

An Analysis of a Macro-Econometric Model with Rational Expectations in the Bond and Stock Markets

By RAY C. FAIR*

The difficulty of accounting for expectational effects in macro-economic models is well known. The standard procedure in dealing with this problem in the construction of large-scale macro-econometric models is to use current and lagged values as "proxies" for expected future values. An alternative procedure is to assume that expectations are rational. Although the assumption of rational expectations has received increased attention lately in work with theoretical and small-scale empirical models,¹ it has not yet been applied to large-scale macro-econometric models. In this paper the assumption that expectations are rational in the bond and stock markets will be applied to a large-scale macro-econometric model. The quantitative effects of monetary and fiscal policies in this model will be compared to those in a similar model without rational expectations. The quantitative sensitivity of monetary and fiscal policy effects to alternative expectational assumptions is clearly an important question in macroeconomics, and the primary purpose of this paper is to provide an estimate of this sensitivity for the assumption of rational expectations in the bond and stock markets.

The econometric model that is used in this study is the one in my 1976 book. Three "versions" of this model are analyzed: the original version and two modified versions.

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¹For one class of models with rational expectations, see Robert E. Lucas, Jr., Thomas J. Sargent (1973, 1976), Sargent and Neil Wallace, and Robert J. Barro. See also my 1978b paper for a criticism of this class of models. For an example of the use of the assumption of rational expectations in a small-scale empirical model (the St. Louis model), see Paul A. Anderson.

The original version, which will be called Model 1, does not have rational expectations in the bond and stock markets. There are two term-structure equations and one stock-price equation in the model, and in these three equations current and lagged values are used as proxies for expected future values. In the first modified version, which will be called Model 2, the two term-structure equations are replaced with a specification that is consistent with the existence of rational expectations in the bond market.² The second modified version, which will be called Model 3, is the same as Model 2 except that the stock-price equation is replaced with a specification that is consistent with the existence of rational expectations in the stock market. In Model 3, therefore, there are rational expectations in both the bond and stock markets, and it is to my knowledge the first example of a large-scale econometric model for which this is true.

It is important to note at the outset that this paper does not contain a test of the assumption of rational expectations in the bond and stock markets. Within the context of the present model it is more difficult to test this assumption than it is to examine its policy implications, and such a test is beyond the scope of the present paper. The way in which this assumption could be tested using the present model is discussed in footnote 13 below. It is also important to note at the outset that Model 3 is not a model in which all expectations are rational. Model 1 contains

²The idea of replacing term-structure equations in macro-econometric models with a specification that is consistent with the existence of rational expectations in the bond market is contained in a paper by William Poole (pp. 477-78). Poole (p. 478) questioned the computational feasibility of this procedure for large-scale models, but, as discussed below, this procedure is in fact computationally feasible.

markets other than the bond and stock markets, such as the labor and goods markets, in which expectations are not rational, and Model 3 differs from Model 1 only with respect to the bond and stock markets. It is again beyond the scope of the present paper to consider an econometric model in which all expectations are rational.

The outline of this paper is as follows. A few of the features of the original version of the model are reviewed in Section I, and then the modifications of it are considered in Section II. The basic experiments that were performed using the three versions are described in Section III, and the results of these experiments are presented and discussed in Section IV. Some further experiments and results are described in Section V. Section VI contains a brief summary of the main conclusions of this study.

I. A Brief Review of Model 1

Model 1 consists of eighty-four equations, twenty-six of which are stochastic. There are five sectors (household, firm, financial, foreign, and government) and five categories of financial securities (demand deposits and currency, bank reserves, member bank borrowing from the Federal Reserve, gold and foreign exchange, and an "all other" category). Since the model is described in detail in my 1976 book, no extensive discussion of it will be presented here. It will be useful for purposes of the following analysis, however, to review briefly the interest rate and wealth effects in the model and the structure of the financial sector.

There are three endogenous interest rates in the model: the three-month Treasury Bill rate (r), an Aaa corporate bond rate (RA), and a mortgage rate (RM);³ the last an explanatory variable in three of the four consumption equations and in one of the three labor supply equations. RA is an explanatory variable in the two interest payment equations, in the stock-price equation, and in the main price equation of the model; and r is an explanatory variable in two of the consump-

tion equations, in the two demand-for-money equations, in an equation explaining member bank borrowing from the Federal Reserve, and in one of the interest payment equations. In addition, there is a loan-constraint variable in the model that is a function of r , and this variable is an explanatory variable in one of the consumption equations and in an equation explaining the dividend payments of the firm sector; r is also an explanatory variable in the term-structure equations for RA and RM .

The variable explained by the stock-price equation is CG , the value of capital gains (+) or losses (-) on corporate stocks held by the household sector. The variable CG is part of the definition of ΔA in the model, where A is the value of securities (other than demand deposits and currency) held by the household sector. I use A lagged one-quarter as an explanatory variable in three of the consumption equations and in one of the labor supply equations.

A key feature of the model regarding the financial sector is that it is closed with respect to the flows of funds in the system. This means that any financial saving or dissaving of a sector in a period results in the change in at least one of its assets or liabilities, and that a financial asset of one sector is a corresponding liability of some other sector. It also means that the government budget constraint is explicitly accounted for and that the amount of government securities outstanding can be taken to be a direct policy variable of the Federal Reserve (henceforth called the Fed). The government budget constraint is

$$(1) \quad 0 = SAVG_t + \Delta VBG_t + \Delta(BR_t - BORR_t) + EXOG_t$$

where $SAVG$ is the financial saving of the government sector (the negative of the government deficit), VBG is the amount of government securities outstanding, BR is the amount of bank reserves, $BORR$ is the amount of member bank borrowing from the Fed, and $EXOG$ denotes the remaining variables in the equation, all of which are exogenous. The terms $SAVG$, BR , and $BORR$ are endogenous variables in the model and are explained by other equations. Equation (1) states that any nonzero level of saving of the government must result in the change in

³In terms of the notation in my 1976 book, $r = RBILL$, $RA = RAAA$, and $RM = RMORT$.

either VBG or nonborrowed reserves. Government securities are included in the "all other" category of securities in the model, and there is an equation in the model that equates the aggregate supply of this category to the aggregate demand. This equation can be written:

$$(2) \quad 0 = \Delta VBG_t - \Delta VBP_t$$

where VBP is the amount of government securities held by the nongovernment sectors.⁴ It is also an endogenous variable and determined elsewhere in the model.

The government budget constraint (1) is redundant, and so it can be dropped from the model. This still leaves equation (2), however, as an "extra" equation. Since VBP is determined elsewhere in the model, if VBG is taken to be exogenous, then some variable not explained by any other equation must be chosen to be endogenous in order to close the model. The variable chosen in this case is the bill rate (r). There is thus no equation in the model in which the bill rate appears naturally on the left-hand side. The bill rate is instead implicitly determined: its solution value each period is (speaking loosely) the value that makes equation (2) hold. As noted above, r is an explanatory variable in a number of the key equations in the model.

If, as just discussed, VBG is taken to be exogenous, then the behavior of the Fed is exogenous. In other words, the behavior of the Fed is not influenced by the state of the economy. In a recent study (1978a) I have estimated an equation explaining Fed behavior, and since this equation is used for some of the experiments below, it will be useful to provide a brief review of it. In this study the Fed was assumed to choose each period an optimal value of the bill rate and then to achieve this value through changes in its three policy variables: the reserve requirement ratio (g_1), the discount rate (RD), and VBG . Based on this assumption, an equation explaining the bill rate was estimated, where the explanatory variables were taken to be variables that seemed likely to affect the Fed's optimal

value of the bill rate. This estimated equation was then interpreted as an explanation of Fed behavior. The equation is⁵

$$(3) \quad r_t = -11.1 + 0.841r_{t-1} \\ (2.93) \quad (16.30) \\ + 0.0497\% PD_{t-1} + 0.0352J_t^* \\ (1.69) \quad (2.97) \\ + 0.0427\% GNPR_t + 0.0188\% GNPR_{t-1} \\ (1.62) \quad (1.36) \\ + 0.0251\% M_{1t-1} \\ (2.10)$$

$$\hat{\rho} = 0.229, S.E. = 0.474, \\ (2.28)$$

$$R^2 = 0.939, D.W. = 1.82$$

Sample Period = 1954I–1976II

where $\%PD$ is the percentage change at an annual rate in the price deflator for domestic sales, J^* is a measure of labor market tightness, $\%GNPR$ is the percentage change at annual rate in real GNP , $\%M_1$ is the percentage change at an annual rate in the money supply, and $\hat{\rho}$ is the estimate of the first-order serial correlation coefficient. The t -statistics in absolute value are in parentheses. Equation (3) states that the current bill rate is a positive function of the lagged rate of inflation, of the current degree of labor market tightness, of the current and lagged rates of growth of real GNP , and of the lagged rate of growth of the money supply. The behavior reflected in this equation is thus behavior in which the Fed "leans against the wind." The wind in this case is composed of the inflation rate, the degree of labor market tightness, the growth rate of real GNP , and the growth rate of the money supply. As these variables rise, so also does the bill rate.

If equation (3) is added to the model, then the behavior of the Fed is endogenous. In this case one of the three policy variables of the Fed (g_1 , RD , or VBG) must be taken to be endogenous in order to close the model. For the experiments below VBG was always chosen to be the endogenous variable in this

⁴Equation (2) is equation 70 in my 1976 book. For purposes of the discussion in this paper, the original notation has been simplified: ΔVBP_t represents all the terms in equation 70 except ΔVBG_t .

⁵Equation (3) was estimated under the assumption of first-order serial correlation of the error term by the two-stage least squares technique described in my 1970 paper. The two endogenous explanatory variables in the equation are J_t^* and $\%GNPR_t$.

case. Thus the solution value for *VBG* each period is (again speaking loosely) the value that makes equation (2) hold.

To summarize this review of the financial sector of Model 1, *r* affects directly *RA*, *RM*, the loan-constraint variable, two consumption variables, two demand-for-money variables, member bank borrowing from the Fed, and one interest payment variable. It affects indirectly through the loan-constraint variable one consumption variable and one dividend variable. It affects indirectly through *RA* and *RM* three consumption variables, one labor supply variable, two interest payment variables, the main price variable in the model (*PF*), and *CG*. In addition, *CG* affects *A*, which affects with a lag of one-quarter three consumption variables and one labor supply variable. These latter effects are wealth effects on the household sector. The variable *PF* affects the level of sales, which affects the level of production, which affects the levels of investment and employment. The bill rate *r* thus has an indirect effect on investment and employment through its indirect effect on *PF* and in turn is affected by all the other variables in the model when it is implicitly determined (*VBG* exogenous). When the behavior of the Fed is endogenous (*VBG* endogenous), then *r* is determined according to the Fed behavioral equation.

II. The Modifications of Model 1

A. The Term Structure Equations

In order to consider the policy implications of the assumption of rational expectations in the bond market, some assumption about the determination of the term structure of interest rates must first be made. The following analysis is based on the assumption that the term structure is determined according to the expectations theory. Although this theory as applied below abstracts from considerations of such things as transactions costs and preferred habitats, the following analysis could be easily modified to incorporate a slightly different theory. What is needed for the work below is some link between long rates and expected future short rates, not that

this link necessarily be the one postulated by the expectations theory.

According to the expectations theory of the term structure of interest rates (which should not be confused with the assumption of rational expectations), the return from holding an *n*-period security is equal to the expected return from holding a series of one-period securities over the *n* periods. Let r_{t+i}^e denote the expected one-period rate of return for period *t* + *i*, the expectation being conditional on information available as of the beginning of period *t*, and let R_t denote the yield to maturity in period *t* on an *n*-period security. Then according to the expectations theory:

$$(4) \quad (1 + R_t)^n = (1 + r_t^e)(1 + r_{t+1}^e) \dots (1 + r_{t+n-1}^e)$$

Since the r_{t+i}^e values in equation (4) are unobserved, some assumption about how expectations are formed must be made in order to implement this theory. Model 1 rests on the assumption that each r_{t+i}^e is a function of r_t , of lagged values of r_t , and of lagged values of the inflation rate.⁶ Given this assumption, R_t is then according to equation (4) a function of these same variables. The estimated equations for RA_t and RM_t in Model 1 that are meant to approximate this function are⁷

⁶The possibility that both lagged values of the nominal rate and lagged values of the inflation rate affect expectations of future nominal rates is discussed by Franco Modigliani and Robert J. Shiller, pp. 19-23.

⁷For purposes of the work in this paper the model in my 1976 book was reestimated through 1976II using the revised national income accounts data. The estimated coefficients in equations (5) and (6) thus differ somewhat from those presented in my 1976 book, Table 2-3. Likewise, the estimates presented in equation (10) below for *CG* differ somewhat from the original estimates. The estimated version of the model used in this study is the same as the one used for the results in my 1978a paper. The $\hat{\rho}$ in equation (6) is the estimate of the first-order serial correlation coefficient. The *t*-statistics in absolute value are in parentheses. Equation (6) was estimated under the assumption of first-order serial correlation of the error term by the technique discussed in my 1970 paper. Equations (5) and (10) were estimated by the standard two-stage least squares technique. The endogenous explanatory variables are r_t in equations (5) and (6) and RA_t and Π_t in equation (10).

$$\begin{aligned}
 (5) \log RA_t = & 0.0695 + 0.915 \log RA_{t-1} \\
 & (4.10) \quad (46.86) \\
 & + 0.1767 \log r_t + 0.1867 \log r_{t-1} \\
 & (3.07) \quad (3.07) \\
 & + 0.0636 \log r_{t-2} + \\
 & (2.34) \\
 & + 1.27 \left(\frac{1}{2} \Delta \log PX_{t-1} \right. \\
 & (2.23) \\
 & \left. + \frac{1}{3} \Delta \log PX_{t-2} + \frac{1}{6} \Delta \log PX_{t-3} \right)
 \end{aligned}$$

$$R^2 = 0.996, S.E. = 0.0223, D.W. = 1.80$$

Sample Period = 1954I-1976II

$$\begin{aligned}
 (6) \log RM_t = & 0.1965 + 0.852 \log RM_{t-1} \\
 & (3.87) \quad (24.33) \\
 & + 0.0297 \log r_t + 0.0854 \log r_{t-1} \\
 & (0.70) \quad (1.56) \\
 & + 0.1138 \log r_{t-2} + 0.0551 \log r_{t-3} \\
 & (3.00) \quad (2.38) \\
 & + 1.59 \left(\frac{1}{2} \Delta \log PX_{t-1} \right. \\
 & (1.87) \\
 & \left. + \frac{1}{3} \Delta \log PX_{t-2} + \frac{1}{6} \Delta \log PX_{t-3} \right)
 \end{aligned}$$

$$\hat{\rho} = 0.247, R^2 = 0.988,$$

(2.41)

$$S.E. = 0.0254, D.W. = 1.93$$

Sample Period = 1954I-1976II

The last term in equations (5) and (6) is a weighted average of the rates of inflation in the past three quarters, with weights of 1/2, 1/3, and 1/6. The term PX is one of the price deflators in the model. Note that each equation includes as explanatory variables both the lagged dependent variable and the current and lagged values of r , which implies a fairly complicated lag structure of r on both the long-term rates.

When considered by themselves, equations (5) and (6) are consistent with the expectations theory in the sense that the current value of r and the lagged values of r and of the inflation rate in (5) and (6) are proxying for the expected future values in (4). When considered as part of the overall model, however, equations (5) and (6) are not consistent with the expectations theory if

expectations of the future values of r are rational. This is because in simulations of the model the predicted values of RA_t , RM_t , r_t , $r_{t+1}, \dots, r_{t+n-1}$ do not in general satisfy equation (4).

It may help in understanding why Model 1 is not consistent with the existence of rational expectations in the bond market to consider how it can be modified to be consistent. This modification consists of dropping the term-structure equations (5) and (6) from the model and requiring instead that the solution values of r and RA and of r and RM satisfy equation (4).⁸ The resulting model, which will be called Model 2, is then consistent with the assumption of rational expectations in the bond market if the following hold:

ASSUMPTION 1: *People believe that Model 2 is the true model and know how to solve it.*

ASSUMPTION 2: *People at any one time have the same set of forecasts regarding the future values of the exogenous variables in*

⁸It is easier to say this than it is to program it. Requiring that the solution values satisfy equation (4) means that the solution values of r for periods $t+1$ and beyond affect the solution values of RA and RM for period t . In this sense future predicted values affect present predicted values, and so the model differs from the typical econometric model, where only present and past values affect present values. In order to solve the model in this case, one must iterate on solution *paths*. For, say, a forty-quarter problem, one first solves the model, given the exogenous variable values, for the forty quarters using guessed values of RA and RM . New values of RA and RM are then computed using equation (4) and the predicted values of r . The model is then resolved for the forty quarters using these new values of RA and RM . New values of RA and RM are then computed from equation (4) using the new predicted values of r , and the model is solved again. There is no guarantee that this process will converge, but for the work in this study it did converge after some damping of some of the solution values. It took an average of about fifteen iterations for this process to converge, which means that the model is about fifteen times more expensive to solve in this case than it is in the regular case. As noted in fn. 2, Poole (p. 478) questioned the computational feasibility of this procedure for large-scale models, but it is in fact not all that expensive. It should also be noted that for the results in this study, values of r beyond the end of the data period were needed. The procedure that was followed to construct these values is discussed in Section III.

the model and the same set of expectations regarding the future values of the error terms.

Given the set of exogenous variable forecasts and the set of expectations of error terms, the solution values of the endogenous variables from the model are also people's expectations of these values.⁹ Three of the endogenous variables in the model are r , RA , and RM , and so if the solution values of these variables satisfy equation (4), then people's expectations are consistent with this equation. Model 1, on the other hand, even if Assumptions 1 and 2 above were true of it, would not be consistent with the rational expectations assumption because the solution (i.e., expected) values would not satisfy equation (4). There would still be, in other words, an inconsistency between the assumption of rational expectations and the postulated link in (4) between long rates and expected future short rates.

B. The Stock-Price Equation

In a manner analogous to the above analysis of the bond market, in order to consider the policy implications of the assumption of rational expectations in the stock market, some assumption about the determination of stock prices must first be made. The following analysis is based on the theory that the price of a stock is the present discounted value of its expected future returns, although again this analysis could be easily modified to incorporate a slightly different theory. All that is needed for the work below is some link between stock prices and expected future returns.

⁹One subtle point should be noted here. When a non-linear model is solved by setting the error terms equal to their expected values, the solution values of the endogenous variables are not in general equal to their expected values. The proper way to solve non-linear models is by means of stochastic simulation, but because of the expense, this is rarely done. Since no stochastic simulation was done in this study, the predicted values of the one-period rates generated by the model are not exactly equal to their expected values. For simplicity, however, no distinction will be made in the text between predicted and expected values.

Let SP_{t-1} denote the value of corporate stocks held by the household sector at the end of period $t - 1$, and let Π_{t+i}^e denote the expected value of after-tax cash flow for period $t + i$, the expectation being conditional on information available as of the beginning of period t . Then according to the above theory:

$$(7) \quad SP_{t-1} = \frac{{}_t\Pi_t^e}{(1 + {}_t r_t^e)} + \frac{{}_t\Pi_{t+1}^e}{(1 + {}_t r_t^e)(1 + {}_t r_{t+1}^e)} + \dots + \frac{{}_t\Pi_{t+T}^e}{(1 + {}_t r_t^e)(1 + {}_t r_{t+1}^e) \dots (1 + {}_t r_{t+T}^e)}$$

where T is large enough to make the last term in (7) negligible. An equation like (7) also holds, of course, for SP_t , with $t + 1$ replacing t :

$$(8) \quad SP_t = \frac{{}_{t+1}\Pi_{t+1}^e}{(1 + {}_{t+1} r_{t+1}^e)} + \frac{{}_{t+1}\Pi_{t+2}^e}{(1 + {}_{t+1} r_{t+1}^e)(1 + {}_{t+1} r_{t+2}^e)} + \dots + \frac{{}_{t+1}\Pi_{t+T+1}^e}{(1 + {}_{t+1} r_{t+1}^e)(1 + {}_{t+1} r_{t+2}^e) \dots (1 + {}_{t+1} r_{t+T+1}^e)}$$

By definition:

$$(9) \quad CG_t = SP_t - SP_{t-1}$$

where CG is, as mentioned in Section I, the value of capital gains or losses on corporate stocks held by the household sector.

Since the expected values in equations (7) and (8) are unobserved, some assumption about how expectations are formed must be made in order to implement the above theory. In Model 1 the current change in RA is used as a proxy for changes in the expected future values of r , and a weighted average of the current and past changes in after-tax cash flow is used as a proxy for changes in expected future after-tax cash flow. The estimated equation for CG_t is

$$(10) \quad CG_t = 13.19 - 124.3\Delta RA_t + 9.824\left(\frac{1}{2}\Delta\Pi_t + \frac{1}{3}\Delta\Pi_{t-1} + \frac{1}{6}\Delta\Pi_{t-2}\right)$$

(2.53) (3.19) (2.12)

$$R^2 = 0.212, S.E. = 44.02, D.W. = 2.33$$

Sample Period = 1954I–1976II

The last term in equation (10) is a weighted average of the change in Π for the current and past two quarters, with weights of $1/2$, $1/3$, and $1/6$, where Π is the value of after-tax cash flow of the firm sector and is an endogenous variable in the model.¹⁰

When considered by itself, equation (10) is consistent with equations (7)–(9) in the sense that ΔRA_t and the weighted average term in (10) are proxying for the changes in expected future interest rates and after-tax cash flow that are implicit in (9). Again, however, when equation (10) is considered as part of the overall model, it is not consistent with equations (7)–(9) if expectations of the future values are rational. This is because in simulations of the model the predicted values of CG_t , $\Pi_t, \dots, \Pi_{t+T+1}$, r_t, \dots, r_{t+T+1} do not in general satisfy (7)–(9).

It is possible to drop equation (10) from the model and to require instead that the solution values of CG , Π , and r satisfy equations (7)–(9).¹¹ If this is done for Model 2, then the resulting model, which will be called Model 3, is consistent with the assumption of rational expectations in both the bond and stock markets, provided that it is also assumed that Assumptions 1 and 2 above are true for Model 3. Note for Model 3 that because r is used as the discount rate in (7) and (8), the expected return on stocks is the same as the expected return on bonds. In other words, there are no arbitrage opportunities in Model 3 between bonds and stocks, just as there are no arbitrage opportunities in either Model 2 or 3 between bonds of different maturities.

III. The Experiments

In order to examine the sensitivity of policy effects to the assumption of rational expecta-

tions in the bond and stock markets, two basic experiments were performed for each of the three models: the first is a fiscal policy action and the second a monetary policy action. For the first experiment, the real value of goods purchased by the government (XG) was permanently increased by \$1.25 billion beginning in 1971I, a quarter that is at or near the bottom of a contraction. The behavior of the Fed was assumed to be endogenous for this experiment: the equation explaining Fed behavior, equation (3), was added to the model, and the amount of government securities outstanding VBG was taken to be endogenous. For the second experiment, VBG was permanently decreased by \$1.25 billion beginning in 1971I. In this case the behavior of the Fed is obviously not endogenous, so equation (3) is not included in the model.

A standard procedure in performing experiments of this type, which was followed here, is first to add to the stochastic equations of the model the residuals obtained in the process of estimating the equations. Doing this means that when the model is simulated using the actual values of all exogenous variables, the predicted values of all endogenous variables are equal to their actual values. In other words, a perfect tracking solution is obtained. These same residuals are then used for all the experiments; they are treated in effect like exogenous variables. This procedure allows the predicted values of the endogenous variables obtained from changing one or more exogenous variables to be compared directly to their actual values in examining the effects of the change. Also, since multipliers in non-linear models are a function of initial conditions and of values of the exogenous variables and error terms, this procedure provides a natural base from which to perform the experiments, namely the actual data.

The experiments for Model 1 are straightforward to perform. The estimated residuals are first added to the model, and then the model is simulated for the exogenous variable changes. As noted above, the simulations began in 1971I. They were dynamic, twelve-quarter simulations. The results for the first experiment are presented in Table 1, and the

¹⁰In terms of the notation in my 1976 book, $\Pi = CF - TAXF$, where CF and $TAXF$ are both endogenous variables.

¹¹In Model 3, unlike in Model 2, future predicted values of Π (as well as r) affect present predicted values. The solution procedure outlined in fn. 8 for Model 2 can, however, with obvious modifications, also be applied to Model 3.

results for the second experiment are presented in Table 2.¹²

The experiments for Models 2 and 3 require that some further assumptions be made. First, some assumption must be made about people's expectations of the future values of the exogenous variables and error terms. For present purposes these expectations were assumed to be perfect. In other words, people were assumed to know the actual future values of all the exogenous variables and all the estimated residuals. As discussed below, this assumption in the present context is actually not very restrictive. Second, some assumption about n in equation (4) and T in equations (7) and (8) has to be made. For the experiments, n was assumed to be 32 for both RA and RM , and T was assumed to be 80. In other words, both RA and RM were assumed to be rates on eight-year securities, and the horizon for determining stock prices was assumed to be twenty years. Third, some assumption has to be made about the predicted (i.e., expected) values of r and Π beyond the end of the period for which there are data on the exogenous variables and estimated residuals. One possibility in this case would be to project the exogenous variables and error terms as far into the future as needed to make the end-point effects have a negligible influence on the period of interest. In this study, however, a somewhat simpler procedure was followed. The expected values of r beyond the end of the data (1976II) were assumed to be equal to the average of the last eight expected values within the data period (i.e., to the average of the expected values of r for the 1974III–1976II period). Similarly, the expected values of Π beyond the end of the data were assumed to be equal to the average of the last eight expected values within the data period. As will be discussed in Section V, the results of the experiments do not appear to be very sensitive to this assumption.

In order to make the results for Models 2 and 3 comparable to those for Model 1, perfect tracking solutions for the two models must first be obtained. This can be done as

follows. First, given a value of n , given the actual data on r , and given the above assumption about the values of r beyond the end of the data, equation (4) can be used to compute predicted values of R , say R^* . The difference between RA_t and R_t^* is the estimated residual in predicting RA_t for period t . If this residual is added to equation (4) in the appropriate way, a perfect fit for RA_t is obtained. Likewise, the difference between RM_t and R_t^* is the estimated residual in predicting RM_t for period t , and if this residual is added to equation (4) in the appropriate way, a perfect fit for RM_t is also obtained. Similarly, given a value of T , given the actual data on r and Π , and given the above assumptions about the values of r and Π beyond the end of the data, equations (7)–(9) can be used to compute predicted values of CG , say CG^* . The difference between CG_t and CG_t^* is the estimated residual in predicting CG_t for period t , and if this residual is added to equation (9), a perfect fit for CG_t is obtained. Perfect tracking solutions for Models 2 and 3 can thus be obtained by using these estimated residuals for equations (4) and (9) along with the estimated residuals from Model 1 for the other equations.

Given the above assumptions, the experiments for Models 2 and 3 can now be described. The estimated residuals are first added to each model, and then the model is simulated for the exogenous variable changes. As was the case for Model 1, the simulations began in 1971I and were dynamic. The simulations were allowed to run to the end of the data (1976II), at which point the assumptions about r and Π beyond the end of the data came into play. Results for the first twelve quarters of the simulation period are presented for each model in Tables 1 and 2.

It should be noted that this simulation procedure implicitly assumes that the changes in XG and VBG that began in 1971I were unanticipated changes. If instead it were assumed that the government announced in, say, 1969I that it would make the changes in XG or VBG beginning in 1971I, then the simulations for Models 2 and 3 would have had to begin in 1969I. In other words, in Models 2 and 3 people would have begun in 1969I adjusting to the announced future

¹²The results in Table 1 for Model 1 are the same as the results presented in my 1978a paper, Table 1, for the endogenous Fed case.

policy changes. Some results are reported in Section V for the case in which the anticipation of the changes precedes the actual changes, but for the basic results in Section IV the changes are assumed to have been unanticipated.

One further point about the solution for Model 3 should also be noted. If, say, the first quarter of the simulation period is quarter t , then for Models 1 and 2, SP_{t-1} , the value of stocks at the beginning of quarter t , is predetermined. For Model 3, however, SP_{t-1} is endogenous. It is determined according to equation (7), where the expected future values of Π and r in the equation are the values predicted by the model. This means that CG_{t-1} is also endogenous for Model 3, since it equals $SP_{t-1} - SP_{t-2}$. Consequently, values of CG are presented in Tables 1 and 2 for quarter $t - 1$ as well as for the other twelve quarters for Model 3.

Before proceeding to the discussion of the basic results, it can now be seen why the assumption that people know the actual future values of all the exogenous variables and all the estimated residuals is not very restrictive in the present context. If instead some different set of values were used, based on extrapolations for the exogenous variables and zeros for the residuals, say, and if this same set were used for all three models, it is unlikely that the comparisons across models would be much different. In this case one would take as the estimates of the effects of the policy change for each model the differences between the predicted values of the endogenous variables before and after the change. The predicted values of the endogenous variables before the change would no longer be the actual values. Because the models are non-linear, the actual numbers in Tables 1 and 2 would be different in this case, but these differences are likely to be fairly similar for each of the three models. Therefore, little is likely to be lost in the present context by running the experiments off of the perfect tracking solution.¹³

¹³Although the assumption that people know the actual future values of all exogenous variables and all estimated residuals does not seem restrictive for present purposes, it would clearly not be reasonable to use it in any test of the

IV. The Basic Results

The effects on seven variables are presented in Tables 1 and 2. Each number in the tables is the difference between the predicted value of the variable for the quarter and the actual value; Y is the key output variable in the models and PF is the key price variable.

Consider the results in Table 1 first. The increase in XG led to an increase in output in all three models. The sum of the output increases over the twelve quarters was 16.81 for Model 1, 9.59 for Model 2, and 10.27 for Model 3. The increases in RA and RM for the first four quarters were much smaller for Model 1 than they were for the other two models. This is, of course, as expected. In Models 2 and 3 people expect that the Fed is going to respond to the fiscal policy stimulus by increasing r (i.e., people know equation (3)), and RA and RM adjust immediately to these expectations. In Model 1, on the other hand, RA and RM adjust only to the current and lagged increases in r . Higher values of RA and RM have, other things being equal, a contractionary effect on the economy, and this is the primary reason for the lower values of Y for Models 2 and 3 compared to those for Model 1. The bond rate RA also has, other things being equal, a positive effect on PF , and this is the main reason for the higher values of PF for Models 2 and 3 compared to those for Model 1.

assumption of rational expectations. One possible test that could be performed using Model 3, which is beyond the scope of this paper, is the following: 1) Choose for each quarter a set of future values of the exogenous variables and error terms that one believes were expected at the time. (In most cases the future values of the error terms would be zero.) 2) Using Model 3, compute for each quarter the predicted values of RM , RA , and CG . The predictions of these variables for each quarter would be based on different initial conditions and a different set of future values of the exogenous variables. 3) Compare the accuracy of these predictions to the accuracy of predictions from other models. The joint hypothesis that would be examined or tested by this procedure is that (a) people know Model 3 and believe it to be true, including equations (4) and (7)–(9) that link expected future values to current values, (b) the chosen exogenous variable values and error terms correctly reflect the expectations at the time, and (c) expectations with respect to the future values of r and Π are rational.

TABLE 1—FISCAL POLICY RESULTS: DIFFERENCE BETWEEN THE PREDICTED VALUE AFTER THE CHANGE AND THE ACTUAL VALUE^a

VARIABLES	QUARTERS											SUM 1-12		
	t	t+1	t+2	t+3	t+4	t+5	t+6	t+7	t+8	t+9	t+10		t+11	
<u>r</u> (bill rate)														
1	0.10	0.20	0.26	0.30	0.31	0.31	0.30	0.28	0.27	0.25	0.24	0.24	0.24	
2	0.08	0.15	0.18	0.19	0.19	0.19	0.18	0.18	0.17	0.16	0.16	0.16	0.16	
3	0.08	0.15	0.18	0.20	0.20	0.20	0.20	0.19	0.19	0.18	0.18	0.18	0.18	
<u>RA</u> (bond rate)														
1	0.03	0.06	0.07	0.11	0.14	0.15	0.16	0.17	0.18	0.18	0.18	0.19	0.19	
2	0.21	0.21	0.22	0.22	0.22	0.22	0.22	0.22	0.23	0.23	0.23	0.23	0.23	
3	0.23	0.24	0.24	0.25	0.25	0.25	0.25	0.26	0.26	0.26	0.27	0.27	0.27	
<u>RM</u> (mortgage rate)														
1	0.01	0.03	0.05	0.07	0.10	0.13	0.14	0.15	0.15	0.16	0.16	0.17	0.17	
2	0.21	0.21	0.22	0.22	0.22	0.22	0.22	0.22	0.23	0.23	0.23	0.23	0.23	
3	0.23	0.24	0.24	0.25	0.25	0.25	0.25	0.26	0.26	0.26	0.27	0.27	0.27	
<u>Y</u> (real output)														
1	1.24	1.77	2.04	2.12	1.95	1.72	1.48	1.25	1.03	0.84	0.71	0.66	16.81	
2	0.98	1.12	1.11	1.08	0.97	0.86	0.77	0.68	0.58	0.49	0.46	0.49	9.59	
3	0.93	1.10	1.12	1.12	1.03	0.94	0.86	0.77	0.67	0.58	0.56	0.59	10.27	
<u>100·PF</u> (price deflator)														
1	0.02	0.05	0.09	0.14	0.19	0.25	0.29	0.33	0.37	0.41	0.45	0.48	0.48	
2	0.07	0.13	0.18	0.23	0.27	0.31	0.34	0.37	0.39	0.42	0.45	0.47	0.47	
3	0.08	0.15	0.20	0.25	0.30	0.34	0.38	0.41	0.44	0.47	0.50	0.53	0.53	
<u>CG</u> (capital gains or losses on stocks)														
	<u>t-1</u>													
1	0.00	-2.14	-3.34	-2.55	-6.19	-5.85	-1.74	-1.47	-0.48	-0.37	0.03	0.09	-0.77	-24.78
2	0.00	-24.40	-1.18	-1.04	-0.82	-0.35	-0.06	0.02	0.06	-0.07	-0.15	-0.14	-0.09	-28.22
3	-3.13	-0.34	-0.05	-0.01	-0.00	0.04	0.00	-0.03	-0.06	-0.08	-0.10	-0.15	-0.16	-4.07
<u>VBG</u> (amount of government securities outstanding)														
1	0.66	1.07	1.34	1.61	1.90	2.29	2.79	3.41	4.12	4.93	5.82	6.75	6.75	
2	0.67	1.29	1.90	2.58	3.29	4.06	4.87	5.72	6.64	7.61	8.62	9.66	9.66	
3	0.68	1.28	1.85	2.46	3.09	3.76	4.46	5.20	6.00	6.85	7.72	8.61	8.61	

Notes: Units of variables are: percentage points at an annual rate for *r*, *RA*, and *RM*; billions of 1972 dollars at a quarterly rate for *XG* and *Y*; 1972 = 1.0 for *PF*; billions of current dollars at a quarterly rate for *CG*; billions of current dollars for *VBG*.

^aEffects of a permanent increase in *XG* of \$1.25 billion beginning in quarter *t* (*t* = 1971). 1 = Model 1 (original version); 2 = Model 2 (rational expectations in bond market); 3 = Model 3 (rational expectations in bond and stock markets).

It is interesting to note that the values of *r* are higher for Model 1 than they are for Models 2 and 3. Since the economy is less expansionary for Models 2 and 3 than it is for Model 1, due to the more rapid response of *RA* and *RM*, the Fed raises *r* less in these two cases than it does in the Model 1 case.

The economy is slightly more expansionary for Model 3 than it is for Model 2, and this is easy to explain. For Model 2 there was a large capital loss on stocks in quarter *t* because of the increase in *RA*. (Remember that the estimated equation for CG_t , equation (10), is

still part of Model 2.) For Model 3, however, the negative effects of the higher expected future values of *r* on the value of stocks were almost completely offset by the positive effects of higher expected future values of after-tax cash flow caused by the increase in economic activity. The capital loss incurred at the beginning of quarter *t* was 3.13 for Model 3, compared to the capital loss in quarter *t* of 24.40 for Model 2. Since capital losses have a negative effect on the economy through the wealth effect on the household sector, the economy was somewhat more expansionary

TABLE 2—MONETARY POLICY RESULTS: DIFFERENCE BETWEEN THE PREDICTED VALUE AFTER THE CHANGE AND THE ACTUAL VALUE^a

VARIABLES	QUARTERS											SUM 1-12		
	t	t+1	t+2	t+3	t+4	t+5	t+6	t+7	t+8	t+9	t+10		t+11	
<u>r</u> (bill rate)														
1	-1.74	0.37	1.41	1.43	0.78	-0.05	-0.55	-0.61	-0.40	-0.13	0.12	0.25		
2	-1.35	-0.50	0.06	0.34	0.35	0.27	0.18	0.10	0.05	0.04	0.04	0.05		
3	-1.35	-0.52	0.04	0.33	0.36	0.30	0.22	0.13	0.08	0.06	0.05	0.05		
<u>RA</u> (bond rate)														
1	-0.73	0.21	0.12	0.19	0.16	-0.01	-0.07	-0.06	-0.03	-0.02	-0.01	0.01		
2	-0.02	0.04	0.05	0.05	0.04	0.03	0.02	0.01	0.01	0.01	0.01	0.01		
3	-0.02	0.04	0.06	0.05	0.04	0.03	0.01	0.01	0.01	0.00	0.00	0.00		
<u>RM</u> (mortgage rate)														
1	-0.14	-0.48	0.21	0.08	0.14	0.11	-0.00	-0.03	-0.02	-0.01	-0.02	-0.01		
2	-0.02	0.04	0.05	0.05	0.04	0.03	0.02	0.01	0.01	0.01	0.01	0.01		
3	-0.02	0.04	0.06	0.05	0.04	0.03	0.01	0.01	0.01	0.00	0.00	0.00		
<u>Y</u> (real output)														
1	0.77	1.98	1.39	0.44	-0.51	-0.85	-0.62	-0.19	0.14	0.27	0.26	0.15	3.23	
2	0.39	0.59	0.54	0.35	0.13	0.00	-0.06	-0.07	-0.06	-0.04	-0.02	-0.02	1.73	
3	0.39	0.57	0.54	0.36	0.15	0.02	-0.04	-0.06	-0.05	-0.03	-0.02	-0.01	1.82	
<u>100-PF</u> (price deflator)														
1	-0.24	-0.10	-0.02	0.06	0.11	0.08	0.03	0.00	-0.00	0.00	0.01	0.02		
2	-0.00	0.02	0.04	0.05	0.06	0.06	0.06	0.05	0.05	0.05	0.04	0.04		
3	-0.00	0.02	0.04	0.05	0.06	0.06	0.06	0.05	0.05	0.04	0.04	0.04		
<u>CG</u> (capital gains or losses on stocks)														
	<u>t-1</u>													
1	0.00	92.54	-117.66	8.89	-11.96	2.57	23.66	10.47	0.13	-3.09	-2.37	-2.63	-2.70	-2.15
2	0.00	2.76	-7.30	-2.35	-0.44	1.19	1.95	1.53	0.93	0.45	0.22	0.07	0.04	-0.95
3	-2.00	-0.78	-0.21	0.15	0.34	0.33	0.21	0.11	0.04	0.02	0.02	0.03	0.04	-1.70
<u>VBG</u> (amount of government securities outstanding)														
1	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	
2	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	
3	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	-1.25	

Notes: See Table 1.

^aEffects of a permanent decrease in *VBG* of \$1.25 billion beginning in quarter *t* (*t* = 1971I). 1 = Model 1 (original version); 2 = Model 2 (rational expectations in bond market); 3 = Model 3 (rational expectations in bond and stock markets).

for Model 3 than it was for Model 2. This difference is, however, much smaller than the difference between the results for Models 1 and 2. In other words, adding rational expectations in the bond market to the original version of the model makes more of a difference than does the further addition of rational expectations in the stock market. In this sense, wealth effects in the model are less important than interest rate effects.

Consider now the monetary policy experiment in Table 2. The experiment itself is not as expansionary as the experiment in Table 1, but the comparison across models in Table 2 is similar to that in Table 1. The economy is

more expansionary for Model 3 than it is for Model 2. For Model 1 the decrease in *VBG* led to a large decrease in *r* in quarter *t* and then a bounce back again in quarter *t* + 1. The rate *RA* was affected in a similar way, which resulted in a large capital gain in quarter *t* and a large capital loss in quarter *t* + 1. For Models 2 and 3, on the other hand, *RA* and *RM* were much less affected by the large initial change in *r*. Rates *RA* and *RM* in fact changed very little for Models 2 and 3 because the long-run effect of the change in *VBG* on *r* was fairly small. In other words, people expect in Models 2 and 3 that the large initial drop in *r* is temporary, and so *RA* and

RM do not respond very much to this event. In Model 1 people do not expect this, and so the initial changes in RA and RM are much larger.

Comparing the results for Models 2 and 3 in Table 2, it can be seen that after the first four quarters Model 2 has a cumulative capital loss of 7.33 compared to a cumulative capital loss of only 2.50 for Model 3. This is the main reason for the slightly more expansionary economy for Model 3. The difference between the results for the two models is, however, quite small, and the cumulative capital loss over twelve quarters is in fact slightly larger for Model 3 than it is for Model 2.

In summary, then, the results in Tables 1 and 2 indicate that the long-run effects of fiscal policy and monetary policy actions on real output are a little over half as large if there are rational expectations in the bond and stock markets than if there are not. For the fiscal policy experiment, the sum of the output increases over twelve quarters for Model 3 is 61.1 percent of that for Model 1 (10.27/16.81). The corresponding figure for the monetary policy experiment is 56.3 percent (1.82/3.23). The results also indicate that the addition of rational expectations in the bond market to the model is quantitatively more important than is the addition of rational expectations in the stock market.

V. Further Results

The results of three other experiments are reported in this section. The first is the same as the first experiment in Section IV except that the behavior of the Fed is assumed to be different. Instead of assuming that the Fed behaves according to equation (3), it was assumed that the Fed behaves by keeping r unchanged each period from its historic value. In this case the behavior of the Fed is exogenous in the sense that its behavior with respect to the value of r each period is not a function of any endogenous or lagged endogenous variables in the model. However, VBG is still endogenous even though equation (3) is dropped from the model, since r is now exogenous.

The results for Models 1 and 2 were nearly

identical for this experiment. The sum of the changes in r over the first twelve quarters was 27.73 for Model 1 and 28.07 for Model 2. For Model 2, RA and RM were completely unchanged, as is obvious from equation (4), but for Model 1 they were slightly higher as a result of the inflation term in equations (5) and (6). The slightly higher values of RA and RM thus led to a slightly smaller output increase in Model 1 than in Model 2. This difference is, however, almost negligible. The main point of this example is that if the Fed keeps r unchanged, then the policy implications of Models 1 and 2 are quite similar. In Model 2 people expect that the short rates will not change in response to the fiscal policy stimulus, and so there is no change in the long rates. In Model 1 people have no explicit expectations of this sort, but a property of the estimated term-structure equations is that long rates do not change much if short rates do not change.

The second experiment is the same as the first experiment in Section IV except that the starting quarter for the change in XG was taken to be 1958I rather than 1971I. As was the case for the results in Section IV, the simulations were run to the end of the data (1976II) for Models 2 and 3, at which point the assumptions about r and Π beyond the end of the data came into play. For this longer period the results for Models 2 and 3 for, say, the first twelve quarters should be less sensitive to the end-point assumptions. It turned out, however, that these results were quite similar to the results for the shorter period presented in Section IV. For the experiment that began in 1958I, the sum of the changes in Y over the first twelve quarters was 16.27 for Model 1 and 8.04 for Model 3. The Model 3 response was thus 49.4 percent of the Model 1 response (8.04/16.27), which is only slightly lower than the figure of 61.1 percent for the first experiment in Section IV. The other results were also similar between the two experiments. It thus appears that the use in Section IV of only ten quarters beyond the basic twelve-quarter prediction period for Models 2 and 3 is enough to capture most of the effects of the future predicted values on the present predicted values.

The third and final experiment is the same

as the first experiment in Section IV except that it was assumed that the government announced the fiscal policy action (to begin in 1971I) in 1958I. In other words, the policy change was assumed to be announced thirteen years before it was actually made. The starting quarter for this experiment was 1958I. For Model 1 the results for the 1971I–1973IV period are exactly as reported in Table 1, since in this model future changes in exogenous variables do not affect current predicted values. For Models 2 and 3, however, the results are different. For Model 3, for example, there was a cumulative output loss between 1958I and 1970IV of 5.70. This output loss was due to the fact that people expected the Fed to raise the bill rate in response to the fiscal policy stimulus, and these expectations got reflected in the values of the long rates before 1971I. The higher long rates then had a contractionary effect on the economy prior to the actual change in XG . After the change in XG in 1971I there was a cumulative output gain during the next twelve quarters (1971I–1973IV) of 10.50. The net cumulative gain through 1973IV was thus 4.80, compared to the 10.27 figure in Table 1 for the unanticipated change. The cumulative output gain that occurs when the policy change is anticipated is thus slightly less than half of the gain that occurs when the policy change is unanticipated.

VI. Summary and Conclusion

This study has demonstrated that it is feasible to analyze a large-scale macro-econometric model with rational expectations in the bond and stock markets. The primary purpose of this paper has been to present some quantitative estimates of the sensitivity of monetary and fiscal policy effects to the assumption of rational expectations in the bond and stock markets. With some qualifications, the results indicate that unanticipated policy actions are about half as effective and anticipated policy actions are about one-fourth as effective (with respect to real output changes) when there are rational expectations in these markets than when there are not. The results also indicate that the existence of rational expectations in the bond market is

quantitatively more important than is the existence of rational expectations in the stock market.

The results of this paper must, however, be interpreted with the usual degree of caution. First, they are dependent on a particular model, and there is at least a small probability that this model is not a good representation of the economy. Second, it is well known that multipliers in non-linear models depend on initial conditions, and the basic starting point for the results in this paper was a quarter that was at or near the bottom of a contraction. Clearly, different results would have been obtained had, say, the starting point been the top of an expansion. Third, it is also well known that fiscal policy effects depend on what one assumes about Fed behavior, and, as reported in Section V, quite different results can be obtained if it is assumed that the Fed behaves differently than is estimated by equation (3).

Given these caveats, it is hoped that this study has made some progress in determining the quantitative importance of the rational expectations assumption with respect to the bond and stock markets. It appears from the present results that this assumption is of considerable quantitative significance.

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