Journal of Monetary Economics 22 (1988) 301–315. North-Holland

OPTIMAL CHOICE OF MONETARY POLICY INSTRUMENTS IN A MACROECONOMETRIC MODEL*

Ray C. FAIR

Yale University, New Haven, CT 06520, USA

Received February 1987, final version received October 1987

This paper uses stochastic simulation and my U.S. econometric model to examine the optimal choice of monetary policy instruments. Are the variances, covariances, and parameters in the model such as to favor one instrument over the other, in particular the interest rate over the money supply? The results for the regular version of the model provide support for what seems to be the Fed's current choice of using the interest rate as its primary instrument. On the other hand, the results support the use of the money supply as the primary instrument if there are rational expectations in the bond market.

1. Introduction

Nearly twenty years ago today Poole (1970) wrote his classic article on the optimal choice of monetary policy instruments in a stochastic IS-LM model. Poole assumed that the monetary authority (henceforth called the Fed) can control the interest rate (r) or the money supply (M) exactly. These are the two 'instruments' of monetary policy. If the aim is to minimize the squared deviation of real output from its target value, Poole showed that the choice of the optimal instrument depends on the variance of the error term in the IS function, the variance of the error term in the LM function, the covariance of the two error terms, and the size of the parameters.

Most people would probably agree that between about October 1979 and October 1982 the Fed put more emphasis on monetary aggregates than it did either before or after. Otherwise, the interest rate has seemed to be the Fed's primary instrument. It is interesting to ask if the use of the interest rate can be justified on the basis of the Poole analysis. Is the economy one in which the variances, covariances, and parameters are such as to lead, a la the Poole analysis, to the optimal instrument being the interest rate?

*The research described in this paper was financed by a grant from the National Science Foundation. I am indebted to Lewis Alexander and James Tobin for helpful discussions regarding the subject matter of this paper. I am also indebted to a referee for helpful comments.

The purpose of this paper is to examine this question using my U.S. econometric model. Are the variances, covariances, and parameters in the model such as to favor one instrument over the other, in particular the interest rate over the money supply? This question can be examined in an econometric model by the use of stochastic simulation. Interestingly enough, Poole's analysis has never been tried on an actual econometric model. The closest study in this respect is that of Tinsley and von zur Muehlen (1983), although they did not analyze the same question that Poole did.[1] Other studies that have extended Poole's work, such as those of Turnovsky (1975) and Yoshikawa (1981), have been primarily theoretical.

Poole also showed that there is a combination policy that is better than either the interest rate policy or the money supply policy. This is the policy where the Fed behaves according to the equation $M=\alpha+\beta r$, where the parameters α and β are chosen optimally.[2] It is possible through repeated stochastic simulation to find the optimal values of α and β for an econometric model, and this is also done in this paper.

Results of a study like this are model-specific. How much confidence one places on the results depends in part on how good an approximation one thinks my model is of the structure of the economy. Since this paper shows that stochastic simulation can be used to examine Poole-like questions in large econometric models, it would be interesting to apply this methodology to other models. For the results in this study I have considered three versions of my model: the regular version, a more interest-sensitive version, and a version in which there are rational expectations in the bond market. It will be seen that the results are somewhat sensitive to which version is used.

2. The model

My model is described in detail in Fair (1984), and it will only be briefly discussed here. The model has been estimated through 1987.I for this study. The beginning quarter is 1954.I. There are 29 structural equations, estimated by two-stage least squares, and 98 identities. When there was evidence of

[1] In their stochastic simulation experiments, Tinsley and von zur Muehlen always used the interest rate (the Federal Funds rate) as the policy instrument. They used this instrument to target a particular variable, called an 'intermediate' target. The intermediate targets they tried are the monetary base, three definitions of the money supply, nominal GNP, and the Federal Funds rate itself. For each of these target choices, they examined how well the choice did in minimizing the squared deviations of the unemployment rate and the inflation rate from their target values. The unemployment rate and the inflation rate are the 'ultimate' targets. In the present study the aim is to see how well the interest rate does when it is used as the policy instrument in minimizing the squared deviations of real output from its target value compared to how well the money supply does when it is used as the policy instrument. This is the question that Poole examined.

[2] See also Tobin (1982) for a discussion of this.

first-order serial correlation of the error term in an equation, the first-order serial correlation coefficient was estimated along with the other coefficients in the equation. The error terms relevant for the stochastic simulations are the error terms *after* elimination of serial correlation, i.e., the error terms that are being dealt with are not serially correlated. The serial correlation coefficients are simply treated as structural coefficients.

The model accounts for all flows of funds among the sectors and all balance-sheet constraints. This is done by linking the National Income Accounts to the Flow of Funds Accounts. This allows one to deal directly with the three 'tools' of the Fed: the discount rate, the reserve requirement rate, and the amount of government securities in the hands of the public. This third tool, denoted AG in the model, is the 'open-market-operations' variable. It is the main variable used by the Fed in practice to manipulate the money supply and interest rates. The discount rate and the reserve requirement rate are minor tools, and they are always taken to be exogenous in the model.

In the basic version of the model there is an estimated interest rate reaction function. The reaction function is an equation with the short-term interest rate (the three-month Treasury bill rate) on the left-hand side and variables that are postulated to affect Fed behavior on the right-hand side. According to this equation, the Fed 'leans against the wind' in the sense that it raises the bill rate as real growth increases, labor markets become tighter, inflation increases, and the lagged growth rate of the money supply increases. In this version of the model both the money supply and the interest rate are endogenous. The money supply is determined by the demand for money equations and the interest rate is determined by the reaction function. Monetary policy (AG) is thus endogenous in this version. The value of AG each quarter is whatever is needed to have the interest rate be the value predicted from the interest rate reaction function.

It is possible to drop the interest rate reaction function from the model and make some other assumption about monetary policy. Three assumptions are considered here. One is that the interest rate is exogenous, one is that the money supply is exogenous, and one is that the Fed behaves according to the rule $M = \alpha + \beta r$. In all three cases AG is still endogenous. Its value each quarter is whatever is needed to have the targets be met.

It will be useful to consider briefly how interest rates enter the model. There are four interest rates in the model: the discount rate, which is always exogenous; the bill rate; and two long-term rates, the AAA corporate bond rate and a mortgage rate. In the regular version the long-term rates are determined by standard term-structure-of-interest-rate equations. Each long rate is a function of current and past values of the short rate.

There are two demand for money equations in the model, one for the household sector and one for the firm sector. The equations are fairly standard. The demand for real money balances is a function of the short-term interest rate, a transactions variable, and the lagged dependent variable. 'Money' includes both demand deposits and currency. There is also a separate demand for currency equation, where the demand for currency is a function of the short-term interest rate, a transactions variable, and the lagged dependent variable. There is a bank borrowing equation in the model, where bank borrowing from the Fed is a positive function of the difference between the bill rate and the discount rate. These four equations will be referred to as the 'money equations'. Of the four, the two demand for money equations are by far the most important; the other two play a fairly minor role in the model.

The bill rate (as a measure of short-term interest rates) appears as an explanatory variable in the service consumption equation, and the mortgage rate (as a measure of long-term interest rates) appears in the durable consumption equation and the housing investment equation. In addition, the mortgage rate appears as an explanatory variable in the demand for imports equation. (The interest rate coefficients are all negative.) The change in the bond rate appears as an explanatory variable in the equation determining the change in stock prices (with a negative sign).

Consider now what happens when the bill rate increases. This raises long-term rates through the term structure equations. These interest rate rises have a direct negative effect on service and durable consumption, housing investment, and imports. The fall in consumption and housing investment has a negative effect on GNP, but the fall in imports has a positive effect. The net effect could thus go either way, but it is in fact negative in the model.

The increase in the bond rate has a negative effect on stock prices, which lowers household wealth. Household wealth is an explanatory variable in the consumption and housing investment equations (with a positive sign), and so the decrease in wealth has a negative effect on consumption and housing investment. Household demand thus falls when interest rates rise for two main reasons. One is the direct negative effect of interest rates on demand, and the other is the indirect effect of interest rates affecting wealth and then wealth affecting demand.

Offsetting these two negative effects in part is the fact that net interest payments to the household sector rise when interest rates rise. Interest payments to the household sector are part of nonlabor income, and nonlabor income is an explanatory variable in the consumption and housing investment equations (with a positive sign). Therefore, a rise in interest payments, other things being equal, leads to an increase in household demand.

So far no mention has been made of plant and equipment (P&E) investment. The equation determining P&E investment is an accelerator-like equation, and the interest rate does not appear in this equation. I have been unable to find significant interest rate effects in this equation, although this is not from lack of trying. Interest rates do, however, have a negative effect on P&E investment in the model because they have a negative effect on output. In other words, interest rates affect P&E investment by first affecting household demand, which affects the level of sales, which affects production, which affects investment.

The version of the model that has rational expectations in the bond market will be called the RE version, although it should be remembered that the bond market is the only place where expectations are assumed to be rational. In the RE version the two term structure equations are replaced with equations that are consistent with there being rational expectations in the bond market. Let R be a long-term rate (either the bond rate or the mortgage rate), and let r be the short-term rate. Assume that R is a five-year (twenty-quarter) rate and that r is a three-month (one-quarter) rate, both at annual rates. According to the expectations theory of the term structure of interest rates,

$$(1+R)^{20}=(1+r)(1+r^{e}_{+1})(1+r^{e}_{+2})\dots(1+r^{e}_{+19}), \tag{1}$$

where r^{e}_{+i} is the expected value of the short-term rate i periods into the future. If expectations are rational in the Muth (1960) sense, the expected values of the future short-term interest rates are equal to the model's predictions of the rates. In the RE version of the model eq. (1) was imposed for both the bond rate and the mortgage rate: the estimated term structure equations were not used.[3]

In the stochastic simulation work account was taken of exogenous-variable uncertainty as well as uncertainty from the 28 stochastic structural equations. (There are 28 rather than 29 stochastic structural equations in the version of the model used in this paper because the interest rate reaction function is dropped.) Autoregressive equations were estimated for 23 exogenous variables in the model. These variables make up the main exogenous variables in the model. The autoregressive equations were eighth-order and contained a constant and time trend. These 23 equations were then added to the model, resulting in a model with 51 stochastic equations for the regular version of the model. For the RE version of the model there are only 49 stochastic equations because the two term structure equations, which are stochastic, are each replaced by (1), which is not stochastic in the sense of having no structural error term.

[3]This is the same procedure that was followed in Fair (1979). The results in Fair (1979) showed that the properties of my model are sensitive to whether or not there are rational expectations in the bond market. The RE version of the model was solved using the method in Fair and Taylor (1983). One eight-quarter solution of the RE version takes about 26 minutes on a VAX 730, which is about 100 times longer than for the regular version.

3. Comparison of the two policy instruments

3.1. The procedure

As noted in section 1, stochastic simulation can be used to estimate variances in econometric models. The appendix describes the procedure that was used in this paper. The simulations were run over the eight-quarter period, 1977.I–1978.IV. A path of values of the interest rate (the bill rate) was chosen for this period, and this path was used for the simulations in which the interest rate was the policy instrument. Similarly, a path of values of the money supply was chosen, and this path was used for all the simulations in which the money supply was the policy instrument.[4] Each stochastic simulation consisted of 1000 trials.

The variance of real GNP for a given quarter corresponds to Poole's loss function if one takes the target value of GNP for that quarter to be the mean value from the stochastic simulation, which is done here. One can compare the variances of GNP for the two policy instruments. If the variance is smaller when the interest rate is the policy instrument, this is evidence in favor of the interest rate, and vice versa if the variance is smaller when the money supply is the policy instrument.

It should be noted that variances are computed for each quarter of the eight-quarter simulation period. The simulations are dynamic, so that, for example, the computed variance for the fourth quarter is the variance of the four-quarter-ahead prediction error. Note also that when the interest rate is the policy instrument, the eight-quarter path for the interest rate is fixed across all simulation trials. The values in the path vary from one quarter to the next (they are the predicted values from the simulation with the error terms set to zero), but for a given quarter the value is the same across all trials. Similarly, when the money supply is the policy instrument, the eight-quarter path for the money supply is fixed across all trials.

In the following discussion $\tilde{\sigma}_{it}^2(r)$ will refer to the stochastic simulation estimate of the variance of variable i for period t when the interest rate is the

[4]The paths were chosen as follows. A dynamic simulation was first run over the eight-quarter period with the error terms set to zero and the interest rate reaction function included in the model. The predicted values of the bill rate from this simulation were then taken as the values for the interest rate path. Likewise, the predicted values of the money supply were taken as the values for the money supply path. To see if the results were sensitive to the choice of the base paths, a stochastic simulation was run with the money supply as the policy instrument in which the base path for the money supply was taken to be a smoothly growing money supply (at an annual rate of 8.0 percent). The results in table 1 below were little affected by this choice. For example, the percentage differences for real GNP for the eight quarters were, respectively, 4.94, 10.99, 12.00, 10.27, 8.91, 4.71, 0.86, and −1.06, which compare closely to the numbers in table 1.

When the money supply is the policy instrument, the question arises as to whether it is the nominal or the real money supply that is the instrument. This question does not arise in Poole's analysis because the price level is exogenous. For purposes of this paper the nominal money supply is taken to be the policy instrument.

Table 1

Percentage differences between the variance under the money supply policy and the variance under the interest rate policy: Regular version of the model.

	Quarters ahead							
	1	2	3	4	5	6	7	8
Real GNP	4.90 (1.55)	10.84 (2.70)	11.99 (3.05)	9.97 (3.28)	8.49 (3.30)	4.04 (3.08)	−0.04 (2.92)	−2.27 (2.92)
Consumption of services	50.74	75.46	59.33	53.47	51.32	43.58	40.46	34.22
Consumption of nondurables	1.90	0.00	0.84	−0.06	−0.44	−1.42	−2.10	−1.21
Consumption of durables	27.11	68.31	87.14	80.98	84.66	69.40	58.35	53.73
Housing investment	0.00	11.39	26.59	27.40	29.45	25.95	21.61	20.18
P&E investment	−1.94	−2.24	−2.61	−2.25	1.21	−1.24	−1.34	−2.19
Inventory investment	−2.31	−1.49	2.33	3.22	5.02	4.28	1.38	1.57
Imports	0.32	10.40	21.44	32.03	46.15	45.79	44.83	46.70
Change in stock prices	90.22	167.48	117.69	161.14	144.95	151.37	162.70	169.95
Inflation rate	0.54	0.15	0.47	0.57	−0.15	0.53	−0.23	−0.49
Unemployment rate	3.70	8.35	9.74	6.93	4.91	3.79	−0.15	−3.14
Profits	1.93	2.73	3.30	3.30	4.35	2.01	−1.44	−0.39

policy instrument. $\tilde{\sigma}_{it}^2(M)$ will refer to the same thing when the money supply is the policy instrument.

3.2. Results for the regular version of the model

The percentage differences between the two variances are presented in table 1 for selected variables in the model. In terms of the above notation, each number in the table is $100 \cdot [\tilde{\sigma}_{it}^2(r) - \tilde{\sigma}_{it}^2(M)]/\tilde{\sigma}_{it}^2(r)$. Remember that for Poole's loss function i is equal to real GNP, and so the results in table 1 for real GNP are the percentage differences between the two loss-function values. The numbers in parentheses for real GNP in table 1 are estimated standard errors of the percent differences. They are a measure of the accuracy of the stochastic simulation estimates.[5] To conserve on space, only the standard errors for GNP are presented.

[5]The appendix discusses the computation of the standard errors. The numbers in parentheses are actually the standard errors of the absolute differences [denoted $\mathrm{var}(\tilde{\delta}_{it})$ in the appendix] divided by $\tilde{\sigma}_{it}^2(r)$.

The results for real GNP show that for the first six quarters the interest rate policy is better, and for the remaining two quarters the money supply policy is better.[6] The largest difference is 11.99 percent in the third quarter. Overall, the differences for GNP are fairly small, and so as a practical matter it does not seem to matter very much which policy is used. The results for GNP do, however, mask some important differences for other variables, as can be seen in the rest of table 1. The four most interest-sensitive components of GNP in the model are consumption of services, consumption of durables, housing investment, and imports, and it is clear for these four variables that the variances are much higher when the money supply is the policy instrument. For example, for the four-quarter-ahead prediction the variance of the consumption of services is 53.47 percent higher for the money supply policy. The variance of the consumption of durables is 80.98 percent higher, the variance of housing investment is 27.40 percent higher, and the variance of imports is 32.03 percent higher. The variances of the change in stock prices are also considerably higher for the money supply policy, by a factor of around 1.5. (The change in stock prices is also interest-sensitive.) Given that the interest-sensitive components of real GNP have considerably larger variances for the money supply policy, it is interesting and perhaps somewhat surprising that the variances of real GNP are so close for the two policies. One of the reasons for this is the following. Consider for the money supply policy a shock to one of the demand for money equations that leads to an increase in the interest rate. This has a direct negative effect on consumption, housing investment, and the demand for imports. The fall in consumption and housing investment has a negative effect on GNP, but the fall in imports has a positive effect. The *net* effect on GNP is thus smaller than would be the case if all the components affected GNP in the same direction. In other words, negative interest rate effects on consumption and housing investment are in part offset by negative effects on imports.

Consider next the case in which there are no shocks to the money equations. If in Poole's model there are no shocks to the LM function, the money supply policy is better. It is interesting to see if something similar holds in my model. This can be done by setting the error terms in the four money equations to zero across all trials and running the stochastic simulations again. The results of doing this are presented in table 2 for real GNP and its components. The differences for GNP are all negative, as expected. The percentage difference for real GNP four quarters out is -5.36 percent, which compares to 9.97 percent in table 1. The difference between these differences is thus about 15 percent. This means that the gain for the money supply policy of there being no errors in the money equations is about 15 percent (four quarters out).

[6]By 'interest rate policy' is meant the case in which the interest rate is the policy instrument, and by 'money supply policy' is meant the case in which the money supply is the policy instrument.

Table 2

Percentage differences between the variance under the money supply policy and the variance under the interest rate policy: Regular version of the model, no shocks to the money equations.

	Quarters ahead							
	1	2	3	4	5	6	7	8
Real GNP	−0.87	−2.08	−3.76	−5.36	−6.80	−8.04	−8.94	−10.24
Consumption of services	−2.20	−3.62	−4.99	−6.57	−8.58	−10.09	−11.61	−13.25
Consumption of durables	−1.00	−2.83	−5.34	−7.65	−10.25	−12.02	−13.88	−15.50
Housing investment	0.00	−0.60	−1.70	−2.77	−3.73	−4.80	−5.71	−6.82
P&E investment	−0.09	−0.36	−0.99	−2.03	−3.34	−4.57	−5.82	−7.35
Inventory investment	0.04	0.10	−0.09	−0.64	−1.23	−1.45	−1.51	−1.01
Imports	−0.02	−0.23	−0.58	−1.67	−3.34	−4.81	−7.40	−8.55

3.3. *Results for a more interest-sensitive version of the model*

One way in which my model may be in error is either to overestimate or underestimate the interest sensitivity of the components of GNP. To see the effects of this in the present context, a version of the model was created in which the coefficients of the interest rate in three equations were doubled in absolute value. The three equations are the consumption of services, consumption of durables, and housing investment equations. The stochastic simulations were run for this more interest-sensitive version of the model. The results are presented in table 3 for GNP and for the three components whose equations were changed.

The differences in table 3 are larger than those in table 1. In other words, given the particular parameter estimates and covariance estimates in the

Table 3

Percentage differences between the variance under the money supply policy and the variance under the interest rate policy: More interest-sensitive version of the model.

	Quarters ahead							
	1	2	3	4	5	6	7	8
Real GNP	8.04	18.86	22.27	21.43	19.73	13.74	7.64	4.65
Consumption of services	53.81	119.67	97.98	88.94	83.98	72.04	66.69	57.15
Consumption of services	41.27	100.55	126.28	119.49	123.38	103.31	87.90	80.95
Housing investment	0.00	18.29	41.46	43.02	45.41	40.58	34.76	32.85

Table 4

Percentage differences between the variance under the money supply policy and the variance under the interest rate policy: RE version of the model.

	Quarters ahead							
	1	2	3	4	5	6	7	8
Real GNP	−2.43 (5.06)	−4.60 (7.79)	−2.55 (8.25)	−4.88 (11.09)	−10.22 (9.57)	−8.86 (8.74)	−10.71 (6.46)	−11.60 (5.75)

model, the interest rate policy gains relative to the money supply policy when the interest sensitivity of the components of GNP is increased.

3.4. *Results for the RE version of the model*

The results for the RE version of the model are presented in table 4. As noted above, the RE version of the model takes about 26 minutes of CPU time on a VAX 730 for one trial. It would thus take about 18 days for 1000 trials. Instead, 40 trials were done, which took about 17 hours. The results in table 4 thus suffer much more from stochastic simulation error than do the results in the other tables. As can be seen, the standard errors for GNP are fairly large relative to the size of the differences, and one can only get a general idea of the properties of the RE version.[7]

The money supply policy does better than the interest rate policy for the RE version: the differences in table 4 are all negative. This is contrary to the case for the regular version of the model. One reason for this is the following. For the money supply policy, shocks to the system affect the short-term interest rate. Consider a shock for the current quarter that affects the short-term interest rate for that quarter. In the regular version of the model this affects the long-term rates through the term structure equations. Because long rates are a function of current and past short-term rates, the initial shock that affected the short-term rate affects the long-term rates over a number of quarters. In the RE version of the model, on the other hand, a shock that affects the short-term rate in a given quarter does not have much affect on the long-term rates. Agents know the shock is only for the current quarter, and

[7]Stochastic simulation is performed for the RE version in the following way for the eight-quarter period. For the solution for quarter 1 for the first trial, a set of error terms for quarter 1 is drawn and the error terms for quarters 2 and beyond are set to zero. Setting future error terms to zero is what agents would do when using the model to form their expectations of the future values of the short-term interest rate. Given the solution for quarter 1 [by the Fair–Taylor (1983) method], quarter 2 is solved, where a set of error terms for quarter 2 is drawn and the error terms for quarters 3 and beyond are set to zero. This process is repeated through quarter 8. At the end of this process one has one set of solution values of the endogenous variables for quarters 1 through 8. This is one trial. For the second trial the entire process begins again.

this does not affect much their expectations (the model's predictions) of the future short-term interest rates. If expectations of the future short-term rates are not much affected, from eq. (1) the long-term rates will not be much affected. Therefore, under the RE version of the model, the long-term interest rates have less variance than they do under the regular version, and this leads to lower variances of real GNP.

Another way of looking at this is that the RE version in effect lessens the sensitivity of some of the components of GNP to the short-term interest rate, which benefits the money supply policy. The sensitivity is lessened if the components are a function of long-term rates because the long-term rates vary less relative to the short-term rates for the RE version.

4. The optimal policy

The optimal policy is defined here to be the policy where the Fed behaves according to the equation

$$\log M = \log M^* + \beta(r - r^*), \tag{2}$$

where M^* and r^* are, respectively, values of the money supply and the interest rate from the base path (values that do not change from trial to trial) and β is the parameter to be determined. The optimal value of β was determined as follows. Eq. (2) was added to the model and a particular value of β was chosen. A stochastic simulation of 1000 trials was run, and the variances of GNP for the eight quarters were recorded. Another value of β was chosen, and a new stochastic simulation was run. This process was repeated for a number of values of β, and the value of β that led to the smallest variances of GNP was taken to be the optimal value. The optimal value was 1.5.[8] The results using this value of β are presented in table 5, where the numbers are the percentage differences between the variance under the optimal policy and the variance under the interest rate policy.

The differences in table 5 are fairly small. For example, for GNP four quarters out the variance under the optimal policy is only 2.54 percent smaller than the variance under the interest rate policy. In other words, the interest rate policy is close to the optimal policy for the regular version of the model. This is because the optimal value of β of 1.5 is fairly high. It says that a one percentage point change in the short-term interest rate leads roughly to a 1.5 percent increase in the money supply.

[8]The value of β of 1.5 gave the smallest variances of real GNP for the second, third, fourth, and sixth quarters ahead. A value of 2.0 gave the smallest variances for the first and fifth quarters ahead, and a value of 1.25 gave the smallest variances for the seventh and eighth quarters ahead. The results between 1.25 and 2.0 were close, and although 1.5 was chosen as the optimal value, any number within this range could have been chosen.

Table 5

Percentage differences between the variance under the optimal policy and the variance under the interest rate policy: Regular version of the model.

	Quarters ahead							
	1	2	3	4	5	6	7	8
Real GNP	−0.05	−0.74	−1.56	−2.54	−2.45	−3.19	−3.85	−4.40
Consumption of services	−1.58	0.00	−1.94	−1.31	−1.44	−0.89	−0.10	0.93
Consumption of nondurables	−0.02	−0.28	−0.41	−0.59	−0.79	−0.93	−1.24	−0.99
Consumption of durables	0.66	1.71	4.93	4.59	7.91	6.29	5.72	7.32
Housing investment	0.00	−1.48	−1.93	−2.26	−0.66	−0.42	−0.68	−0.18
P&E investment	−0.32	−0.62	−1.34	−2.33	−2.25	−3.20	−3.11	−3.37
Inventory investment	−0.44	−0.37	−0.26	−0.47	−0.56	−0.36	−0.63	−0.98
Imports	0.01	0.36	0.42	0.42	3.09	3.49	3.16	4.23

5. Conclusion

This study has shown that stochastic simulation can be used to consider the optimal choice of monetary policy instruments in econometric models. The present results obviously depend on the properties of my model, and it would be of interest to see if similar results hold for other models. The results for the regular version of the model provide some support for what seems to be the Fed's current choice of using the interest rate as its primary instrument. The support is even greater for the more interest-sensitive version. On the other hand, the results provide support for the use of the money supply as the primary instrument if there are rational expectations in the bond market.

Appendix

The use of stochastic simulation to estimate variances in nonlinear econometric models is discussed in this appendix. Write the model as

$$f_i(y_t, x_t, \alpha_i) = u_{it}, \qquad i = 1, \ldots, n, \quad t = 1, \ldots, T, \tag{A.1}$$

where y_t is an n-dimensional vector of endogenous variables, x_t is a vector of predetermined variables, α_i is a vector of unknown coefficients, and u_{it} is an error term. The first m equations are assumed to be stochastic, with the remaining u_{it} $(i = m+1, \ldots, n)$ identically zero for all t. It is assumed that $u_t = (u_{1t}, \ldots, u_{mt})$ is independently and identically distributed as multivariate normal $N(0, \Sigma)$.[9] It is also assumed that consistent estimates of α_i, denoted $\hat{\alpha}_i$, are available for all i. Given these estimates, consistent estimates of u_{it}, denoted $\hat{u}_{it}$, can be computed as $f_i(y_t, x_t, \hat{\alpha}_i)$. The covariance matrix Σ can then be estimated as $(1/T)\hat{U}\hat{U}$, where $\hat{U}$ is the $m \times T$ matrix of the values of $\hat{u}_{it}$.

Let u_t^* denote a particular draw of the m error terms for period t from the $N(0, \hat{\Sigma})$ distribution. Given u_t^* and given $\hat{\alpha}_i$ for all i, one can solve the model for period t. This is merely a deterministic simulation for the given values of the error terms and coefficients. Call this simulation a 'trial'. Another trial can be made by drawing a new set of values of u_t^* and solving again. This can be done as many times as desired. From each trial one obtains a prediction of each endogenous variable. Let y_{it}^j denote the value on the jth trial of variable i for period t. For J trials, the stochastic simulation estimate of the expected value of variable i for period t, denoted $\tilde{\mu}_{it}$, is

$$\tilde{\mu}_{it} = (1/J) \sum_{j=1}^{J} y_{it}^j. \tag{A.2}$$

Let

$$\sigma_{it}^{2j} = \left(y_{it}^j - \tilde{\mu}_{it} \right)^2. \tag{A.3}$$

The stochastic simulation estimate of the variance of variable i for period t, denoted $\tilde{\sigma}_{it}^2$, is then

$$\tilde{\sigma}_{it}^2 = (1/J) \sum_{j=1}^{J} \sigma_{it}^{2j}. \tag{A.4}$$

Given the data from the trials, it is also possible to compute the variances of the stochastic simulation estimates. The variance of $\tilde{\mu}_{it}$, for example, is $\tilde{\sigma}_{it}^2/J$. The variance of $\tilde{\sigma}_{it}^2$, denoted $\operatorname{var}(\tilde{\sigma}_{it}^2)$, is

$$\operatorname{var}\left(\tilde{\sigma}_{it}^2 \right) = (1/J)^2 \sum_{j=1}^{J} \left(\sigma_{it}^{2j} - \tilde{\sigma}_{it}^2 \right)^2. \tag{A.5}$$

For some work, as in this paper, one is interested in the difference between two estimated variances. Let $\tilde{\sigma}_{it}^2(a)$ be one estimated variance, let $\tilde{\sigma}_{it}^2(b)$ be

[9]Although the normality assumption is used in this paper, other assumptions could be used. This would simply change the way the error terms are drawn.

another, and let $\tilde{\delta}_{it}$ be the difference between the two:

$$\tilde{\delta}_{it} = \tilde{\sigma}_{it}^2(a) - \tilde{\sigma}_{it}^2(b). \tag{A.6}$$

a and b correspond to two different experiments – for example, one in which the interest rate is the policy instrument and one in which the money supply is the policy instrument.

It is also possible to compute the variance of the difference, denoted $\text{var}(\tilde{\delta}_{it})$. First, let

$$d_{it}^j = \sigma_{it}^{2j}(a) - \sigma_{it}^{2j}(b). \tag{A.7}$$

From (A.4), (A.6), and (A.7), $\tilde{\delta}_{it}$ can be written

$$\tilde{\delta}_{it} = (1/J) \sum_{j=1}^{J} d_{it}^j. \tag{A.8}$$

The variance of $\tilde{\delta}_{it}$ is then

$$\text{var}(\tilde{\delta}_{it}) = (1/J)^2 \sum_{j=1}^{J} \left(d_{it}^j - \tilde{\delta}_{it}\right)^2. \tag{A.9}$$

Given $y_{it}^j(a)$ and $y_{it}^j(b)$, $j = 1, \dots, J$, all the above values can be computed.

In many applications, as in the present study, one is interested in predicted values more than one period ahead, i.e., in predicted values from dynamic simulations. The above discussion can be easily modified to incorporate this case. One simply draws values for u_t for each period of the simulation. Each trial is one dynamic simulation over the period of interest. For, say, an eight-quarter period, each trial yields eight predicted values, one per quarter, for each endogenous variable.

Although not done in this paper, it is also possible to draw coefficients for the trials. Given an estimate of the distribution of the coefficient estimates, which one has from the estimation of the model, coefficient values can be drawn. In this case each trial consists of draws of error terms and coefficients.

Regarding exogenous variables, if the exogenous-variable values are the same from trial to trial, then the estimated variances are conditional on fixed values of the exogenous variables. It is also possible, however, to take into account exogenous-variable uncertainty. There are a number of ways to do this. For purposes of this paper, equations explaining the main exogenous variables in model were added to the model. An eighth-order autoregressive equation (with a constant term and time trend included) was estimated for each exogenous variable of interest and these equations were added to the model. Stochastic simulation can then be done for this expanded version of the model. By drawing error terms from the equations explaining the exogenous variables, exogenous-variable uncertainty is taken into account.

Assume that there are q exogenous-variable equations added to the model. This means that the covariance matrix Σ is now $(m+q)\times(m+q)$. In estimating this matrix one may want to take Σ to be block-diagonal, where the first block is the original $m\times m$ matrix and the second block is the $q\times q$ estimated covariance matrix of the error terms in the exogenous-variable equations. This procedure is consistent with the assumption upon which the estimation of the model is based. This procedure was used for the results in this paper.

Stochastic-simulation error can be large when comparing differences of variances. In the present case 1000 trials was enough to make var($\tilde{\sigma}_{it}^2$) acceptably small, but without any tricks, it was not enough to make var($\tilde{\delta}_{it}$) anywhere close to being acceptably small. Fortunately, there is an easy trick available. The variance of $\tilde{\delta}_{it}$ is equal to the variance of $\tilde{\sigma}_{it}^2(a)$ plus the variance of $\tilde{\sigma}_{it}^2(b)$ minus twice the covariance. The trick is to make the covariance high, which can be done by using the same draws of the error terms for the computation of both $\tilde{\sigma}_{it}^2(a)$ and $\tilde{\sigma}_{it}^2(b)$. Any one equation of the model, for example, requires 8000 draws of its error term for 1000 trials for a forecast horizon of eight quarters. If these same 8000 numbers are used to compute both $\tilde{\sigma}_{it}^2(a)$ and $\tilde{\sigma}_{it}^2(b)$, the covariance between them will be increased. When this trick is used, 1000 trials leads to values of var($\tilde{\delta}_{it}$) that are acceptably small. Each eight-quarter simulation of 1000 trials for the regular version of my model takes about five hours of CPU time on a VAX 730.

References

Fair, Ray C., 1984, Specification, estimation, and analysis of macroeconometric models (Harvard University Press, Cambridge, MA).

Fair, Ray C., 1979, An analysis of a macro-econometric model with rational expectations in the bond and stock markets, American Economic Review 69, 539–552.

Fair, Ray C. and John B. Taylor, 1983, Solution and maximum likelihood estimation of dynamic rational expectations models, Econometrica 51, 1169–1185.

Muth, John F., 1961, Rational expectations and the theory of price movements, Econometrica 29, 315–335.

Poole, William, 1970, Optimal choice of monetary policy instruments in a simple stochastic macro model, Quarterly Journal of Economics 84, 197–216.

Tinsley, P.A. and P. von zur Muehlen, 1983, Conditional intermediate targeting, Mimeo.

Tobin, James, 1982, Financial structure and monetary rules, Geld, Banken und Versicherungen, Band I (VVW, Karlsruhe).

Turnovsky, S., 1975, Optimal choice of monetary instrument in a linear economic model with stochastic coefficients, Journal of Money, Credit, and Banking 7, 51–80.

Yoshikawa, H., 1981, Alternative monetary policies and stability in a stochastic Keynesian model, International Economic Review 22, 541–565.