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THE SENSITIVITY OF FISCAL POLICY EFFECTS TO ASSUMPTIONS ABOUT THE BEHAVIOR OF THE FEDERAL RESERVE[1]

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The purpose of this paper is to examine within the context of a particular U.S. econometric model the sensitivity of fiscal policy effects to alternative assumptions about the behavior of the Federal Reserve. Five cases are considered, four in which Fed behavior is exogenous and one in which Fed behavior is endogenous. In each of the four exogenous cases the Fed is assumed to control a particular variable, which is then taken to be exogenous for purposes of the fiscal-policy experiments. For the endogenous case an estimated equation explaining Fed behavior is added to the model, and the expanded model is used to perform the experiments. The results of some optimal control experiments are also reported in this paper. These latter experiments are designed to examine the sensitivity of optimal fiscal policies to alternative assumptions about Fed behavior. The main conclusion of this paper is that fiscal policy effects and optimal fiscal policies are quite sensitive to assumptions about the behavior of the Fed.

1. INTRODUCTION

MOST EXAMINATIONS OF FISCAL POLICY EFFECTS in U.S. econometric models are based on the assumption that the behavior of the Federal Reserve (henceforth called the "Fed") is exogenous, i.e., that the behavior of the Fed is not influenced by the state of the economy. The typical procedure is to assume that the Fed has control over a particular variable in the model and then to take this variable as exogenous for purposes of the fiscal policy experiments. An alternative procedure, if one believes that the behavior of the Fed is not exogenous, is to estimate an equation explaining Fed behavior (i.e., explaining the variable that the Fed is assumed to control), add this equation to the model, and use this expanded model to perform the fiscal policy experiments.

The purpose of this paper is to examine within the context of a particular U.S. econometric model the sensitivity of fiscal policy effects to alternative assumptions about Fed behavior. Five cases are considered, four in which Fed behavior is exogenous and one in which Fed behavior is endogenous. In each of the four exogenous cases the Fed is assumed to control a particular variable, which is then taken to be exogenous for purposes of the fiscal policy experiments. The control variables in the four cases are: (1) the amount of government securities outstanding; (2) the money supply; (3) nonborrowed reserves; and (4) the bill rate. For the endogenous case an estimated equation explaining Fed behavior is added to the model, and the expanded model is used to perform the fiscal policy experiments.

Section 2 contains a brief description of the econometric model used for purposes of this paper. The model, which is described in detail in Fair **[9]**, is particularly suited for examining the effects of monetary and fiscal policies

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because it is closed with respect to the flows of funds in the system. This means, among other things, that the government budget constraint is accounted for and that the amount of government securities outstanding can be taken to be a direct policy variable of the Fed. The equation explaining Fed behavior is presented and discussed in Section 3, and the results of the various fiscal policy experiments are presented in Section 4. Some optimal control results are then reported in Section 5. Given that an equation explaining Fed behavior has been estimated, it is possible to conduct optimal control experiments in which the fiscal authority maximizes an objective function taking into account the behavior of the Fed. A comparison can then be made, as is done in Section 5, between how well the fiscal authority does in this case versus the case in which the behavior of the Fed is exogenous.

The main conclusion of this paper is that fiscal policy effects are quite sensitive to alternative assumptions about the behavior of the Fed. When, for example, the Fed behaves as estimated in Section 3, the effect on real output of an increase in government expenditures on goods after eight quarters is less than half of the effect that occurs when the Fed behaves by keeping the bill rate unchanged. After twelve quarters, the effect is less than one-third. When the Fed behaves by keeping the money supply unchanged, fiscal policy is effective at all only for about the first eight quarters after the policy change. With respect to the optimal control results, when the Fed behaves as estimated in Section 3, the fiscal authority cannot achieve as low a value of loss as it can when the Fed behaves by keeping the bill rate unchanged. Also, much more fiscal stimulus is required in the case in which the Fed behaves as estimated in Section 3, even to obtain the somewhat higher value of the loss, because of the need to offset the negative response of the Fed to the stimulus. When the Fed behaves by keeping the money supply unchanged, the fiscal authority's ability to lower the value of loss at all is severely limited.

Before proceeding with the discussion of the model, mention should be made about how this study relates to the literature. There have been a number of studies concerned with estimating equations to explain the behavior of the Fed. These include Dewald and Johnson [**4**], Goldfeld [**13**], Wood [**20**], Havrilesky [**15**], Christian [**3**], Teigen [**19**], Keran and Babb [**16**], Friedlaender [**11**], and Froyen [**12**]. The work in Section 3 of this paper is an addition to this literature. The main difference between the present work and previous work is the treatment in the present case of the bill rate as the policy variable of the Fed. With the exception of one set of results in Dewald and Johnson [**4**] and Christian [**3**], previous studies have not done this.[2]

Work similar to that in Section 4 does not appear to have been done previously. While it is well known that multipliers in models can be quite different depending on whether or not reaction functions of the monetary and fiscal authorities are postulated, there has not been to the author's knowledge any previous study in which this question is examined using an actually estimated

[2] See, for example, Froyen [**12**, Table 3] for a list of the various policy variables used in five of the above studies.

reaction function and an actual econometric model. Goldfeld and Blinder [**14**], for example, in their important paper in this area use an actual econometric model in some of their work (the Moroney-Mason model [**17**]), but the reaction functions that they use are made up. Teigen [**19**] estimated an equation explaining Fed behavior within the context of a complete model, but he did not simulate the model to examine its properties. The work in Section 4 thus appears to be an attempt for the first time to gauge the actual quantitative importance of endogenous Fed behavior on fiscal multipliers.

The optimal control work in Section 5 is perhaps most closely related to the work of Pindyck [**18**], although there are significant differences between the approach here and Pindyck's approach. Pindyck sets up the problem of two independent authorities with conflicting objectives as a differential game and applies Nash solution strategies to it. In the present case the problem is set up as one in which the fiscal authority solves an optimal control problem given the behavior or reaction function of the Fed: in the terminology of the Stackelberg duopoly model, the fiscal authority is the leader and the Fed is the follower.

2. THE ECONOMETRIC MODEL

The econometric model in [**9**] consists of 84 equations, 26 of which are stochastic. There are five sectors (household, firm, financial, foreign, and government) and five categories of financial securities (demand deposits and currency, bank reserves, member bank borrowing from the Fed, gold and foreign exchange, and an "all other" category). Since the model is described in detail in [**9**], no extensive discussion of it will be presented here. It will be useful for purpose of the following analysis, however, to discuss briefly the financial sector and the effects of the Fed in the model.

There are three policy variables of the Fed: the reserve requirement ratio (g_1), the discount rate (RD), and the amount of government securities outstanding (VBG). The following four equations in the model will help to explain the effects of these three variables:

$$BR = g_1 \cdot DDB, \tag{45}$$

$$BORR/BR = 0.0121 + 0.0106(RBILL - RD), \tag{20}$$

$$0 = SAVG + \Delta VBG + \Delta(BR - BORR) + \Delta CURR - \Delta GFXG - DISG, \tag{69}$$

$$0 = \Delta VBG - \Delta VBP, \tag{70}$$

where the additional variables are, in alphabetic order:

- $BORR$: member bank borrowing from the Fed (endogenous);
- BR: bank reserves (endogenous);
- $CURR$: currency outstanding (exogenous);
- DDB: demand deposits held in the financial sector (endogenous);
- $DISG$: a statistical discrepancy relating to the government sector (exogenous);

GFXG: holdings of gold and foreign exchange of the government sector (exogenous);
RBILL: three-month treasury bill rate (endogenous);
SAVG: financial saving of the government sector (endogenous);
VBP: amount of government securities held by the non-government sectors (endogenous).

The above equation numbers correspond to the numbers in [9, Table 2-2].

Equation (45) relates bank reserves to demand deposits. Since g_1 is the reserve requirement ratio and *BR* is the actual level of reserves, equation (45) reflects the assumption that the banking system holds no excess reserves in the aggregate.[3] Equation (20) explains member bank borrowing from the Fed. It is a stochastic equation in the model, and the two coefficients in the equation are estimated coefficients. The equation states that the ratio of borrowed to total reserves is a function of the difference between the bill rate and the discount rate. Equation (69) is the government budget constraint. It states that any nonzero level of saving of the government must result in the change in at least one of the following: government securities outstanding, nonborrowed reserves, currency outstanding, or the government's holdings of gold and foreign exchange. Equation (70) equates the supply of government securities to the demand for them.[4]

The government budget constraint is redundant, and so it can be dropped from the model. This still leaves equation (70), however, as an "extra" equation. *VBP* is determined elsewhere in the model, and so if *VBG* is taken to be exogenous, then some variable not explained by any other equation must be chosen to be endogenous in order to close the model. The variable chosen in this case is the bill rate (*RBILL*). There is thus no equation in the model in which the bill rate appears naturally on the left hand side. The bill rate is instead implicitly determined: its solution value each period is the value that makes equation (70) hold. The bill rate is an explanatory variable in a number of the key equations in the model.

Although the above four equations are not sufficient for demonstrating this, the main way that the Fed affects the economy is through the bill rate. By changing *VBG*, g_1, or *RD*, the Fed affects the amount of funds in the system and thus the bill rate, and the bill rate in turn affects the other variables in the model.[5] It makes little difference in the model which of the three variables the Fed uses as a control variable with respect to its ability to control the economy.

[3] More precisely, equation (45) reflects the assumption that the ratio of excess reserves to total reserves is exogenous. See the discussion in [9, pp. 135–137] for more details.

[4] Government securities are actually included in the "all other" category of securities in the model, and equation (70) is the equation that equates the aggregate supply of this category of securities to the aggregate demand. For purposes of the discussion in this paper, the notation for equation (70) has been simplified: ΔVBP represents all the terms in equation 70 in [9, Table 2-2] except ΔVBG.

[5] Because the model is simultaneous, this statement is not quite right. It is meant to give a general idea of how the Fed affects the economy, but it is not a rigorous description of causality in the model.

For the empirical work in Sections 4 and 5 the model was re-estimated through 1976 II using the revised national income accounts data. A few small changes had to be made in the model to account for some definitional changes in the new data. The model was estimated by the two-stage least squares technique, as described in [9, Chapter 3]. The quantitative properties of the re-estimated model differed little from those of the original model. When, for example, the properties of the re-estimated model were examined in the manner described for the original model in [9, Chapter 9], the new results were quite similar to the old. The reader is thus referred to this chapter for a fairly accurate description of the properties of the re-estimated model.[6]

3. AN EXPLANATION OF FED BEHAVIOR

For the work in this section the Fed is assumed to take the bill rate as its control variable. In other words, the Fed is assumed to choose each period a desired or optimal value of the bill rate and then to achieve this value through changes in *VBG*, g_1, and *RD*. Given this assumption, the purpose of this section is to determine the variables that explain the optimal value of the bill rate for each period.

If the Fed behaves by solving an optimal control problem each period (with the bill rate as its control variable), then the optimal value of the bill rate will be a function of the initial state of the economy, the Fed's expectation of future values of the exogenous variables (including the policy variables of the fiscal authority), and the Fed's objective function and model.[7] This thus suggests for empirical work that one regress the bill rate on variables representing the initial state of the economy and on variables that one believes affect the Fed's expectation of future values. Any estimated equation of this kind is, of course, only an approximation to the actual behavior of the Fed, and some may object that it is not likely to be a very good approximation. In principle, however, this procedure is no different from that followed in macroeconometric models in the estimation of behavioral equations for, say, the household and firm sectors. On a priori

[6] It is interesting to note that the two demand-for-money equations in the model (explaining *DDH* and *DDF*, demand deposits and currency of the household and firm sectors, respectively) do not appear to break down in the 1974–75 period. This is in contrast, for example, to the demand-for-money equation in the *MPS* model as reported in Enzler, Johnson, and Paulus [5]. The *MPS* equation, when estimated through 1972IV and simulated dynamically for the 1973I–1976I period, shows a percentage error (predicted minus actual) for 1976I of 14.6 per cent [5, p. 264]. When the *DDH* and *DDF* equations in the present model were estimated through 1972IV and simulated dynamically for the 1973I–1976I period, the combined percentage error for 1976I was only 2.6 per cent. When various modified versions of the *MPS* equation were estimated through 1974II and simulated dynamically for the 1974III–1976I period, the percentage error for 1976I ranged from 5.8 to 10.6 per cent [5, p. 276]. This compares to a combined percentage error of 1.9 per cent for the *DDH* and *DDF* equations in the present model for the same experiment.

[7] In the simple linear-quadratic optimal control problem in Chow [2, Chapter 7], for example, $x_t^* = G_t y_{t-1} - g_t$, where x_t^* is a vector of the optimal values of the control variables, y_{t-1} represents the initial state of the economy, G_t is a matrix of coefficients that depend on the model and the objective function, and g_t is a vector of values that depend, among other things, on the expected future values of the exogenous variables.

grounds, it is not obvious that the application of this procedure to the Fed is any worse than its application to other sectors of the economy. At any rate, the procedure followed here was to regress the bill rate on variables that seemed likely to affect the Fed's solution of its control problem. As is usual in econometric work of this kind, current and lagged values of the variables were meant in part to be serving as "proxies" for expected future values.[8]

After some experimentation, the estimated equation that was chosen as the explanation of Fed behavior is the following:[9]

$$
\begin{aligned}
(1)\qquad RBILL_t = & -\underset{(3.8)}{11.1} + \underset{(0.052)}{0.841}\, RBILL_{t-1} + \underset{(0.0293)}{0.0497}\, \%PD_{t-1} + \underset{(0.0118)}{0.0352}\, J_t^* \\
& + \underset{(0.0264)}{0.0427}\, \%GNPR_t + \underset{(0.0138)}{0.0188}\, \%GNPR_{t-1} + \underset{(0.0120)}{0.0251}\, \%M_{1t-1}; \\
& \hat{\rho} = \underset{(0.103)}{0.229}, \quad SE = 0.474, \quad R^2 = 0.939, \quad DW = 1.82.
\end{aligned}
$$

The sample period is 1954I–1976II. *RBILL* is the three-month treasury bill rate, percentage points; $\%PD$ is the percentage change at an annual rate in the price deflator for domestic sales, percentage points; J^* is a measure of labor market tightness in the model in [**9**]; $\%GNPR$ is the percentage change at an annual rate in real *GNP*, percentage points; $\%M_1$ is the percentage change at an annual rate in the money supply, percentage points; $\hat{\rho}$ is the estimate of the first-order serial correlation coefficient. Standard errors are in parentheses.

Equation (1) states that the current bill rate is a positive function of the lagged rate of inflation, of the current degree of labor market tightness, of the current and lagged rates of growth of real *GNP*, and of the lagged rate of growth of the money supply. The behavior reflected in this equation is thus behavior in which the Fed "leans against the wind." The wind in this case is composed of the inflation rate, the degree of labor market tightness, the growth rate of real *GNP*, and the growth rate of the money supply. As these variables rise, so also does the bill rate.

The fit of equation (1) is fairly good, with an estimated standard error of 0.474 percentage points, and the estimate of the serial correlation coefficient of 0.229 is not very large. Also, the coefficient estimates of the equation are not highly sensitive to the use of alternative sample periods. A Chow test, for example, accepted the hypothesis that the coefficients are the same for the periods before and after 1969.[10] In other words, the test accepted the hypothesis that there was no structural change in Fed behavior with the advent of Arthur Burns. On statistical grounds, therefore, the equation does not seem too bad as an explana-

[8] In the estimation of the behavioral equations for the household and firm sectors in [**9**], lagged values were freely used in an attempt to account for expectational effects. As is the case for the Fed behavioral equation in this paper, the behavioral equations for the household and firm sectors in [**9**] were assumed to be derived from the solutions of optimal control problems.

[9] Equation (1) was estimated under the assumption of first-order serial correlation of the error term by the two-stage least squares technique described in Fair [**6**]. The two endogenous explanatory variables in the equation are J_t^* and $\%GNPR_t$.

[10] The F value was 1.06, which compares to the critical F value at the 95 per cent confidence level of 2.08. The Chow test is only approximate in this case because of the endogenous explanatory variables in the equation and the existence of serial correlation of the error term.

tion of Fed behavior, although, as discussed above, the approximate nature of any equation of this kind should be kept in mind.[11]

Equation (1) can be easily added to the econometric model in [**9**]. The price deflator for domestic sales, *PD*, is one of the deflators in the model, and J^* is the primary measure of labor market tightness used in the model. J^* is a detrended ratio of total hours paid for in the economy to the total population 16 and over.[12] The money supply variable, M_1, is equal to $DDB + CURR$ + a few small exogenous terms. *DDB* and *CURR* are defined in Section 2. Since *CURR* is always taken to be exogenous in the model,[13] it does not matter for purposes of the results in the next two sections whether *DDB* or M_1 is taken as the measure of the money supply. In any experiment, the endogenous change in M_1 will be the same as the endogenous change in *DDB*. In the discussion of the results in the next two sections, *DDB* will be referred to as the money supply. Real *GNP*, *GNPR*, does not appear explicitly in the model in [**9**], but a definitional equation explaining it can be easily added to the model. Real *GNP* is the sum of the real outputs of the firm, financial, and government sectors in the model.

4. SIMULATION RESULTS

The results of five experiments are reported in this section. Each experiment corresponds to a sustained increase in the real value of goods purchased by the government (denoted as *XG* in the model) of 1.25 billion dollars beginning in 1971I, a quarter that is at or near the bottom of a contraction. All flow variables in the model are at quarterly rates, so that the 1.25 billion dollar increase is an increase of 5.0 billion dollars at an annual rate. The experiments are based on different assumptions about the behavior of the Fed.

The experiments were performed as follows. The residuals obtained in the process of estimating each equation of the model were first added to the equations. This means that when the model is simulated using the actual values of all exogenous variables, the predicted values of all endogenous variables are equal to their actual values. In other words, a perfect tracking solution is obtained. These residuals were then used for all of the experiments. All simulations were dynamic, 12-quarter simulations for the period 1971I–1973IV. For all experiments, the reserve requirement ratio, g_1, and the discount rate, *RD*,

[11] It may be, of course, that equation (1) is completely spurious and does not at all reflect the behavior of the Fed. The equation may instead be picking up some of the effects of the bill rate on the right-hand-side variables. It is true that the bill rate affects J_t^* and $\%GNPR_t$ in the model in [**9**], but this simultaneity problem has in theory been accounted for in this study by the use of the two-stage least squares technique mentioned in footnote 9 to estimate equation (1). Whether in practice one wants to interpret the equation as reflecting the behavior of the Fed is perhaps open to question, but for purposes of the work in this paper it will be so interpreted.

[12] Both in the model in [**9**] and in equation (1), the use of J^* as the measure of labor market tightness gave somewhat better results than did the use of one minus the unemployment rate. In all cases, however, the results using the two measures were fairly close.

[13] It should be noted that the treatment of *CURR* as exogenous does *not* mean that the money supply is treated as exogenous. The holdings of demand deposits *and* currency of the household and firm sectors (*DDH* and *DDF*) are explained by stochastic equations in the model. What is not explained is the division of these holdings between demand deposits and currency.

were taken to be exogenous. For the first experiment, the amount of government securities outstanding, VBG, was also taken to be exogenous. This means, from equation (69), that any dissaving of the government that results from the increase in XG is financed by an increase in nonborrowed reserves, $BR-BORR$. In other words, any deficit is financed by an increase in high-powered money: the Fed buys the securities that the Treasury issues to finance the deficit.

For the other four experiments, VBG was taken to be endogenous. This requires, in order to close the model, either that one variable that was endogenous in the first experiment be taken to be exogenous or that an extra equation be added in which no new endogenous variable is introduced. For the second experiment the money supply (DDB) was taken to be exogenous; for the third experiment the level of nonborrowed reserves ($BR-BORR$) was taken to be exogenous; and for the fourth experiment the bill rate ($RBILL$) was taken to be exogenous. "Exogenous" here means that in the simulation runs the values of these variables for each period were kept unchanged from their actual (historic) values. For the fifth experiment no extra variable was taken to be exogenous, but instead the equation explaining Fed behavior from Section 3 was added to the model. This equation introduces no new endogenous variable, and so it meets the requirement for the model to be closed.

To summarize, the behavior of the Fed in response to the increase in XG in the five experiments is as follows: (1) The Fed allows any government deficit resulting from the increase in XG to be financed by an increase in nonborrowed reserves; (2) the Fed allows no change in the money supply to take place; (3) the Fed allows no change in nonborrowed reserves to take place; (4) the Fed allows no change in the bill rate to take place; (5) the Fed behaves as estimated in Section 3.

The results for the five experiments are presented in Table I. The effects on seven endogenous variables in the model are presented. Each number in the table is the difference between the predicted value of the endogenous variable for the quarter and the actual value. Y is the key output variable in the model, and PF is the key price variable.

The results in Table I clearly show that fiscal policy effects are sensitive to assumptions about the behavior of the Fed. The most expansionary experiment with respect to real output changes is the first, where Y is 2.39 billion dollars higher in quarter $t+11$ than it was historically, and the least expansionary is the second, where Y is actually 0.09 billion dollars lower in quarter $t+11$ than it was historically. For purposes of explaining the different effects in the table, it will be useful to concentrate on the results for the bill rate for the first quarter. In the first experiment, the bill rate fell in the first quarter as a result of the increase in XG. This is because the dissaving of the government ($-SAVG_t$) of 0.62 was financed by an increase in nonborrowed reserves. The government action in this case effectively increases the amount of funds in the system, which causes the bill rate to fall. Experiment 1 is thus doubly expansionary in the sense that sales are higher because of the increased purchase of goods by the government and the bill rate is lower because of the increase in funds in the system.

TABLE I

RESULTS FOR FIVE EXPERIMENTS—EFFECTS OF A PERMANENT INCREASE IN *XG* OF 1.25 BILLION DOLLARS BEGINNING IN QUARTER *t*.[a]

Variables[b]	Quarters											
	t	$t+1$	$t+2$	$t+3$	$t+4$	$t+5$	$t+6$	$t+7$	$t+8$	$t+9$	$t+10$	$t+11$
RBILL (bill rate)												
1	−0.86	−0.35	0.41	0.83	0.79	0.34	−0.18	−0.50	−0.54	−0.53	−0.28	−0.17
2	0.13	0.27	0.42	0.41	0.34	0.37	0.40	0.45	0.47	0.53	0.59	0.58
3	0.03	0.08	0.14	0.19	0.21	0.25	0.27	0.28	0.31	0.33	0.34	0.37
4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	0.10	0.20	0.26	0.30	0.31	0.31	0.30	0.28	0.27	0.25	0.24	0.24
Y (real output)												
1	1.60	2.92	3.43	3.27	2.62	2.10	1.98	2.14	2.36	2.44	2.45	2.39
2	1.23	1.73	1.92	1.89	1.67	1.42	1.13	0.83	0.52	0.26	0.06	−0.09
3	1.27	1.88	2.25	2.41	2.32	2.14	1.92	1.66	1.40	1.11	0.89	0.74
4	1.28	1.93	2.38	2.66	2.73	2.75	2.72	2.62	2.47	2.24	2.04	1.91
5	1.24	1.77	2.04	2.12	1.95	1.72	1.48	1.25	1.03	0.84	0.71	0.66
100 · *PF* (price deflator)												
1	−0.091	−0.062	0.005	0.097	0.197	0.243	0.254	0.261	0.286	0.326	0.391	0.448
2	0.022	0.060	0.104	0.157	0.215	0.269	0.318	0.367	0.408	0.448	0.481	0.516
3	0.012	0.037	0.067	0.109	0.164	0.217	0.266	0.315	0.363	0.413	0.457	0.502
4	0.009	0.028	0.049	0.079	0.119	0.158	0.198	0.241	0.289	0.349	0.409	0.464
5	0.019	0.052	0.087	0.135	0.195	0.246	0.291	0.334	0.373	0.412	0.446	0.482
SAVG (financial saving of the government)[c]												
1	−0.62	0.18	0.41	0.30	0.06	−0.27	−0.37	−0.28	−0.13	−0.06	0.03	0.01
2	−0.61	−0.38	−0.30	−0.35	−0.42	−0.54	−0.67	−0.81	−0.97	−1.11	−1.24	−1.37
3	−0.61	−0.33	−0.17	−0.13	−0.12	−0.21	−0.32	−0.44	−0.56	−0.69	−0.80	−0.90
4	−0.61	−0.32	−0.12	−0.03	0.05	0.06	0.02	−0.02	−0.07	−0.14	−0.19	−0.25
5	−0.61	−0.37	−0.26	−0.25	−0.28	−0.41	−0.52	−0.63	−0.74	−0.84	−0.91	−0.96
DDB (money supply)												
1	0.95	1.42	1.36	1.04	0.67	0.73	1.23	2.03	2.65	3.22	3.49	3.87
2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	0.08	0.23	0.41	0.57	0.68	0.77	0.85	1.00	1.08	1.18	1.25	1.36
4	0.11	0.32	0.58	0.89	1.20	1.50	1.78	2.16	2.40	2.66	2.84	3.15
5	0.02	0.08	0.17	0.25	0.28	0.32	0.39	0.53	0.64	0.78	0.91	1.08
BR − *BORR* (nonborrowed reserves)												
1	0.62	0.44	0.03	−0.28	−0.34	−0.07	0.31	0.59	0.72	0.79	0.75	0.74
2	−0.07	−0.15	−0.22	−0.23	−0.20	−0.21	−0.21	−0.25	−0.26	−0.28	−0.36	−0.35
3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4	0.02	0.06	0.10	0.17	0.22	0.28	0.30	0.33	0.38	0.41	0.48	0.52
5	−0.05	−0.10	−0.10	−0.12	−0.13	−0.11	−0.09	−0.07	−0.04	−0.01	0.00	0.03
VBG (amount of government securities outstanding)												
1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.67	1.13	1.51	1.87	2.26	2.81	3.49	4.33	5.32	6.45	7.76	9.12
3	0.61	0.94	1.11	1.24	1.36	1.57	1.89	2.33	2.89	3.58	4.38	5.27
4	0.59	0.87	0.95	0.92	0.81	0.70	0.65	0.64	0.66	0.78	0.91	1.12
5	0.66	1.07	1.34	1.61	1.90	2.29	2.79	3.41	4.12	4.93	5.82	6.75

[a] t = 1971I (bottom of a contraction). Assumptions about Fed behavior: 1—*VBG* exogenous, 2—*DDB* exogenous, 3—*BR* − *BORR* exogenous, 4—*RBILL* exogenous, 5—Fed behaves according to equation (1) in Section 3.

[b] Units of variables are: percentage points for *RBILL*; billions of 1972 dollars at a quarterly rate for *XG* and *Y*; 1972 = 1.0 for *PF*; billions of current dollars at a quarterly rate for *SAVG*; billions of current dollars for *DDB*, *BR* − *BORR*, and *VBG*.

[c] From equation (69), the effect on *SAVG* equals, except for rounding, minus the effect on *BR* − *BORR* minus the effect on *VBG*. (*CURR*, *GFXG*, and *DISG* in equation (69) are exogenous.)

Consider now the fourth experiment, where the Fed prevents the bill rate from changing. In the first quarter the Fed prevents the bill rate from falling (as the bill rate did in experiment 1) by selling government securities. *VBG* increased by 0.59 in this case, which met all but 0.02 of the dissaving of the government of 0.61. (The 0.02 was met by an increase in nonborrowed reserves of this amount.) Experiment 4 is thus less expansionary than is experiment 1 because the bill rate is not allowed to fall: fewer funds are released to the system.

Even fewer funds are released to the system in experiment 2, where the money supply is prevented from changing. In this case the bill rate must rise to choke off any increase in the demand for money caused by an increase in income, the rise being 0.13 percentage points for the first quarter. This required an increase in *VBG* of 0.67. Over time, the higher bill rates in experiment 2 had a gradual contractionary effect on the economy until, as mentioned above, by quarter $t+11$ real output was actually lower.

Experiment 3, where nonborrowed reserves are prevented from changing, is more expansionary than is experiment 2. Keeping nonborrowed reserves unchanged allows the money supply to increase through an increase in bank borrowing. The money supply is linked directly to *BR*, and *BR* can increase to the extent that *BORR* does while still keeping unborrowed reserves unchanged. There are thus somewhat more funds allowed in the system in experiment 3 than in experiment 2. The increase in *VBG* in the first quarter was 0.61 in experiment 3, compared to 0.67 in experiment 2.

In experiment 5, where the behavior of the Fed is endogenous, the Fed responds to the increase in economic activity by increasing the bill rate. This experiment is thus less expansionary than is experiment 4, where the bill rate remained unchanged. It is, however, more expansionary than is experiment 2: the money supply does increase some in experiment 5. The results for this experiment are closest to the results for experiment 3, where the level of nonborowed reserves remained unchanged. Comparing experiments 4 and 5, it can be seen that the effects on real output are about the same for the first three or four quarters, and then they start to diverge as the contractionary effects of the higher past bill rates in experiment 5 begin to be felt.

In summary, then, the results in Table I show that fiscal policy effects are quite sensitive to assumptions about the behavior of the Fed.[14] The most expansionary case is where the government deficit is financed by an increase in nonborrowed reserves, and the least expansionary case is where the Fed keeps the money supply unchanged. This latter case is in fact expansionary only for about the first eight quarters. The second most expansionary case is where the Fed keeps the bill rate unchanged. The case in which the Fed behaves as estimated in Section 3 is considerably less expansionary after the first few quarters than is the case in

[14] If this conclusion is valid, it implies that the measures of fiscal policy reported in Blinder and Goldfeld [1] should be interpreted with caution. The measures may be accurate for the particular assumption about Fed behavior that Blinder and Goldfeld used, namely that nonborrowed reserves plus currency is exogenous, but they are not likely to be accurate for other assumptions about Fed behavior. In other words, Blinder and Goldfeld are likely to have gotten quite different measures of fiscal policy had they assumed something different about Fed behavior.

which the bill rate is kept unchanged. The Fed as estimated in Section 3 responds to the increase in XG by increasing the bill rate. By quarter $t+11$, the increase in Y is only 0.66 billion dollars in this case compared to 1.91 billion dollars in the constant bill rate case. The case in which nonborrowed reserves are kept unchanged is similar to the case in which the Fed behaves as estimated in Section 3.

Two further points about the results in Table I should be made. First, the bill rate has in the model a positive effect on the bond rate, and the bond rate in turn has a positive effect on the price level.[15] This explains the fall in PF in the first two quarters in experiment 1 in Table I. In this case the negative effects of the decrease in the bill rate were large enough to offset any positive effects of an increase in aggregate demand on the price level. This is a good illustration of the fact, as discussed in [**9**], that there is no stable relationship between aggregate demand and inflation in the model. Second, the fact that $SAVG$ did not fall by the full current dollar[16] amount of the increase in XG is explained by endogenous government expenditures and tax receipts. When the economy expands, tax receipts increase and some transfer payments decrease. Government interest payments also change as interest rates change and as VBG changes. The net result of all these effects is that $SAVG$ fell in the first quarter by only about half of the increase in the value of goods purchased by the government.

5. OPTIMAL CONTROL RESULTS

For the present model it is obvious from the results in the previous section that the performance of the fiscal authority with respect to maximizing some objective function will depend significantly on the behavior of the Fed. In order to gauge the quantitative effects of this dependency, the results of solving three control problems for the fiscal authority are presented in this section. The problems correspond to three different assumptions about the behavior of the Fed. The control period is 1969I–1976II, for a total of 30 quarters.

The objective function that was used for the fiscal authority targets a given level of real output and a zero rate of inflation for each quarter. It is easiest to consider the objective function to be a loss function that is to be minimized. This loss function is:

$$(2) \qquad L=\sum_{t=1969\text{I}}^{1976\text{II}}\left[\left|\frac{Y_t-Y_t^*}{Y_t^*}\right|^2+(\%PF_t)^2\right],$$

[15] In the theoretical model [**8**] that was used to guide the specification of the econometric model [**9**], a higher interest rate causes a firm to contract, and the primary way that a firm contracts in this model is to raise its price, which lowers expected sales, and then to lower production, investment, and employment. The econometric work appeared to confirm this property of a higher interest rate leading to a higher price in that the bond rate turned out to be a significant variable in the price equation of the firm sector.

[16] For the simulation period the current dollar increases in XG were close to the constant dollar increase of 1.25 billion dollars. Constant dollars are 1972 dollars in the newly revised national income accounts data.

where Y_t^* is the target level of Y_t, $\%PF_t$ is the percentage change in PF_t at an annual rate, and

$$\left|\frac{Y_t - Y_t^*}{Y_t^*}\right|^2 = \begin{cases} \left(\dfrac{Y_t - Y_t^*}{Y_t^*}\right)^2 & \text{if } Y_t < Y_t^*, \\ 0 & \text{if } Y_t \geq Y_t^*. \end{cases}$$

The loss function penalizes rates of inflation that are both above and below the target value of zero, but it only penalizes values of Y that are below the target. The construction of the target values for real output is explained in [**9**, Chapter 10]. The values are meant to correspond to high levels of economic activity.[17]

The three assumptions about the behavior of the Fed are: (i) the Fed behaves as estimated in Section 3, (ii) the Fed allows no change in the bill rate, and (iii) the Fed allows no change in the money supply. By "no change" here is meant that the value of the variable for each quarter was not changed from its historic value when solving the control problem. For all three problems *VBG* is endogenous. *XG* was used as the control variable of the fiscal authority. The error terms in the model were set equal to their estimated values, and the resulting deterministic control problem was solved as described in [**7**].[18] A summary of the results for the first 24 quarters of the 30-quarter period is shown in Table II. Only the results for the first 24 quarters are summarized here because the optimal values for the last few quarters may be heavily influenced by the fact that there is no tomorrow after 30 quarters, especially since in the model inflation responds with a longer lag to current stimulative measures than does output.

TABLE II

	Actual	Optimal when Fed behaves endogenously	Optimal when bill rate is kept unchanged	Optimal when money supply is kept unchanged
Value of loss function	0.2279	0.1785	0.1685	0.1901
Average value of the bill rate over the first 24 quarters	6.1	7.5	6.1	5.2
Sum of Y over the first 24 quarters[a]	5946.3	6085.3	6092.8	5910.1
Average rate of inflation over the first 24 quarters (annual rate)	6.0	6.6	6.3	5.7
Sum of ΔXG over the first 24 quarters[b]	0.0	169.9	86.3	−58.5

[a] Sum of Y^* over the first 24 quarters = 6135.0.
[b] ΔXG = difference between the optimal and actual values of XG.

[17] The values for Y^* used in this study differ from those presented in [**9**, Table 10-3] because of the use here of the revised national income accounts data.

[18] See also [**9**, Chapter 10, pp. 198–203], for a discussion of the solution of optimal control problems using the present model.

The fiscal authority does best in the case in which the Fed keeps the bill rate unchanged. The optimum in this case corresponds to more output and more inflation than existed historially. Inflation is, however, only slightly higher than existed historically, whereas output is much higher. The output target is in fact close to being met. This example reflects an important characteristic of the model: the extra inflation cost of increasing output is generally less than the lost output cost of lowering inflation for loss functions like (2).[19]

The fiscal authority does not do as well when the Fed behaves as estimated in Section 3. The Fed responds to stimulative measures by increasing the bill rate, which, other things being equal, has a positive effect on the rate of inflation. The optimum in this case corresponds to slightly less output and somewhat more inflation than existed in the unchanged bill rate case. It is also important to note that the increase in XG is substantially larger in this case than it is in the unchanged bill rate case. The higher bill rates that the Fed sets when it behaves endogenously cause a contraction in private demand, which, since output is nearly the same in the two cases, the fiscal authority nearly completely offsets by increasing XG. The optimum in the endogenous Fed case thus corresponds to a substantially larger government sector than does the optimum in the unchanged bill rate case. This difference would, of course, not exist if, say, the personal income tax were used as the control variable of the fiscal authority instead of XG.

The fiscal authority does worst in the unchanged money supply case. Output is not only lower in this case than it was in the other two cases, but it is also slightly lower than existed historically. Inflation is, however, also lower. The optimal strategy in this case was for the fiscal authority to lower on average XG, which, with an unchanged money supply, causes the bill rate to fall. The lower bill rate offsets to some extent the lower output caused by the fall in XG, and it also leads to somewhat less inflation. The main point of this example is not, however, that the optimal level of output is slightly lower than existed historically; with less weight on inflation in the loss function this result is likely to be reversed. The main point is that when the Fed keeps the money supply unchanged, the fiscal authority has little room to maneuver. It can increase output by increasing XG, but only at the expense of a substantially higher bill rate and thus higher inflation. It can lower the bill rate and thus inflation by decreasing XG, but the net result of this policy is also to lower output. The optimal policy may go either way, but except for small changes, the fiscal authority can do little about changing the output path once the money supply path is fixed.

The results in this section are thus as expected, given the results in Section 4. The optimal performance of the fiscal authority depends significantly on the behavior of the Fed. When the Fed behaves endogenously, the fiscal authority does not do as well as when the Fed behaves by keeping the bill rate unchanged. It does not do as well in terms of lowering the value of loss, and the optimal policy also calls for about twice as much fiscal stimulus to offset the increases in the bill rate by the Fed.

[19] See Fair [**10**] for further discussion of this point.

As a final point, it should be noted that, given the model and the loss function in (2), it would not be reasonable to solve an optimal control problem in which the fiscal authority and the Fed cooperate, i.e., in which, for example, both XG and VBG were used as control variables. With only the two arguments in the loss function, the optimal values of XG and the bill rate would probably be close to zero. Since lower bill rates decrease inflation, the optimum is likely to correspond to a very low bill rate and a value of XG low enough to offset any "undesired" increase in output caused by the low bill rate's positive effect on private demand. One would need other arguments in the loss function, such as a target size of the government sector, before it would be reasonable to use both XG and VBG as control variables.

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