An Analysis of the Accuracy of Four Macroeconometric Models

Ray C. Fair

Yale University

The primary purpose of this paper is to compare the predictive accuracy of four models: (1) Sargent's classical macroeconometric model, (2) Sims's six-equation unconstrained vector autoregression model, (3) a "naive" eighth-order autoregressive model, and (4) my model. A recent method that I have proposed for estimating the predictive accuracy of a model, which takes account of the four main sources of uncertainty of a forecast, is used for the comparisons. The results indicate that Sargent's and Sims's models are the same as or less accurate than the naive model, depending on the variable, and that my model is more accurate for real GNP, the GNP deflator, and the unemployment rate and less accurate for the money supply and the wage rate than the naive model. A secondary purpose of the paper is to point out some econometric mistakes that Sargent made in his empirical work and to propose an alternative technique that can be used to estimate a rational expectations model like his.

I. Introduction

There is currently in macroeconomics a considerable difference of opinion as to what the true structure of the economy is like. One way in which this difference manifests itself is in the wide variety of macroeconometric models that are in existence. One might have thought at the beginning of large-scale model construction in the early 1950s that by the late 1970s the debate would be over fairly minor specification issues. This is, of course, not the case, as any casual glance at a number of models will reveal. There is also little sign that the range of differences is narrowing.

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One reason for this lack of agreement is the difficulty of testing and comparing alternative models. Models differ, among other things, in the number and types of variables that are taken to be exogenous, and it is difficult to compare models with different degrees of exogeneity. In a recent study (Fair 1978*a*), however, I have proposed a method for estimating the uncertainty of a forecast from an econometric model that does allow comparisons across models to be made, and the primary purpose of this paper is to use this method to compare the accuracy of four models. The four models are (1) Sargent's (1976) classical macroeconometric model; (2) Sims's (1977) six-equation unconstrained vector autoregression model; (3) a "naive" model in which each variable is regressed on a constant, a time trend, and its first eight lagged values; and (4) my model (Fair 1974, 1976, 1978*b*).

A secondary purpose of this paper is to discuss the estimation of Sargent's model. Sargent made a few econometric mistakes in his empirical work, and these mistakes must be corrected before his model can be compared with the others. A rational expectations model like Sargent's poses some interesting econometric issues, and one way in which this kind of model can be estimated is proposed and used in this paper.

It should be stressed at the outset that the results of the present comparison are highly tentative and must be interpreted with considerable caution. First of all, Sargent's model itself is quite tentative, and an advocate of this model is likely to want to work more on its specification before rendering a final judgment about its accuracy. A second and related point is that an advocate of Sargent's or Sims's model may want to use at least slightly different data to estimate and analyze the model from what have been used in the present study. Also, the procedure that has been used to estimate Sargent's model is not the only procedure that one might try, and an advocate of this model may want to experiment with other estimation procedures. Finally, the method that is used to compare the models rests on one fairly strong assumption, and it is an open question whether the present results are sensitive to this assumption. Even given these caveats, however, the results in this paper should convey some information about the relative accuracy of the models, and at the least it is hoped that this study will provide an example of how models can be tested and compared in the future.

Sargent's model is discussed in Section II, and the other three models are discussed in Section III. The method and its application to the four models are described in Section IV, and the results are presented and discussed in Section V. Section VI contains a summary of the main conclusions of this study.

II. Sargent's Model

The following discussion relies heavily on the material in tables 1–3. The model as Sargent estimated it is presented in table 1. The main econometric mistake that Sargent made was to include variables in the regression to obtain $E_{t-1}P_t$ and in the first-stage regressions of the two-stage least squares (2SLS) technique that are not in the model. A related problem is that equation (5*c*) is not identified unless one assumes that the error terms in equations (4) and (5*c*) are uncorrelated. If this assumption is made, then R_t can be treated as predetermined in the estimation of equation (5*c*). Sargent did not treat R_t as predetermined in this case, and although he should not have been able to estimate the equation by 2SLS, he did not encounter any difficulties because he used more variables in the first stage regression for R_t than he should have.

One way of dealing with the above problems would be to expand the model to include more variables. For those who are interested in this kind of model, this would be interesting future work. The aim of this paper, however, is to examine the accuracy of the current version, and so I have chosen not to expand the model. I have instead concentrated on obtaining estimates under the assumption that the model as presented in table 1 is correctly specified.

The procedure that is proposed in this paper for estimating Sargent's model is presented in table 2. It is briefly as follows. First, the

TABLE 1

SARGENT'S MODEL AS ORIGINALLY ESTIMATED

Equation Number	LHS Variable	RHS Variables
(1)	Un_t	$1, t, p_t - E_{t-1}p_t, Un_{t-i}$ $(i = 1, \ldots, 4)$
(2)	nf_t	$1, t, p_t - E_{t-1}p_t, Un_t, nf_{t-i}$ $(i = 1, \ldots, 4)$
(3)	y_t	$1, t, n_t, n_{t-i}$ $(i = 1, \ldots, 4)$; filter: $(1 - .6L)^2$
(4)	R_t	$1, t, R_{t-i}$ $(i = 1, \ldots, 4)$
(5c)	$m_t - p_t$	$1, t, R_t, R_{t-i}$ $(i = 1, \ldots, 7), y_t, y_{t-i}$ $(i = 1, \ldots, 7)$; filter: $(1 - .8L)^2$
(6)	n_t	$nf_t - Un_t + pop_t$

NOTE:

i) $E_{t-1}p_t$ was obtained from a regression of p_t on $1, t$, three seasonal dummies, p_{t-i} $(i = 1, \ldots, 4)$, w_{t-i} $(i = 1, \ldots, 4)$, nf_{t-i} $(i = 1, \ldots, 4)$, and Un_{t-i} $(i = 1, \ldots, 4)$.

ii) The equations were estimated by the two-stage least squares (2SLS) technique. The explanatory variables used in the first-stage regressions were those variables listed in i plus pop_t, m_t, g_t, $surp_t$, and the log of current government employment. The RHS endogenous variables in the structural equations are p_t in eq. (1), p_t and Un_t in eq. (2), n_t in eq. (3), and R_t and y_t in eq. (5c).

iii) The filter $(1 - .6L)^2$ means that each variable z_t in the equation was transformed into $z_t^* = z_t - 1.2z_{t-1} + 0.36z_{t-2}$ before estimation. For the filter $(1 - .8L)^2$, the transformation is $z_t^* = z_t - 1.6z_{t-1} + 0.64z_{t-2}$.

Variables: Un_t = unemployment rate, nf_t = log of labor force participation rate, y_t = log of real GNP, R_t = long-term interest rate (Moody's Baa rate), m_t = log of the money supply, p_t = log of the GNP deflator, pop_t = log of population, n_t = log of employment (approximately), w_t = log of an index of a straight-time manufacturing wage, g_t = log of government purchases of goods and services in real terms, and $surp_t$ = government surplus in real terms.

TABLE 2

SARGENT'S MODEL AS ESTIMATED IN THIS PAPER

i) In place of using the filters, eqq. (3) and (5c) were estimated under the assumption of first- and second-order serial correlation of the error terms.
ii) The error terms in eqq. (4) and (5c) were assumed to be uncorrelated, and R_t was taken to be predetermined in the estimation of eq. (5c).
iii) There are two exogenous variables in the model, m_t and pop_t. Each of these was regressed on $1, t$, and its first eight lagged values, and predicted values, $\hat{m}_t$ and $\widehat{pop}_t$, from these two regressions were obtained.
iv) An iterative procedure was used to estimate the model. First, given an initial guess for $E_{t-1}p_t$, eqq. (1), (2), (3), and (5c) were estimated by 2SLS. The explanatory variables used in the first-stage regressions were all the predetermined variables in the model: $1, t, E_{t-1}p_t, Un_{t-i}$ $(i = 1, \ldots, 4)$, nf_{t-i} $(i = 1, \ldots, 4)$, n_{t-i} $(i = 1, \ldots, 6)$, R_t, R_{t-i} $(i = 1, \ldots, 9)$, y_{t-i} $(i = 1, \ldots, 9)$, $m_{t-1} - p_{t-1}$, $m_{t-2} - p_{t-2}$, pop_t, and m_t. Given these 2SLS estimates and the OLS estimates of eq. (4), the model was solved for $E_{t-1}p_t$, with $\hat{m}_t$ replacing m_t and $\widehat{pop}_t$ replacing pop_t. This new set of values for $E_{t-1}p_t$ was then used to obtain a new set of 2SLS estimates, which was then used to obtain a new set of values for $E_{t-1}p_t$, and so on. The iterative procedure was stopped when successive estimates of $E_{t-1}p_t$ (for each t) were within a prescribed tolerance level.
v) Data:

Current Name	Variable(s) in Fair (1978*b*)
Un_t	UR_t
nf_t	Log $[(TLF_{1t} + TLF_{2t} - JOBGM_t)/(POP_t - JOBGM_t)]$
y_t	Log $GNPR_t$
R_t	$RAAA_t$
m_t	Log $M1_t$
p_t	Log $GNPD_t$
pop_t	Log $(POP_t - JOBGM_t)$

variables that Sargent used in the first-stage regressions that are not in the model were excluded from consideration. Second, the error terms in equations (4) and (5*c*) were assumed to be uncorrelated and R_t was taken to be predetermined in the estimation of equation (5*c*). Third, in place of using the filters for equations (3) and (5*c*), the equations were estimated under the assumption of first- and second-order serial correlation of the error terms. Sargent's use of the filters is equivalent to constraining the first- and second-order serial correlation coefficients to be particular numbers, and so the approach taken in this study is less restrictive than Sargent's approach.

The most interesting and difficult question about the estimation of Sargent's model, and of rational expectations models in general, is how to treat a variable like $E_{t-1}P_t$. The first assumption that must be made in the present case is what people know at the beginning of period t about the values of the two exogenous variables, m_t and pop_t. For present purposes, each of these variables was regressed on a constant, a time trend, and its first eight lagged values; and the predicted values from these regressions, denoted $\hat{m}_t$ and $\widehat{pop}_t$, were assumed to be what people expect the values to be. Given these predicted values for each period, the model was then estimated by the iterative procedure outlined in point iv in table 2. This procedure constrains $E_{t-1}p_t$ to be the value of p_t predicted by the model when m_t and pop_t are replaced by $\hat{m}_t$ and $\widehat{pop}_t$. This is the value that would be computed at the beginning of period t by someone who knew the model and had as expectations for m_t and pop_t the predicted values $\hat{m}_t$ and $\widehat{pop}_t$. (Note that the predicted value of $p_t - E_{t-1}p_t$ in the Un_t and nf_t equations is always zero in this case.) Within the context of this iterative procedure, the 2SLS technique was used for all but equation (4), which requires only the ordinary least squares (OLS) technique.[1]

The data that were used to estimate Sargent's model were taken from the data base for my model. The matching of the variables in Sargent's model to those in mine is presented at the bottom of table 2. The data are explained in Fair (1976, 1978*b*). There are at least some minor differences between the data used in this study and the data that Sargent used; and, as mentioned in the Introduction, in future work with Sargent's model one may want to use slightly different data than were used here.

The basic sample period that was used to estimate the model was 1954III–1977IV, although for the application of the method below the model was estimated over 35 other sample periods as well. Coefficient estimates for the basic sample period and for two of the others are presented in table 3, along with Sargent's original estimates. One of the other two sample periods (1954III–1968IV) is the shortest of the periods, and the other (1954III–1973III) is the period that most closely matches the period that Sargent used (1951I–1973III).

It is an unfortunate characteristic of macroeconometric models that their coefficient estimates can change substantially as the sample pe-

[1] With respect to the assumption of first- and second-order serial correlation in eqq. (3) and (5*c*), see Fair (1973) for a discussion of the use of the 2SLS technique under this assumption. In the initial stages of the estimation work for Sargent's model, I tried substituting the LIVE ("limited information instrumental variables") technique of Brundy and Jorgenson (1971) (or, in the case of eqq. [3] and [5*c*], the LIVER ["limited information instrumental variables with autoregressive errors"] technique in Fair [1972]) for the 2SLS technique within the context of the above iterative procedure. Since the model was being used to solve for $E_{t-1}p_t$ (with $\hat{m}_t$ and $\widehat{pop}_t$ replacing m_t and pop_t), it seemed reasonable to use the model also to solve for the instrumental variables (with the actual values of m_t and pop_t used). This attempt failed, however. Some of the estimates after one iteration were completely unreasonable, and there was no evidence after a few iterations that the procedure was converging. A similar failure occurred when an attempt was made to estimate the model by the full-information maximum likelihood (FIML) technique using the method described in Chow and Fair (1973). Although the present attempts failed, it may be with more diligence that one can obtain LIVE and FIML estimates for Sargent's model within the context of the above iterative procedure. This is clearly a possible area for future work on the model.

TABLE 3

COEFFICIENT ESTIMATES FOR SARGENT'S MODEL

Sample Period	RHS Variables							SE
	1	t	$p_t - E_{t-1}p_t$	Un_{t-1}	Un_{t-2}	Un_{t-3}	Un_{t-4}	
LHS variable Un_t:								
Original (1951I–1973III)	.0043	.0000007	−.287	1.47	−.59	−.03	.04	.00371
	(2.5)	(.5)	(2.0)	(12.8)	(2.9)	(.1)	(.3)	
1954III–1968IV	.0045	−.0000302	−.014	1.63	−.80	−.12	.22	.00268
	(1.68)	(1.34)	(.14)	(12.96)	(3.36)	(.50)	(1.88)	
1954III–1973III	.0040	−.0000047	−.037	1.63	−.79	−.06	.14	.00260
	(2.06)	(.35)	(.56)	(14.39)	(3.67)	(.29)	(1.37)	
1954III–1977IV	.0012	−.0000076	1.52	1.79	−1.10	.10	.18	.00267
	(.87)	(.69)	(3.97)	(18.57)	(5.90)	(.58)	(1.91)	
	1	t	$p_t - E_{t-1}p_t$	Un_t	nf_{t-1}	nf_{t-2}	nf_{t-3}	nf_{t-4}	
LHS variable nf_t:									
Original (1951I–1973III)	−.038	.00004	.149	−.075	.94	−.11	−.02	.12	.0040
	(1.3)	(2.1)	(.9)	(1.9)	(8.2)	(.7)	(.2)	(1.0)	
1954III–1968IV	−.084	−.000037	−.086	−.100	.88	−.04	−.08	.06	.00351
	(2.33)	(1.21)	(.68)	(1.92)	(6.81)	(.26)	(.46)	(.50)	
1954III–1973III	−.012	.000014	−.189	−.086	.88	.02	−.04	.11	.00336
	(.44)	(.68)	(2.14)	(2.03)	(7.45)	(.15)	(.29)	(.94)	
1954III–1977IV	.027	.000016	−.087	−.076	.92	.05	.02	.06	.00334
	(1.00)	(.76)	(1.65)	(2.08)	(8.10)	(.36)	(.13)	(.54)	

	1	t	n_t	n_{t-1}	n_{t-2}	n_{t-3}	n_{t-4}	ρ_1	ρ_2	
LHS variable y_t:										
Original (1951I–1973III)	.35	.0009	1.09	.24	−.24	−.14	−.02	1.20	−.36	.00964
	(1.8)	(4.5)	(3.5)	(1.0)	(1.0)	(.5)	(.1)			
1954III–1968IV	4.23	.0103	1.29	−.25	−.57	.03	−.33	.94	.0	.00653
	(1.22)	(3.99)	(4.88)	(1.11)	(2.62)	(.15)	(1.63)	(6.10)	(.03)	
1954III–1973III	5.85	.0092	.98	−.19	−.41	−.09	−.25	1.07	−.14	.00724
	(1.39)	(4.91)	(3.33)	(.85)	(1.83)	(.40)	(1.25)	(8.23)	(1.18)	
1954III–1977IV	4.95	.0076	1.14	−.23	−.44	−.13	−.22	1.14	−.17	.00735
	(1.40)	(4.51)	(4.68)	(1.19)	(2.36)	(.72)	(1.24)	(9.80)	(1.51)	

	1	t	R_{t-1}	R_{t-2}	R_{t-3}	R_{t-4}	
LHS variable R_t:							
Original (1951I–1973III)	.15	.0034	1.52	−.77	.44	−.24	.158
	(1.8)	(2.0)	(13.1)	(3.7)	(2.4)	(2.1)	
1954III–1968IV	.07	.0035	1.77	−.70	.30	−.03	.127
	(1.03)	(1.32)	(10.09)	(2.96)	(1.24)	(.23)	
1954III–1973III	.17	.0047	1.44	−.80	.49	−.20	.159
	(2.09)	(2.03)	(12.74)	(4.19)	(2.57)	(1.76)	
1954III–1977IV	.19	.0055	1.47	−.85	.56	−.26	.165
	(2.76)	(2.43)	(14.76)	(4.93)	(3.24)	(2.65)	

	1	t	R_t	R_{t-1}	R_{t-2} ...	R_{t-7}	y_t	y_{t-1}	y_{t-2} ...	y_{t-7}	ρ_1	ρ_2	
LHS variable $m_t - p_t$:													
Original (1951I–1973III)	−.22	.0003	−.0060	−.0059	−.0091 ...		.45	.16	.19 ...		1.60	−.64	.00561
	(2.6)	(2.1)	(.5)	(1.2)	(1.7)		(3.3)	(1.9)	(2.3)				
1954III–1968IV	5.15	.0147	−.0101	−.0162	−.0072 ...		.12	.04	.06 ...		1.02	−.04	.00541
	(2.19)	(.89)	(1.44)	(2.26)	(.87)		(.87)	(.37)	(.49)		(7.35)	(.31)	
1954III–1973III	2.85	.0026	−.0097	−.0078	−.0025 ...		.48	−.06	.03 ...		.80	.51	.00671
	(1.73)	(.72)	(1.91)	(1.28)	(.38)		(3.14)	(.50)	(.26)		(6.60)	(1.24)	
1954III–1977IV	−4.33	−.0127	−.0075	−.0045	.0090 ...		1.39	−.53	.26 ...		.58	.15	.01075
	(3.99)	(6.96)	(1.04)	(.48)	(.85)		(5.10)	(1.74)	(1.18)		(3.66)	(.98)	

riod changes, and this is clearly true of a number of the coefficient estimates in Sargent's model. Two of the key coefficients in the model are the coefficients for the $p_t - E_{t-1}p_t$ term in the Un_t and nf_t equations, and the estimates of these two coefficients are not particularly stable. For the Un_t equation, the estimate for the basic sample period is of the wrong expected sign, and the estimates for the other two periods are not significant. For the nf_t equation, the estimates for all three periods are of the wrong expected sign, although in this case not even Sargent's original estimate was significant. The coefficient estimates for the Un_t and nf_t equations are affected by the entire model, since $E_{t-1}p_t$ is generated from the model, and what the present results seem to suggest to advocates of Sargent's model is that more work on the overall model is needed before one can hope to get good estimates of the $p_t - E_{t-1}p_t$ coefficients.

III. The Other Three Models

The variables and data that were used for Sims's model are presented in table 4. For the estimation of this model, each of the six variables was regressed on a constant, a time trend, three seasonal dummy variables (although the data are seasonally adjusted), and the first four lagged values of each of the six variables. This meant that there were 29 coefficients to estimate in each equation. The naive model has already been described in the Introduction. Each variable is simply regressed on a constant, a time trend, and its first eight lagged values. My model is described elsewhere (Fair 1974, 1976, 1978*b*), and no discussion of it will be presented here.

The sample periods that were used for these three models were the same as those used for Sargent's model except for slight differences in the beginning quarters. The beginning quarters were 1954I for Sims's model and for my model and 1954II for the naive model. The data

TABLE 4

VARIABLES AND DATA FOR SIMS'S MODEL

LHS Variable	Variable in Fair (1978*b*)
Log of the money supply	Log $M1_t$
Log of real GNP	Log $GNPR_t$
Unemployment rate	UR_t
Log of the wage rate	Log WFF_t
Log of the price level	Log $GNPD_t$
Log of import prices	Log PIM_t

NOTE.—The explanatory variables in each equation are: 1, t, three seasonal dummies, and the first four lagged values of each of the six variables.

base began in 1952I, and slightly different beginning quarters had to be used for the models because of different lag lengths.

IV. The Method and Its Application

The method that is proposed in Fair (1978*a*) for estimating the uncertainty of a forecast from a model accounts for the four main sources of uncertainty: uncertainty due to (1) the error terms, (2) the coefficient estimates, (3) the exogenous variable forecasts, and (4) the possible misspecification of the model. It also accounts for the fact that the variances of forecast errors are not constant across time. Because the method accounts for all four sources of uncertainty, it can be used to make comparisons across models.

Since the method is discussed in detail in Fair (1978*a*), it will be only briefly described here. The uncertainty from the error terms and coefficient estimates is estimated by means of stochastic simulation. Given estimates of the relevant variance-covariance matrices, this is a fairly straightforward procedure. The uncertainty from the exogenous variable forecasts is also estimated by means of stochastic simulation, although what is first required in this case is an estimate of the uncertainty of the exogenous variable forecasts themselves. The procedure that was followed in this study for the 60 exogenous variables in my model and the two exogenous variables in Sargent's model was to regress each variable on a constant, a time trend, and its first eight lagged values and then to take the estimated standard error from this regression as the estimate of the uncertainty attached to forecasting the change in this variable for each quarter.

Estimating the uncertainty from the possible misspecification of the model is the most difficult and costly part of the method, and it also rests on one fairly strong assumption. This part of the method requires successive reestimation and stochastic simulation of the model. It is based on a comparison of estimated variances computed by means of stochastic simulation with estimated variances computed from outside-sample forecast errors. The strong assumption is that the model is misspecified in such a way that, for each variable and length of forecast, the expected value of the difference between the two estimates of the variance is constant across time. This part of the method accounts for misspecification effects, if any, that are not already reflected in the variances that are estimated by means of stochastic simulation.

The results of applying the method to the four models are presented in table 5. The results in table 5 for the naive model and for my model are the same as those in Fair (1978*a*). Since the same procedure was followed for Sargent's and Sims's models as was followed for these

TABLE 5

ESTIMATED STANDARD ERRORS OF FORECASTS

VARIABLE AND MODEL	1978			1979				1980				1981			
	II	III	IV	I	II	III	IV	I	II	III	IV	I	II	III	IV
Real GNP:															
Naive:															
a	.61	1.02	1.34	1.64	1.84	1.94	2.01	2.03	2.04	2.03	2.04	2.04	2.03	2.03	2.03
b, c	.67	1.13	1.53	1.90	2.20	2.38	2.50	2.59	2.64	2.68	2.74	2.77	2.81	2.84	2.87
d	1.09	1.93	2.72	3.45	4.01	4.32	4.58	4.74	...	...	...	...	...	...	...
Sargent:															
a	.82	1.35	1.72	1.91	2.09	2.19	2.27	2.38	2.45	2.51	2.62	2.67	2.69	2.72	2.77
b, c	.98	1.59	2.05	2.39	2.64	2.86	3.04	3.25	3.49	3.79	4.11	4.37	4.61	4.83	5.09
d	1.30	2.23	3.00	3.72	4.23	4.61	4.93	5.10	...	...	...	...	...	...	...
Sims:															
a	.64	.91	1.07	1.29	1.53	1.72	1.93	2.12	2.25	2.33	2.35	2.36	2.35	2.33	2.36
b, c	.88	1.32	1.64	2.13	2.72	3.23	3.68	4.02	4.32	4.58	4.73	4.91	5.03	5.14	5.25
d	1.30	2.29	3.04	4.04	5.26	6.24	7.13	7.79	...	...	...	...	...	...	...
Fair:															
a	.65	.88	1.03	1.15	1.25	1.30	1.35	1.34	1.36	1.40	1.43	1.44	1.47	1.46	1.43
b	.67	.95	1.19	1.38	1.49	1.59	1.66	1.69	1.77	1.81	1.82	1.84	1.88	1.88	1.94
c	.74	1.09	1.37	1.63	1.76	1.94	2.04	2.08	2.15	2.18	2.22	2.30	2.34	2.36	2.43
d	.80	1.23	1.54	1.96	2.27	2.51	2.48	2.27	...	...	...	...	...	...	...
GNP deflator:															
Naive:															
a	.20	.36	.53	.71	.90	1.08	1.24	1.37	1.49	1.58	1.65	1.71	1.76	1.80	1.83
b, c	.24	.45	.70	1.00	1.36	1.73	2.10	2.48	2.84	3.18	3.52	3.85	4.17	4.48	4.80
d	.45	.94	1.53	2.25	3.12	4.05	5.10	6.20	...	...	...	...	...	...	...
Sargent:															
a	1.02	1.34	1.67	1.84	2.05	2.24	2.46	2.71	2.95	3.15	3.33	3.43	3.52	3.59	3.72
b	1.32	1.80	2.29	2.62	3.03	3.36	3.71	4.15	4.58	5.06	5.54	5.94	6.33	6.69	7.09
c	1.58	2.20	2.75	3.13	3.56	3.92	4.29	4.73	5.17	5.54	5.92	6.28	6.68	7.00	7.38
d	2.00	2.95	4.00	4.70	5.54	6.20	7.32	8.53	...	...	...	...	...	...	...

Sims:															
a	.22	.30	.42	.56	.70	.84	.96	1.05	1.13	1.19	1.26	1.30	1.36	1.42	1.49
b, c	.28	.42	.63	.88	1.16	1.44	1.74	2.03	2.31	2.58	2.86	3.12	3.39	3.66	3.92
d	.50	.77	1.34	2.07	2.87	3.89	5.01	6.26	...	...	...	...	...	...	...
Fair:															
a	.28	.35	.42	.47	.51	.55	.59	.61	.64	.65	.65	.65	.66	.67	.68
b	.31	.47	.58	.71	.83	.93	1.02	1.10	1.19	1.28	1.37	1.44	1.50	1.57	1.63
c	.44	.67	.84	1.04	1.21	1.36	1.49	1.62	1.75	1.88	1.98	2.09	2.23	2.35	2.43
d	.53	.93	1.37	1.87	2.33	2.74	3.15	3.48	...	...	...	...	...	...	...
Unemployment rate:															
Naive:															
a	.28	.55	.77	.94	1.02	1.08	1.12	1.14	1.15	1.16	1.16	1.16	1.15	1.15	1.16
b, c	.29	.58	.84	1.04	1.17	1.27	1.34	1.40	1.44	1.48	1.52	1.55	1.59	1.63	1.66
d	.36	.74	1.12	1.48	1.73	1.91	2.07	2.19	...	...	...	...	...	...	...
Sargent:															
a	.26	.55	.78	.93	1.01	1.05	1.08	1.09	1.13	1.16	1.20	1.22	1.24	1.25	1.27
b, c	.28	.58	.85	1.05	1.16	1.22	1.26	1.31	1.37	1.45	1.54	1.62	1.69	1.75	1.81
d	.42	.83	1.23	1.57	1.76	1.82	1.88	1.88	...	...	...	...	...	...	...
Sims:															
a	.21	.35	.43	.49	.56	.63	.69	.75	.82	.87	.90	.91	.90	.89	.90
b, c	.28	.48	.59	.68	.84	1.01	1.16	1.28	1.38	1.48	1.55	1.61	1.65	1.68	1.71
d	.44	.85	1.19	1.30	1.45	1.64	1.96	2.23	...	...	...	...	...	...	...
Fair:															
a	.27	.45	.57	.64	.71	.77	.80	.82	.82	.85	.90	.92	.93	.96	.97
b	.36	.58	.76	.92	1.03	1.12	1.16	1.23	1.28	1.34	1.38	1.42	1.50	1.56	1.62
c	.36	.60	.80	.95	1.08	1.17	1.24	1.31	1.35	1.41	1.47	1.50	1.55	1.59	1.64
d	.35	.60	.77	.82	.85	.83	.77	.71	...	...	...	...	...	...	...
Money supply:															
Naive:															
a	.54	.67	.81	.88	.97	1.02	1.10	1.15	1.22	1.28	1.33	1.40	1.45	1.49	1.51
b, c	.62	.81	1.03	1.21	1.36	1.53	1.72	1.93	2.12	2.34	2.51	2.73	2.96	3.19	3.42
d	1.38	1.64	1.95	2.32	2.62	2.91	3.30	3.70	...	...	...	...	...	...	...

TABLE 5 (*Continued*)

Variable and Model	1978			1979				1980				1981			
	II	III	IV	I	II	III	IV	I	II	III	IV	I	II	III	IV
Sims:															
a	.69	.86	1.02	1.17	1.29	1.46	1.56	1.71	1.82	1.94	2.00	2.09	2.15	2.21	2.31
b, c	.92	1.19	1.56	1.92	2.33	2.72	3.08	3.44	3.80	4.19	4.57	4.93	5.28	5.66	6.05
d	1.37	1.53	2.29	3.29	4.18	5.23	6.07	6.79	...	...	...	...	...	...	...
Fair:															
a	.83	1.09	1.29	1.47	1.62	1.76	1.84	1.92	1.98	2.03	2.11	2.13	2.17	2.19	2.23
b	.91	1.31	1.63	1.87	2.13	2.36	2.56	2.79	2.96	3.16	3.35	3.55	3.72	3.94	4.15
c	.91	1.33	1.69	1.98	2.34	2.68	3.06	3.45	3.79	4.14	4.51	4.88	5.28	5.63	5.97
d	1.29	2.16	2.95	3.75	4.62	5.50	6.49	7.50	...	...	...	...	...	...	...
Wage rate:															
Naive:															
a	.30	.40	.48	.53	.59	.61	.67	.72	.76	.81	.85	.88	.91	.95	.98
b, c	.36	.48	.59	.75	.86	.97	1.15	1.29	1.46	1.64	1.81	1.99	2.19	2.39	2.59
d	.63	.84	1.04	1.26	1.41	1.56	1.81	2.04	...	...	...	...	...	...	...
Sims:															
a	.47	.62	.71	.77	.82	.92	.95	1.02	1.10	1.15	1.17	1.22	1.26	1.30	1.34
b, c	.63	.90	1.08	1.23	1.35	1.50	1.65	1.85	2.05	2.25	2.42	2.59	2.75	2.95	3.12
d	1.23	2.20	3.02	3.71	4.26	4.70	5.12	5.69	...	...	...	...	...	...	...
Fair:															
a	.60	.77	.88	.89	.96	1.01	1.03	1.05	1.07	1.10	1.08	1.07	1.08	1.04	1.05
b	.70	.93	1.12	1.34	1.52	1.65	1.76	1.82	1.94	2.04	2.15	2.27	2.35	2.45	2.51
c	.67	.95	1.16	1.35	1.53	1.66	1.80	1.94	2.08	2.20	2.32	2.40	2.52	2.61	2.69
d	.65	1.06	1.45	2.01	2.53	3.07	3.59	4.16	...	...	...	...	...	...	...

NOTE.—Forecast period = 1978II–1981IV; row *a* = uncertainty due to error terms; row *b* = uncertainty due to error terms and coefficient estimates; row *c* = uncertainty due to error terms, coefficient estimates, and exogenous variable forecasts; row *d* = uncertainty due to error terms, coefficient estimates, exogenous variable forecasts, and possible misspecification of the model. For the unemployment rate, the errors are in percentage points; for the other variables, the errors are expressed as percentages of the forecast means (in percentage points).

other two models, little further discussion of the calculations for these models are needed here. The basic forecast period was 1978II–1981IV, and for the misspecification calculations the first of the 35 sample periods ended in 1968IV and the last ended in 1977II.

A few points about the estimation of the variance-covariance matrices for Sargent's model should be noted. First, for the estimation of these matrices for the coefficient estimates, no account was taken of the fact that $E_{t-1}p_t$ is estimated along with the coefficients. In other words, for purposes of estimating the matrices, the $E_{t-1}p_t$ series was treated as if it were known with certainty. With respect to the estimates of the first- and second-order serial correlation coefficients in equations (3) and (5*c*), on the other hand, the correlation between these estimates and the other coefficient estimates in the equation was taken into account in the estimation of the variance-covariance matrices. The procedure that was followed in this case is analogous to the procedure that was followed in my model for those equations in which the first-order serial correlation coefficient was estimated (see Fair 1978*a*). It should be noted, finally, that for the estimation of the variance-covariance matrix of the error terms for Sargent's model the error term in equation (4) was assumed to be uncorrelated with the other four error terms in the model.

The number of trials used for each of the *a*, *b*, and *c* rows in table 5 was 2,000 for the Sargent, Sims, and naive models and 1,000 for my model. For each of the 35 stochastic simulations that were needed in the computation of the *d*-row values, the number of trials was 500 for the Sargent, Sims, and naive models and 100 for my model. The results in table 5 are based on a total of 77,000 solutions of the models. For the *a*-, *b*-, and *c*-row calculations, each solution is a dynamic simulation of the model over the 15 quarters, 1978II–1981IV; and for the *d*-row calculations, each solution is a dynamic simulation of the model over differing 8-quarter periods.[2]

[2] At the risk of being pedantic, it should perhaps be noted what is meant by a dynamic simulation for Sargent's model. First, the values of m_t, pop_t, $\hat{m}_t$, and $\widehat{pop}_t$ for all t are taken to be exogenous, where $\hat{m}_t$ and $\widehat{pop}_t$ are static predictions from the two regressions mentioned above. (It would not be appropriate to take for $\hat{m}_t$ and $\widehat{pop}_t$ the dynamic predictions from the two regressions because of the exogeneity of m_t and pop_t themselves.) Given these values, the model is first solved for the initial quarter of the prediction period, say quarter t, using $\hat{m}_t$ and $\widehat{pop}_t$ in place of m_t and pop_t. This produces a value for $E_{t-1}p_t$, namely, the predicted value of p_t from this solution. (The term $p_t - E_{t-1}p_t$ in the Un_t and nf_t equations does not play any role in these calculations because its solution value in this case is always zero.) The model is then solved again for quarter t using this value for $E_{t-1}p_t$ and the actual values of m_t and pop_t. This completes the predictions for quarter t. The model is next solved for quarter $t+1$ using the (second) predicted values of the lagged endogenous variables for period t and $\hat{m}_{t+1}$ and $\widehat{pop}_{t+1}$ in place of m_{t+1} and pop_{t+1}. This produces a value for E_tp_{t+1}. The model is then solved again for quarter $t+1$ using this value for E_tp_{t+1} and the actual values of m_{t+1} and pop_{t+1}. This process continues through the end of the prediction period. It should finally be

TABLE 6

ROOT MEAN SQUARED ERRORS OF OUTSIDE-SAMPLE FORECASTS

VARIABLE AND MODEL	NUMBER OF QUARTERS AHEAD							
	1	2	3	4	5	6	7	8
Real GNP:*								
Naive	1.11	1.96	2.76	3.51	4.09	4.42	4.70	4.91
Sargent	1.31	2.26	3.04	3.77	4.27	4.59	4.89	5.00
Sims	1.42	2.54	3.54	4.79	6.34	7.79	9.36	10.98
Fair	.79	1.26	1.63	2.12	2.59	2.97	3.24	3.52
GNP deflator:*								
Naive	.47	.98	1.59	2.36	3.26	4.23	5.35	6.52
Sargent	2.89	4.93	7.02	8.81	10.58	12.22	14.05	15.87
Sims	.54	.90	1.53	2.39	3.36	4.56	5.90	7.42
Fair	.50	.93	1.43	1.97	2.49	2.95	3.43	3.83
Unemployment rate:†								
Naive	.36	.75	1.13	1.49	1.73	1.89	2.03	2.14
Sargent	.44	.87	1.30	1.66	1.87	1.94	2.01	2.00
Sims	.47	.91	1.30	1.46	1.64	1.89	2.31	2.74
Fair	.36	.60	.75	.80	.79	.79	.77	.77
Money supply:*								
Naive	1.41	1.71	2.10	2.52	2.89	3.23	3.64	4.09
Sims	1.47	1.95	3.05	4.40	5.80	7.41	9.04	10.82
Fair	1.34	2.12	2.93	3.81	4.74	5.72	6.82	7.97
Wage rate:*								
Naive	.67	.94	1.21	1.47	1.71	2.01	2.40	2.83
Sims	1.33	2.37	3.32	4.15	4.91	5.60	6.37	7.36
Fair	.78	1.25	1.71	2.31	2.89	3.49	4.06	4.65

NOTES.—These results are based on 35 sets of estimates of each model. Each 8-quarter outside-sample forecast began 2 quarters after the end of the estimation period. The first estimation period ended in 1968IV, and the last (the thirty-fifth) ended in 1977II. Data through 1977IV were used, which allowed 35 1-quarter-ahead errors to be computed for each variable, 34 2-quarter-ahead errors, and so on. The predicted values for these calculations were the mean values from the 35 stochastic simulations that were generated for the *d*-row results in table 5.

*In percent.

†In percentage points.

The method that I have proposed is meant to replace the traditional way of estimating the accuracy of a model by computing root mean squared errors (RMSEs). Computing RMSEs does not account for the fact that the variances of forecast errors vary across time, and it does not account for the uncertainty due to the exogenous variable forecasts. Since computing RMSEs is such a wide-spread procedure, however, I have for purposes of this study also computed RMSEs for the four models. These results are presented in table 6. They are based on the same 35 sets of estimates and sample periods that were used for the *d*-row calculations in table 5. The forecasts upon which

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noted that each of the 35 sets of estimates of Sargent's model includes estimates of the m_t and pop_t equations. These two equations were treated like the five structural equations of Sargent's model in computing the 35 sets of estimates.

these results are based are all outside-sample forecasts and use the actual values of all the exogenous variables.

V. The Results

In order to simplify the discussion, it will be useful to concentrate on the 4- and 8-quarter-ahead results in table 5. For real GNP, the estimated standard errors of the 4-quarter-ahead forecast, taking into account all four sources of uncertainty, are (in percentages) 3.45 for the naive model, 3.72 for Sargent's model, 4.04 for Sims's model, and 1.96 for my model. The corresponding 8-quarter-ahead errors are 4.74, 5.10, 7.79, and 2.27. For the GNP deflator, the 4-quarter-ahead errors are 2.25, 4.70, 2.07, and 1.87; and the 8-quarter-ahead errors are 6.20, 8.53, 6.26, and 3.48. For the unemployment rate, the 4-quarter-ahead errors are (in percentage points) 1.48, 1.57, 1.30, and 0.82; and the 8-quarter-ahead errors are 2.19, 1.88, 2.23, and 0.71. The results for these three variables thus show that my model is the most accurate for all three. For the unemployment rate, the other three models are about the same. For real GNP, the naive model and Sargent's model are about the same and are better than Sims's model. For the GNP deflator, the naive model and Sims's model are about the same and are better than Sargent's model. Sargent's and Sims's models are thus either about the same or not as good as the naive model.

For the money supply, the naive model is better than my model and Sims's model, and Sims's model is slightly better than mine. The 8-quarter-ahead errors are 3.70 percent for the naive model, 6.79 for Sims's model, and 7.50 for my model. (The money supply is exogenous in Sargent's model.) For the nominal wage rate, the naive model is better than my model and Sims's model, and my model is better than Sims's. The 8-quarter-ahead errors are 2.04 for the naive model, 5.69 for Sims's model, and 4.16 for my model. (The wage rate is not a variable in Sargent's model.) The naive model is thus better than both Sims's model and my model for the money supply and the nominal wage rate.

The same qualitative conclusions emerge from the RMSE results in table 6, except that my model is now more accurate than Sims's model with respect to the money supply. This difference is explained in part by the fact that the calculations in table 6 do not take into account the uncertainty from the exogenous variable forecasts, whereas those in table 5 do. There are no exogenous variables in Sims's model, unlike in my model, and so any comparison of the two models that does not take into account the exogenous variable uncertainty is likely to be biased in favor of my model.

The *a*-, *b*-, *c*-, and *d*-row comparisons in table 5 are self-explanatory, and so no discussion of these results is needed here.[3] One can determine from these comparisons how much of the total error is due to each of the four sources, which may be useful information regarding possible future work on the models.

One further set of results should be mentioned, which has to do with the treatment of degrees of freedom. No corrections for degrees of freedom were made for the results in table 5: the estimated variances and covariances that were used for the stochastic simulations were based on division of the appropriate sums of squares and cross products by the number of observations, not the number of observations less the number of coefficients estimated. This was done in part because, as is discussed in Fair (1978*a*), there are no obvious corrections to make for my model. In particular, when estimating the variance-covariance matrix of the error terms for my model, there are no obvious corrections to make for the off-diagonal terms because in general the number of coefficients estimated per equation differs across equations. This is also true of Sargent's model.

For Sargent's model, the naive model, and my model, the number of coefficients per equation is fairly small relative to the number of observations, and so whether or not one corrects for degrees of freedom is not likely to make much difference to the results. This is not necessarily true of Sims's model, however, where there are 29 coefficients per equation. (The number of observations ranges from 60 for the shortest sample period used to 96 for the longest.) To examine the sensitivity of the results to the treatment of degrees of freedom, the results for the naive model and Sims's model were redone with the corrections made. (The corrections are obvious for these two models.)

With respect to the effect of these corrections on the results in table 5, one would expect them to lead to larger *a*- and *b*-row values because the stochastic simulations are now based on larger estimated variances and covariances. Also, one would expect the *d*-row values to be somewhat smaller, for the following reason. First, the differences between

[3] One point about the *c*-row calculations for Sargent's model should be noted. If, as was assumed for the present calculations, the uncertainty attached to forecasting m_t is the same as that attached to forecasting $\hat{m}_t$, then the predicted values of Un_t, nf_t, and y_t are unaffected by this uncertainty. In other words, if in the stochastic simulations the draws for the m_t and $\hat{m}_t$ errors are the same, then $E_{t-1}p_t$ and the "second" predicted value of p_t are affected in the same way, and so these draws have no effect on the real variables. Only the predictions of p_t are affected by the uncertainty in forecasting the money supply. Also, the uncertainty attached to forecasting pop_t is so small as to be negligible, which means that there is in effect no exogenous variable uncertainty in forecasting the real variables in Sargent's model. Consequently, the *b* and *c* rows in table 5 for real GNP and the unemployment rate have not been listed separately for Sargent's model.

the *d*- and *b*-row values are based on the differences between variances estimated from outside-sample errors and variances estimated by stochastic simulation. The former are affected by the corrections only to the extent that the forecast means are affected, which in practice is very little. The latter, however, are affected as the *b*-row values in table 5 are (since they are merely *b*-row values for a different sample period). Therefore, the net effect of the corrections on the differences between the *d*- and *b*-row values in table 5 is expected to be negative. Second, the *b*-row values in table 5 are based on a larger sample period than are the *b*-row values used in estimating the above-mentioned differences, and so the corrections affect the former less than the latter. This means that the differences are negatively affected more than the *b*-row values in table 5 are positively affected, and so the net effect on the *d*-row values is at least slightly negative.

The results of the corrections were as expected, although, as will be discussed, one problem did arise for Sims's model. For the naive model, the effects were small and in the expected directions. To cite two examples, the 8-quarter-ahead values for real GNP were 2.15, 2.75, and 4.69, compared with 2.03, 2.59, and 4.74 in table 5, and the 8-quarter-ahead values for the GNP deflator were 1.45, 2.63, and 6.11, compared with 1.37, 2.48, and 6.20 in table 5. For Sims's model, the effects were larger than they were for the naive model and in the expected directions. The problem that arose pertains to the 7- and 8-quarter-ahead results. For these two horizons, the variances estimated by stochastic simulation for a few of the sample periods were quite large compared with the variances computed from the outside-sample errors, and these few observations had a large effect on the averages of the estimated differences. (The 7- and 8-quarter-ahead results are based on only 29 and 28 observations, respectively, and so a few outliers can dominate the results.) This led in the present case to quite small *d*-row values for some variables for the two horizons. This problem appeared to be much less important for the other horizons. Again, to cite two examples, the 4-quarter-ahead values for real GNP were 1.54, 2.61, and 3.17, compared with 1.29, 2.13, and 4.04 in table 5, and the 4-quarter-ahead values for the GNP deflator were 0.68, 1.07, and 1.78, compared with 0.56, 0.88, and 2.07 in table 5.[4] It should finally be noted that the corrections had little effect on the RMSE results in table 6 since, as noted above, they had little effect on the forecast means.

[4] I am indebted to Robert Litterman and Christopher Sims for helpful discussions regarding the degrees-of-freedom issue. In as yet unpublished work, Litterman has also obtained results for Sims's model with corrections made for degrees of freedom.

VI. Summary and Conclusions

The main results of this study are easy to summarize. Sargent's model and Sims's model are no more accurate than the naive model, and for some variables they are less accurate. My model is more accurate than the other models for real GNP, the GNP deflator, and the unemployment rate. It is less accurate than the naive model for the money supply and the nominal wage rate, and it is slightly less accurate than Sims's model for the money supply.

The tentative nature of the results in this paper has been mentioned in the Introduction, and this point should again be stressed. Clearly, advocates of Sargent's and Sims's models are likely to want to work more on the models before reaching a final judgment about their accuracy. In addition, one may want to try other estimation techniques for Sargent's model than the one used in this study. Finally, there appear to be some small sample problems associated with Sims's model with respect to the results in table 5, and more observations are needed before much confidence can be placed on the results. Nevertheless, the present results do indicate that Sargent's and Sims's models are not very accurate, and so the present prognosis for these two models is not encouraging.

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