtained in Equation 18.

The other variables in the government sector that are endogenous are the price of goods purchased, PG_t, the civilian wage rate, WGC_t, the military wage rate, WGM_t, the net value of taxes paid, TAX_t, and the saving of the government sector, $SAVG_t$. PG_t is determined in Equation 36 in Table 2–2 as a function of PX_t, and WGC_t and WGM_t are determined in Equations 38 and 39 as functions of WF_t. TAX_t is defined in Equation 67 in Table 2–2 as the sum of the net taxes paid by the household, firm, and financial sectors. $SAVG_t$ is determined in Equation 68 as the difference between TAX_t and the expenditures of the government sector.

Some experimentation was done to see if other variables of the government sector ought to be treated as endogenous. The main question considered was whether state and local government expenditures on goods and/or services are responsive to interest rates and therefore should be treated as endogenous. Little evidence could be found that these expenditures are responsive to interest rates, and in the end it was decided not to endogenize them. There seemed to be little harm in treating all government expenditures on goods (in real terms) and all government jobs and hours paid per job as exogenous.

Table 7–1. The Exogenous Variables in the Government Sector

†$CURR_t$ = value of currency outstanding less the value of demand deposits of the government sector, *BCURT*.
†d_{1t} = profit tax rate
†d_{3t} = one of the two personal income tax rates
†d_{4t} = indirect business tax rate
†d_{5t} = employer social security tax rate
†d_{6t} = employee social security tax rate
†DEP_t = depreciation of the firm sector, *BCURT*
†$DTAXCR_t$ = investment tax credit variable
$DISG_t$ = discrepancy of the government sector, *BCURT*
g_{1t} = reserve requirement ratio
$GFXG_t$ = value of gold and foreign exchange of the government sector, *BCURT*
$GHSUB_t$ = net subsidies of government enterprises, *BCURT*
$GHWLD_t$ = wage accruals less disbursements of the government sector, *BCURT*
$GRTRP_t$ = transfer payments from the government sector to the foreign sector, *BCURT*
$HPGC_t$ = average number of hours paid per civilian job per quarter by the government sector
$HPGM_t$ = average number of hours paid per military job per quarter by the government sector
†$JOBGC_t$ = number of civilian jobs in the government sector
$JOBGM_t$ = number of military jobs in the government sector
†RD_t = the discount rate
†VBG_t = value of government securities, *BCURT*
†XG_t = purchases of goods of the government sector, B1958
†YG_t = transfer payments from the government sector to the household sector, not counting TPU_t, *BCURT*
τ = progressivity tax parameter in the personal income tax equation

Note: A † denotes that the effects on the economy of changing this variable are examined in section 9.3.

The exogenous variables in the government sector are listed in Table 7–1 in alphabetic order. The important variables affecting the household sector directly are the personal income tax rates (d_{3t} and τ), the indirect business tax rate (d_{4t}), the social security tax rates (d_{5t} and d_{6t}), the employment variables ($HPGC_t$, $HPGM_t$, $JOBGC_t$, and $JOBGM_t$), and the level of transfer payments (YG_t). The important variables affecting the firm sector directly are the profit tax rate (d_{1t}), depreciation (DEP_t), the investment tax credit variable ($DTAXCR_t$), and the purchase of goods (XG_t). The important variables affecting the financial sector directly are the reserve requirement ratio (g_{1t}), the discount rate (RD_t), and the value of government securities outstanding (VBG_t). The depreciation of the firm sector is considered to be an exogenous variable in the government sector in the sense that the government controls by law the various allowable depreciation rates. The effects on the economy of changing the various exogenous variables of the government will be examined in Chapters Nine and Ten.

Chapter Eight

The Predictive Accuracy of the Model

8.1 INTRODUCTION

The predictive accuracy of the empirical model is examined in this chapter. In the next section the predictive accuracy of my forecasting model [14] is compared to the predictive accuracy of other models. It is argued in this section that the forecasting model appears, from previous results, to be at least as accurate as other models. In section 8.3 the predictive accuracy of the empirical model is compared to the predictive accuracy of the forecasting model. The results presented in this section indicate that the empirical model is more accurate than the forecasting model. This indirect comparison of the empirical model with other models thus indicates that the empirical model is more accurate. This conclusion is, as mentioned in section 1.1, clearly tentative, and further tests and comparisons are needed before one can hold this conclusion with much confidence.

The following comparisons are of ex post predictive accuracy, not ex ante. An ex post forecast is a forecast that has been generated from a model in a mechanical way (no subjective constant-term adjustments) using the actual values of the exogenous variables. An ex ante forecast is an actual forecast released by a model builder for a future period. It is based on guessed values of the exogenous variables and may also have been generated from a model to which subjective constant-term adjustments were applied.

Comparisons of the ex ante forecasting records of model builders, which have been made in two recent studies by McNees [34], [35], are not valid comparisons of the models. Because of the extensive use of subjective constant-term adjustments by most model builders in actual forecasting situations, the accuracy of the ex ante forecasts may not be at all a good indicator of the accuracy of the models. Even if subjective constant-term adjustments are not applied to a model, as in the case of my work with the forecasting model, the accuracy of the ex ante forecasts from the model is still affected by the use of guessed rather than actual values of the exogenous variables. In evaluating the accuracy of a model qua model it is clear that actual rather than guessed values of the exogenous variables should be used. This is not to say, however, that the kinds of comparisons that McNees has made are not of interest. They are clearly of interest to people who want to know who are currently the most accurate forecasters.

8.2 A COMPARISON OF THE FORECASTING MODEL WITH OTHER MODELS

The results in Fromm and Klein [26] indicate that my forecasting model is at least as accurate, if not more so, than other models. Fromm and Klein's results cover nine quarterly econometric models, four variables (GNP in current dollars, GNP in constant dollars, the GNP deflator, and the unemployment rate), two error measures (root mean square errors of levels and first differences), and both within sample and outside sample forecasts (all ex post forecasts). The within sample forecast periods are roughly the same for all of the models, although the outside sample forecast periods are not.

One can get an indication of how the forecasting model performed relative to the other eight models from the results in Table 8–1. Table 8–1 presents the rank of the model against the other models for each possible category. For the within sample results, which are based on roughly the same period, the model is generally at or near the top. The results are not particularly good for the three- through five-quarter-ahead forecasts of the unemployment rate, but are quite good for the three- through five-quarter-ahead forecasts of the other three variables. For the outside sample results, the model is always best for the two GNP variables. The model is not only best for these variables, but is best by a substantial amount, as can be seen from examining Tables 1 and 2 in [26]. For the GNP deflator, the outside sample results deteriorate after three quarters ahead, and for the unemployment rate the outside sample results are not very good.

The outside sample results, while providing a more stringent test of the models, must be interpreted with some caution here because of the different forecast periods used. It is also the case that the forecasting model was reestimated up to the first quarter being forecast for each set of outside sample forecasts, so that, for example, each five-quarter-ahead forecast was never more than five quarters away from the end of the estimation period. This was not true for the other models, although the outside sample forecast period for each model did always begin with the quarter immediately following the end of its (one) estimation period.

Although the overall results in [26] are not completely unambig-

Table 8–1. The Ranking of the Forecasting Model Against Eight Other Quarterly Models (Results Are from Fromm and Klein [26], Tables 1, 2, 3, and 4)

RMSE — Root Mean Square Error of Level Predictions
RMSEΔ = Root Mean Square Error of First Difference Predictions

Variable	*One Quarter Ahead*		*Two Quarters Ahead*		*Three Quarters Ahead*		*Four Quarters Ahead*		*Five Quarters Ahead*	
	RMSE	*RMSEΔ*	*RMSE*	*RMSEΔ*	*RMSE*	*RMSEΔ*	*RMSE*	*RMSEΔ*	*RMSE*	*RMSEΔ*
	Within Sample									
Current Dollar GNP	4 (9)	4 (8)	2 (9)	1 (8)	1 (9)	3 (8)	1 (9)	3 (8)	1 (8)	3 (7)
Constant Dollar GNP	3 (9)	3 (8)	6 (9)	1 (8)	3^T(9)	3 (8)	2 (9)	3 (8)	1 (8)	2 (7)
GNP Deflator	2 (9)	NA	4 (9)	NA	2 (9)	NA	1 (9)	NA	1 (8)	NA
Unemployment Rate	2^T(9)	NA	3^T(9)	NA	5 (9)	NA	6 (9)	NA	6 (8)	NA
	Outside Sample									
Current Dollar GNP	1 (8)	1 (7)	1 (8)	1 (7)	1 (8)	1 (7)	1 (8)	1 (7)	1 (6)	1 (6)
Constant Dollar GNP	1 (8)	1 (7)	1 (8)	1 (7)	1 (8)	1 (7)	1 (8)	1 (7)	1 (6)	1 (6)
GNP Deflator	1 (8)	NA	1 (8)	NA	1 (8)	NA	3^T(8)	NA	6 (6)	NA
Unemployment Rate	6 (8)	NA	6 (8)	NA	6 (8)	NA	6 (8)	NA	4 (6)	NA

Notes: 1. NA = Not Available.
2. A superscript ^{T} denotes a tie.
3. The two annual models and the one monthly model considered in Fromm and Klein [26] were excluded from the rankings because of lack of comparability.
4. The number in parentheses for each rank is the number of models used for the ranking. A complete set of results was not available for every model.
5. The eight other models are (1) BEA (Bureau of Economic Analysis), (2) Brookings, (3) DHL III (University of Michigan), (4) DRI-71 (Data Resources, Inc.), (5) Federal Reserve Bank of St. Louis, (6) MPS (University of Pennsylvania), (7) Wharton Mark III (University of Pennsylvania), (8) Wharton Mark III Anticipations Version.
6. The within sample prediction period was 1961I–1967IV for all models except Brookings and the forecasting model. For Brookings the period was 1959I–1965IV, and for the forecasting model the period was 1962I–1967IV.

uous in indicating that the forecasting model is the most accurate of the models, they certainly do indicate that the model is at least no less accurate than any of the other models. Another encouraging set of results about the forecasting model is presented in Fair [13], where the ex ante forecasting performance of the model is examined for the 1970III–1973II period. The results in [13] indicate that the ex ante forecasts from the model, which are never subjectively adjusted before being released, are nearly (but not quite) as accurate as subjective ex ante forecasts.[a] The model appears to be the first model that can be used in a mechanical way and produce reasonably accurate results.

Although, as mentioned in the previous section, the ex ante performance of a model cannot be used in a rigorous way to evaluate its accuracy, the result just cited is at least encouraging as to the model's accuracy. This is especially true in the present case because the results in [13] also show that the forecasting accuracy of the model would generally have been improved had the actual values of the exogenous variables been known (rather than guessed) at the time the forecasts were made. This latter conclusion is certainly what one would expect from a model, but, as discussed in [13], it does not appear to be true of other models. If the forecasting accuracy of a model is not generally improved when actual values of exogenous variables are substituted for guessed values, this both indicates the important role that subjective adjustments play in the release of the ex ante forecasts and, unless the differences are fairly small, calls into question the basic accuracy of the model.

There are some negative results regarding the accuracy of the forecasting model that have occurred since the evaluation in [13] was completed. The model does not predict 1973 and 1974 nearly as well as it predicts earlier years. This is true of both the ex ante forecasts that have been released by me and the ex post forecasts that have been generated since the data on the exogenous variables for 1973 and 1974 became available. The three equations that perform the worst for 1973 and 1974 are the price equation, the inventory equation, and the import equation.

The price equation substantially underpredicts the inflation that occurred during 1973 and 1974, and the inventory and import equations do not capture very well the large changes in inventory investment and imports that occurred during these years. The other equations of the model appear to have held up much better during 1973 and 1974. Their coefficient estimates for the most part have not changed very much as the observations for 1973 and 1974 have been added to the sample period, and the residual estimates for the quarters of 1973 and 1974 are not noticeably larger than the estimates for earlier quarters.

The 1973–1974 period is not an easy period to predict, and it appears to be the case that other models also do not predict this period nearly as well as they predict earlier periods. The periods considered for the results in Fromm and Klein [26] do not include 1973 and 1974, and at the time of this writing there are no similar comparisons of the models for the 1973–1974 period. It is thus unknown whether the predictive accuracy of my forecasting model deteriorated more in 1973 and 1974 than it did for other models. The conclusion of this section is thus that the forecasting model appears to be at least as accurate as other models for the period prior to 1973, but that it is unknown whether this result is also true for the 1973–1974 period.

Because of the uncertainty as to whether the accuracy of the forecasting model deteriorated more in 1973 and 1974 than it did for other models and because the poorer performance of the forecasting model in 1973 and 1974 can be traced in large part to the price, inventory, and import equations, it was decided for the comparison of the empirical model and the forecasting model in the next section to drop these three equations from the forecasting model. The price level, inventory investment, and imports were thus taken to be exogenous in the forecasting model. The empirical model was not changed, so that these three variables remained endogenous in the empirical model.

This procedure clearly biases the results in favor of the forecasting model and thus provides a more stringent test of the empirical model. If the empirical model is more accurate than this less endogenous version of the forecasting model, then the conclusion that the empirical model is also more accurate than other models can be held with more confidence than it could be if the empirical model were merely more accurate than the complete version of the forecasting model. Although it may be that the complete version of the forecasting model is less accurate than other models for the 1973–1974 period, it seems unlikely that the less endogenous version is also less accurate.

8.3 A COMPARISON OF THE EMPIRICAL MODEL AND THE FORECASTING MODEL

For a comparison of the predictive accuracy of two models to be fair, the prediction periods should be the same for both models, and both models should be of the same degree of endogeneity. Requiring the prediction periods to be the same rules out the obvious possibility that one model will perform better than another merely because of an easier prediction period used. Requiring the models to be of the same degree of endogeneity rules out the possibility of one model performing better merely because it treats important endogenous variables as exogenous. One model should not treat as exogenous any variable that the other model treats as endogenous and that most people would agree is in fact truly endogenous.

It is also desirable if possible for the predictions to be outside sample and dynamic. Requiring the predictions to be outside sample rules out the possibility of a model performing well merely because of much diligence on the part of a model builder in getting her or his model to fit the estimation period well. This requirement, in other words, lessens the possibility that a model will perform well merely because of data mining. Since lagged endogenous variables play an important role in most macroeconometric models, requiring the predictions to be dynamic provides a good way of testing whether the dynamic structure of the economy has been captured adequately in the model.

The empirical model and the complete version of the forecasting model are not of the same degree of endogeneity. Both take government variables, population, and exports as exogenous, but the forecasting model also takes as exogenous the mortgage rate, deposit flows into Savings and Loan Associations and Mutual Savings Banks, a consumer sentiment variable, and a variable measuring plant and equipment investment expectations. The empirical model takes as exogenous relative prices and the price of imports, which are not part of the forecasting model and therefore not taken as exogenous.

Overall, it is clear that the forecasting model is of a lesser degree of endogeneity than is the empirical model. This is, of course, even more true for the less endogenous version of the forecasting model. The following comparison of the empirical model and the forecasting model is thus not ideal, even though all the other requirements discussed above have been met, and, as discussed at the end of the previous section, one should consider the comparison as being somewhat biased in favor of the forecasting model.

The empirical model was estimated through 1974II. Data through 1975I were collected for this study, and so there are three quarters available for outside sample comparisons. Two prediction periods were considered for the comparisons in this section: a within sample period of 46 observations (1963I–1974II) and the outside sample period of 3 observations (1974III–1975I). The estimates of the forecasting model that were used for the results in this section are presented in Appendix B to this volume. The forecasting model was also estimated through 1974II to put it on a comparable basis with the empirical model. Both static and dynamic predictions were generated for the two models.

The accuracy of the two models is compared in Table 8–2. The table is fairly self-explanatory, and so the following is only a brief discussion of the results. Consider first the static, within sample results. For these results the two models are quite similar. The forecasting model is slightly more accurate with respect to predictions of current dollar GNP, but some-

Table 8–2. The Predictive Accuracy of the Empirical Model versus the Forecasting Model

EM = Empirical Model, TSLS Estimates
FM = Forecasting Model with Inventory Investment, Imports, and the Price Level Exogenous
RMSE = Root Mean Square Error of Level Predictions
RMSEΔ = Root Mean Square Error of First Difference Predictions

		1963I–1974II (*within sample*) *DYNAMIC*				1963I–1974II (*within sample*) *STATIC*		1974III–1975I (*outside sample*) *DYNAMIC*				1974III–1975I (*outside sample*) *STATIC*	
		RMSE		*RMSEΔ*		*RMSE*		*RMSE*		*RMSEΔ*		*RMSE*	
Variable in EM	*Variable in FM*	*EM*	*FM*	*EM*	*FM*	*EM*	*FM*	*EM*	*FM*	*EM*	*FM*	*EM*	*FM*
GNP_t	GNP_t	9.10	9.87	7.60	5.47	5.19	5.09	10.64	26.84	12.85	25.95	6.83	23.87
Y_t	$GNPR_t$	9.12	7.74	5.20	4.13	3.66	3.84	5.95	15.59	7.66	15.35	6.12	14.01
$100 \cdot UR_t$	$100 \cdot UR_t$	0.437	0.860	0.264	0.234	0.227	0.262	0.760	0.982	0.745	0.742	0.374	0.667
$PCS_t CS_t$	CS_t	4.55	1.24	1.50	1.15	1.21	1.11	2.78	3.55	1.99	3.33	2.05	3.19
$PCN_t CN_t$	CN_t	3.62	7.59	2.12	2.31	1.76	2.25	1.68	3.46	2.68	4.62	2.92	4.93
$PCD_t CD_t$	CD_t	4.23	3.21	3.09	2.60	2.37	2.44	7.73	12.67	12.19	14.97	11.79	13.76
$PIH_t IH_t$	IH_t	3.78	2.85	2.76	1.30	1.36	0.89	9.86	9.41	6.28	4.76	5.20	3.22
TLF_{1t}	LF_{1t}	77.	104.	51.	50.	51.	50.	132.	189.	184.	203.	166.	201.
TLF_{2t}	LF_{2t}	827.	296.	215.	209.	182.	193.	488.	370.	294.	249.	237.	264.
$MOON_t$	D_t	314.	336.	242.	254.	211.	235.	130.	96.	124.	121.	59.	203.
$PFF_t INV_t$	IP_t	2.26	3.06	1.33	1.46	1.44	1.41	3.83	7.42	2.29	4.01	2.22	4.25
$JOBF_t$	M_t	1126.	909.	257.	269.	235.	248.	459.	806.	582.	758.	436.	660.
$EMPL_t$	E_t	966.	730.	285.	249.	234.	224.	540.	876.	670.	868.	492.	755.

Note: See Appendix B for a definition of the variables in the forecasting model. Root mean square errors for flow variables are at *annual* rates.

what less accurate with respect to predictions of constant dollar GNP and the unemployment rate. For the components of current dollar GNP, the empirical model does worst relative to the forecasting model with respect to the predictions of housing investment, which reflects in large part the fact that the forecasting model takes the mortgage rate and deposit flows into Savings and Loan Associations and Mutual Savings Banks as exogenous. Somewhat surprisingly, the two models are of about the same degree of accuracy with respect to predictions of plant and equipment investment, even though the forecasting model takes plant and equipment investment expectations as exogenous.

Consider next the dynamic, within sample results in Table 8–2. The discussion here will concentrate on the RMSE results. The empirical model is slightly more accurate with respect to predictions of current dollar GNP, somewhat less accurate with respect to predictions of constant dollar GNP, and considerably more accurate with respect to predictions of the unemployment rate. Even though the forecasting model is less accurate with respect to predictions of the unemployment rate, it is more accurate with respect to predictions of the level of employment ($EMPL_t$ or E_t) and the level of nonprime-age-male labor force (TLF_{2t} or LF_{2t}). With respect to the components of GNP, the empirical model is better for nondurable consumption and plant and equipment investment and worse for service consumption, durable consumption, and housing investment.

Consider finally the outside sample results in Table 8–2. It is obvious from these results that the empirical model is more accurate than the forecasting model for the outside sample period considered here. The three quarters that comprise this period are not easy quarters to predict, and the empirical model clearly does a better job in predicting them than does the forecasting model. When the forecasting model is made more endogenous by adding back in, in various combinations, the price, inventory, and import equations, the results are worse than those presented in Table 8–2. Consequently, the poorer results for the forecasting model in Table 8–2 are not due to an unfortunate exclusion of equations that cause the overall model to be less accurate than it would be if the equations were not excluded.

Although the outside sample results are based on only three observations, the overall results in Table 8–2 clearly indicate that the empirical model is more accurate than the forecasting model. The within sample results are about the same for the two models, and the outside sample results are considerably better for the empirical model. Since the forecasting model appears from the results in the previous section to be at least as accurate as other models, the tentative conclusion here is that the empirical model is more accurate than other models. This conclusion is tentative because of the uncertainty as to whether even the less endogenous version of the forecasting model is as accurate as other models for the 1973–1974 period. Clearly more comparisons are needed before any definitive conclusions can be drawn. It should be noted, however, that even if it turns out that the less endogenous version of the forecasting model is less accurate than other models for the 1973–1974 period, it may still be the case that the empirical model is more accurate. The empirical model is substantially more accurate than the forecasting model for the outside sample results in Table 8–2, not just marginally so.

8.4 FURTHER RESULTS ON THE PREDICTIVE ACCURACY OF THE EMPIRICAL MODEL

The purpose of this section is to consider the predictive accuracy of the empirical model in somewhat more detail. Two of the questions considered in this section are how the accuracy of the model estimated by TSLS compares to the accuracy of the model estimated by FIML, and how accurate the model is during recessionary periods and other hard-to-forecast periods.

Results that are relevant to answering the first question are presented in Table 8–3. Two prediction periods are considered in the table: a within sample period of 82 observations (1954I–1974II) and the outside sample period of 3 observations (1974III–1975I)[b]. Results for both static and dynamic predictions and for both the TSLS and FIML estimates are presented in the table. The results in Table 8–3 are again fairly self-explanatory, and the discussion here will only highlight some of the more interesting ones.

First, a comparison of the TSLS and FIML results in the table yields no obvious winner. The results are generally fairly close for the two sets of estimates, and there are no strong grounds for arguing that one set of results is better than the other. One would, of course, expect the results to be fairly close because of the closeness of the TSLS and FIML estimates themselves. As discussed in Chapter Three, it is not clear how close the FIML estimates obtained in this study are to the true FIML estimates, and so the FIML results in Table 8–3 must be interpreted with some caution. It may be, as the results in Table 8–3 indicate, that the predictive accuracy of the model is about the same for both the TSLS and FIML estimates, but one should probably reserve judgment on this until further experimentation is done on trying to obtain true FIML estimates of the model.

Consider next the accuracy of the model regarding the predictions of the bill rate. For the TSLS results, the root mean square errors of the level predictions of the bill rate range from 1.81 percentage points for the static, within sample results to 4.14 percentage points for the dynamic,

Table 8–3. Further Results on the Predictive Accuracy of the Empirical Model

FIML = FIML Estimates Used
TSLS = TSLS Estimates Used
RMSE = Root Mean Square Error of Level Predictions
RMSEΔ = Root Mean Square Error of First Difference Predictions

Equation No. in Table 2–2	*Variable*	1954I–1974II (*within sample*) *DYNAMIC*				1954I–1974II (*within sample*) *STATIC*		1974III–1975I (*outside sample*) *DYNAMIC*				1974III–1975I (*outside sample*) *STATIC*	
		RMSE		*RMSEΔ*		*RMSE*		*RMSE*		*RMSEΔ*		*RMSE*	
		FIML	*TSLS*	*FIML*	*TSLS*	*FIML*	*TSLS*	*FIML*	*TSLS*	*FIML*	*TSLS*	*FIML*	*TSLS*
10.	Y_t	8.15	6.24	5.88	6.28	3.45	3.47	5.88	5.95	6.40	7.66	6.16	6.12
9.	PF_t	0.0156	0.0147	0.0060	0.0056	0.0051	0.0049	0.0046	0.0097	0.0063	0.0086	0.0052	0.0016
	GNP_t	7.84	9.71	7.30	8.03	4.90	4.93	7.88	10.64	10.23	12.85	6.20	6.83
83.	UR_t	0.574	0.529	0.352	0.356	0.240	0.238	0.945	0.760	0.737	0.745	0.462	0.374
70.	$RBILL_t$	3.04	2.59	2.91	3.03	1.84	1.81	4.75	4.14	6.44	6.48	4.26	2.97
1.	CS_t	6.81	4.20	1.23	1.09	0.81	0.80	2.27	1.13	1.23	1.15	1.57	1.29
2.	CN_t	5.43	4.65	1.42	1.42	1.23	1.22	1.39	1.45	2.48	2.55	2.01	2.03
46.	CD_t	2.78	2.56	2.68	2.62	1.83	1.84	6.53	6.16	9.52	9.98	9.60	9.29
47.	IH_t	3.12	3.26	2.62	3.37	1.13	1.12	4.70	4.88	2.57	2.98	2.77	2.53
5.	TLF_{1t}	169.	132.	65.	65.	58.	58.	129.	132.	183.	184.	165.	166.
6.	TLF_{2t}	462.	407.	258.	253.	221.	223.	362.	488.	234.	294.	222.	237.
7.	$MOON_t$	319.	292.	256.	258.	218.	219.	102.	130.	105.	124.	60.	59.
8.	DDH_t	3.92	3.27	2.70	2.35	1.75	1.70	5.53	7.67	2.91	4.32	2.35	2.89
11.	INV_t	2.00	1.71	1.00	0.96	0.96	0.95	2.09	2.31	1.03	1.18	1.25	1.24
12.	$JOBF_t$	876.	691.	319.	310.	264.	263.	594.	459.	613.	582.	497.	436.
13.	HPF_t	2.04	1.97	1.99	1.97	1.56	1.55	2.68	2.33	1.23	1.16	1.97	2.01
14.	$HPFO_t$	3.09	3.04	2.29	2.37	2.59	2.60	3.41	3.46	4.00	4.24	3.30	3.31

15.	WF_t	0.60	0.98	0.24	0.25	0.26	0.26	0.54	0.79	0.37	0.41	0.35	0.38
16.	DDF_t	2.20	1.98	0.84	0.80	0.68	0.69	0.17	0.23	0.17	0.27	0.20	0.12
17.	$DIVF_t$	4.70	3.18	0.48	0.44	0.38	0.38	0.42	0.50	0.67	0.43	0.49	0.41
18.	$INTF_t$	6.89	6.49	0.46	0.45	0.32	0.32	0.89	1.25	0.67	0.89	0.28	0.32
19.	IVA_t	4.09	3.83	5.41	5.24	3.71	3.67	16.59	16.11	17.56	18.79	17.84	16.29
20.	$BORR_t$	0.97	0.86	0.65	0.64	0.49	0.51	2.22	2.12	2.56	2.65	2.46	2.18
21.	$RAAA_t$	0.85	0.81	0.81	0.80	0.48	0.47	1.07	1.27	1.50	1.93	0.96	0.86
22.	$RMORT_t$	0.79	0.72	0.66	0.62	0.19	0.19	0.51	0.43	0.63	0.51	0.89	0.86
23.	CG_t	310.	309.	540.	533.	213.	211.	696.	816.	848.	1009.	585.	587.
24.	IM_t	1.48	1.46	0.93	0.93	0.80	0.80	4.14	3.96	3.35	3.19	3.36	3.36
25.	TPU_t	0.57	0.50	0.35	0.36	0.35	0.35	2.99	2.59	2.74	2.75	1.66	1.37
26.	$INTG_t$	1.22	1.05	0.55	0.50	0.39	0.38	0.48	0.84	0.84	1.06	0.62	0.54
45.	BR_t	0.87	0.63	0.56	0.48	0.34	0.33	0.99	1.44	0.51	0.78	0.41	0.52
48.	X_t	7.83	5.93	5.14	5.67	3.18	3.15	7.33	7.12	12.13	12.65	12.35	12.11
51.	$V_t - V_{t-1}$	3.05	3.10	3.33	3.39	2.51	2.47	6.68	6.88	10.87	11.12	7.95	7.68
52.	πF_t	7.99	7.30	8.97	9.46	6.12	5.88	19.46	19.97	15.50	18.56	16.85	12.95
53.	CF_t	6.34	5.81	4.08	4.92	3.11	2.96	11.31	10.59	16.46	14.77	15.06	14.94
55.	LF_t	40.74	38.46	1.64	1.53	0.97	0.97	2.90	3.33	3.37	3.28	4.31	4.24
58.	YH_t	9.42	11.56	3.57	3.62	3.33	3.38	9.45	12.90	10.64	11.52	7.84	8.89
60.	$SAVH_t$	8.10	7.47	4.74	5.62	3.00	2.89	11.97	13.17	12.47	12.96	11.75	10.96
61.	A_t	104.83	97.19	76.41	76.49	52.43	51.96	175.88	208.13	173.50	202.79	146.70	146.72
62.	DDB_t	4.74	3.46	3.10	2.65	1.83	1.81	5.41	7.88	2.95	4.55	2.15	2.77
64.	$LBVBB_t$	3.69	2.99	2.44	2.07	1.48	1.47	5.67	6.81	3.59	4.49	3.65	3.52
65.	$SAVR_t$	1.78	1.78	1.27	1.27	1.06	1.05	9.80	10.27	8.01	7.83	7.83	8.15
67.	TAX_t	4.47	4.69	4.94	5.24	3.26	3.24	10.88	10.94	11.77	13.38	10.20	9.09
68.	$SAVG_t$	3.85	3.65	4.48	4.79	2.61	2.62	10.33	11.09	11.26	13.10	9.26	8.53
81.	$EMPL_t$	721.	597.	305.	320.	236.	246.	685.	540.	683.	670.	545.	492.
82.	U_t	422.	394.	259.	263.	187.	185.	883.	717.	679.	688.	417.	333.

Note: Root mean square errors for flow variables are at *annual* rates.

outside sample results. Given the way the bill rate is determined in the model, these errors seem fairly reasonable, although they are by no means as small as one might hope.

I thought in the initial phases of this study that the FIML estimates would lead to more accurate predictions of the bill rate than would the TSLS estimates. As mentioned in Chapter Three, the FIML estimator does not require that there be a natural left-hand side variable for each equation, and since there is no equation in which the bill rate appears naturally as a left-hand side variable, the FIML estimator appears to be the natural estimator to use for the model. Since the FIML estimator, unlike the TSLS estimator, takes into account in an explicit way the fact that the bill rate is implicitly determined in the model, one might expect the predictions of the bill rate to be more accurate for the FIML estimates than for the TSLS estimates. This unfortunately is not the case for the results in Table 8–3. Again, however, this may be due to a failure to obtain estimates that are close to the true FIML estimates, and so one should probably reserve judgment on this issue as well until more experimentation is done.

The FDYN estimator discussed in Chapter Three also takes into account the fact that the bill rate is implicitly determined in the model, and if in future work it is possible to obtain FDYN estimates of the model, it will be of interest to see if these estimates lead to more accurate predictions of the bill rate than have been obtained so far.

Two final points about the results in Table 8–3 will be made here. First, the accuracy with which the model predicts $SAVR_t$ is the accuracy with which it predicts the U.S. balance of payments on current account. The RMSEs for $SAVR_t$ in Table 8–3 range from about a billion dollars (at an annual rate) for the static, within sample results to about ten billion dollars for the dynamic, outside sample results. Second, the accuracy with which the model predicts $SAVG_t$ is the accuracy with which it predicts the budget deficit or surplus of the government. The RMSEs for $SAVG_t$ range from about three billion dollars to about ten billion dollars. The RMSEs for $SAVG_t$ are always quite close to the RMSEs for the total net tax collections of the government (TAX_t).

Results that pertain to the question of the accuracy of the model during hard-to-predict periods are presented in Tables 8–4, 8–5, and 8–6. The three periods considered in these tables are 1955I–1962IV, which encompasses the 1958 and 1960 recessions; 1968I–1974II, which encompasses the 1970 recession and the beginning of the 1974 recession; and 1974III–1975I, which is the outside sample period considered in Tables 8–2 and 8–3. The period between 1962IV and 1968I was not considered because it is a period of fairly smooth growth.

The results in Tables 8–4 and 8–5 are taken from the dynamic simulation using the TSLS estimates that began in 1954I. The predictions in these two tables are within sample predictions. The results in Table 8–6 are taken from the dynamic simulation using the TSLS estimates that began in 1974III. The predictions in this table are outside sample predictions. Predictions for five variables are presented in the tables: Y_t, PF_t, GNP_t, UR_t, and $RBILL_t$. Again, the results in the three tables are fairly self-explanatory, and the following discussion will only highlight a few of them.

There is no question that the model stays roughly on track over time. The model ends the dynamic 82-quarter simulation in 1974II (Table 8–5) with an error for Y_t of only 11.2 billion dollars and an error for the unemployment rate of only 0.51 percentage points. The error for the price level is -0.025, or about -1.8 percent. The fact that the model ends the simulation in this way means that any large errors that it might make along the way tend to get corrected over time.

Consider now the results in Table 8–4 and concentrate on those quarters in which the error in predicting Y_t is greater than 10.0 billion dollars in absolute value. The first such quarter is 1958I. Y_t decreased by 18.1 billion dollars from 1957III to 1958I (from 406.8 to 388.7), whereas the model predicted a decrease of only 7.0 billion dollars (from 406.9 to 399.9). The prediction error in 1958I, the trough for Y_t, is 11.2 billion dollars. The model predicted the trough for Y_t to occur two quarters later than it did. The predicted value of Y_t for this quarter (1958III) is 392.4 billion dollars, which compares closely to the actual trough value of 388.7 billion dollars. The model thus caught the magnitude of the 1958 recession fairly well, but missed some of the timing.

Regarding the predictions of the unemployment rate during the 1958 recession, the model had it peaking in 1958III at 7.24 percent, which compares to the actual peak a quarter earlier of 7.38 percent. The bill rate predictions for 1958I and 1958II are both much too high. The prediction for 1958II is 7.77 percent, which compares to the actual rate of only 1.02 percent. There are a number of quarters in Tables 8–4, 8–5, and 8–6 in which very large errors are made in predicting the bill rate, and 1958I and 1958II are clearly two of them.

The next large errors for Y_t occur in 1959IV and 1960I, where errors of 19.2 and 13.8 billion dollars are made. The economy is difficult to predict for 1959IV and 1960I because of the effects of the 1959 steel strike, and not much importance should be attached to the results for these two quarters.

The model caught the 1960 recession about as well as it caught the 1958 recession. Y_t reached a trough of 429.2 billion dollars in 1961I. The model predicted the trough to occur a quarter later. The predicted value of Y_t for this quarter (1961II) is 429.7 billion dollars, which compares almost exactly to the actual trough value of 429.2 billion dollars. The model predicted the unemployment rate very well during this period. The unemploy-

Table 8–4. Predicted and Actual Values for Five Variables for the 1955I–1962IV Period

Dynamic Predictions Using TSLS Estimates
Prediction Period Began in 1954I
P = Predicted Value
A = Actual Value
$E = P - A$

Quarter	Y_t			PF_t			GNP_t			$100 \cdot UR_t$			$RBILL_t$		
	P	A	E	P	A	E	P	A	E	P	A	E	P	A	E
1955I	380.1	381.7	−1.6	0.825	0.825	0.000	385.6	386.2	−0.6	4.73	4.75	−0.02	0.36	1.26	−0.90
II	391.2	389.3	1.9	0.828	0.825	0.003	398.0	394.4	3.6	3.98	4.42	−0.44	0.68	1.61	−0.93
III	395.8	395.7	0.1	0.830	0.831	−0.001	402.8	402.5	0.3	3.61	4.15	−0.54	0.85	1.86	−1.01
IV	400.3	399.7	0.6	0.831	0.839	−0.008	406.8	408.7	−2.0	3.38	4.25	−0.87	0.57	2.35	−1.78
1956I	405.9	397.2	8.7	0.841	0.846	−0.004	417.5	410.6	6.9	3.12	4.07	−0.96	3.30	2.38	0.93
II	404.8	398.1	6.7	0.849	0.851	−0.002	421.8	416.2	5.6	3.23	4.23	−1.00	3.82	2.60	1.23
III	402.5	397.3	5.2	0.857	0.859	−0.002	424.9	420.7	4.3	3.68	4.17	−0.49	6.16	2.60	3.57
IV	399.1	403.0	−3.9	0.860	0.867	−0.008	422.2	429.4	−7.2	4.35	4.14	0.21	1.79	3.06	−1.27
1957I	409.3	405.5	3.7	0.868	0.880	−0.012	435.5	437.2	−1.7	4.29	3.99	0.30	2.50	3.17	−0.67
II	410.5	405.0	5.5	0.878	0.883	−0.005	442.7	439.7	2.9	4.26	4.10	0.15	5.55	3.16	2.39
III	406.9	406.8	0.0	0.885	0.889	−0.004	444.3	446.2	−1.9	4.75	4.25	0.50	4.55	3.38	1.17
IV	406.1	399.6	6.5	0.892	0.894	−0.002	446.0	441.4	4.6	5.47	4.96	0.51	5.15	3.34	1.81

1958I	399.9	388.7	11.2	0.898	0.896	0.002	445.6	434.8	10.8	6.23	6.30	−0.07	5.90	1.84	4.06
II	395.3	390.2	5.1	0.905	0.897	0.008	446.3	438.6	7.8	6.84	7.38	−0.54	7.77	1.02	6.75
III	392.4	401.3	−8.9	0.898	0.901	−0.003	441.0	451.6	−10.6	7.24	7.33	−0.09	0.60	1.71	−1.11
IV	410.8	411.9	−1.1	0.901	0.906	−0.006	460.8	464.4	−3.5	6.57	6.38	0.19	1.25	2.79	−1.53
1959I	419.1	418.7	0.4	0.899	0.912	−0.013	468.1	473.9	−5.9	5.90	5.83	0.07	0.88	2.80	−1.92
II	430.1	430.2	−0.1	0.899	0.915	−0.016	479.2	486.6	−7.4	5.20	5.14	0.07	1.22	3.02	1.80
III	430.8	423.4	7.4	0.885	0.919	−0.034	475.5	483.8	−8.3	6.04	5.32	0.72	0.07	3.53	−3.46
IV	448.6	429.4	19.2	0.902	0.920	−0.019	500.7	490.2	10.5	4.95	5.62	−0.68	9.60	4.30	5.30
1960I	452.9	439.1	13.8	0.908	0.922	−0.014	509.9	502.9	7.0	5.29	5.19	0.10	11.89	3.94	7.95
II	428.8	437.0	−8.2	0.906	0.924	−0.018	487.3	504.7	17.4	6.09	5.26	0.83	2.19	3.09	−0.90
III	437.5	435.0	2.5	0.909	0.925	−0.015	499.6	504.2	−4.6	6.65	5.58	1.07	2.07	2.39	−0.32
IV	443.0	430.7	12.2	0.917	0.928	−0.011	510.2	503.2	7.0	6.51	6.28	0.23	6.66	2.36	4.30
1961I	438.2	429.2	9.0	0.924	0.929	−0.005	509.7	503.6	6.1	6.60	6.80	−0.20	9.09	2.38	6.71
II	429.7	439.6	−9.9	0.924	0.931	−0.007	501.0	514.9	−13.9	7.07	6.99	0.07	2.28	2.32	−0.04
III	441.8	447.8	−6.1	0.918	0.930	−0.012	512.3	524.2	−11.9	6.71	6.77	−0.07	0.42	2.32	−1.91
IV	452.8	457.2	−4.4	0.924	0.934	−0.010	527.9	537.7	−9.8	5.75	6.20	−0.45	2.30	2.48	−0.18
1962I	461.8	464.3	−2.5	0.925	0.935	−0.010	539.9	547.8	−8.0	5.20	5.64	−0.44	2.57	2.74	−0.17
II	475.3	472.0	3.2	0.930	0.938	−0.008	556.0	557.2	−1.2	4.83	5.51	−0.68	5.01	2.72	2.29
III	478.0	476.9	1.1	0.936	0.939	−0.003	563.2	564.4	−1.2	4.96	5.57	−0.61	8.23	2.86	5.38
IV	478.7	482.2	−3.6	0.936	0.942	−0.006	564.8	571.9	−7.2	5.49	5.54	−0.05	3.05	2.80	0.25

Notes: Values for Y_t and GNP_t are at annual rates.
E does not always equal $P - A$ in the table because of rounding.

Table 8–5. Predicted and Actual Values for Five Variables for the 1968I–1974II Period

Dynamic Predictions Using TSLS Estimates
Prediction Period Began in 1954I
P = Predicted Value
A = Actual Value
$E = P - A$

	Y_t			PF_t			GNP_t			$100 \cdot UR_t$			$RBILL_t$		
Quarter	*P*	*A*	*E*	*P*	*A*	*E*	*P*	*A*	*E*	*P*	*A*	*E*	*P*	*A*	*E*
1968I	613.5	622.3	−8.9	1.057	1.053	0.004	826.5	834.0	−7.5	3.66	3.77	−0.10	4.62	5.06	−0.44
II	627.2	633.9	−6.6	1.055	1.062	−0.007	845.1	857.4	−12.3	3.40	3.58	−0.18	0.77	5.51	−4.74
III	639.4	640.0	−0.6	1.069	1.071	−0.002	873.3	875.2	−1.9	2.83	3.55	−0.72	4.93	5.23	−0.30
IV	647.1	644.3	2.8	1.072	1.082	−0.011	887.0	890.2	−3.2	2.78	3.43	−0.66	1.88	5.58	−3.70
1969I	654.5	650.1	4.3	1.088	1.093	−0.005	908.9	906.9	2.1	2.56	3.42	−0.86	6.01	6.14	−0.13
II	653.1	652.8	0.4	1.098	1.105	−0.007	919.3	923.5	−4.2	2.77	3.46	−0.69	5.84	6.24	−0.40
III	655.9	655.4	0.5	1.109	1.118	−0.009	936.5	941.8	−5.3	3.11	3.63	−0.52	8.46	7.05	1.41
IV	656.0	651.5	4.5	1.119	1.130	−0.011	946.5	948.9	−2.4	3.60	3.62	−0.02	6.55	7.32	−0.76
1970I	652.5	647.5	5.0	1.131	1.143	−0.012	956.5	958.5	−2.1	4.23	4.19	0.04	9.21	7.26	1.94
II	651.8	647.7	4.2	1.137	1.155	−0.018	963.2	970.6	−7.4	4.87	4.76	0.11	4.01	6.75	−2.74
III	657.5	653.0	4.6	1.148	1.164	−0.016	981.0	987.4	−6.3	5.21	5.20	0.01	4.24	6.37	−2.13
IV	653.6	644.8	8.7	1.158	1.185	−0.027	983.2	991.8	−8.6	5.60	5.84	−0.24	4.90	5.36	−0.46
1971I	669.8	662.4	7.5	1.164	1.195	−0.031	1014.0	1027.8	−13.9	5.64	5.96	−0.33	2.78	3.86	−1.09
II	677.0	667.1	9.9	1.174	1.207	−0.033	1034.3	1047.3	−13.0	5.46	5.92	−0.47	4.56	4.21	0.36
III	687.6	671.2	16.4	1.185	1.215	−0.030	1058.7	1061.3	−2.6	5.39	5.97	−0.58	6.43	5.05	1.38
IV	693.4	682.5	10.9	1.196	1.216	−0.021	1080.4	1083.2	−2.8	5.53	5.97	−0.45	7.75	4.23	3.51
1972I	694.5	694.3	0.2	1.204	1.232	−0.028	1090.6	1115.0	−24.4	5.81	5.82	−0.01	5.14	3.44	1.70
II	704.3	709.8	−5.5	1.208	1.237	−0.029	1110.8	1143.0	−32.2	5.97	5.68	0.29	1.68	3.75	−2.06
III	719.4	720.7	−1.3	1.214	1.244	−0.030	1142.6	1169.3	−26.7	5.65	5.56	0.08	1.24	4.24	−3.00
IV	739.7	736.1	3.6	1.228	1.253	−0.025	1186.6	1204.7	−18.1	5.05	5.31	−0.26	3.55	4.85	−1.30

1973I	758.6	754.9	3.7	1.239	1.264	-0.025	1232.8	1248.9	-16.1	4.58	4.99	-0.42	2.87	5.64	-2.77
II	764.0	758.4	5.5	1.261	1.281	-0.020	1267.5	1277.9	-10.3	4.28	4.91	-0.63	5.44	6.61	-1.17
III	759.2	762.0	-2.8	1.289	1.298	-0.009	1297.5	1308.9	-11.3	4.64	4.76	-0.12	12.37	8.39	3.98
IV	759.3	766.6	-7.4	1.310	1.328	-0.018	1316.6	1344.0	-27.5	5.13	4.75	0.38	2.97	7.46	-4.49
1974I	766.0	751.3	14.7	1.352	1.374	-0.022	1361.0	1358.8	2.2	5.28	5.14	0.14	7.02	7.60	-0.58
II	758.7	747.6	11.2	1.399	1.424	-0.025	1380.7	1383.8	-3.1	5.65	5.15	0.51	10.27	8.27	2.00

Notes: See notes to Table 8–4.

Table 8–6. Predicted and Actual Values for Five Variables for the 1974III–1975I Period

Dynamic Predictions Using TSLS Estimates
Prediction Period Began in 1974III
P = Predicted Value
A = Actual Value
$E = P - A$

		Y_t			PF_t			GNP_t			$100 \cdot UK_t$			$RBILL_t$	
Quarter	*P*	*A*	*E*	*P*	*A*	*E*	*P*	*A*	*E*	*P*	*A*	*E*	*P*	*A*	*E*
1974III	734.5	743.5	-8.9	1.469	1.468	0.000	1404.8	1416.3	-11.5	5.85	5.51	0.34	8.41	8.29	0.12
IV	722.5	724.0	-1.5	1.501	1.515	-0.014	1417.8	1430.9	-13.1	6.49	6.60	-0.10	2.60	7.34	-4.74
1975I	703.6	698.7	4.9	1.546	1.555	-0.009	1422.5	1416.6	5.9	7.08	8.35	-1.27	11.26	5.87	5.38

Notes: See notes to Table 8–4.

ment rate reached a peak of 6.99 percent in 1961II, which compares almost exactly to the predicted peak in the same quarter of 7.07 percent.

Consider next the results in Table 8–5. The first errors for Y_t of greater than 10.0 billion dollars occur in 1971III and 1971IV. The model predicted that Y_t would increase more in the last half of 1971 than it actually did. The model was back on track in 1972I, however, and it stayed fairly much on track until 1974I, where it failed to predict the decrease in Y_t that occurred in that quarter. The unemployment rate predictions are all fairly good in Table 8–5. All the errors are less than a percentage point, with the largest error of 0.86 percentage points occurring in 1969I. The largest value of the bill rate for the period considered in this study is 8.39 percent in 1973III, and it is interesting to note that the largest predicted value of the bill rate also occurs in this quarter, 12.37 percent.

All the errors in predicting the price level in Table 8–5 are negative except for the first one. The errors are not, however, particularly large. The largest error occurs in 1971II (−0.033), where the actual value is about 2.8 percent larger than the predicted value. The largest three errors in predicting the *rate* of inflation in Table 8–5 occur in 1968II, 1970IV, and 1973III. The actual rates in these three quarters (at an annual rate) are 3.5 percent, 7.4 percent, and 5.4 percent, respectively, while the predicted rates are −0.8 percent, 3.5 percent, and 9.2 percent, respectively.

The final predictions to consider are the ones in Table 8–6. These predictions are outside sample predictions for a fairly difficult period, and so they provide a good test of the model. Y_t decreased by 48.9 billion dollars from 1974II to 1975I (from 747.6 to 698.7). The model predicted a decrease in this period of 44.2 billion dollars (from 747.6 to 703.6). Not a bad prediction. The price level increased at an annual rate of 12.5 percent in this three-quarter period (from 1.424 to 1.555). The model predicted an increase of 11.6 percent (from 1.424 to 1.546). Not bad again. This is clearly a remarkable performance by the model given the extreme behavior of the economy during this period and the fact that the predictions are outside sample predictions.

The unemployment rate increased from 5.15 percent in 1974II to 8.35 in 1975I. The model predicted an increase to 7.08 percent in 1975I, and so it underpredicted the increase by 1.27 percentage points. The model predicted the bill rate almost exactly in 1974III, but it underpredicted the bill rate by 4.74 percentage points in 1974IV and overpredicted the bill rate by 5.38 percentage points in 1975I.

This completes the examination of the predictive accuracy of the model. While some of the above discussion has concentrated on the more negative results, the overall performance of the model appears quite good. There are only a few cases in which the model does not appear capable of tracking well the quarter-to-quarter performance of the economy. The outside sample predictions for 1974III, 1974IV, and 1975I in Table 8–6 are particularly encouraging regarding the model's accuracy. The predictions of the bill rate are clearly in the most need of improvement. At times very large errors are made by the model in predicting the bill rate. As mentioned above, these errors may be lessened by the use of estimates like FIML and FDYN, but as of now this is only a conjecture.

NOTES

[a] The results of the two studies of McNees [34], [35], are consistent with this conclusion. The ex ante performance of the model is not generally as good as the ex ante performance of the other (subjectively adjusted) models, but it is not too far below the others.

[b] Regarding the 1954I–1974II period, data on one endogenous variable, $HPFO_t$, are only available beginning in 1956I. Since $HPFO_t$ is endogenous and enters the model only contemporaneously, the lack of data on $HPFO_t$ causes no problem except in computing its error measure. For purposes of computing RMSE and RMSEΔ for $HPFO_t$ in Table 8–3 for the 82-observation period, the predictions of $HPFO_t$ for the first eight quarters (1954I–1955IV) were compared to the single-equation predicted values of $HPFO_t$ generated from Equation 14 in Table 2–3 using the actual values of HPF_t.

Chapter Nine

The Properties of the Model

9.1 INTRODUCTION

Section 9.3 contains a detailed examination of the properties of the model. The properties are examined by observing how the model responds to changes in the exogenous variables. The results in section 9.3 are useful not only in showing the quantitative properties of the model, but also in pointing out the various asymmetrical properties of the model, in pointing out the various tax leakages that occur when a government policy variable is changed, and in indicating what the consequences are of the fact that the model is closed with respect to the flows of funds. Before proceeding to the detailed examination in section 9.3, the model will be briefly reviewed in the next section.

9.2 A BRIEF REVIEW OF THE MODEL

The important variables affecting the household sector are: the various price deflators, the wage rate, nonlabor income, the marginal personal income tax rate, the bill and mortgage rates, the value of assets of the previous period, the hours constraint variable, and the loan constraint variable. Nonlabor income includes transfer payments from the government. The seven main decision variables of the household sector are: expenditures on services, nondurable goods, durable goods, and housing, the labor force participation of men 25–54 and of all persons 16 and over except men 25–54, and the percentage of people moonlighting. These latter three variables are referred to as "work effort" variables.

When prices rise relative to the wage rate, this has a negative effect on consumption and work effort. The negative effect on work effort means that a rise in prices relative to the wage rate has, other things being equal, a negative effect on the unemployment rate. The effect on the unemployment rate is negative in this case because the size of the labor force has decreased. The interest rates have a negative effect on consumption and a slight positive effect on work effort. This latter effect means that a rise in the interest rates has a direct positive effect on the unemployment rate.

Raising net taxes either by increasing the marginal tax rate or by decreasing the level of transfer payments has a negative effect on consumption, the effect of the decrease in transfer payments working through the nonlabor income variable. Increasing the marginal tax rate has, however, a negative effect on work effort, while decreasing the level of transfer payments has a positive effect. Therefore, raising net taxes by increasing the *marginal tax rate has a direct negative effect on the unemployment rate,* whereas raising net taxes by decreasing the level of transfer payments has a direct positive effect.

The value of assets of the previous period (A_{t-1}) has a positive effect on consumption and a negative effect on work effort. Much of the variance of A_{t-1} is due to the variance of CG_{t-1}, the variable measuring capital gains or losses on corporate stocks held by the household sector. Consequently, much of the effect of A_{t-1} on the household sector is reflecting the effect of CG_{t-1}. Since A_{t-1} has a negative effect on work effort, this means that an increase in stock prices in period $t - 1$ has a direct negative effect on the unemployment rate in period t.

The five main decision variables of the firm sector are its price, production, investment, employment demand, and wage rate. The important variables affecting this sector are: the price of imports, the bond rate, the investment tax credit, the level of sales, the amounts of excess labor and capital on hand, the variable measuring labor market tightness (J_t^*), the labor constraint variable, and lagged values of the price level, the wage rate, production, and the stock of inventories.

The bond rate has a contractionary effect on the firm sector. An increase in the bond rate causes the firm sector to raise its price, thus lowering sales. Lower sales lead the firm sector to decrease its production, investment, and employment demand. In a similar manner, a decrease in the investment tax credit has a contractionary effect on the firm sector, since it causes the firm sector to raise its price. The same also holds true for an increase in the price of imports.

With respect to the various stock variables in the firm sector, the stock of inventories of the previous period has a negative effect on current production; the amount of excess capital on hand at the end of the previous period has a negative effect on current investment; and the amount of excess labor on hand at the end of the previous period has a negative effect on the current number of jobs and hours paid per job.

Labor market conditions have two main effects on the firm sector. One is that J_t^* has a direct positive effect on the wage rate that the firm sector sets. The other effect is through the labor constraint variable, ZJ'_t. If the firm sector does not get in a period as much labor as it expected that it would at the wage rate that it set, then it raises its price and contracts. In this case the firm and household sectors are assumed to interact a number of times within the quarter, with the effect in the end being that the price and wage rate are raised enough so that the final employment demand from the firm sector is equal to the amount that the household sector is willing to supply. These interactions are assumed to be captured in the model through the specification of simultaneous equations.

Regarding the relationship between the price level and the wage rate, the current price level has a positive effect on the current wage rate, but not vice versa. The wage rate instead affects the price level with a lag of one quarter. As discussed in Chapter Five, the inclusion of the wage rate in the price equation is designed to pick up expectational effects, whereas the inclusion of the price level in the wage rate equation is more designed to reflect the assumption that the firm and household sectors bargain over the real wage.

The two main links between the household and firm sectors are through the price level and wage rate, and through the hours and labor constraint variables. The firm sector sets the price level and the wage rate, and the household sector responds negatively to the former and positively to the latter. The firm sector constrains the household sector through the hours constraint variable, and the household sector constrains the firm sector through the labor constraint variable. In theory, when the hours constraint is binding, the labor constraint should not be, and vice versa. This is not quite true in the empirical model, however, because of the approximations that have been used.

Since the bill rate is implicitly determined in the model, all sectors contribute to its determination. The bill rate results from equating the aggregate constrained demand for funds to the aggregate constrained supply. The effect of the financial sector on the firm and household sectors is assumed to be reflected in the loan constraint variable. The net effect of the loan constraint variable is to make the model more nonlinear in the bill rate when the loan constraint is binding than it otherwise would be.

9.3 THE RESPONSE OF THE MODEL TO CHANGES IN VARIOUS EXOGENOUS VARIABLES

The analysis in this section is based on the results of a number of experiments. Each experiment corresponds to changing the value of at least one exogenous variable. The effects of fifteen exogenous variables are examined, the variables being exports, the price of imports, and the thirteen government variables with a † beside them in Table 7–1. Two periods were used for the experiments, a period beginning in 1969I and a period beginning in 1971I. 1969I is at or near the top of an expansion, and 1971I is at or near the bottom of a contraction.

The experiments were performed as follows. Consider the period beginning in 1969I. The model was first simulated (dynamically) beginning in 1969I for ten quarters using the actual values of the exogenous variables. The predicted values of the endogenous variables from this simulation were recorded. Other simulations were then run for the ten quarters using different values of the exogenous variables, and the predicted values of the endogenous variables from these simulations were compared to the predicted values from the base simulation. When a value of an exogenous variable was changed, it was changed for the entire ten quarters, not for just the first quarter.

Most of the experiments corresponded to changing the value of only one exogenous variable. The individual effects of fourteen of the fifteen exogenous variables were examined in this way. Both positive and negative changes were considered for the two periods, which resulted in 58 experiments. The other experiments corresponded to changing more than one exogenous variable at a time.

A Decrease in XG_{t+i} of $1.25/PG_{t+i}$ —No Change in VBG_{t+i}

It will be useful to examine the results of five experiments in detail and then to examine the other results in a more summary fashion. The results for the first experiment are presented in Table 9–1. This experiment is for the second period and corresponds to decreasing government purchases of goods by 1.25 billion dollars (5.0 billion dollars at an annual rate) in each quarter from the level that actually prevailed in that quarter. This was accomplished by decreasing XG_{t+i}, government purchases of goods in real terms in quarter $t+i$, by $1.25/PG_{t+i}$ $(i = 0, 1, \ldots, 9)$, where PG_{t+i} is the actual value of the price deflator for government purchases of goods in quarter $t+i$. Since PG_{t+i} is generally rising over time, this procedure means that the changes in XG_{t+i} are generally getting smaller over time. Because PG_{t+i} is an endogenous variable, this procedure is not quite equivalent to decreasing government purchases by 1.25 billion dollars each quarter, but it is quite close. (Note that the actual values of PG_{t+i} were used for the deflation, not the predicted values.)

Results for 46 variables are presented in Table 9–1. The figure for each variable and time period in the table is the difference between the predicted value of the variable that resulted from the simulation with XG_{t+i} changed and the predicted value of the variable that resulted from the base simulation.

Consider the results for quarter t first. The fact that no variable except XG_t was changed for this experiment means that any surplus that the government ran because of the decrease in XG_t resulted in a change in either bank reserves (BR_t) or bank borrowing ($BORR_t$). The saving of the government ($SAVG_t$) increased by 0.69 billion dollars in quarter t, which took the form of a decrease in BR_t of 0.40 billion dollars and an increase in $BORR_t$ if 0.29 billion dollars. The decrease in XG_t led to a decrease in Y_t of 1.40 billion dollars (in real terms) and a decrease in GNP_t of 1.37 billion dollars (in current dollar terms). The unemployment rate increased by 0.13 percentage points.

The bill rate rose by 0.81 percentage points. Loosely speaking, the bill rate rose because of the funds taken out of economy by the increased saving of the government. The increase in the bill rate is the reason for the increase in bank borrowing. The increase in the bill rate also caused the bond rate and mortgage rate to increase. The increase in the bond rate then resulted in the price level being higher. The decrease in XG_t thus resulted in an initial increase in the price level because of the higher interest rates that the decrease caused.

Although government expenditures on goods decreased by roughly 1.25 billion dollars in quarter t, the saving of the government only increased by 0.69 billion dollars. Much of this discrepancy of 0.56 can be explained by the 0.41 billion dollar decrease in net tax collections (TAX_t) that occurred in quarter t as a result of the contraction in the economy. The 0.41 figure includes a 0.05 increase in unemployment insurance benefits (TPU_t) that resulted from the increase in unemployment. The rest of the discrepancy can be explained by the other endogenous changes in government spending that occur when the economy changes. The endogenous variables that are relevant in explaining the rest of the discrepancy are $INTG_t$, WGC_t, WGM_t, and PG_t. $INTG_t$, the interest paid by the government, for example, increased by 0.09 billion dollars as a result of the higher bill and bond rates.

The contraction of the firm sector in quarter t took the form, in addition to a higher price level and a lower level of production, of a decrease in investment (INV_t) of 0.04 billion dollars in real terms, a decrease in the number of jobs ($JOBF_t$) of 129 thousand, a decrease in the average number of hours paid per job for the quarter (HPF_t) of 0.68 hours, and a decrease in the wage rate (WF_t) of 0.013 points. The positive effect that the higher price level had on the wage rate was offset by the negative effect of fewer worker hours needed. The fact that the number of jobs and hours paid per job decreased meant that the hours constraint on the household sector became more restrictive. The hours constraint was already binding in quarter t because the quarter (1970I) is at or near the bottom of a contraction. The level of profits of the firm sector was lower by 0.31 billion dollars, and its cash flow was lower by 0.78 billion dollars.

Table 9–1. Detailed Experimental Results: A Decrease in XG_{t+i} of $1.25/PG_{t+i}$. No Change in VBG_{t+i} ($t = 1971$II [bottom of contraction])

Equation No. in Table 2–2	*Change in:*	t	$t+1$	$t+2$	$t+3$	$t+4$	$t+5$	$t+6$	$t+7$	$t+8$	$t+9$
10.	Y	−1.40	−2.44	−2.75	−2.75	−2.30	−1.88	−1.63	−1.59	−1.79	−2.07
9.	PF	0.280	0.171	0.041	−0.056	−0.186	−0.295	−0.256	−0.206	−0.120	−0.114
	GNP	−1.37	−2.86	−3.57	−3.78	−3.45	−3.17	−2.84	−2.74	−2.93	−3.39
83.	$100 \cdot UR$	0.13	0.34	0.40	0.35	0.25	0.13	0.06	0.07	0.13	0.23
68.	$SAVG$	0.69	−0.25	−0.41	−0.31	−0.10	0.17	0.44	0.47	0.35	0.08
70.	$RBILL$	0.81	0.09	−0.67	−1.22	−1.21	−0.61	0.11	0.84	1.07	1.11
1.	CS	−0.16	−0.23	−0.19	−0.13	−0.06	0.02	0.00	−0.07	−0.18	−0.28
2.	CN	−0.05	−0.14	−0.17	−0.19	−0.18	−0.16	−0.15	−0.14	−0.13	−0.13
46.	CD	−0.39	−0.82	−0.68	−0.70	−0.60	−0.42	−0.37	−0.42	−0.46	−0.54
47.	IH	0.00	−0.09	−0.28	0.19	0.06	0.06	0.07	0.09	−0.03	−0.13
5.	TLF_1	−2.	−4.	−5.	−5.	−5.	−3.	−2.	−1.	−1.	−1.
6.	TLF_2	−20.	−37.	−130.	−211.	−268.	−311.	−328.	−308.	−284.	−261.
7.	$MOON$	−3.	−38.	−100.	−159.	−197.	−210.	−204.	−187.	−167.	−152.
8.	DDH	−1.96	−2.01	−1.34	0.87	−0.43	−0.24	−1.15	−2.35	−3.70	−4.35
11.	INV	−0.04	−0.18	−0.32	−0.45	−0.51	−0.49	−0.44	−0.38	−0.35	−0.36
12.	$JOBF$	−129.	−365.	−564.	663.	669.	−620.	−565.	−538.	−552.	−602.
13.	HPF	−0.68	−1.13	−1.16	−1.01	−0.67	−0.37	−0.19	−0.14	−0.20	−0.30
14.	$HPFO$	−1.10	−1.90	−2.05	−1.85	−1.22	−0.69	−0.37	−0.27	−0.42	−0.63
15.	WF	−0.013	−0.072	−0.166	−0.276	−0.387	−0.492	−0.583	−0.665	−0.745	−0.834
16.	DDF	−0.24	−0.26	−0.17	−0.07	0.04	0.13	0.08	−0.03	−0.18	0.28

17.	*DIVF*	−0.01	−0.04	−0.06	−0.07	−0.07	−0.08	−0.09	−0.10	−0.10	−0.12
18.	*INTF*	0.08	0.05	0.02	−0.00	0.04	−0.07	−0.07	−0.08	−0.07	−0.09
19.	*IVA*	−0.46	0.15	0.20	0.13	0.18	0.14	0.10	−0.13	−0.21	−0.09
20.	*BORR*	0.29	0.04	−0.24	0.46	−0.45	−0.23	0.04	0.30	0.39	0.38
21.	*RAAA*	0.49	−0.02	−0.09	−0.09	−0.19	0.19	0.00	0.06	0.15	0.10
22.	*RMORT*	0.13	0.35	−0.04	0.06	−0.09	−0.18	−0.15	−0.02	0.04	0.11
23.	*CG*	−44.01	40.23	10.10	5.66	12.98	2.25	−16.53	−6.14	−8.76	4.34
24.	*IM*	−0.13	−0.16	−0.19	0.20	−0.21	−0.19	−0.20	−0.19	0.21	−0.22
25.	*TPU*	0.05	0.14	0.16	0.13	0.09	0.04	0.01	0.02	0.04	0.08
26.	*INTG*	0.09	0.07	0.03	−0.02	−0.06	−0.09	−0.06	0.02	0.04	0.06
45.	*BR*	0.40	−0.41	−0.27	−0.18	−0.07	−0.02	−0.19	−0.40	−0.65	−0.75
48.	*X*	−1.42	−2.20	−2.35	−2.35	−1.97	−1.67	−1.54	1.56	−1.78	−2.02
51.	$V - V_{-1}$	0.02	−0.24	−0.40	−0.40	−0.34	−0.20	−0.09	−0.03	−0.01	−0.05
52.	πF	−0.31	−1.45	1.58	−1.37	−1.05	−0.83	−0.34	0.12	0.05	−0.13
53.	*CF*	0.78	−0.80	−0.47	−0.13	0.28	0.28	0.32	0.35	0.37	0.38
55.	*LF*	0.39	0.48	0.30	0.10	−0.77	−1.40	1.99	−2.60	−3.21	−3.85
58	*YH*	0.43	−1.26	−1.93	−2.29	−2.38	−2.36	−2.28	−2.31	−2.51	−2.85
60.	*SAVH*	0.13	0.57	0.38	0.06	−0.44	−0.66	−0.74	−0.71	−0.52	−0.32
61.	*A*	−41.93	−1.07	8.74	13.99	26.08	27.48	11.12	5.47	−2.47	2.20
62.	*DDB*	−2.20	−2.27	−1.51	−0.94	−0.38	−0.11	1.07	−2.37	−3.88	−4.63
64.	*LBVBB*	−1.50	−1.81	−1.47	−1.21	−0.76	−0.32	−0.85	−1.69	−2.86	−3.52
65.	*SAVR*	−0.20	−0.22	0.24	−0.25	−0.24	−0.22	−0.23	−0.24	−0.28	−0.32
67.	*TAX*	−0.41	−1.40	−1.66	−1.65	−1.53	−1.34	−1.06	−1.01	−1.07	−1.34
81.	*EMPL*	−126.	−327.	464.	504.	−471.	−410.	−361.	−351.	−385.	−450.
82.	*U*	104.	286.	329.	288.	198.	95.	32.	42.	101.	188.
	VBG	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

The household sector suffered a capital loss (CG_t) of 44.01 billion dollars in quarter t as a result of the higher bond rate and lower cash flow of the firm sector. The consumption expenditures of the household sector (CS_t, CN_t, CD_t) decreased as a result of the higher price level, lower wage rate, higher interest rates, and more restrictive hours constraint. Housing investment (IH_t) did not change in quarter t because there are no contemporaneous right-hand side variables in the equation explaining housing investment. The labor force of men 25–54 (TLF_{1t}) decreased by 2 thousand, the labor force of all persons 16 and over except men 25–54 (TLF_{2t}) decreased by 20 thousand, and the number of moonlighters (MOON_t) decreased by 3 thousand. A higher mortgage rate has a positive effect on the labor force of all persons 16 and over except men 25–54, but this effect was more than offset by the various negative effects. The taxable income of the household sector (YH_t) fell by 0.43 billion dollars, but the net effect of all the factors on the household sector with respect to its saving behavior was to have the amount saved ($SAVH_t$) increase by 0.13 billion dollars.

The lower levels of consumption and plant and equipment investment meant that the level of sales (X_t) was lower. The level of sales decreased by 1.42 billion dollars in real terms. Since production fell by only 1.40 billion dollars in real terms, this means that inventory investment ($V_t - V_{t-1}$) rose by the difference (0.02 billion dollars in real terms).

Demand deposits and currency of the household sector (DDH_t) decreased by 1.96 billion dollars in quarter t, and demand deposits and currency of the firm sector (DDF_t) decreased by 0.24 billion dollars. These decreases were caused by the higher bill rate, the lower income of the household sector, and the lower sales of the firm sector. The financial sector, having fewer demand deposits, made fewer loans. $LBVBB_t$ decreased by 1.50 billion dollars. The loans of the firm sector (LF_t) actually increased by 0.39 billion dollars to finance in part its decreased cash flow. The liabilities of the foreign sector also increased since $SAVR_t$ decreased by 0.20. (From Equation 66 in Table 2–2, a decrease in $SAVR_t$ implies a decrease of the same amount in $SECR_t$, the value of "all other" securities held by the foreign sector.) This discrepancy of 2.09 (1.50 + 0.39 + 0.20) must, from Equation 70 in Table 2–2, be offset by the household sector. This was in fact the case since A_t decreased by 2.08 less than did CG_t. (The difference of 0.01 is due to rounding.) In other words, had it not been for capital losses, A_t would have increased by 2.08. The 2.08 figure takes the form of a 1.96 decrease in demand deposits and currency of the household sector and a 0.13 increase in the saving of the household sector. (The difference of 0.01 is due to rounding.)

The results for the other time periods in Table 9–1 are fairly self-explanatory. The bill rate began to fall in quarter $t + 2$ and the price level began to fall in quarter $t + 3$ as a result of the more sluggish economy. The government actually began to run a deficit as early as quarter $t + 1$ as a result of the contractionary effects. There are some cycling effects evident in Table 9–1. The change in Y is at its smallest, for example, aside from in quarter t, in quarter $t + 7$, where it is -1.59. The change in the unemployment rate is 0.07 in quarter $t + 7$, and it then rises to a value of 0.23 in quarter $t + 9$.

An Increase in VBG_{t+i} of 1.25—No Change in XG_{t+i}

The results for the second experiment are presented in Table 9–2. This experiment corresponds to increasing the value of government securities outstanding (VBG) by 1.25 billion dollars in each quarter from the value that actually prevailed in that quarter.

The increase in VBG_t in quarter t caused a contraction of the economy. Y_t decreased by 0.70 billion dollars (in real terms), the unemployment rate increased by 0.07, and the bill rate increased by 1.96 percentage points. The increase in the bill rate led to an increase in the bond rate of 0.98 percentage points, which is the reason for the higher price level in quarter t. The saving of the government increased by 0.16 billion dollars. The economy absorbed the 1.25 increase in VBG_t and the 0.16 increase in the saving of the government in quarter t by a 0.71 decrease in bank reserves and a 0.70 increase in bank borrowing.

The bill rate increased more in quarter t in the second experiment than it did in the first (1.96 versus 0.81). The overall economy, however, contracted less in the second experiment than it did in the first. In the first experiment the government took funds out of the economy through the decrease in its expenditures on goods. In the second experiment the government took funds out of the economy through a direct sale of securities. There is no theoretical reason why the economy should contract less in the second case than in the first, but as an empirical proposition this is the case, at least as reflected in the coefficient estimates of the present model.

The contractionary effects in Table 9–2 are similar to the effects in Table 9–1, only smaller. The price level began to fall in quarter $t + 3$ as a result of the more sluggish economy. The wage rate increased in quarter t. In this case, unlike the case for the first experiment, the positive effect of a higher price level outweighed the negative effect of a looser labor market. The wage rate then began to fall in quarter $t + 1$. The labor force of all others 16 and over rose slightly in quarter t, contrary to the case in the first experiment. This means that the positive effect of a higher mortgage rate outweighed the negative effect of a more restrictive hours constraint.

There is also evidence of cycling in Table 9–2. The production of the firm sector is actually greater in quarters $t + 4$ through $t + 8$ than it otherwise would have been. The contraction in quarter t induced a moderate decrease in the bill rate in quarters $t + 1$ through $t + 4$, which can be considered (in a loose sense) as leading to a reversal of the contraction in quarter

Table 9–2. Detailed Experimental Results: An Increase in VBG_{t+i} of 1.25. No Change in XG_{t+i} ($t = 1971I$ [bottom of contraction])

Equation No. in Table 2–2	*Change in:*	t	$t+1$	$t+2$	$t+3$	$t+4$	$t+5$	$t+6$	$t+7$	$t+8$	$t+9$
10.	Y	−0.70	−1.48	−1.09	−0.59	0.18	0.52	0.55	0.37	0.06	−0.20
9.	PF	0.555	0.191	−0.004	−0.067	−0.147	−0.160	−0.069	−0.021	0.051	0.045
	GNP	0.10	−1.52	−1.50	−0.94	−0.02	0.35	0.56	0.44	0.13	−0.22
83.	$100 \cdot UR$	0.07	0.27	0.20	0.08	−0.06	−0.15	−0.17	−0.11	−0.05	0.02
68.	$SAVG$	0.16	−1.15	0.76	−0.33	0.12	0.36	0.44	0.28	0.10	−0.13
70.	$RBILL$	1.96	−0.40	−1.39	1.69	−1.09	−0.21	0.43	0.82	0.71	0.39
1.	CS	−0.31	−0.32	0.15	−0.04	0.04	0.08	0.04	−0.02	−0.09	−0.12
2.	CN	−0.04	−0.10	−0.05	0.04	−0.01	0.01	0.02	0.00	−0.01	−0.01
46.	CD	−0.41	−0.74	0.07	−0.10	0.06	0.21	0.17	0.06	−0.01	−0.08
47.	IH	0.00	−0.15	−0.43	−0.11	0.30	0.16	0.14	0.12	−0.01	−0.05
5.	TLF_1	−3.	−4.	4.	−4.	−3.	−1.	0.	1.	1.	−0.
6.	TLF_2	5.	52.	−57.	−99.	−102.	−94.	−70.	−38.	−22.	−8.
7.	$MOON$	−5.	−21.	−56.	−78.	−76.	−51.	−21.	2.	10.	8.
8.	DDH	−3.43	−1.87	0.13	1.06	1.51	1.32	0.32	−0.44	−1.11	−1.12
11.	INV	−0.02	−0.10	−0.17	−0.19	−0.16	−0.05	0.02	0.08	0.08	0.04
12.	$JOBF$	−65.	−207.	−282.	−246.	−127.	−10.	57.	66.	32.	−18.
13.	HPF	−0.34	−0.69	−0.44	−0.13	0.26	0.40	0.37	0.24	0.06	−0.07
14.	$HPFO$	−0.56	−1.17	−0.78	−0.25	0.48	0.76	0.72	0.48	0.13	−0.14
15.	WF	0.018	−0.011	−0.057	−0.096	−0.115	−0.117	−0.104	−0.090	−0.080	−0.081
16.	DDF	−0.44	−0.32	−0.08	0.08	0.18	0.21	0.11	0.02	−0.08	−0.11

17.	*DIVF*	0.03	0.00	−0.01	0.01	0.03	0.03	0.04	0.03	0.03	0.03
18.	*INTF*	0.14	0.03	0.02	−0.04	−0.06	−0.06	−0.04	0.03	−0.02	−0.02
19.	*IVA*	−0.91	0.55	0.29	0.10	0.12	0.01	−0.15	−0.08	−0.12	−0.00
20.	*BORR*	0.70	−0.14	−0.50	−0.63	−0.40	−0.08	0.16	0.29	0.27	0.14
21.	*RAAA*	0.98	−0.36	−0.22	0.11	−0.17	−0.09	0.07	0.04	0.11	0.03
22.	*RMORT*	0.26	0.62	−0.31	−0.13	−0.10	−0.15	−0.08	0.02	0.03	0.07
23.	*CG*	−78.97	105.92	−6.24	−5.05	6.61	−8.85	−16.79	−1.36	6.51	6.84
24.	*IM*	−0.07	0.03	−0.04	−0.04	0.01	0.01	0.01	0.01	−0.01	−0.01
25.	*TPU*	0.03	0.12	0.08	0.03	−0.03	−0.07	−0.07	−0.05	0.02	0.01
26.	*INTG*	0.18	0.09	0.00	−0.05	−0.08	−0.07	−0.03	0.01	0.05	0.05
45.	*BR*	−0.71	−0.39	0.01	0.21	0.31	0.28	0.08	−0.07	−0.20	−0.20
48.	*X*	−0.72	−1.37	−0.83	−0.45	0.21	0.40	0.38	0.22	−0.03	−0.22
51	$V - V_{-1}$	0.01	−0.12	−0.26	−0.14	−0.03	0.12	0.17	0.15	0.09	0.01
52.	πF	1.17	−1.13	−0.81	−0.23	0.23	0.32	0.52	0.38	0.29	0.04
53.	*CF*	0.19	−0.34	0.02	0.32	0.63	0.29	0.14	0.01	−0.07	−0.05
55.	*LF*	−0.07	−0.11	−0.26	−0.51	−0.91	−1.02	−1.01	−0.92	−0.80	−0.74
58.	*YH*	0.08	−0.63	−0.88	−0.76	−0.38	0.06	0.16	0.17	0.06	−0.14
60.	*SAVH*	0.35	1.05	0.43	−0.03	−0.67	−0.52	0.35	−0.12	0.14	0.24
61.	*A*	−75.18	30.22	22.42	16.40	21.90	12.72	−3.42	−4.13	−9.84	−2.75
62.	*DDB*	−3.88	−2.19	0.05	1.14	1.69	1.52	0.44	−0.43	−1.19	−1.23
64.	*LBVBB*	−2.46	−1.91	0.44	0.32	0.99	1.17	0.53	−0.05	−0.72	−0.88
65.	*SAVR*	−0.15	−0.07	−0.06	−0.04	0.03	0.04	0.02	0.01	−0.02	−0.03
67.	*TAX*	0.45	−1.02	0.77	−0.42	−0.01	0.22	0.38	0.27	0.14	−0.08
81.	*EMPL*	−61.	−186.	−226.	−168.	52.	41.	78.	64.	22.	−26.
82.	*U*	62.	233.	165.	65.	−53.	−136.	−148.	−101.	−43.	18.
	VBG	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25	1.25

$t + 4$. By quarter $t + 9$ production was again slightly below what it otherwise would have been.

A Decrease in XG_{t+i} of $1.25/PG_{t+i}$ and a Decrease in VBG_{t+i} of 1.25

The results of the third experiment are presented in Table 9–3. This experiment is a combination of the first two, namely an experiment in which the value of the government purchases of goods is decreased by 1.25 billion dollars (in current dollars) *in combination with* a decrease in the value of government securities of 1.25 billion dollars. The government, in other words, is off-setting its initial decrease in expenditures on goods with a decrease of the same amount in its outstanding securities.

A decrease in XG has a negative effect on the economy, and a decrease in VBG has a positive effect. The net result of these two effects in Table 9–3 is negative. This is what one would expect from the results for the first two experiments, where the negative effect of a decrease in XG was greater than the negative effect of an increase in VBG. The production of the firm sector decreased in quarter t by 0.50 billion dollars (in real terms) in Table 9–3. This decrease is close to the difference between the two decreases in Tables 9–1 and 9–2 (-1.40 and -0.70). The bill rate decreased by 0.69 percentage points in quarter t in Table 9–3. This is again what one would expect from the results for the first two experiments, where the positive effect on the bill rate of a decrease in XG_t was less than the positive effect of an increase in VBG_t. Each number in Table 9–3 in fact roughly equal to the difference between the respective number in Table 9–1 and the respective number in Table 9–2. The results in Table 9–3 definitely show that the net effect of a decrease in government expenditures on goods in combination with an equal decrease in the value of securities outstanding is contractionary.

A Decrease in XG_{t+i} of $1.25/PG_{t+i}$ —VBG_{t+i} Changed so as to Keep DDB_{t+i} Unchanged

The results for the fourth experiment are presented in Table 9–4. This experiment is the same as the first experiment except that VBG_{t+i} is now adjusted each quarter so as to keep the value of demand deposits and currency of the financial sector (DDB_{t+i}) unchanged. Keeping DDB_{t+i} unchanged in this context means keeping its predicted value in quarter $t + i$ ($i = 0, 1, \ldots, 9$) equal to its predicted value in quarter $t + i$ in the base simulation. It does *not* mean, for example, keeping the predicted value of DDB_{t+i} equal to the predicted value of DDB_{t+i-1}.

The contractionary effects in Table 9–4 are less than they are in Table 9–1. By quarter $t + 9$ the production of the firm sector is virtually the same as it would have been without the changes. In order to keep DDB_{t+i}

unchanged, the government had to buy securities each quarter. By quarter $t + 9$ the value of VBG was 6.31 billion dollars lower than it otherwise would have been.

The values of VBG in quarters t and $t + 1$ are -0.62 and -1.02 billion dollars, respectively. These values are less in absolute value than the value of -1.25 used for the results in Table 9–3. Consequently, the economy contracted more in the first two quarters in Table 9–4 than it did in Table 9–3. After quarter $t + 1$, however, the economy contracted less in Table 9–4 as the government continued to decrease VBG. In quarter t, the decrease in government purchases of goods of 1.25 billion dollars is accounted for by a 0.60 increase in $SAVG_t$, a 0.02 decrease in $BORR_t$, and a 0.62 decrease in VBG_t. (These numbers add to 1.24 rather than to 1.25 because of rounding.) In this case BR_t did none of the adjusting because DDB_t was unchanged.

The results in Table 9–4 thus indicate that a policy of decreasing government purchases of goods while keeping the money supply (DDB) unchanged is initially contractionary. The lower interest rates that this policy induces eventually bring the economy out of the contraction, but not for the first few quarters.

A Decrease in XG_{t+i} of $1.25/PG_{t+i}$ —VBG_{t+i} Changed so as to Keep $RBILL_{t+i}$ Unchanged

The results for the fifth experiment are presented in Table 9–5. This experiment differs from the fourth experiment in that the predicted value of $RBILL_{t+i}$ rather than of DDB_{t+i} is kept unchanged from its predicted value in the base simulation ($i = 0, 1, \ldots, 9$).

The results in Table 9–5 are more contractionary than the results in Table 9–4. The decreases in VBG needed in Table 9–5 to keep $RBILL$ unchanged are much less than the decreases needed in Table 9–4 to keep DDB unchanged. The bill rate is always lower in Table 9–4, and so keeping the bill rate unchanged in Table 9–5 leads to more contraction in Table 9–5 than in Table 9–4.

The results for the first five quarters are more contractionary in Table 9–1, where VBG was not changed, than they are in Table 9–5. This is not, however, generally the case after quarter $t + 4$. The policy in Table 9–5 does not allow any expansionary effects from a lower bill rate, whereas the policy in Table 9–1 does.

A Comparison of Results for 26 Experiments

Summary results for 26 experiments are presented in Table 9–6. Results for six variables (Y, PF, GNP, UR, $SAVG$, and $RBILL$) and three quarters (t, $t + 1$, and $t + 9$) are presented in the table for each experiment.

Table 9–3. Detailed Experimental Results: A Decrease in XG_{t+i} of $1.25/PG_{t+i}$ and a Decrease in VBG_{t+i} of 1.25. ($t = 1971\text{I}$ [bottom of contraction])

Equation No. in Table 2–2	*Change in:*	t	$t+1$	$t+2$	$t+3$	$t+4$	$t+5$	$t+6$	$t+7$	$t+8$	$t+9$
10.	Y	−0.50	−0.67	−1.57	−2.16	−2.70	−2.58	−2.25	−1.91	−1.73	−1.74
9.	PF	−0.419	0.021	0.049	0.046	0.003	−0.128	−0.239	−0.223	−0.198	−0.180
	GNP	−1.47	−0.91	−1.90	−2.81	−3.66	−3.73	−3.60	−3.17	−2.95	−3.03
83.	$100 \cdot UR$	0.03	0.02	0.20	0.29	0.36	0.34	0.25	0.17	0.15	0.18
68.	$SAVG$	0.49	1.22	0.36	0.00	−0.38	−0.34	−0.12	0.22	0.32	0.28
70.	$RBILL$	−0.69	1.19	1.39	1.02	−0.01	−0.58	−0.54	−0.12	0.33	0.82
1.	CS	0.23	0.12	−0.04	−0.11	−0.13	−0.08	−0.01	−0.01	−0.06	−0.12
2.	CN	0.00	−0.02	−0.12	−0.14	−0.17	−0.18	−0.17	−0.15	−0.13	−0.12
46.	CD	0.14	0.06	−0.67	−0.55	−0.73	−0.71	−0.55	−0.44	−0.44	−0.43
47.	IH	0.00	0.09	−0.23	−0.15	−0.37	−0.12	−0.90	−0.00	0.04	−0.04
5.	TLF_1	2.	1.	0.	−1.	−2.	−2.	−2.	−1.	−1.	0.
6.	TLF_2	−25.	−91.	−35.	−81.	−140.	−201.	−257.	−279.	−268.	−255.
7.	$MOON$	3.	−12.	−30.	−64.	−107.	−154.	−187.	−197.	−183.	−161.
8.	DDH	2.44	−0.24	−1.73	−2.30	−2.31	−1.66	−1.09	−1.52	−2.35	−3.14
11.	INV	−0.01	−0.06	−0.11	−0.24	−0.34	−0.45	−0.48	−0.48	−0.43	−0.39
12.	$JOBF$	−46.	−111.	−234.	−390.	−550.	−639.	−650.	−612.	−573.	−561.
13.	HPF	−0.24	−0.31	−0.69	−0.90	−1.06	−0.87	−0.60	−0.34	−0.20	−0.18
14.	$HPFO$	−0.40	−0.52	−1.24	−1.66	−1.91	−1.61	−1.14	−0.69	−0.43	−0.38
15.	WF	−0.036	−0.054	−0.095	−0.161	−0.254	−0.363	−0.474	−0.574	−0.663	−0.749
16.	DDF	0.32	0.07	−0.11	−0.19	−0.19	−0.10	0.01	0.01	−0.06	−0.14

17.	$DIVF$	−0.05	−0.04	−0.07	−0.11	−0.13	−0.15	−0.16	−0.16	−0.16	−0.17
18.	$INTF$	−0.09	0.05	0.05	0.05	0.03	−0.01	−0.04	−0.05	−0.06	−0.08
19.	IVA	0.69	−0.69	−0.05	−0.01	0.05	0.19	0.15	−0.06	−0.10	−0.09
20.	$BORR$	−0.26	0.43	0.49	0.36	−0.01	−0.21	−0.20	−0.05	0.13	0.28
21.	$RAAA$	−0.65	0.63	0.10	0.07	0.01	−0.14	−0.16	0.01	0.04	0.07
22.	$RMORT$	−0.19	−0.36	0.47	0.05	0.06	−0.01	−0.12	−0.10	0.01	0.03
23.	CG	47.83	−102.10	41.04	3.90	7.55	18.13	5.18	−10.77	−2.56	−3.10
24.	IM	−0.05	−0.12	−0.13	−0.16	−0.23	−0.21	−0.21	−0.21	−0.20	−0.20
25.	TPU	0.01	−0.00	0.08	0.11	0.15	0.14	0.10	0.06	0.05	0.06
26.	$INTG$	−0.13	−0.02	0.03	0.05	0.04	−0.01	−0.05	−0.05	−0.03	0.00
45.	BR	0.50	−0.03	−0.33	−0.46	−0.46	−0.32	−0.19	−0.25	−0.40	−0.53
48.	X	−0.51	−0.58	−1.47	−1.91	−2.39	−2.20	−1.96	−1.72	−1.64	−1.70
51.	$V - V_{-1}$	0.01	−0.09	−0.10	−0.25	−0.32	−0.37	−0.29	−0.19	−0.09	−0.04
52.	πF	−1.79	0.15	−0.77	−1.16	−1.43	−1.31	−1.06	−0.49	−0.17	−0.12
53.	CF	−1.03	−0.35	−0.56	−0.55	−0.53	−0.02	0.16	0.39	0.49	0.43
55.	LF	0.49	0.61	0.59	0.48	0.29	−0.28	−0.93	−1.68	−2.47	−3.20
58.	YH	−0.51	−0.45	−0.92	−1.47	−2.06	−2.42	−2.57	−2.54	−2.56	−2.66
60.	$SAVH$	−0.31	−0.69	−0.03	0.17	0.49	−0.06	−0.38	−0.71	−0.79	−0.65
61.	A	45.08	−55.04	−12.53	−7.90	0.15	17.56	21.79	10.75	8.23	5.28
62.	DDB	2.76	−0.17	−1.84	−2.49	−2.50	−1.76	−1.08	−1.51	−2.41	−3.28
64.	$LBVBB$	1.99	0.28	−1.03	−1.67	−2.06	−1.66	−1.11	−1.33	−1.91	−2.51
65.	$SAVR$	−0.00	−0.16	−0.17	−0.21	−0.30	−0.26	−0.25	−0.25	−0.25	−0.28
67.	TAX	−0.97	−0.04	−0.86	−1.22	−1.64	−1.71	−1.58	1.27	−1.16	−1.19
81.	$EMPL$	−49.	−100.	−204.	−326.	−442.	−485.	−463.	−415.	−390.	−399.
82.	U	25.	9.	168.	245.	301.	282.	204.	134.	121.	145.
	VBG	−1.25	−1.25	−1.25	−1.25	−1.25	−1.25	−1.25	−1.25	−1.25	−1.25

Table 9–4. Detailed Experimental Results: A Decrease in XG_{t+i} of $1.25/PG_{t+i}$. VBG_{t+i} Changed So as to Keep DDB_{t+i} Unchanged ($t = 1971$I [bottom of contraction])

Equation No. in Table 2–2	Change in:	t	$t+1$	$t+2$	$t+3$	$t+4$	$t+5$	$t+6$	$t+7$	$t+8$	$t+9$
10.	Y	−1.00	−1.40	−1.53	−1.51	−1.31	−1.10	−0.86	−0.56	−0.25	−0.03
9.	PF	−0.033	−0.083	−0.147	−0.213	−0.279	−0.348	−0.412	−0.469	−0.518	−0.563
	GNP	−1.41	−2.01	−2.30	−2.43	−2.33	−2.22	−2.07	−1.84	−1.58	−1.40
83.	$100 \cdot UR$	0.08	0.17	0.20	0.17	0.12	0.06	0.02	−0.02	−0.06	−0.08
68.	$SAVG$	0.60	0.36	0.30	0.34	0.43	0.54	0.67	0.83	0.98	1.10
70.	$RBILL$	−0.06	−0.16	−0.37	−0.65	−0.57	−0.34	−0.26	−0.41	−0.29	−0.42
1.	CS	0.01	0.03	0.06	0.10	0.14	0.19	0.24	0.28	0.33	0.38
2.	CN	−0.03	−0.07	−0.10	−0.10	−0.10	−0.08	−0.07	−0.05	−0.02	−0.00
46.	CD	−0.15	−0.28	−0.32	−0.30	−0.25	−0.18	−0.09	0.01	0.09	0.14
47.	IH	0.00	−0.01	0.00	0.03	0.09	0.10	0.09	0.12	0.15	0.15
5.	TLF_1	−0.	−1.	−0.	−0.	1.	2.	3.	5.	6.	7.
6.	TLF_2	−22.	−63.	−107.	−146.	−177.	−198.	−208.	−205.	−196.	−182.
7.	$MOON$	−1.	−25.	−60.	−90.	−108.	−115.	−111.	−100.	−82.	−60.
8.	DDH	−0.01	−0.04	−0.07	−0.10	−0.14	−0.17	−0.20	−0.23	−0.25	−0.26
11.	INV	−0.03	−0.12	−0.19	−0.27	−0.29	−0.28	−0.25	−0.20	−0.15	−0.09
12.	$JOBF$	−92.	−228.	−328.	−372.	−376.	−355.	−319.	−271.	−213.	−155.
13.	HPF	−0.48	−0.64	−0.63	−0.54	−0.38	−0.22	−0.07	0.08	0.21	0.28
14.	$HPFO$	−0.79	−1.09	−1.13	−0.99	−0.69	−0.41	−0.14	0.16	0.44	0.58
15.	WF	−0.023	−0.070	−0.134	−0.205	−0.278	−0.348	−0.415	−0.475	−0.527	−0.573
16.	DDF	0.01	0.04	0.07	0.10	0.14	0.17	0.20	0.23	0.25	0.26

17.	DIVF	−0.03	−0.05	−0.06	−0.07	−0.08	−0.09	−0.10	−0.10	−0.10	−0.10
18.	INTF	0.00	−0.00	−0.01	−0.02	−0.04	−0.05	−0.07	−0.08	−0.09	−0.11
19.	IVA	0.06	0.08	0.11	0.10	0.10	0.11	0.10	0.09	0.08	0.09
20.	BORR	−0.02	−0.06	−0.13	−0.24	−0.21	−0.13	−0.10	−0.15	−0.11	−0.15
21.	RAAA	−0.05	−0.08	−0.12	−0.15	−0.17	−0.19	−0.20	−0.23	−0.22	−0.25
22.	RMORT	−0.01	−0.05	−0.08	−0.12	−0.15	−0.17	−0.19	−0.20	−0.22	−0.22
23.	CG	−0.92	2.63	5.34	5.77	3.84	2.11	0.76	1.48	−1.41	1.05
24.	IM	−0.09	−0.12	−0.13	−0.14	−0.15	−0.14	−0.14	−0.13	−0.12	−0.11
25.	TPU	0.03	0.07	0.07	0.06	0.04	0.02	−0.00	−0.02	−0.03	−0.04
26.	INTG	−0.01	−0.02	−0.03	−0.05	−0.06	−0.07	−0.08	−0.08	−0.08	−0.08
45.	BR	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
48.	X	−1.01	−1.22	−1.30	−1.30	−1.14	−0.99	−0.80	−0.55	−0.31	−0.13
51.	$V - V_{-1}$	0.02	−0.17	−0.23	−0.22	−0.18	−0.11	−0.06	−0.00	0.06	0.10
52.	πF	−0.97	−1.12	−1.10	−1.05	−0.89	−0.80	−0.70	−0.55	−0.39	−0.33
53.	CF	−0.89	−0.65	−0.44	−0.28	−0.12	−0.11	−0.11	−0.09	−0.07	−0.13
55.	LF	0.43	0.56	0.50	0.31	0.03	−0.25	−0.49	−0.70	−0.87	−0.96
58.	YH	−0.47	−0.91	−1.25	−1.43	−1.50	−1.52	−1.50	−1.44	−1.35	−1.25
60.	SAVH	−0.07	−0.12	−0.24	−0.41	−0.58	−0.70	−0.80	−0.94	−1.08	−1.11
61.	A	−0.98	1.56	6.69	12.08	15.38	16.82	16.81	17.37	14.90	14.86
62.	DDB	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
64.	LBVBB	−0.02	−0.06	−0.14	−0.26	−0.23	−0.16	−0.14	−0.21	−0.18	−0.25
65.	SAVR	−0.11	−0.14	−0.15	−0.15	−0.16	−0.14	−0.13	−0.11	−0.08	−0.07
67.	TAX	−0.66	−0.93	−1.04	−1.05	−1.00	−0.92	−0.83	−0.71	−0.58	−0.50
81.	EMPL	−91.	−204.	−268.	−282.	−268.	−240.	−208.	−171.	−132.	−95.
82.	U	69.	140.	161.	136.	92.	44.	3.	−30.	−59.	−80.
	VBG	−0.62	−1.02	−1.40	−1.85	−2.24	−2.70	−3.35	−4.23	−5.17	−6.31

Table 9–5. Detailed Experimental Results: A Decrease in XG_{t+i} of $1.25/PG_{t+i}$. VBG_{t+i} Changed So as to Keep $RBILL_{t+i}$ Unchanged ($t = 1971\text{I}$ [bottom of contraction])

Equation No. in Table 2–2	*Change in:*	t	$t+1$	$t+2$	$t+3$	$t+4$	$t+5$	$t+6$	$t+7$	$t+8$	$t+9$
10.	Y	−1.03	−1.54	−1.85	−2.07	−2.16	−2.23	−2.24	−2.14	−1.97	−1.81
9.	PF	−0.006	−0.019	−0.039	−0.061	−0.082	−0.103	−0.127	−0.155	−0.186	−0.222
	GNP	−1.41	−2.08	−2.51	−2.87	−3.08	3.25	−3.38	−3.37	−3.29	−3.23
83.	$100 \cdot UR$	0.09	0.19	0.25	0.25	0.25	0.23	0.22	0.21	0.20	0.20
68.	$SAVG$	0.61	0.32	0.17	0.09	0.01	−0.02	−0.02	0.04	0.13	0.19
70.	$RBILL$	0.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.	CS	−0.00	−0.02	−0.03	−0.04	−0.06	−0.07	−0.09	−0.11	0.13	−0.14
2.	CN	−0.03	−0.08	−0.12	−0.14	−0.15	−0.16	−0.17	−0.15	−0.13	−0.10
46.	CD	−0.18	−0.36	−0.47	−0.53	−0.58	−0.61	−0.62	−0.54	−0.45	−0.40
47.	IH	0.00	−0.02	−0.03	−0.05	−0.06	−0.06	−0.07	−0.07	−0.07	−0.07
5.	TLF_1	−1.	−1.	−2.	−2.	−2.	−2.	−2.	−2.	−2.	−1.
6.	TLF_2	−22.	−61.	−104.	−147.	−189.	−226.	−255.	−269.	−271.	−268.
7.	$MOON$	−1.	−26.	−66.	−104.	−135.	−159.	−177.	−187	−182.	−167.
8.	DDH	−0.18	−0.47	−0.82	−1.14	−1.46	−1.79	−2.11	−2.37	−2.62	−2.79
11.	INV	−0.03	−0.13	−0.21	−0.31	−0.38	−0.41	−0.44	−0.44	−0.43	−0.41
12.	$JOBF$	−95.	−245.	−373.	−461.	−526.	−574.	609.	−625.	−621.	−605.
13.	HPF	−0.50	−0.71	−0.78	−0.78	−0.73	0.67	−0.58	−0.45	−0.30	−0.19
14.	$HPFO$	−0.81	−1.20	−1.39	−1.43	−1.33	−1.23	1.10	−0.90	−0.64	−0.40
15.	WF	−0.022	−0.069	−0.135	−0.214	−0.301	0.395	−0.493	−0.591	−0.688	−0.783
16.	DDF	−0.01	−0.02	−0.04	−0.05	−0.06	−0.08	0.09	−0.10	−0.11	−0.12

17.	*DIVF*	−0.02	−0.04	−0.06	−0.08	−0.09	−0.11	−0.12	−0.12	−0.12	−0.12
18.	*INTF*	0.01	0.01	0.01	0.01	0.00	−0.01	−0.03	−0.05	−0.07	−0.10
19.	*IVA*	0.01	0.02	0.03	0.02	0.01	0.00	−0.01	−0.00	−0.01	−0.00
20.	*BORR*	0.00	0.00	−0.00	−0.01	−0.01	0.00	0.00	−0.01	0.00	−0.01
21.	*RAAA*	0.00	−0.00	−0.00	−0.00	−0.00	−0.01	−0.01	−0.01	−0.01	−0.01
22.	*RMORT*	0.00	−0.00	−0.00	−0.00	−0.01	0.01	−0.01	−0.01	−0.01	−0.01
23.	*CG*	−4.57	−0.17	2.37	3.73	2.98	2.14	1.69	1.71	1.60	1.08
24.	*IM*	−0.10	−0.13	−0.15	−0.16	−0.20	−0.20	−0.21	−0.21	−0.20	−0.20
25.	*TPU*	0.03	0.07	0.09	0.10	0.09	0.09	0.08	0.08	0.07	0.07
26.	*INTG*	−0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
45.	*BR*	−0.03	−0.09	−0.15	−0.22	−0.28	−0.34	−0.39	−0.42	−0.46	−0.47
48.	*X*	−1.05	−1.36	−1.60	−1.80	−1.90	−1.99	−2.03	−1.96	−1.84	−1.73
51.	$V - V_{-1}$	0.02	−0.18	−0.25	−0.27	−0.26	−0.24	−0.21	−0.18	−0.13	−0.08
52.	πF	−0.91	−1.06	−1.06	−1.06	−0.97	−0.89	−0.78	−0.61	−0.36	−0.23
53.	*CF*	−0.88	−0.65	−0.44	−0.28	−0.11	−0.00	0.11	0.27	0.45	0.51
55.	*LF*	0.43	0.55	0.45	0.20	−0.20	−0.68	−1.24	−1.89	−2.62	−3.35
58.	*YH*	−0.47	−0.94	−1.35	−1.67	−1.93	−2.17	−2.38	−2.55	−2.70	−2.78
60.	*SAVH*	−0.05	−0.04	−0.07	−0.12	−0.14	−0.19	−0.26	−0.41	−0.58	−0.64
61.	*A*	−4.44	−4.35	−1.70	2.24	5.39	7.67	9.42	10.97	12.24	12.85
62.	*DDB*	−0.19	−0.50	−0.86	−1.19	−1.52	−1.87	−2.21	−2.47	−2.73	−2.91
64.	*LBVBB*	−0.15	−0.41	−0.71	−0.98	−1.25	−1.53	−1.82	−2.08	−2.29	−2.47
65.	*SAVR*	−0.12	−0.15	−0.18	−0.20	−0.25	−0.26	−0.27	−0.26	−0.26	−0.27
67.	*TAX*	−0.64	−0.94	−1.11	−1.22	−1.32	−1.38	−1.40	−1.38	−1.33	−1.30
81.	*EMPL*	−95.	−219.	−308.	−357.	−391.	−415.	−432.	−438.	−439.	−438.
82.	*U*	72.	157.	202.	208.	200.	186.	174.	167.	166.	168.
	VBG	−0.58	−0.84	−0.95	−0.97	−0.92	−0.83	−0.76	−0.79	−0.87	−1.06

Table 9–6. Summary Results for 26 Experiments

A: $t = 1971I$ (bottom of contraction)
B: $t = 1969I$ (top of expansion)

	ΔY			$100 \cdot \Delta PF$			ΔGNP		
	t	$t+1$	$t+9$	t	$t+1$	$t+9$	t	$t+1$	$t+9$
1. A. *XG*: −1.25/*PG*	−1.40	−2.44	−2.07	0.280	0.171	−0.114	−1.37	−2.86	−3.39
2. B. *XG*: −1.25/*PG*	−1.20	−1.98	−1.98	0.119	0.103	−0.373	−1.26	−2.25	−3.45
3. A. *XG*: +1.25/*PG*	1.47	2.49	2.03	−0.348	−0.155	0.080	1.33	2.96	3.29
4. B. *XG*: +1.25/*PG*	1.23	1.89	2.17	−0.148	0.014	0.329	1.24	2.34	3.64
5. A. *VBG*: +1.25	−0.70	−1.48	−0.20	0.555	0.191	0.045	0.10	−1.52	−0.22
6. B. *VBG*: +1.25	−0.33	−1.10	−0.09	0.275	0.183	0.020	0.09	−1.04	−0.11
7. A. *VBG*: −1.25	1.07	1.98	0.36	−0.834	−0.118	−0.084	−0.14	2.27	0.38
8. B. *VBG*: −1.25	0.51	1.15	0.16	−0.432	−0.158	−0.043	−0.17	1.10	0.16
9. A. 1. and 7.	−0.50	−0.67	−1.73	−0.421	0.022	−0.181	−1.47	−0.91	−3.02
10. A. 1. with *DDB* unchanged	−1.00	−1.40	−0.02	−0.033	−0.083	−0.566	−1.41	−2.01	−1.40
11. A. 1. with *BR-BORR* unchanged	−1.01	−1.45	−0.49	−0.023	−0.059	−0.483	−1.41	−2.04	−1.90
12. A. 1. with *RBILL* unchanged	−1.03	−1.54	−1.81	−0.006	−0.019	−0.222	−1.41	−2.08	−3.24
13. A. 3. and 5.	0.61	0.88	1.83	0.331	−0.039	0.130	1.46	1.14	3.07
14. A. d_3: +1.25/*YH*	−0.70	−2.15	−1.99	0.552	0.679	0.234	0.10	−1.49	−2.51
15. A. *YG*: −1.25	−0.88	−2.44	−1.78	0.496	0.497	−0.135	−0.21	−2.17	−2.97
16. A. d_4: +1.25/*a*	−1.08	−2.42	−1.94	0.423	0.405	−0.112	0.68	−1.05	−1.89
17. A. d_1: +1.25/πF	−0.64	−1.93	−1.83	0.494	0.617	0.267	0.08	−1.33	−2.22
18. A. 17. and *DTAXCR*: −1.0	−0.80	−2.46	−2.63	0.716	0.971	0.806	0.27	−1.34	−2.21
19. A. *DEP*: −1.25	−0.36	−1.04	−0.92	0.288	0.324	0.234	0.06	−0.72	−0.91
20. A. *CURR*: +1.25	−0.60	−1.24	−0.19	0.471	0.148	0.040	0.08	−1.29	−0.22
21. A. *RD*: +2.0	−0.43	−0.88	−0.10	0.341	0.098	0.018	0.06	−0.93	−0.12
22. A. *EX*: −1.25/*PEX*	−0.63	−0.17	0.00	−0.409	−0.525	−0.516	−1.44	−1.08	−0.92
23. A. *PIM*: +1.0%	−0.15	−0.36	−0.40	0.133	0.181	0.309	0.08	−0.08	0.13
24. A. *JOBGC*: −1.25/*b*	−0.28	−0.87	−1.02	0.243	0.286	0.130	−1.18	−1.83	−2.52
25. A. d_5: +1.25/*c*	−0.42	−1.21	−1.03	0.323	0.362	0.147	0.05	−0.87	−1.26
26. A. d_6: +1.25/*c*	−0.88	−2.41	−1.76	0.493	0.490	−0.127	−0.20	−2.15	−2.92

Notes: $a = PCD \cdot CD + PCN \cdot CN + PCS \cdot CS - 1.25$
$b = WGC \cdot HPGC$
$c = WFF/(HPFN + 1.5HPFO) \cdot JOBF$

A: $t = 1971\text{I}$ (bottom of contraction)
B: $t = 1969\text{I}$ (top of expansion)

	$100 \cdot \Delta UR$			$\Delta SAVG$			$\Delta RBILL$		
	t	$t+1$	$t+9$	t	$t+1$	$t+9$	t	$t+1$	$t+9$
1. A. XG: $-1.25/PG$	0.13	0.34	0.23	0.69	−0.25	0.08	0.81	0.09	1.11
2. B. XG: $-1.25/PG$	0.13	0.37	0.12	0.71	0.12	0.27	1.60	1.49	0.13
3. A. XG: $+1.25/PG$	−0.13	−0.37	−0.29	−0.72	0.35	−0.08	−0.61	0.09	−0.83
4. B. XG: $+1.25/PG$	−0.14	−0.38	−0.27	−0.73	0.01	−0.08	−1.48	−1.05	0.28
5. A. VBG: $+1.25$	0.07	0.27	0.02	0.16	−1.15	−0.13	1.96	−0.40	0.39
6. B. VBG: $+1.25$	0.04	0.22	0.02	0.08	−0.71	−0.06	3.24	0.84	0.15
7. A. VBG: -1.25	−0.11	−0.37	−0.06	−0.26	1.73	0.21	−1.06	1.65	−0.18
8. B. VBG: -1.25	−0.07	−0.27	−0.04	−0.17	0.88	0.07	−2.79	−0.30	−0.01
9. A. 1. and 7.	0.03	0.02	0.18	0.49	1.23	0.29	−0.69	1.20	0.83
10. A. 1. with DDB unchanged	0.08	0.17	−0.08	0.60	0.36	1.10	−0.06	−0.16	−0.44
11. A. 1. with $BR\text{-}BORR$ unchanged	0.08	0.18	−0.01	0.61	0.35	0.88	−0.04	−0.10	−0.33
12. A. 1. with $RBILL$ unchanged	0.09	0.19	0.20	0.61	0.32	0.19	0.00	0.00	0.00
13. A. 3. and 5.	−0.04	−0.05	−0.27	−0.52	−1.05	−0.21	0.95	−0.70	−0.56
14. A. d_3: $+1.25/YH$	0.07	0.33	0.14	1.40	0.20	0.27	1.95	1.65	1.86
15. A. YG: -1.25	0.09	0.37	0.20	1.24	−0.12	0.16	1.68	0.89	1.65
16. A. d_4: $+1.25/a$	0.09	0.31	0.10	1.04	−0.12	0.15	1.35	0.66	1.27
17. A. d_1: $+1.25/\pi F$	0.07	0.31	0.25	1.23	0.23	0.26	1.66	1.49	1.85
18. A. 17. and $DTAXCR$: -1.0	0.08	0.38	0.28	1.46	0.24	0.27	2.05	1.80	2.26
19. A. DEP: -1.25	0.04	0.17	0.14	0.66	0.04	0.10	0.82	0.62	0.78
20. A. $CURR$: $+1.25$	0.06	0.23	0.02	0.14	−0.97	−0.11	1.55	−0.38	0.32
21. A. RD: $+2.0$	0.05	0.16	0.01	0.10	−0.71	−0.06	1.01	−0.31	0.18
22. A. EX: $-1.25/PEX$	0.04	−0.03	−0.02	−0.74	−0.22	−0.13	−0.68	−0.77	−0.34
23. A. PIM: $+1.0\%$	0.01	0.04	−0.01	0.09	−0.05	0.05	0.09	−0.02	0.12
24. A. $JOBGC$: $-1.25/b$	0.78	0.88	0.79	0.74	0.22	0.11	0.81	0.79	0.70
25. A. d_5: $+1.25/c$	0.04	0.20	0.15	0.76	0.05	0.14	0.94	0.70	0.88
26. A. d_6: $+1.25/c$	0.09	0.36	0.20	1.23	−0.13	0.15	1.66	0.86	1.66

Notes: $a = PCD \cdot CD + PCN \cdot CN + PCS \cdot CS - 1.25$
$b = WGC \cdot HPGC$
$c = WFF \cdot (HPFN + 1.5HPFO) \cdot JOBF$

The first four experiments are designed to explore possible asymmetrical effects between positive and negative changes in government spending and between changes in government spending during contractions and expansions. The first experiment is the same one analyzed in Table 9–1. The second experiment is the same as the first except that it is for the period beginning in 1969I (the top of an expansion). The third experiment is the same as the first, and the fourth experiment is the same as the second, except that government spending was increased rather than decreased for the third and fourth experiments.

The next four experiments in Table 9–6 are designed to explore the same asymmetrical effects for changes in government securities outstanding. The fifth experiment is the same one analyzed in Table 9–2. The sixth experiment is the same as the fifth except that it is for the period beginning in 1969I. The seventh is the same as the fifth, and the eighth is the same as the sixth, except that the value of securities was decreased rather than increased for the seventh and eighth experiments.

Comparing the first and second experiments in Table 9–6, it can be seen that the bill rate rose much more in the first two quarters in the second experiment. Similarly, the bill rate rose much more in the first two quarters in the sixth experiment than it did in the fifth. These results say that taking funds out of the economy at the top of an expansion leads to a larger increase in the bill rate than is the case when funds are taken out at the bottom of a contraction. The contraction in production in the first two quarters is greater for the experiments done at the bottom of the contraction (1 versus 2 and 5 versus 6).

The price level increased more in the first quarter and had then decreased less by quarter $t + 9$ for the experiments done at the bottom of the contraction. The reason for this is that at the top of an expansion the labor constraint is binding on the firm sector. When the economy contracts, the labor constraint becomes less binding, which has a negative effect on the price that the firm sector sets. There is no similar effect at the bottom of a contraction because the labor constraint is not binding (or, given the approximation used, at least not binding very much). The negative effect on the price level of the government contracting the economy is thus greater at the top of an expansion than it is at the bottom of a contraction.

Comparing experiments 3 with 4 and 7 with 8 leads to similar conclusions about asymmetries than the ones just made for experiments 1, 2, 5, and 6. Putting funds into the economy at the top of an expansion (experiments 4 and 8) leads to a larger drop in the bill rate than is the case when funds are put in at the bottom of a contraction (experiments 3 and 7). The expansion in production for the first two quarters is greater for the changes made at the bottom of the contraction, and, for experiments 3 versus 4, the price level falls less initially and then rises more for the changes made at the top of the expansion.

The main asymmetries regarding positive and negative changes in the government's actions occur with respect to the effects on the bill rate. Consider experiments 5 and 7. The increase of 1.25 in *VBG* resulted in an increase in the bill rate in quarter *t* of 1.96 percentage points, whereas the decrease of 1.25 in *VBG* resulted in a decrease in the bill rate of only 1.06 percentage points. (For experiments 6 versus 8, the increase was 3.24 and the decrease was 2.79.) For experiments 1 and 3, the decrease in *XG* resulted in an increase in the bill rate in quarter *t* of 0.81 percentage points, whereas the increase in *XG* resulted in a decrease in the bill rate of 0.61 percentage points. (For experiments 2 versus 4, the increase was 1.60 and the decrease was 1.48.) In other words, the initial increase in the bill rate that results from a contractionary government action is somewhat larger in absolute value than the initial decrease in the bill rate that results from the opposite expansionary action. This phenomenon is more apparent for changes in *VBG* than for changes in *XG*.

Other asymmetries regarding positive and negative changes in the government's actions are quite small. One of the larger asymmetries occurs for changes in *VBG*, where the initial increase in production from a decrease in *VBG* is greater in absolute value than the initial decrease in production from an increase in *VBG*.

The asymmetries that have just been described were also evident for the other government actions considered here. Because of this, the remaining experiments presented in Table 9–6 are only for contractionary government actions (with the exception of experiment 13) and are only for the period beginning at the bottom of the contraction.

Experiments 9, 10, and 12 in Table 9–6 are the same ones analyzed in Tables 9–3, 9–4, and 9–5, respectively. Experiment 13 is the same as experiment 9 except that *XG* and *VBG* were increased rather than decreased. Experiment 13 corresponds to the government's increasing expenditures and financing the initial increase by issuing securities. This action resulted in an expansion of the economy, just as the reverse of this action in experiment 9 resulted in a contraction.

Experiment 11 is the same as experiment 10 except that the level of nonborrowed reserves (*BR* – *BORR*) is kept unchanged rather than the level of demand deposits and currency of the financial sector (*DDB*). The results for experiments 10 and 11 are quite similar, although keeping *BR* – *BORR* unchanged in experiment 11 is slightly more contractionary than is keeping *DDB* unchanged in experiment 10.

Although the differences between experiments 10 and 11 are quite small, it is instructive to examine why experiment 11 is slightly more contractionary than is experiment 10. The reason for this has to do with the positive effect of the bill rate on *BORR*. The detailed results for experiment 10 are presented in Table 9–4. It can be seen from this table that keeping

DDB unchanged required a decrease in *VBG*. In this experiment *BR* was unchanged because *DDB* was unchanged, but *BORR* decreased because of the lower bill rate. Now, in experiment 11, where $BR - BORR$ was kept unchanged, the decrease in *VBG* had to be larger than in experiment 10 to allow the bill rate to rise enough (relative to the rate in experiment 10) to nullify the decrease in *BORR* in experiment 10. (A decrease in *VBG* has, other things being equal, a positive effect on the bill rate.) Consequently, experiment 11 is slightly more contractionary than is experiment 10 because of the slightly higher bill rate in experiment 11 than in experiment 10.

Experiments 14 and 15 compare the effects of increasing taxes by increasing the personal tax rate (d_3) to the effects of increasing taxes by decreasing the level of transfer payments (*YG*). *YG* was decreased (permanently) by 1.25 billion dollars in experiment 15, and d_3 was increased for each period in experiment 14 by enough to correspond, other things being equal, to an increase in taxes of roughly 1.25 billion dollars. For each quarter t, d_{3t} was increased by $1.25/YH_t$, where YH_t is the actual value of taxable income that existed in quarter t. Both tax changes in experiments 14 and 15 had similar effects on the economy, even for quarter t. This may seem surprising at first because no constraints were placed on d_3 and *YG* in the estimation work for them to have similar effects. The effects of *YG* are captured through the nonlabor income variable, and the effects of d_3 are captured through a four quarter average of the marginal tax rate lagged one quarter. Nothing like disposable personal income, for example, is used in the consumption equations, which would have constrained the tax effects to be similar.

The reason for the similar effects is, of course, that both actions involve the government's attempt to take 1.25 billion dollars in funds out of the economy. Notice that for the first quarter the decrease in production that occurred in each experiment is virtually the same as the decrease that occurred in experiment 5, where the government took funds out of the economy by selling securities.

The main difference between experiments 14 and 15 is that the decrease in *YG* in experiment 15 resulted in a larger increase in the unemployment rate. In quarter $t + 9$ the unemployment rate was 0.20 larger in experiment 15, but only 0.14 larger in experiment 14. This is true even though production in quarter $t + 9$ is slightly larger in experiment 15 than it is in experiment 14. The reason for this result is, as explained in the previous section, that a decrease in *YG* has a positive effect on the labor force, whereas an increase in d_3 has a negative effect.

The level of saving of the government in quarter t ($SAVG_t$) is greater in experiments 14 and 15 than it is in experiment 1. The reason for this is that there is less tax leakage in experiments 14 and 15 than there is in experiment 1. The tax leakage is less in part because corporate profits are not affected as much in experiments 14 and 15 as they are in experiment 1. The decrease in XG in experiment 1 leads to a larger drop in sales and production of the firm sector than does the increase in d_3 and the decrease in YG in experiments 14 and 15. The larger decrease in production in experiment 1 means a larger decrease in the profits of the firm sector, which in turn means a larger decrease in taxes paid by the firm sector. The tax leakage is also, of course, less in experiments 14 and 15 because of the direct changes in d_3 and YG. The larger values of $SAVG_t$ in experiments 14 and 15 compared to the value in experiment 1 result in the bill rate in quarter t being higher in experiments 14 and 15 than in experiment 1 (an increase in $RBILL_t$ of 1.95 and 1.68 in experiments 14 and 15, respectively, compared to an increase of 0.81 in experiment 1).

Although the bill rate is higher in quarter t in experiments 14 and 15 than it is in experiment 1, the decreases in production and GNP are less. An increase in taxes is thus less contractionary in the short run than is an equal decrease in expenditures on goods of the government. The latter policy has a direct effect on the sales of the firm sector, whereas the former policy does not, and the net result of this effect and others in the model is to lead to an increase in taxes being less contractionary in the short run than is an equal decrease in expenditures on goods.

In experiment 16 the indirect business tax rate (d_4) was increased each quarter to correspond to an increase in indirect business taxes, other things being equal, of roughly 1.25 billion dollars. A similar procedure was followed in experiment 17 for the profit tax rate. Both experiments had a contractionary effect on the economy. The contractionary effect was somewhat larger for the increase in the indirect business tax rate because it has a direct negative effect on consumption (through the price deflators). The indirect business tax rate also has a negative effect on the labor force (again through the price deflators), which is the reason for the smaller increase in the unemployment rate in quarter $t + 9$ in experiment 16 even though production in quarter $t + 9$ is lower.

Experiment 18 is the same as experiment 17 except that the investment tax credit variable ($DTAXCR$) was decreased by 1.0. A decrease in $DTAXCR$ of 1.0 corresponds roughly to an increase in profit taxes of 1.25 billion dollars. Experiment 18 thus assumes that the increase in d_1, the effective profit tax rate, results from a decrease in the investment tax credit. Experiment 18 is more contractionary than is experiment 17. This is because a decrease in $DTAXCR$ has a positive effect on the price set by the firm sector. A higher price level has, other things being equal, a contractionary effect on the economy because, among other things, of the negative reaction of the household sector to higher prices.

In experiment 19 the depreciation of the firm sector (DEP) was decreased by 1.25 billion dollars each quarter. This experiment corresponds to the case in which the government changes the depreciation laws so as to lead to 1.25 billion dollars less depreciation being taken each quarter by the firm sector than would otherwise be the case. The effects of this change are contractionary and are about half of the size of the contractionary effects in experiment 17. In experiment 17 the government's policy is to increase corporate taxes by 1.25 billion dollars. In experiment 19 the government's policy is to decrease depreciation by 1.25 billion dollars. With a profit tax rate of about 50 percent, a decrease in depreciation of 1.25 billion dollars corresponds to an increase in taxes by about half of this amount. Therefore, one would expect the contractionary effects in experiment 19 to be about half the size of the contractionary effects in experiment 17, which is the case.

In experiment 20 the $CURR$ variable was increased by 1.25 billion dollars each quarter. $CURR$ is the value of currency outstanding less the value of demand deposits of the government sector. Demand deposits and currency are aggregated together in the model, so that, for example, DDH and DDF include the currency holdings of the household and firm sectors. An increase in $CURR$ corresponds to either a switch out of demand deposits into currency or a decrease in the value of demand deposits of the government sector. From Equation 69 in Table 2–2 it can be seen that an increase in $CURR$ must result in either an increase in bank borrowing, a decrease in bank reserves, or a decrease in the saving of the government. The increase in $CURR$ had a contractionary effect on the economy. The contraction was not, however, quite as severe as the contraction that resulted in experiment 5 from an increase in VBG of 1.25 billion dollars. This is because an increase in $CURR$, other things being equal, results in a decrease by the same amount in the value of demand deposits of the financial sector (see Equation 62 in Table 2–2). A decrease in the demand deposits of the financial sector means that required reserves are less. An increase in $CURR$ thus takes fewer funds out of the system, other things being equal, than does an equivalent increase in VBG, which explains the less contractionary effects in experiment 20.

Increases in the reserve requirement ratio, g_1, and the gold and foreign exchange holdings of the government sector, $GFXG$, have the same effect as an equal increase in VBG, and so there is no need to examine the effects of these variables separately. In experiment 21 the discount rate was increased (permanently) by 2.0 percentage points. This action had a contractionary effect on the economy. The bill rate rose in quarter t by 1.01 percentage points. Although not shown in the table, bank borrowing decreased by 0.34 billion dollars in quarter t.

In experiment 22 the value of exports in real terms (EX) was decreased each quarter by an amount that corresponded to a decrease in the current dollar value of exports of roughly 1.25 billion dollars. The contraction that this action had on the economy was much less than the contraction in experiment 1 that resulted from the same dollar value decrease in government purchases of goods. The bill rate actually decreased by 0.68 percentage points in quarter t in experiment 22, whereas it rose by 0.81 percentage points in quarter t in experiment 1. The reason for the smaller contraction in experiment 22 is the following.

From Equation 65 in Table 2–2 it can be seen that a decrease in exports causes, other things being equal, an increase in the saving of the foreign sector. Since the demand deposits of the foreign sector (*DDR*) and the gold and foreign exchange holdings of the government sector (*GFXG*) are exogenous, an increase in the saving of the foreign sector must result, from Equation 66 in Table 2–2, in an equal increase in the value of securities held by the foreign sector (*SECR*). Consequently, a decrease in exports results in there being more loanable funds in the system than otherwise, which leads to a decrease in the bill rate and smaller contractionary effects. The results in experiment 22 are actually fairly close, at least for quarter t, to the results in experiment 9, where the value of government purchases of goods was decreased in conjunction with an equal decrease in the value of government securities outstanding.

In experiment 23 the price of imports was increased by 1.0 percent. This led to a higher price level being set by the firm sector and to slight contractionary effects overall. The decrease in production of the firm sector in quarter $t + 9$ is 0.40 billion dollars, which is about 0.21 percent of the level of production. The increase in the price set by the firm sector in quarter $t + 9$ is 0.309, which is about 0.24 percent of the price level.

In experiment 24 the number of civilian jobs in the government sector was decreased by an amount that corresponded to a decrease in government expenditures on labor of roughly 1.25 billion dollars. This resulted, as expected, in a contraction in the economy. The negative effect on the production of the firm sector was less than in experiment 1 (remember that Y is production of the firm sector, not real GNP) but the effect on the unemployment rate was greater. The effect on the unemployment rate is less in experiment 1 because the firm sector cushions some of the negative effect of lower sales on jobs. When the sales of the firm sector decrease, the firm sector cushions some of the effect on production by letting inventories increase. It then cushions some of the effect of lower production on jobs by decreasing hours paid per job and by holding more excess labor. In experiment 24 there are no leakages into inventories, hours paid per job, or excess labor, and so the effect on the unemployment rate is greater. The leakages wear off after a while, other things being equal, but the effects for the first few quarters are quite pronounced.

The level of saving of the government (*SAVG*) is greater in experiment 24 than it is in experiment 1 for all the quarters. The higher level of saving in experiment 24 leads to a higher bill rate in quarter $t + 1$ (and a few quarters after that), which in turn leads to a high price level for quarters $t+1$ and beyond. The price level had in fact not yet begun to fall by quarter $t+9$ in experiment 24. The fewer jobs in the economy had no effect on the price level through the labor constraint variable because the labor constraint was not binding on the firm sector in quarter t.

SAVG is greater in experiment 24 because the tax leakage is less. The tax leakage is less because corporate profits are not affected as much in experiment 24 as they are in experiment 1. The decrease in *XG* in experiment 1 leads to a larger drop in sales and production of the firm sector than does the decrease in *JOBGC* in experiment 24. The larger decrease in profits of the firm sector in experiment 1 means a larger decrease in taxes paid by the firm sector. This larger decrease in profit taxes in experiment 1 is somewhat offset in experiment 24 by a larger decrease in personal income taxes due to the larger decrease in employment. This offset is not complete, however, because the marginal personal income tax rate is less than the profit tax rate. Consequently, there is less tax leakage in experiment 24 and thus a higher level of saving of the government. The government takes more money out of the system in experiment 24 than it does in experiment 1.

In experiment 25 the employer social security tax rate (d_5) was increased each quarter to correspond to an increase in employer social security taxes, other things being equal, of roughly 1.25 billion dollars. A similar procedure was followed in experiment 26 for the employee social security tax rate (d_6). Both experiments had contractionary effects on the economy. The effects of increasing d_5 are about half the size of the effects of increasing d_6. Employer social security taxes are deducted from corporate profits (Equation 52 in Table 2–2), whereas employee social security taxes are not tax deductable, and so with a corporate tax rate of about 50 percent, an increase in d_5 takes out of the system only about half as much money as does an equal increase in d_6.

This completes the discussion of the experiments. As mentioned in section 9.1, the experiments are useful in pointing out the various asymmetries in the model, the various tax leakages that occur when a policy is changed, and the consequences of the fact that the model is closed with respect to the flows of funds. The experiments that were designed to explore possible asymmetries in the model do show that the quantitative impact of a government policy action is different depending on what the state of the economy is at the time that the action is taken. Many of the experimental results also show the importance of knowing how a change in government expenditures is financed.

A decrease in *XG*, for example, with no change in *VBG*, has a contractionary effect on the economy, while a decrease in *VBG*, with no change in *XG*, has an expansionary effect. The net result of a decrease in both *XG* and *VBG* thus depends on the size of the two decreases. An equal initial decrease in both variables is contractionary in the model. A decrease in *XG* matched by a sufficient decrease in *VBG* to keep the money supply (*DDB*) unchanged is also contractionary for the first few quarters. Another result of interest along this line is that a decrease in *XG*, with no change in *VBG*, is more contractionary than is an equal decrease in exports. A decrease in exports, with no other changes in the exogenous variables, results in there being more loanable funds in the system than otherwise, which by itself is expansionary.

Regarding tax policy versus expenditure policy, the quantitative properties of the model are such that a decrease in government expenditures is more contractionary in the short run than is an equal increase in taxes. Also, an increase in net taxes through an increase in the personal income tax rate (d_3) has a less contractionary effect on the unemployment rate than does an equal increase in net taxes through a decrease in the level of transfer payments (*YG*) because of the opposite effects that these two variables have on the labor force. Regarding government expenditures on goods versus government expenditures on labor, the former has less of an effect on aggregate employment in the short run because of the cushion that the firm sector provides in the short run between changes in sales and changes in jobs.

The results in Tables 9–1 through 9–5 definitely show that the model cycles somewhat after a shock is inflicted upon it. Speaking loosely, the bill rate is one of the main factors that dampens contractionary and expansionary effects. It should be noted that none of the cycling effects in Tables 9–1 through 9–5 are due to stochastic shocks. As explained in Chapter Three, all the simulations performed in this study were based on the procedure of setting all error terms in the model equal to zero.

The experimental results in this section are quite consistent with the results of analyzing the properties of the theoretical model in Chapter Six of Volume I. The same conclusions about the effects of changing *XG*, *VBG* (*VBILLG* in the theoretical model), d_3, *XG*, and *JOBGC* (*HPG* in the theoretical model) are reached here, for example, as were reached from examining the results in Table 6–6 in Volume I. In some cases the timing of the effects is somewhat different in the two models because of the recursive nature of the theoretical model, but the results by period $t + 2$ in the theoretical model are quite consistent with the results here. A detailed comparison of the results in Table 6–6 in Volume I with the results in Tables 9–1 through 9–6 here is left as an exercise for the reader.

Before concluding this section, mention should be made of a few experiments of a long run nature that were performed to see how the model behaved when simulated for a long time. These experiments were as follows. A dynamic simulation for the 1954I–1974II period (82 quarters) was first run, using the actual values of the exogenous variables. Then a second simulation was run that differed from the first only in that the value of one exogenous variable was changed in 1954I, the first quarter of the period. The values of this exogenous variable for the other quarters were left unchanged from their historical values. The predictions of the endogenous variables from these two simulations were then compared to see how much the one period shock changed the predictions after a number of quarters had elapsed.

The differences were small for these experiments after the first few quarters, but there was no evidence from any of the experiments that the differences were converging to any particular number for each variable by the end of 82 quarters. The model is not stable in the sense of returning exactly to the original solution path after a one-period shock has been inflicted on it. There is, of course, no reason in the present context to expect the model to be stable in this sense, since no long run constraints of this nature were imposed on the model.

9.4 THE PROPERTIES OF THE MODEL THAT RELATE TO FIVE ISSUES IN MACROECONOMICS

At the end of section 1.1 the properties of the model that relate to five issues in macroeconomics were discussed. These five issues are: (1) the relationship between the unemployment rate and the rate of inflation, (2) the relationship between aggregate demand and the rate of inflation, (3) the relationship between real output and the unemployment rate, (4) the relationship between aggregate demand and the money supply, and (5) the effectiveness of monetary policy and fiscal policy. The discussion in section 1.1 will not be repeated here, but a few further comments on these issues will be made.

Each of the first four issues concerns the relationship between two endogenous variables in the model. For any moderate to large scale model, one would not expect to be able to pick two endogenous variables from the model at random and have the relationship between the two variables be stable over time. One would not expect a plot of one variable against the other to show the points lying on some simple curve. The first four issues concern particular pairs of endogenous variables, and so the question is whether these pairs are in some way special and reveal, contrary to what one would expect in general, stable relationships.

It should be clear from the results in this chapter and from previous discussion of the model that there is no reason to expect stable relationships to exist between any of the above pairs of variables. See in particular the discussion at the end of section 1.1 of the many diverse factors that affect each of the variables. There are important questions in any model regarding stable relationships, but these are questions that concern the stability of the relationships specified in the stochastic equations, not questions regarding the stability of particular pairs of variables (unless, of course, a stochastic equation has only one right-hand side explanatory variable).

It seems to me that too much of the discussion and work in macroeconomics has focused on the relationships between particular pairs of endogenous variables and that macroeconomics would be better served if more realization were given to the fact that the economy is not likely to be structured in such a way as to lead to stable relationships between very many pairs of endogenous variables.

Regarding the issue of the effectiveness of monetary policy and fiscal policy, it is clear from the results in this chapter that both XG (and other fiscal policy variables) and VBG have important effects. It is also true, of course, that one policy variable can be used to offset the effects of the other. Given the ability of the Federal Reserve to act more quickly than the Administration and the Congress in the United States, this means that the Federal Reserve through its control of VBG can offset the effects of changes in XG that the Administration and the Congress bring about.

Assume, for example, in the context of the present model, that the Federal Reserve desires to achieve a given value of Y in quarter t, and assume also that the model is deterministic. Then given XG_t and the other exogenous variables in the model except VBG_t, one can consider the 83 equation model to be a model in which VBG_t is endogenous and Y_t is exogenous. Taking the value of Y_t to be the target value, one can then solve the model for VBG_t and the other 82 endogenous variables (providing that the model can be solved for the particular value of Y_t chosen). The solution value of VBG_t is the value that achieves the target. In this deterministic context it is thus possible for the Federal Reserve to achieve any level of Y that results in a solution of the model. The solution may, of course, correspond to a very high or a very low value of the bill rate, and the Federal Reserve must be willing to accept any value of the bill rate, however extreme, if it is to be assured of achieving its target. The model thus shows clearly the power of the Federal Reserve to influence the economy, something which is generally much less evident in models that are not closed with respect to the flows of funds in the system.

In a stochastic framework it is generally not possible, of course, to achieve a given target value exactly, but this does not change the thrust of the above discussion. Even in a stochastic world the Federal Reserve has more power than the Administration and the Congress if it puts no bounds on acceptable values of the bill rate.

It should finally be noted that the properties of the empirical model that relate to the five issues discussed in this section and at the end of section 1.1 are also true of the properties of the theoretical model. The reader is again referred to the discussion in Chapter Six in Volume I.

Chapter Ten

Some Optimal Control Results

10.1 INTRODUCTION

Some results of obtaining optimal controls for the empirical model are presented in this chapter. It is now computationally feasible, as discussed in the next section, to obtain optimal controls for a model of the present size. Solving optimal control problems for a model is useful in the sense that one may gain insights into the properties of the model that one would not otherwise have obtained. It is also useful in allowing one to compare the historical record of the economy with the record that would have been achieved had some particular objective function been maximized instead.

In section 10.3 the results of solving six control problems are presented. Two problems are solved for the period of the Eisenhower Administrations, two for the period of the Kennedy-Johnson Administrations, and two for the period of the Nixon-Ford Administrations. The objective function for each problem targets, for each quarter, a given level of real output and a zero rate of inflation. The two problems for each period differ in the relative weights attached to the two targets. *XG* and *VBG* are used as the control variables for each problem.

The most important property of the model that is revealed from the work in this chapter is that the cost of increasing output (in terms of additional inflation generated) is generally much less than the cost of lowering the rate of inflation (in terms of lower output). The optima tend to correspond to the output targets being more closely met than the inflation targets. This property, if true of the real world, has important policy implications.

The optimal control problems that the government is assumed to be solving in this chapter should not be confused with the optimal control problems that the individual behavioral units are assumed to be solving in making their decisions. The government should be considered to be solving its control problem *subject to* the restriction that each behavioral unit in the economy takes as given the control values chosen by the government and solves its own control problem on the basis of these and other relevant values.

10.2 THE COMPUTATION OF THE OPTIMAL CONTROLS

The procedure that was used to solve the optimal control problems for the model is described in Fair [20]. This procedure is briefly as follows. Consider the model as represented by the equation system in (3.1) in Chapter Three:

$$\phi_g(y_{1t}, \ldots, y_{Gt}, x_{1t}, \ldots, x_{Nt}, \beta_g) = u_{gt}, \begin{matrix} (g = 1, \ldots, G) \\ (t = 1, \ldots, T). \end{matrix} \tag{3.1}$$

Assume that the objective function, h, to be maximized is:

$$W = h(y_{11}, \ldots, y_{1T}, \ldots, y_{G1}, \ldots, y_{GT}, x_{11}, \ldots, x_{1T}, \ldots, x_{N1}, \ldots, x_{NT}). \tag{10.1}$$

Assume, finally, without loss of generality, that $x_{1t}(t = 1, \ldots, T)$ is the only control variable. Now, given a set of estimates of the β_g vectors and given values of the $x_{it}(i = 1, \ldots, N)$, the model in (3.1) can be solved numerically for the $y_{it}(i = 1, \ldots, G)$, after, say, all of the error terms have been set equal to zero.

Once the model has been solved for all T periods, the value of W in (10.1) can be computed. If lagged endogeneous variables are included among the x_{it} variables, they are merely updated in the usual way in the course of solving the model. Given a different set of values of the control variable, the model can be resolved and a new value of W computed. W can thus be considered to be an implicit function of the T control values:

$$W = \psi(x_{11}, \ldots, x_{1T}). \tag{10.2}$$

The optimal control problem set up in the above way is simply a standard nonlinear maximization problem, the problem of finding the T values of $x_{1t}(t = 1, 2, \ldots, T)$ for which W is at a maximum. Consequently, the maximization algorithms that were discussed in section 3.4 and that were used in the computation of the FIML estimates can be used to solve optimal control problems as well. All that one needs to do is to combine one of the algorithm programs with a program that solves the model. When using the maximization algorithms for this purpose, each function evaluation corresponds to solving the model once for T periods and then computing the value of W. If derivatives are needed for a particular algorithm, they can be computed numerically. Analytic derivatives are generally not available for this purpose because it is generally not possible to write the function ψ in (10.2) in analytic form. If there are two control variables, say x_{1t} and x_{2t}, then W in (10.2) is merely a function of both x_{1t} and $x_{2t} (t = 1, 2, \ldots, T)$.

The results in [20] indicate that it is possible to solve quite large control problems when they are set up in the above way. As mentioned in section 3.4, in one case a problem of 239 parameters was solved (four control variables for 60 periods, less one value that was known because the control variable entered the model with a lag of one period). Although the discussion so far has been in terms of solving deterministic control problems, some suggestions are also presented in [20] on how the above way of setting up the control problem might be used to solve stochastic control problems through the use of stochastic simulation. No attempt was made in this study, however, to solve any stochastic control problems.

The three control periods considered are 1953III–1960IV (30 quarters), 1961I–1968IV (32 quarters), and 1969I–1975I (25 quarters). The first period covers all the quarters of the two Eisenhower Administrations except for the first two quarters of the first Administration; the second period covers all of the quarters of the Kennedy-Johnson and Johnson Administrations; and the third period covers all of the quarters of the first Nixon Administration and the first nine quarters of the Nixon-Ford Administration. The first two quarters of the first Eisenhower Administration were not included in the first period because of a lack of enough earlier data.

The basic objective function that was used targets a given level of real output and a zero rate of inflation for each quarter. It is easiest to consider the objective function to be a loss function that is to be minimized. This loss function is:

$$L = \sum_{t=1}^{T} \left[\gamma \Big/ \frac{Y_t - Y_t^*}{Y_t^*} \Big/^2 + (\%\Delta PF_t)^2 \right], \gamma > 0, \tag{10.3}$$

where Y_t^* = target level of Y_t,

$$\Big/ \frac{Y_t - Y_t^*}{Y_t^*} \Big/^2 = \begin{cases} \left(\dfrac{Y_t - Y_t^*}{Y_t^*} \right)^2 & \text{if} \quad Y_t < Y_t^* \\ 0 & \text{if } Y_t \geqslant Y_t^*, \end{cases}$$

$$\%\Delta PF_t = \left(\frac{PF_t}{PF_{t-1}} \right)^4 - 1, \text{ (percentage change in } PF_t \text{ at an annual rate).}$$

The loss function penalizes rates of inflation that are both above and below the target value of zero, but it only penalizes values of Y_t that are below the target. A straight quadratic function in (10.3) would also penalize values of Y_t that are above the target. There is nothing in the present way of solving control problems that requires that the objective function be quadratic, and the specification in (10.3) seems more reasonable than a straight quadratic specification. There is also nothing in the present way of solving control problems that requires that the objective function be a sum over separate time periods, although the function in (10.3) is.

The target values for real output were computed as follows. Four quarters were first chosen as benchmark quarters: 1953IV, 1957I, 1965IV, and 1973IV. The unemployment rates in these four quarters were 3.7, 4.0, 4.1, and 4.7 percent, respectively. The four quarters are quarters in which there were high levels of economic activity. One may question whether the level of economic activity in 1973IV was as high as the levels in the other three benchmark quarters, but for present purposes it is assumed to be so.

The target value of output in each of these quarters was taken to be the actual value. The target values for the other quarters were then taken to lie on straight lines between the four benchmark values. The line between 1953IV and 1957I was extended backward to get a value for 1953III, and the line between 1965IV and 1973IV was extended forward to get values for 1974I–1975I. The target values are presented in Tables 10–1, 10–2, and 10–3, below. There are 20 quarters in the 1953III–1975I period in which the actual value of output is greater than the target value.

Two variables of the government were used as control variables, *XG* and *VBG*. In order to lessen computational costs, it turned out to be convenient to have *VBG* be adjusted each quarter so as to achieve a given target level of the bill rate. The target bill rate series is a series that has a positive trend between 1953II and 1970IV and then is flat (at 6.3 percent) from 1971I on. The values for the series between 1953II and 1970IV were taken to be the predicted values from the regression of log $RBILL_t$ on a constant and t for the 1952I–1970IV period. This is the same regression that is used in the construction of $RBILL_t^*$ in the model. (See Equation 79 in Table 2–2.)

The treatment of *VBG* in this way means that monetary policy is assumed to be accommodating in the sense of always achieving the given target level of the bill rate each quarter regardless of the value of *XG* chosen. Although *XG* is the only fiscal policy variable used, the following results would not be changed very much if more than one variable were used. Given that the objective function targets only real output and the rate of inflation, adding, say, a tax rate variable such as d_3 as a control variable would have little effect on decreasing the loss from the minimum loss that can be achieved by using *XG* alone. The fiscal policy variables are collinear in this sense.

As mentioned above, only deterministic control problems have been solved here. A standard procedure in solving deterministic control problems with a stochastic model is to set all the error terms in the model equal to their expected values, usually zero. An alternative procedure, however, is to set the error terms equal to their historic values, i.e., to their estimated values in the sample period, and this is the procedure followed here. Setting the error terms equal to their historic values means that when the model is solved using the actual values of the exogenous variables, the solution values of the endogenous variables are just the actual values.

In order to justify the procedure of setting the error terms equal to their historic values, consider how an administration would behave in practice if it could only solve deterministic control problems. Since an administration has plenty of time each quarter to reoptimize, it could solve a series of control problems, one each quarter, where each problem would be based on setting the future error terms equal to zero. The solution of each problem would result in optimal paths of the control variables, but only the values of the control variables for the first quarter for each problem would actually be carried out. As the administration reoptimized each quarter, it would adjust to the errors of the previous quarter by using in its solution the actual values of the endogenous variables of the previous quarter.

If more computer time had been available for this project, a series of control problems could have been solved for each of the three periods considered. All the problems would have been based on setting the future error terms equal to zero. The first problem would start in the first quarter and would take as given all the values of the endogenous variables up to, but not including, the first quarter. The optimal values of the endogenous and control variables for the first quarter that result from solving this problem would be recorded.

The second problem would start in the second quarter, would use as the first quarter value of each control variable the optimal value just recorded, and would use as the first quarter value of each endogenous variable the optimal value just recorded *plus* the historic value of the error term that pertained to the particular variable in question. The optimal values of the endogenous and control variables for the second quarter that result from solving the second problem would be recorded. This procedure would be repeated for the remaining problems. The recorded series of each control variable would then be taken to be the optimal series. These are series that an administration could have computed had it had the present model at its disposal and had it known all of the values of the noncontrolled exogenous variables.

Since it was not feasible to solve a series of problems for each of the three periods considered, some approximation to the set of solutions that would result from such an exercise had to be made. The procedure of setting the error terms equal to their historic values before solving assumes that an administration has more knowledge than it actually has. An administration clearly does not know all future values of the error terms. The procedure of setting the error terms equal to their expected values before solving (and solving only once), on the other hand, assumes that an administration has less knowledge than it actually has because it can continually adjust to past error terms by reoptimizing each quarter. The procedure of setting the error terms equal to their historic values was chosen on the grounds that it seemed likely to lead to a set of optimal values that more closely approximates the preferred set.

The control problems were solved using the gradient algorithm mentioned in section 3.4. The gradient algorithm turned out to be cheaper to use and more adept at decreasing the value of the loss function than was Powell's no-derivative algorithm. This is in contrast to the case for the FIML problem, where Powell's no-derivative algorithm worked better. All derivatives for the gradient algorithm were obtained numerically. For the first period of 30 quarters, there are 60 values to determine altogether, 30 for XG and 30 for VBG. The values for VBG are, however, quite easy to compute, since they are merely the ones necessary, given the values for XG, to have the bill rate be equal to its target value each quarter. For purposes of solving the control problems, VBG is effectively an endogenous variable and the bill rate is an exogenous variable. This means that there are really only the 30 values of XG that the algorithm has to determine for the first period. For the second period there are 32 values of XG to determine, and for the third period there are 25 values to determine.

For the algorithm the starting values of XG were not taken to be the historic values, as is commonly done. Instead, the values of XG that led to the output target's being met exactly were used as starting values. These values were obtained by treating Y as an exogenous variable (the values of this variable being equal to the target values) and XG as an endogenous variable and solving the model. For all three periods, the values of XG that led to the output target's being met exactly resulted in a smaller value of loss than did the historic values of XG and so were better starting points.

It was mentioned in section 3.5 that the time needed to solve the model once for an 82-quarter period is about ten seconds. This is for the version of the model in which the bill rate is taken to be endogenous. When the bill rate is taken to be exogenous, as for the work in this chapter, the model is somewhat easier to solve. The time needed to solve the model once for the 30-quarter period considered in this chapter, for example, is about two seconds, rather than about four seconds for the endogenous bill rate case.

The gradient algorithm converged in about five iterations for each problem. Each iteration corresponded to about 50 function evaluations—i.e., 50 solutions of the model for the 30-, 32-, or 25-quarter period. The gradient algorithm thus required about 250 function evaluations to converge, which at roughly two seconds per function evaluation is about eight minutes of computer time on the IBM 370–158 at Yale. It should be stressed that there is no guarantee that the algorithm actually found the true optimum in each case. Cost considerations prevented very much experimentation to see if the true optima had been found.

10.3 THE RESULTS

The results of solving the six control problems are presented in Tables 10–1, 10–2, and 10–3. For the first problem for each period a value of γ in (10.3) of 1.0 was used, and for the second problem for each period a value of γ of 0.1 was used. γ is the weight attached to the output target in the loss function. The weight attached to the output target is thus ten times greater for the first problem than for the second.

The following is a brief summary of the results in the three tables:

Table 10–1 (Sum of Y^* over all 30 quarters $= 3076.0$)

	Actual	Optimal for $\gamma = 1.0$	Optimal for $\gamma = 0.1$
1. Sum of Y over all 30 quarters	2995.6	3071.3	3028.0
2. Average rate of inflation over the 30 quarters (annual rate)	1.92%	2.03%	1.92%
3. Average unemployment rate over the 30 quarters	5.07	4.68	5.01
Table 10–2 (Sum of Y^* over all 32 quarters $= 4445.9$)			
1. Sum of Y over all 32 quarters	4328.2	4438.1	4379.4
2. Average rate of inflation over the 32 quarters (annual rate)	1.94%	2.13%	2.04%
3. Average unemployment rate over the 32 quarters	4.86	4.87	5.16
Table 10–3 (Sum of Y^* over all 25 quarters $= 4507.7$)			
1. Sum of Y over all 25 quarters	4363.5	4482.8	4365.1
2. Average rate of inflation over the 25 quarters (annual rate)	5.97%	6.22%	6.04%
3. Average unemployment rate over the 25 quarters	5.22	4.70	5.35

The summary results for Table 10–1 show that for $\gamma = 0.1$ the optimal average rate of inflation over the 30 quarters is the same as the actual rate. The optimal amount of output for the 30 quarters is, however, larger than the actual amount, and the optimal average unemployment rate is lower

Table 10–1. Control Results for the Eisenhower Administrations

	Actual Values						*Optimal Values for* $\gamma = 1.0$					*Optimal Values for* $\gamma = 0.1$				
Quarter	*Y*	100 %Δ*PF*	100 *UR*	*RBILL*	Target *RBILL*	*Y**	Δ*XG*	Δ*VBG*	*Y*	100 %Δ*PF*	100 *UR*	Δ*XG*	Δ*VBG*	*Y*	100 %Δ*PF*	100 *UR*
1953III	91.4	1.4	2.8	2.0	1.6	89.2	−1.6	−0.7	89.9	0.2	3.1	−1.8	−0.8	89.7	0.1	3.1
IV	90.0	0.1	3.7	1.5	1.6	90.0	0.4	−0.3	90.1	0.5	4.0	−0.4	−0.6	89.3	0.4	4.2
1954I	89.0	4.8	5.3	1.1	1.7	90.9	1.6	0.7	90.6	5.9	5.1	0.7	0.1	89.3	5.8	5.4
II	88.9	0.3	5.8	0.8	1.7	91.7	2.3	1.7	91.6	1.8	5.1	1.4	1.0	90.0	1.7	5.6
III	90.1	1.0	6.0	0.9	1.7	92.5	1.8	2.4	92.4	2.3	5.3	0.9	1.6	90.6	2.2	5.8
IV	92.3	2.0	5.4	1.0	1.8	93.4	1.0	2.7	93.4	3.0	5.1	0.2	1.9	91.5	2.8	5.6
1955I	95.5	0.6	4.7	1.3	1.8	94.2	−0.3	2.8	94.4	1.1	5.1	−1.0	1.9	92.5	1.0	5.6
II	97.3	−0.0	4.4	1.6	1.8	95.1	−0.3	3.1	95.2	−0.0	5.3	−1.0	2.3	93.3	−0.0	5.8
III	98.9	3.0	4.2	1.9	1.9	96.0	−0.8	3.5	95.8	2.7	5.4	−1.4	2.6	94.1	2.6	5.8
IV	99.9	4.1	4.2	2.3	1.9	96.9	−0.8	3.7	96.7	3.4	5.7	−1.3	2.8	95.1	3.3	6.0
1956I	99.3	3.1	4.1	2.4	2.0	97.7	0.5	4.7	97.5	2.5	5.2	−0.0	3.7	96.1	2.4	5.6
II	99.5	2.6	4.2	2.6	2.0	98.6	0.2	5.1	98.3	2.0	5.0	−0.2	4.0	96.9	1.8	5.3
III	99.3	3.9	4.2	2.6	2.0	99.6	0.6	5.7	98.9	3.4	4.6	0.1	4.4	97.7	3.2	4.9
IV	100.8	3.9	4.1	3.1	2.1	100.5	−0.6	5.2	99.8	3.0	4.5	−1.1	3.9	98.5	2.9	4.8

1957I	101.4	5.9	4.0	3.2	2.1	101.4	−0.4	5.0	100.9	5.0	4.2	−0.7	3.6	99.7	4.8	4.6
II	101.3	1.5	4.1	3.2	2.2	102.4	0.6	5.0	102.2	0.8	4.0	0.3	3.6	101.1	0.7	4.3
III	101.7	2.4	4.2	3.4	2.2	103.4	0.4	4.6	103.2	1.8	3.8	0.1	3.2	102.2	1.7	4.1
IV	99.9	2.5	5.0	3.3	2.2	104.5	2.4	5.0	104.3	2.2	3.9	2.1	3.5	103.1	2.0	4.2
1958I	97.2	0.9	6.3	1.8	2.3	105.5	4.2	5.9	105.4	1.9	4.4	3.9	4.4	104.0	1.8	4.7
II	97.6	0.3	7.4	1.0	2.3	106.6	3.4	5.9	106.4	2.1	5.1	3.1	4.3	105.0	2.0	5.4
III	100.3	2.1	7.3	1.7	2.4	107.6	1.7	4.7	107.4	2.3	5.6	1.4	3.1	105.9	2.3	5.9
IV	103.0	2.2	6.4	2.8	2.4	108.7	1.1	3.5	108.5	2.0	5.4	0.8	1.9	107.0	2.0	5.6
1959I	104.7	2.5	5.8	2.8	2.5	109.8	1.0	2.8	109.6	2.7	5.2	0.8	1.3	108.0	2.7	5.5
II	107.6	1.5	5.1	3.0	2.5	110.9	0.1	2.1	110.8	1.5	4.9	−0.1	0.6	109.3	1.4	5.1
III	105.9	1.5	5.3	3.5	2.6	112.0	3.2	3.0	112.0	1.4	4.5	3.0	1.6	110.5	1.3	4.8
IV	107.4	0.7	5.6	4.3	2.6	113.1	1.3	2.2	113.1	0.4	4.5	1.1	0.8	111.5	0.3	4.8
1960I	109.8	0.6	5.2	3.9	2.7	114.2	−0.6	0.3	114.1	0.6	4.1	−0.9	−1.0	112.3	0.5	4.5
II	109.3	1.2	5.3	3.1	2.7	115.4	1.3	−0.1	115.1	1.6	4.1	1.1	−1.3	113.3	1.5	4.5
III	108.7	0.1	5.6	2.4	2.8	116.5	2.7	0.2	116.2	1.1	4.0	2.5	−1.0	114.6	0.9	4.4
IV	107.7	1.3	6.3	2.4	2.8	117.7	4.3	1.0	117.5	2.2	4.1	4.1	−0.2	115.9	2.0	4.5

Notes: ΔXG = difference between the optimal and actual values of XG.
ΔVBG = difference between the optimal and actual values of VBG.
For both problems the optimal bill rate series is the target $RBILL$ series.

Table 10–2. Control Results for the Kennedy-Johnson Administrations

Quarter	Actual Values						Optimal Values for $\gamma = 1.0$					Optimal Values for $\gamma = 0.1$				
	Y	100 $\%\Delta PF$	100 UR	$RBILL$	Target $RBILL$	Y^*	ΔXG	ΔVBG	Y	100 $\%\Delta PF$	100 UR	ΔXG	ΔVBG	Y	100 $\%\Delta PF$	100 UR
1961I	107.3	0.7	6.8	2.4	2.9	118.9	10.3	5.2	118.8	1.5	5.3	9.9	5.0	118.4	1.5	5.4
II	109.9	0.7	7.0	2.3	3.0	120.1	3.8	4.3	119.9	1.7	4.6	3.0	3.8	118.9	1.6	4.8
III	112.0	−0.5	6.8	2.3	3.0	121.3	3.5	3.9	121.1	0.4	4.8	2.6	3.1	119.7	0.3	5.1
IV	114.3	2.0	6.2	2.5	3.1	122.5	2.8	3.5	122.2	2.7	5.0	1.8	2.4	120.6	2.6	5.4
1962I	116.1	0.5	5.6	2.7	3.1	123.7	3.3	4.0	123.5	1.1	4.7	2.5	2.8	121.9	1.0	5.2
II	118.0	1.3	5.5	2.7	3.2	124.9	3.2	4.7	124.7	2.0	4.7	2.4	3.4	123.4	1.9	5.1
III	119.2	0.2	5.6	2.9	3.3	126.2	3.3	5.7	126.0	0.9	4.8	2.7	4.2	124.7	0.8	5.1
IV	120.6	1.1	5.5	2.8	3.3	127.5	2.8	6.1	127.2	1.8	4.8	2.0	4.4	125.7	1.7	5.1
1963I	121.3	0.9	5.8	2.9	3.4	128.7	3.4	7.0	128.5	1.5	5.2	2.7	5.2	126.9	1.4	5.5
II	122.4	2.0	5.7	2.9	3.5	130.0	3.4	7.9	129.8	2.7	5.0	2.7	5.9	128.2	2.6	5.3
III	124.5	0.4	5.5	3.3	3.5	131.3	2.4	8.1	131.1	0.8	4.9	1.8	6.0	129.5	0.7	5.2
IV	126.3	1.7	5.6	3.5	3.6	132.6	2.1	8.2	132.4	2.0	5.1	1.3	6.0	130.7	1.9	5.4
1964I	128.4	1.4	5.5	3.5	3.7	134.0	1.1	7.8	133.7	1.7	5.3	0.4	5.5	131.8	1.6	5.6
II	130.2	1.4	5.3	3.5	3.8	135.3	1.2	8.0	135.0	1.7	5.1	0.5	5.6	133.1	1.6	5.5
III	131.8	1.7	5.0	3.5	3.8	136.6	1.0	8.1	136.4	2.0	5.1	0.5	5.8	134.5	1.9	5.4
IV	132.5	1.3	5.0	3.7	3.9	138.0	1.9	8.9	137.8	1.6	5.0	1.4	6.6	136.1	1.5	5.2

1965I	135.9	1.1	4.9	3.9	4.0	139.4	−0.3	8.0	139.2	1.2	5.0	−0.8	5.8	137.5	1.1	5.2
II	137.8	2.0	4.7	3.9	4.1	140.8	0.1	8.0	140.6	2.1	5.0	−0.3	5.8	139.1	2.1	5.2
III	140.5	1.3	4.4	3.9	4.1	142.2	−1.1	7.3	142.0	1.4	4.8	−1.4	5.1	140.5	1.3	5.0
IV	143.6	1.7	4.1	4.2	4.2	143.6	−2.2	6.3	143.4	1.5	4.9	−2.6	4.0	141.7	1.4	5.1
1966I	146.6	2.6	3.9	4.6	4.3	144.9	−3.2	4.7	144.7	2.1	5.0	−3.8	2.4	142.8	2.0	5.2
II	147.7	3.8	3.8	4.6	4.4	146.2	−2.6	3.9	146.0	3.4	5.0	−3.2	1.5	144.0	3.3	5.3
III	148.7	2.6	3.8	5.0	4.5	147.5	−2.5	3.0	147.3	2.0	4.9	−3.2	0.5	145.1	1.9	5.2
IV	150.5	4.4	3.7	5.2	4.6	148.9	−3.3	1.2	148.6	3.8	4.8	−4.1	−1.3	146.3	3.7	5.1
1967I	149.7	3.7	3.8	4.5	4.7	150.2	−1.3	1.2	150.0	3.7	4.7	−1.9	−1.3	147.8	3.6	5.0
II	150.9	1.6	3.8	3.7	4.8	151.6	−1.9	0.6	151.3	2.0	4.5	−2.6	−1.9	149.1	1.9	4.8
III	152.6	3.5	3.8	4.3	4.9	153.0	−2.2	−0.8	152.7	3.3	4.5	−3.0	−3.3	150.2	3.2	4.8
IV	153.7	3.4	4.0	4.8	5.0	154.4	−2.0	−2.0	154.0	3.3	4.6	−2.7	−4.5	151.5	3.2	5.0
1968I	155.6	2.8	3.8	5.1	5.1	155.8	−2.8	−3.5	155.4	2.6	4.6	−3.5	−6.0	152.8	2.4	4.9
II	158.5	3.5	3.6	5.5	5.2	157.2	−4.2	−6.0	156.8	3.0	4.6	−5.1	−8.5	154.0	2.9	5.0
III	160.0	3.4	3.6	5.2	5.3	158.6	−4.0	−7.8	158.3	3.1	4.8	−4.7	−10.2	155.5	3.0	5.1
IV	161.1	4.3	3.4	5.6	5.4	160.0	−3.6	−9.7	159.7	3.7	4.7	−3.9	−11.6	157.4	3.6	5.0

Notes: See notes to Table 10-1.

Table 10–3. Control Results for the Nixon-Ford Administrations

Quarter	*Actual Values*						*Optimal Values for* $\gamma = 1.0$					*Optimal Values for* $\gamma = 0.1$				
	Y	100 $\%\Delta PF$	100 UR	$RBILL$	Target $RBILL$	Y^*	ΔXG	ΔVBG	Y	100 $\%\Delta PF$	100 UR	ΔXG	ΔVBG	Y	100 $\%\Delta PF$	100 UR
1969I	162.5	4.1	3.4	6.1	5.5	161.5	−1.2	−1.0	161.5	3.8	3.5	−1.0	−0.9	161.6	3.8	3.5
II	163.2	4.4	3.5	6.2	5.6	162.9	−0.4	−1.2	162.8	4.2	3.6	−1.2	−1.7	162.0	4.1	3.7
III	163.8	4.7	3.6	7.0	5.7	164.4	−0.0	−1.6	164.2	4.3	3.7	−1.1	−2.7	162.9	4.1	3.9
IV	162.9	4.4	3.6	7.3	5.8	165.9	1.9	−0.9	165.5	4.4	3.3	0.5	−2.6	163.8	4.1	3.6
1970I	161.9	4.5	4.2	7.3	5.9	167.4	3.3	0.3	167.0	4.9	3.3	1.8	−2.1	165.0	4.5	3.8
II	161.9	4.5	4.8	6.8	6.0	168.9	3.6	1.1	168.5	5.0	3.4	1.8	−2.0	166.1	4.7	3.9
III	163.2	3.2	5.2	6.4	6.2	170.5	2.6	0.6	169.8	3.6	3.7	0.3	−3.5	166.6	3.4	4.4
IV	161.2	7.2	5.8	5.4	6.3	172.0	5.6	2.6	171.1	7.9	4.2	3.2	−2.5	167.4	7.7	4.9
1971I	165.6	3.6	6.0	3.9	6.3	173.6	2.1	1.7	173.0	4.8	4.5	0.2	−3.7	169.4	4.7	5.2
II	166.8	4.1	5.9	4.2	6.3	175.1	4.3	2.9	174.8	4.8	4.8	2.7	−2.9	171.6	4.7	5.4
III	167.8	2.6	6.0	5.1	6.3	176.7	4.5	3.7	176.4	3.0	4.9	3.1	−2.2	173.4	3.0	5.4
IV	170.6	0.4	6.0	4.2	6.3	178.3	3.4	4.6	177.9	1.4	5.1	1.9	−1.6	174.9	1.4	5.5
1972I	173.6	5.3	5.8	3.4	6.3	179.9	3.5	6.5	179.4	6.7	5.3	2.1	−0.1	176.6	6.6	5.7
II	177.5	1.7	5.7	3.7	6.3	181.6	2.7	8.1	181.2	2.6	5.5	1.7	1.4	178.7	2.5	5.8
III	180.2	2.2	5.6	4.2	6.3	183.2	3.0	10.7	182.9	2.9	5.6	1.9	3.8	180.3	2.9	5.9
IV	184.0	3.0	5.3	4.9	6.3	184.9	1.7	12.6	184.5	3.4	5.6	0.4	5.3	181.7	3.4	6.0
1973I	188.7	3.3	5.0	5.6	6.3	186.5	−0.4	13.2	186.2	3.5	5.7	−2.1	5.4	182.6	3.4	6.1
II	189.6	5.7	4.9	6.6	6.3	188.2	1.3	15.9	187.8	5.5	5.7	−0.8	7.4	183.4	5.4	6.2
III	190.5	5.4	4.8	8.4	6.3	189.9	0.9	17.1	189.0	4.7	5.5	−2.3	7.4	183.0	4.5	6.1
IV	191.7	9.5	4.7	7.5	6.3	191.7	−0.3	18.1	189.6	9.3	5.3	−5.4	6.0	180.7	9.0	6.4
1974I	187.8	14.5	5.1	7.6	6.3	193.4	3.8	22.7	190.2	14.1	5.4	−2.6	7.8	178.4	13.8	6.8
II	186.9	15.5	5.1	8.3	6.3	195.1	3.6	25.2	191.3	15.0	4.8	−3.1	8.4	177.7	14.5	6.5
III	185.9	13.0	5.5	8.3	6.3	196.9	5.1	28.5	193.2	12.7	4.6	−0.6	11.3	179.9	12.1	6.4
IV	181.0	13.3	6.6	7.3	6.3	198.7	10.6	37.6	195.9	13.4	4.9	6.8	21.2	185.3	13.0	6.3
1975I	174.7	11.0	8.3	5.9	6.3	200.5	15.1	47.9	199.1	11.6	5.5	13.8	34.7	192.1	11.3	6.4

than the actual average rate. The optimal output series is smoother than the actual output series, which, because of the nonlinearities in the model, allows more output to be produced on average with the same average rate of inflation.

For $\gamma = 1.0$ in Table 10–1, the optimum corresponds to more output, but also to a higher average rate of inflation. Comparing the two sets of optimal results in Table 10–1, it can be seen that the optimum for $\gamma = 1.0$ corresponds to 43.3 billion dollars more in output being produced over the 30 quarters and to a higher average rate of inflation of 0.11 percent per year. The difference between the optimal average unemployment rates over the 30 quarters is 0.33 percentage points.

The summary results for Table 10–2 show that both optima correspond to more output and more inflation than actually existed. Comparing the two sets of optimal results, it can be seen that the optimum for $\gamma = 1.0$ corresponds to 58.7 billion dollars more in output over the 32 quarters, to a higher average rate of inflation of 0.09 percent per year, and to a lower average unemployment rate of 0.29 percentage points. It is interesting to note that the average unemployment rate for both optima are higher than the actual rate, even though both optima correspond to more output being produced. There are two main reasons for this. The first is that the bill rates that were targeted for the two runs are generally larger than the actual bill rates. Interest rates have a positive effect on the work effort of the household sector; in particular the mortgage rate has a positive effect on the labor force participation of all persons 16 and over except men 25–54. The higher interest rates for the optimal runs thus cause the labor force to be larger than otherwise, which in turn causes the unemployment rate to be larger than otherwise.

The other main reason for the higher unemployment rates for the optimal runs is that the optima correspond to higher real wages. When the economy expands in the model, the money wage rate, *WF*, rises faster initially than does the price level. The real wage thus increases initially, which has a positive effect on the labor force and thus on the unemployment rate. It was mentioned in section 1.1 and in Chapter Nine that there are many factors that have an effect on the unemployment rate, and the results in Table 10–2 provide a good example of how the unemployment rate can be higher in one run than in another even though real output is also higher.

The summary results for Table 10–3 show that the optimal values for $\gamma = 0.1$ are close to the actual values. The optimal value of output over the 25 quarters is only 1.6 billion dollars higher than the actual value. It is again the case that the optimal average unemployment rate is larger than the actual rate even though the optimal value of output is greater than the actual value. Comparing the two sets of optimal results, the optimum for $\gamma = 1.0$ corresponds to 117.7 billion dollars more in output over the 25 quarters, to a higher average rate of inflation of 0.18 percent per year, and to a lower average unemployment rate of 0.65 percentage points.

An important feature of the results in the three tables is that for $\gamma = 1.0$ the optimal output series correspond closely to the target series. In Table 10–1, for example, the difference between the sum of Y^* and the sum of the optimal output values over the 30 quarters is only 4.7 billion dollars. In Tables 10–2 and 10–3 the respective differences are 7.8 and 24.9 billion dollars. Since the starting values used for XG corresponded to the output targets being achieved exactly, this closeness may be due merely to a failure on the part of the algorithm to find the true optima. This, however, did not appear to be the case from some experimentation that was carried out to see if the true optima had been attained.

What these and other results show is that the model has the property that output can be increased to some reasonable target value (from a lower value) without having too serious an affect on the rate of inflation. It is not, however, generally possible to decrease the rate of inflation to, say, zero percent (from a higher rate) without having serious effects on the level of output. Consequently, when a loss function like (10.3) is minimized, with equal weights attached to the output and inflation targets, the optimum tends to correspond more closely to the output target being achieved than it does to the inflation target being achieved. Even when the weight on the output target is only one-tenth of the weight on the inflation target, it is still the case that the inflation target of zero percent is not close to being achieved.

It is possible to use the results in Tables 10–1, 10–2, and 10–3 to examine the question of the "trade-off" between, say, the rate of inflation and the level of output. One must be very careful in doing this, however, because of the many diverse factors that affect both variables. It was argued in section 9.4 that there is no reason to expect there to be a stable relationship between the rate of inflation and the level of output, and this holds true whether the values of the policy variables are historic or optimal values. The trade-off that one observes in tables like 10–1, 10–2, and 10–3 for one control period and one set of problems may not hold true for other control periods and other sets of problems.

Comparing the two sets of optimal results in Table 10–1 shows that a yearly gain of output of 5.8 billion dollars (43.3 ÷ 7.5 years) is achieved at a cost of an extra 0.11 percent inflation per year. In Table 10–2 the yearly gain is 7.3 billion dollars (58.7 ÷ 8 years) at a cost of 0.09 percent inflation per year, and in Table 10.3 the yearly gain is 18.8 billion dollars (117.7 ÷ 6.25 years) at a cost of 0.18 percent inflation per year. These figures show, as already mentioned, that the trade-off in the three tables is such that it is costly in terms of lost output to lower the rate of inflation, or, the other way around, that it is not costly in terms of extra inflation to increase the level of output.

It should be stressed again, however, that these figures should not necessarily be extrapolated to other periods. Because of the nonlinearities in the model, the figures in particular should not be extrapolated to situations in which the two sets of optimal results that are compared correspond to much larger differences in the state of the economy than the differences in the current three tables.

It is finally of interest to note that the optimal values for $\gamma = 0.1$ in the three tables correspond more closely to the actual values than do the optimal values for $\gamma = 1.0$. One possible conclusion from this fact is that the people who were responsible for controlling the economy weighted inflation more heavily than output in their loss functions. This would be true, however, only if the people believed that the trade-offs between inflation and output were similar to those in the present model and they had targets for inflation and output that were similar to the targets in the loss function (10.3).

Chapter Eleven

Conclusion

The model is summarized in section 1.1, and so it will not be summarized again here. This chapter instead contains a brief discussion of possible future research topics on the model and some closing remarks.

It should be clear that this study has been restricted in important ways by the use of a relatively slow computer and by a relatively small computer budget. It would definitely be of interest in future work to do more experimentation in trying to obtain true FIML estimates. It might also be of interest, as discussed in section 3.6, to try to obtain FDYN estimates of the model and see how these estimates compare and perform relative to the FIML and TSLS estimates. Finally, it might be of interest with more computer resources to do further experimenting on obtaining optimal controls for the model. All three of these problems are similar (and expensive) in that they involve solving fairly large nonlinear maximization problems by the use of algorithms like the ones discussed in section 3.4.

There are a number of areas in which one might consider trying to improve the specification of the model. Some suggestions are presented in section 5.3, for example, on possible alternative ways of accounting for the hours constraint on the household sector and the labor constraint on the firm sector. The approach taken in this study regarding these two constraints does not necessarily use all the information on the labor market that is available. There may also be other approaches than the one taken in this study for trying to pick up loan constraint effects on the household and firm sectors.

The model could be disaggregated in a number of ways. Possible variables to disaggregate include the labor force variables, the consumption and investment variables, and the asset and liability variables. The division of the model into sectors and the closed nature of the model with respect to the flows of funds should enable this type of disaggregation to be carried out without any major changes in the basic structure of the model.

It is also possible within the basic structure of the model to consider alternative lag structures for the stochastic equations and alternative functional forms. The stochastic equations are clearly only approximations to the way that the decision variables are actually determined, and experimenting with alternative lag structures, alternative functional forms, and even alternative variables that are designed to pick up expectational effects is certainly within the spirit of the model.

One of the most important questions about the current version of the model is whether the properties of the model reported in the last chapter regarding the "trade-off" between inflation and output are true of the real world. The model does have the characteristic that it is generally possible to achieve a fairly high level of output without causing very much additional inflation. In the price equation (Equation 9 in Table 2–3), PF_t is not very sensitive to recent changes in economic activity, especially if these changes are from a low level of activity. Whether this is also true of the real world is perhaps unclear, but from the experimentation done in this study it does not appear possible to pick up in the data very strong effects of the level of economic activity on the price level. Since this is such an important question for policy purposes, however, more experimentation should be done to see if the actual effects are stronger than the effects currently in the model.

Another important question about the current version of the model is whether the predictive accuracy of the model regarding the bill rate can be improved by the use of other estimation techniques or by slight changes in the specification of some of the equations. Some of the predictions of the bill rate in Chapter Eight are quite wide of the mark, and one would hope in the future to be able to improve upon this performance. As discussed in section 8.4, it may be that truer FIML estimates or FDYN estimates will lead to better predictions. Given the key role that the bill rate plays in the model, this is certainly an important area for future work.

This completes the discussion of possible future research topics on the model, and I would like to conclude this study on a personal note. It seems to me that a long run goal of macroeconometric model building ought to be the development of models that when used in a nonsubjective way for policy purposes, via the computation of optimal controls, result, on average, in better policies (i.e., result, on average, in a larger value of the objective function) than any other approaches. Numerical methods and computer technology have now advanced to the point where computational problems no longer appear to be a serious constraint to the attainment of this goal. It now appears feasible to obtain full information estimates and optimal controls for almost any model. Although this study was hindered somewhat by a slow computer and a tight computer budget, in an actual policy making situation the cost of a few hours of computer time to estimate a model and compute optimal controls for it is trivial compared to the billions of 1958 dollars that might be saved from the implementation of better policies. The remaining constraints to the attainment of the above goal are, it seems to me, the quality of the data and the accuracy of the specifications of the equations. Some would argue, however, that this goal will never be achieved because the structure of the economy is not stable enough to allow models to be used in mechanical ways. My work in these two volumes is based on the premise that this argument is not true, and my primary aim has been to try to make some contribution toward the development of more accurate models.

Appendix A

Some Results for the Alternative Technology

The purpose of this appendix is to show that the two technologies discussed in section 5.2 lead to similar results. The first technology is represented by Equations (5.1) and (5.2), and the second technology is represented by Equations (5.3) and (5.4). The measurement of excess capital and excess labor for both technologies is described in section 5.2. Both technologies lead to estimates of the capital stock (K_t^a), of the minimum amount of capital needed to produce the output of the period ($KMIN_t$), of the physical depreciation of the capital stock during the period (denoted here as $DEPK_t$), and of the number of worker hours required to produce the output of the period ($M_t H_t^M$).

$DEPK_t$ for the first technology is simply $\delta_K K_{t-1}^a$. $DEPK_t$ for the second technology can be obtained as $INV_t - (K_t^a - K_{t-1}^a)$, where INV_t is gross investment for period t and $K_t^a - K_{t-1}^a$ is net investment. As discussed in section 5.2, K_t^a is obtained for this technology by summing past values of gross investment back to the age of the oldest machine in existence (m_t in section 5.2). Two sets of estimates of $M_t H_t^M$ were obtained for the second technology, one for values of $\bar{\mu}\bar{H}/\bar{\lambda}$ and δ_λ of 118894.4 and 0.005204, respectively, and one for values of $\bar{\mu}\bar{H}/\bar{\lambda}$ and δ_λ of 121927.8 and 0.005602, respectively.

The results of estimating the investment equation for the two technologies are presented in Table A-1. The estimates for the first technology are the same as the ones presented in Table 2–3. The estimates are TSLS estimates for the 1954I–1974II period. The equations for the two technologies differ only in the values used for K_{t-1}^a, $KMIN_{t-1}$, and $DEPK_t$. As can be seen in the table, the results for the two technologies are close, with the results for the first technology being slightly better.

The results of estimating the employment and hours equations for the two technolgies are presented in Tables A–2 and A–3. The estimates

Table A–1. Estimates of the Investment Equation for the Two Technologies (The top set of estimates is for the alternative technology)

		DW	R^2
	-0.000469 $+0.0236$		
	(0.39) (0.69)	1.86	0.567
$INV_t - INV_{t-1} =$	$-0.00256(K^a_{t-1} - KMIN_{t-1}) + 0.0272(Y_t - Y_{t-1})$	1.89	0.579
	(0.80) (0.78)		
	$+0.0797$ $+0.0257$ $+0.0566$		
	(3.21) (1.18) (2.68)		
	$+0.0782(Y_{t-1} - Y_{t-2}) + 0.0241(Y_{t-2} - Y_{t-3}) + 0.0558(Y_{t-3} - Y_{t-4})$		
	(3.11) (1.09) (2.52)		
	-0.0115 -1.07 $+0.498$		
	(1.01) (3.87) (1.68)		
	$-0.0155(INV_{t-1} - DEPK_t) - 1.04D704_t + 0.509D711_t$		
	(0.82) (3.74) (1.75)		

Table A–3. Estimates of the Hours Equation for the Two Technologies (The top set of estimates is for the alternative technology)

		$\hat{\rho}$	DW	R^2
	1.90 -0.345	-0.195	1.93	0.374
	(4.35)(4.26)	(1.80)		
$\log HPF_t - \log HPF_{t-1} =$	$1.42 - 0.269 \log HPF_{t-1}$	-0.221	1.96	0.345
	(4.15)(4.15)	(2.06)		
	-0.0427			
	(2.97)			
	$-0.0438(\log JOBF_{t-1} - \log M_{t-1}H^M_{t-1})$			
	(2.70)			
	$-0.000377 + 0.138$			
	(4.26) (4.28)			
	$-0.000253t + 0.162(\log Y_t - \log Y_{t-1})$			
	(4.20) (5.22)			

for the first technology are also the same as the ones presented in Table 2–3, and both sets of estimates are TSLS estimates for the 1954I–1974II period. The equations for the two technologies differ only in the values used for $M_{t-1}H^M_{t-1}$. The values used for $M_{t-1}H^M_{t-1}$ for the second technology are the ones based on values of $\bar{\mu}\bar{H}/\bar{\lambda}$ and δ_λ of 118894.4 and 0.005204.

Table A–2. Estimates of the Employment Equation for the Two Technologies (The top set of estimates is for the alternative technology)

		$\hat{\rho}$	DW	R^2
	$-0.181\quad -0.0292$	0.340	1.95	0.715
	$(1.40)\quad (1.38)$	(3.28)		
$\log JOBF_t - \log JOBF_{t-1} =$	$-0.489 - 0.0780(\log JOBF_{t-1} - \log M_{t-1}H^M_{t-1})$	0.307	1.96	0.737
	$(2.86)\quad (2.85)$	(2.92)		
	$+0.0000293\quad +0.211$			
	$(1.07)\qquad (3.53)$			
	$+0.0000971t + 0.215(\log Y_t - \log Y_{t-1})$			
	$(2.97)\qquad (3.67)$			
	$+0.195$			
	(4.27)			
	$+0.172(\log Y_{t-1} - \log Y_{t-2})$			
	(3.84)			
	$+0.0810\qquad -0.0109$			
	$(1.88)\qquad (2.94)$			
	$+0.0725(\log Y_{t-2} - \log Y_{t-3}) - 0.00945\, D593_t$			
	$(1.79)\qquad (2.22)$			
	$+0.00142$			
	(0.34)			
	$+0.00196\, D594_t$			
	(0.49)			

The results for the two technologies are again close, with the results for the first technology being slightly better for the employment equation in Table A–2 and slightly worse for the hours equation in Table A–3. When the alternative values of $\bar{\mu}\bar{H}/\bar{\lambda}$ and δ_λ were used for the second technology, the results were little changed. The estimate for the coefficient of the excess labor variable was -0.0298 in the employment equation (versus -0.0292 in Table A–2) and -0.0431 in the hours equation (versus -0.0427 in Table A–3.)

It appears to be fairly clear from the results in Tables A–1, A–2, and A–3 that the properties of the model would be little changed regardless of which technology was used. The first technology is computationally easier to work with, since it does not require keeping track of as many past values of investment, and this is the primary reason for its use in this study.

Appendix B

The Forecasting Model Used for Comparison Purposes

The forecasting model was estimated through 1974II for the comparisons in Chapter Eight. The notation for the model is presented in Table B–1, and the model is presented in Table B–2. The same techniques were used to estimate the model for the work in this study that were used in Fair [14] and that have been used all along to reestimate the model each quarter. One of the techniques described in Fair and Jaffee [24] for estimating markets in disequilibrium is used to estimate the monthly housing starts sector. The two stage least squares technique described in Fair [22] for estimating simultaneous equations models with lagged endogenous variables and first order serially correlated errors is used to estimate the equations in the money GNP sector and Equation (9.12). The other equations in the model are estimated by ordinary least squares under the assumption of first order serial correlation of the error terms.

Strikes were handled in [14] by excluding the strike observations from the estimation periods. For the work in this study, however, no observations have been excluded from the estimation periods, and strikes have been handled by adding dummy variables to the equations most affected by the strikes. This was done to allow the models to be simulated over a longer period than would have been possible with gaps in the estimation period. For the forecasting model, four dummy variables were added to the equation explaining consumer durable expenditures (CD_t); two to the equation explaining plant and equipment investment (IP_t); three to the equation explaining inventory investment ($V_t - V_{t-1}$); five to the equation explaining imports (IMP_t); and two to the equation explaining employment (M_t). These are the same dummy variables that were added to the corresponding equations in the empirical model.

For the sake of completeness, the results of estimating the inventory, import, and price equations are presented in Table B–2, even though these equations are not used for the comparisons in Chapter Eight.

These equations are (6.15), (7.3), and (10.7). Since the forecasting model is described in detail elsewhere, no further discussion of it will be presented here.

Table B–1. The List of Variables in the Forecasting Model in Alphabetic Order by Sector

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The Monthly Housing Starts Sector

†$DHF3_t$ = Three-month moving average of the flow of advances from the Federal Home Loan Bank to Savings and Loan Associations in millions of dollars
†$DSF6_t$ = Six-month moving average of private deposit flows into Savings and Loan Associations and Mutual Savings Banks in millions of dollars
HS_t = Private nonfarm housing starts in thousands of units
†RM_t = *FHA* mortgage rate series on new homes in units of 100
†W_t = Number of working days in month t
†$/\Delta RM_t/$ = [see Equation (8.21) in [14]]
†$\backslash\Delta RM_t\backslash$ = [see Equation (8.22) in [14]]

The Money *GNP* Sector

CD_t = Consumption expenditures for durable goods, *SAAR*
CN_t = Consumption expenditures for nondurable goods, *SAAR*
CS_t = Consumption expenditures for services, *SAAR*
†EX_t = Exports of goods and services, *SAAR*
†G_t = Government expenditures plus farm residential fixed investment, *SAAR*
GNP_t = Gross National Product, *SAAR*
HSQ_t = Quarterly nonfarm housing starts, seasonally adjusted at quarterly rates in thousands of units
IH_t = Nonfarm residential fixed investment, *SAAR*
IMP_t = Imports of goods and services, *SAAR*
IP_t = Nonresidential fixed investment, *SAAR*
†$MOOD_t$ = Michigan Survey Research Center index of consumer sentiment in units of 100
†$PE2_t$ = Two-quarter-ahead expectation of plant and equipment investment, *SAAR*
$V_t - V_{t-1}$ = Change in total business inventories, *SAAR*

The Price Sector and the Employment and Labor Force Sector

†AF_t = Level of the armed forces in thousands
D_t = Difference between the establishment employment data and household survey employment data, seasonally adjusted in thousands of workers
E_t = Total civilian employment, seasonally adjusted in thousands of workers
†GG_t = Government output, *SAAR*
$GNPR_t$ = Gross National Product, seasonally adjusted at annual rates in billions of 1958 dollars
†$GNPR^*$ = Potential GNP, seasonally adjusted at annual rates in billions of 1958 dollars
LF_{1t} = Level of the labor force of males 25–54, seasonally adjusted in thousands
LF_{2t} = Level of the labor force of all others 16 and over, seasonally adjusted in thousands
M_t = Private nonfarm employment, seasonally adjusted in thousands of workers
†MA_t = Agricultural employment, seasonally adjusted in thousands of workers
†MCG_t = Civilian government employment, seasonally adjusted in thousands of workers

M_rH_t = Worker hour requirements in the private nonfarm sector, seasonally adjusted in thousands of worker hours per week
†P_{1t} = Noninstitutional population of males 25–54 in thousands
†P_{2t} = Noninstitutional population of all others 16 and over in thousands
PD_t = Private output deflator, seasonally adjusted in units of 100
UR_t = Civilian unemployment rate, seasonally adjusted
Y_t = Private nonfarm output, seasonally adjusted at annual rates in billions of 1958 dollars
†YA_t = Agricultural output, seasonally adjusted at annual rates in billions of 1958 dollars
†YG_t = Government output, seasonally adjusted at annual rates in billions of 1958 dollars

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Notes: † Exogenous variable.

SAAR: Seasonally adjusted at annual rates in billions of current dollars.

The following dummy variables are also used in the model: $D593_t$, $D594_t$, $D601_t$, $D644_t$, $D651_t$, $D652_t$, $D691_t$, $D692_t$, $D693_t$, $D704_t$, $D711_t$, $D714_t$, $D721_t$. (See Table 2–1 for a list of these variables.)

Table B–2. The List of Equations in the Forecasting Model by Sector

Notes:
1. Absolute values of the t-statistics are in parentheses.
2. DW = Durbin-Watson statistic.
3. R^2 = coefficient of determination.
4. $\hat{\rho}$ = estimate of the first order serial correlation coefficient for the equation. "1.0" means the coefficient was constrained to be one.
5. When $\hat{\rho} \neq 0$, DW and R^2 are computed using the estimates of the transformed residuals.
6. logs are natural logs.
7. α_t = production function coefficient obtained from peak-to-peak interpolations.

Equation Number in [14]		$\hat{\rho}$	DW	R^2
	The Monthly Housing Starts Sector			
(8.23)	$HS_t = \underset{(2.14)}{114.4} + \underset{(3.93)}{1.77W_t} - \underset{(1.27)}{0.0191}\sum_{i=1}^{t-1} HS_i + \underset{(1.55)}{2.98t} - \underset{(1.65)}{0.151RM_{t-2}}$ $-\underset{(0.94)}{0.104/\Delta RM_t/}$	0.921 (31.85)	2.54	0.874
(8.24)	$HS_t = \underset{(1.58)}{41.4} + \underset{(3.78)}{1.91W_t} + \underset{(0.01)}{0.000760t} + \underset{(8.08)}{0.0258DSF6_{t-1}}$ $+\underset{(1.91)}{0.0137DHF3_{t-2}} + \underset{(0.25)}{0.0109RM_{t-1}} - \underset{(0.94)}{0.104/\Delta RM_t/}$	0.661 (11.86)	2.16	0.880

(3.3)	$CD_t = \underset{(4.53)}{-38.5} + \underset{(30.82)}{0.108}GNP_t + \underset{(1.31)}{0.0820}MOOD_{t-1} + \underset{(3.23)}{0.229}MOOD_{t-2}$ $\underset{(1.50)}{-2.47}D644_t + \underset{(1.29)}{2.12}D651_t - \underset{(3.65)}{6.03}D704_t + \underset{(0.90)}{1.48}D711_t$	0.783 (10.83)	1.96	0.997
(3.7)	$CN_t = \underset{(2.55)}{0.0380}GNP_t + \underset{(14.05)}{0.876}CN_{t-1} - \underset{(0.03)}{0.000662}MOOD_{t-2}$	0.140 (1.07)	2.02	0.999
(3.11)	$CS_t = \underset{(2.05)}{0.0121}GNP_t + \underset{(44.24)}{0.979}CS_{t-1} - \underset{(4.32)}{0.0145}MOOD_{t-2}$	−0.269 (2.41)	1.87	0.9999
(4.4)	$IP_t = \underset{(3.23)}{-7.98} + \underset{(10.07)}{0.0727}GNP_t + \underset{(6.12)}{0.521}PE2_t - \underset{(3.51)}{3.33}D704_t$ $\underset{(1.87)}{-1.79}D711_t$	0.862 (14.66)	1.88	0.999
(5.5)	$IH_t = \underset{(5.15)}{-17.2} + \underset{(8.85)}{0.0281}GNP_t + \underset{(7.03)}{0.0235}HSQ_t + \underset{(8.70)}{0.0291}HSQ_{t-1}$ $\underset{(4.18)}{+0.0141}HSQ_{t-2}$	0.897 (15.31)	2.17	0.995
(6.15)	$V_t - V_{t-1} = \underset{(3.11)}{-59.6} + \underset{(4.13)}{0.471}(CD_{t-1} + CN_{t-1}) - \underset{(3.81)}{0.310}V_{t-1}$ $\underset{(0.10)}{-0.0140}(CD_{t-1} + CN_{t-1} - CD_t - CN_t) - \underset{(2.42)}{7.32}D593_t$ $\underset{(0.61)}{-2.14}D594_t + \underset{(1.27)}{3.83}D601_t$	0.925 (20.94)	2.00	0.622

Table B–2. (continued)

Equation Number in [14]		$\hat{\rho}$	*DW*	R^2
(7.3)	$IMP_t = \underset{(6.83)}{0.146}GNP_t - \underset{(0.93)}{2.21}D651_t + \underset{(0.33)}{0.78}D652_t - \underset{(1.98)}{4.96}D691_t$ $+\underset{(0.77)}{2.23}D692_t - \underset{(0.13)}{0.34}D693_t - \underset{(1.88)}{4.46}D714_t + \underset{(1.29)}{3.04}D721_t$	1.0	0.55	0.989
Income Identity	$GNP_t = CD_t + CN_t + CS_t + IP_t + IH_t + V_t - V_{t-1} - IMP_t$ $+EX_t + G_t$			
	The Price Sector			
(10.5)	$GAP2_t = GNPR_t^* - GNPR_{t-1} - (GNP_t - GNP_{t-1})$			
(10.7)	$PD_t - PD_{t-1} = \underset{(2.38)}{2.20} - \underset{(1.13)}{0.0285}\left(\frac{1}{20}\sum_{i=1}^{20} GAP2_{t-i+1}\right)$	0.937 (23.14)	2.54	0.784
(10.8)	$GNPR_t = 100\,\frac{GNP_t - GG_t}{PD_t} + YG_t$			
(10.9)	$Y_t = GNPR_t - YA_t - YG_t$			
	The Employment and Labor Force Sector			
(9.2)	$M_tH_t = \frac{1}{\alpha_t}\,Y_t$			

(9.8)	$\log M_t - \log M_{t-1} = \underset{(3.47)}{-0.416} + \underset{(3.31)}{0.000106_t}$ $\underset{(3.44)}{-0.113}(\log M_{t-1} - \log M_{t-1}H_{t-1})$ $\underset{(1.92)}{+0.0814}(\log Y_{t-1} - \log Y_{t-2})$ $\underset{(7.89)}{+0.288}(\log Y_t - \log Y_{t-1})$ $\underset{(0.55)}{-0.00192}D593_t \underset{(0.23)}{- 0.000782}D594_t$	0.400 (3.75)	1.92	0.737
(9.10)	$D_t = \underset{(7.61)}{-11353} \underset{(6.10)}{- 71.1}t \underset{(8.55)}{+ 0.321}M_t$	0.624 (6.87)	2.04	0.930
(9.9)	$E_t = M_t + MA_t + MCG_t - D_t$			
(9.11)	$\frac{LF_{1t}}{P_{1t}} = \underset{(234.04)}{0.991} \underset{(6.64)}{- 0.000349}t$	0.810 (11.89)	2.13	0.942
(9.12)	$\frac{LF_{2t}}{P_{2t}} = \underset{(3.76)}{0.233} \underset{(6.48)}{+ 0.000827}t \underset{(2.79)}{+ 0.315} \frac{E_t + AF_t}{P_{1t} + P_{2t}}$	0.912 (19.14)	2.11	0.987
(9.14)	$UR_t = 1 - \frac{E_t}{LF_{1t} + LF_{2t} - AF_t}$			

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