2

Theory

2.1 One Country

2.1.1 Background

The theory that has guided the specification of the US model was first presented in Fair (1974) and then in Chapter 3 in Fair (1984). This work stresses three ideas: 1) basing macroeconomics on solid microeconomic foundations, 2) allowing for the possibility of disequilibrium in some markets, and 3) accounting for all balance-sheet and flow of funds constraints. The implications of the first two ideas were first worked on by Patinkin (1956), Chapter 13, and Clower (1965). This was followed by the work of Barro and Grossman (1971). By requiring that the decisions of agents be "choice theoretic" (i.e., based on the maximizing postulates of microeconomics) and by relaxing the assumption that markets are always in equilibrium, this work provided a more solid theoretical basis for the existence of the Keynesian consumption function and for the existence of unemployment. The existence of excess supply in the labor market is a justification for including income as an explanatory variable in the consumption function, and the existence of excess supply in the goods market is a justification for the existence of unemployment.

The problem with these early disequilibrium studies is that they did not provide an explanation of why prices and wages may not always clear markets. Prices and wages were either taken to be exogenous or determined in an ad hoc manner. This was also true of the related literature on fixed price equilibria (see Grandmont (1977) for a survey of this literature). The treatment of prices and wages as exogenous or in an ad hoc manner is particularly restrictive in a disequilibrium context because disequilibrium questions are inherently concerned with whether prices always get set in such a way as to clear markets.

The theoretical work in Fair (1974) and Fair (1984), Chapter 3, provides an explanation of disequilibrium. This explanation draws heavily on the studies in Phelps et al. (1970), which in turn were influenced by Stigler's classic article (1961) on imperfect information and search. In these studies prices and wages are part of the decision variables of firms. If a firm raises its price above prices of other firms, this does not result in an immediate loss of all its customers, and if a firm lowers its price below prices of other firms, this does not result in an immediate gain of everyone else's customers. There is, however, a tendency for high price firms to lose customers over time and for low price firms to gain customers. A similar statement holds for wages. This feature is likely to be true if customers and workers have imperfect information about prices and wages, hence the relevance of Stigler's article.

If a firm's market share is a function of its price relative to the prices of other firms, then a firm's optimal price strategy is a function of this relationship. Models of this type are in Phelps and Winter (1970) for prices and in Phelps (1970) and Mortensen (1970) for wages. Disequilibrium can occur in models of this type. In the Phelps and Winter model, for example, disequilibrium occurs if the average price set by firms differs from the expected average price (1970, p. 335).

In the model in Fair (1974) and Fair (1984), Chapter 3, prices and wages are decision variables of firms, along with investment, employment, and output. Firms choose these variables in a multiperiod profit maximization context. The maximization problems require that firms form expectations of various variables before the problems are solved, and expectation errors may lead to the setting of prices and wages that do not clear markets. In other words, disequilibrium can occur because of expectation errors. Errors can occur if firms do not know the exact processes that generate the variables for which they must form expectations, and these errors can persist for more than one period.

This model thus expands on the fixed price disequilibrium studies by adding prices and wages as decision variables, and it expands on the studies in Phelps et al. (1970) by adding investment, employment, and output as decision variables. The model is also more general in its treatment of household behavior and the behavior of financial markets. Also, as noted above, the model accounts for all balance-sheet and flow of funds constraints among the sectors. This means that the government budget constraint is automatically accounted for. Christ (1968) was one of the first to emphasize the government budget constraint.

The properties of the theoretical model were analyzed in Fair (1974) and Fair (1984), Chapter 3, using simulation techniques. The following is a brief outline of the model and the simulation results. Regarding the use of simulation techniques in this context, note that the simulation of theoretical models is not a test of the models in any way. This is in contrast to the stochastic simulation of econometric models, which, as discussed in Chapter 7, can be used in the testing of models. Simulating a theoretical model is simply a way of learning about its properties for the particular set of parameters used.

2.1.2 Household Behavior

The household maximization problem is similar to the example presented in Section 1.2. Taxes and transfers have been added to the problem, and a relationship has been added between the level of money holdings and time spent taking care of these holdings. There are thus three things a household can do with its time: work, take care of money holdings, and engage in leisure. The treatment of money holdings provides a choice theoretic explanation of the interest sensitivity of the demand for money. There is also a possible labor constraint on a household, which is that it may not be able to work as many hours as it would like (i.e., as much as the solution of its maximization problem implies).

The variables that influence how much a household consumes and works include current and expected future values of 1) the wage rate, 2) the price level, 3) the interest rate, 4) the tax rate, and 5) the level of transfer payments. The initial and terminal values of wealth also affect these decisions. If the substitution effect dominates the income effect, which it does in the simulation runs, then an increase in the after tax wage rate relative to the price level leads a household to work and consume more. An increase in transfer payments or in the initial value of wealth leads it to work less and consume more. An increase in the interest rate, other things being equal, has a positive effect on the household's saving rate at the beginning of the horizon and a negative effect near the end.

The possible labor constraint can also affect household behavior. If the labor constraint is binding after the household has solved its maximization problem ignoring the constraint, the household reoptimizes subject to the constraint.[1] This leads the household to consume less than it otherwise would. It works as much as the labor constraint allows. A household's "unconstrained"

[1]It was assumed for the simulation exercises that households do not expect the labor constraint to be binding in future periods.

decision is defined to be the decision it would make if the labor constraint were not binding, and a household's "constrained" decision is defined to be the decision it actually makes taking into account the possible labor constraint. If the labor constraint is not binding, then the unconstrained and constrained decisions are the same.

Regarding money demand, time spent taking care of money holdings responds negatively to the wage rate and positively to the interest rate. In other words, a household spends more time keeping money balances low when the wage rate is low or the interest rate is high. The demand for money is also a function of the level of transactions.

Note that *real* interest rate effects on household behavior are accounted for in the model. A household solves its multiperiod optimization problem based on expectations of future prices and wages (as well as of nominal interest rates), and so future inflation effects are captured.

2.1.3 Firm Behavior

As noted above, firms solve profit maximization problems in which prices, wages, investment, employment, and output are decision variables. The technology of a firm is assumed to be of a "putty-clay" type, where at any one time there are a number of different types of machines that can be purchased. The machines differ in price, in the number of workers that must be used per machine per unit of time, and in the amount of output that can be produced per machine per unit of time. The worker-machine ratio is assumed to be fixed for each type of machine.

There are assumed to be costs involved in changing the size of the work force and the size of the capital stock. Because of these adjustment costs, it may be optimal for a firm to operate some of the time below capacity and "off" its production function. This means that some of the time the number of worker hours paid for may be greater than the number of hours that the workers are effectively working. Similarly, some of the time the number of machine hours available for use may be greater than the number of machine hours actually used. The difference between hours paid for by a firm and hours worked is "excess labor," and the difference between the number of machines on hand and the number of machines required to produce the output is "excess capital."

A firm expects that it will gain customers by lowering its price relative to the expected prices of other firms. The main expected costs from doing this, aside from the lower price it is charging per good, are the adjustment costs.

The firm also expects that other firms over time will follow it if it lowers its price, and so it does not expect to be able to capture an ever increasing share of the market with its lower price. Conversely, a firm expects that it will lose customers by raising its price relative to the expected prices of other firms. The main costs from doing this, aside from the lost customers, are again the adjustment costs. On the plus side, the firm expects that other firms over time will follow it if it raises its price, and so it does not expect to lose an ever increasing share of the market with its higher price.

Similar reasoning holds for wages. A firm expects that it will gain (lose) workers if it raises (lowers) its wage rate relative to the expected wage rates of other firms. The firm also expects that other firms will follow it if it raises (lowers) its wage rate, and so it does not expect to capture (lose) an ever increasing share of the market with its wage rate increase (decrease).

Because of the adjustment costs, a firm, if it chooses to lower output, may choose in the current period not to lower its employment and capital stock to the minimum levels required. In other words, it may be optimal for the firm to hold either excess labor or excess capital or both during certain periods.

Some of the main properties of the model of firm behavior that result from the simulation runs are the following. 1) A change in the expected prices (wages) of other firms leads the given firm to changes its own price (wage) in the same direction. 2) Excess labor on hand has a negative effect on current employment decisions, and excess capital on hand has a negative effect on current investment decisions. 3) An increase (decrease) in the interest rate leads to a substitution away from (toward) less labor intensive machines and a decrease (increase) in investment expenditures. 4) A firm responds to a decrease in demand by lowering its price and contracting, and it responds to an increase in demand by raising its price and expanding.

Similar to household behavior, *real* interest rate effects on firm behavior are accounted for in the model through the multiperiod optimization problem of a firm.

2.1.4 Bank and Government Behavior

The model in Fair (1974) allowed for the possible existence of credit rationing, and so both labor and *loan* constraints were considered. I did not find in the empirical work much evidence of the effects of loan constraints on the economy, and so in presenting the model in Fair (1984), Chapter 3, the possible existence of loan constraints was dropped. This considerably simplifies the model. The household and firm maximization problems are easier to specify, and it no longer necessary to specify a maximization problem for banks. This simpler model of bank behavior is outlined here.

Banks receive money from households and firms in the form of demand deposits. They must hold a certain portion of these deposits in the form of bank reserves, and they are assumed never to hold excess reserves. The percent of bank reserves borrowed from the monetary authority is a function of the spread between the bank loan rate and the discount rate. Banks loan money to households, firms, and the government.

The fiscal authority in the government sets the tax rates and collects taxes from households, firms, and banks. It chooses its spending for goods and labor, and it pays interest on its debt. The monetary authority earns interest on its loans to the banks. It sets the reserve requirement ratio and the discount rate. It can also engage in open market operations by buying and selling government securities.[2] In Fair (1974) the amount of government securities outstanding was taken as exogenous, i.e., as a policy variable of the monetary authority. In Fair (1984), Chapter 3, on the other hand, an interest rate reaction function was postulated for the monetary authority, where the interest rate implied by the reaction function was attained through open market operations. In this version the amount of government securities outstanding is endogenous—the amount is whatever is needed to have the interest rate value from the reaction function be met. The addition of an interest rate reaction function to the theoretical model grew out of empirical work I had done in estimating a reaction function of the Federal Reserve [Fair (1978)]. As will be seen in Chapter 4, an estimated reaction function is part of the US model.

All the flows of funds among the sectors are accounted for, and so the government budget constraint is met. The constraint states that any nonzero level of saving of the government must result in the change in non borrowed reserves or the amount of government securities outstanding.

2.1.5 The Complete Model

The complete model consists of the maximization problems of households and firms and the specification of how households, firms, banks, and the government interact. Some of the main properties of the complete model as gleaned from the simulation results are the following.

[2]The phrase "government securities" is used for convenience here and in what follows even though there is no distinction in the model between government securities and other types of securities.

1. If the quantity of labor demanded from firms and the government (banks do not demand labor) is less than the quantity of labor that households want to supply from their unconstrained maximization problems, the labor constraint is binding on households. When households reoptimize subject to the labor constraint, they consume less than they otherwise would. This lowers firms' sales, and firms respond in the next period by lowering output and their demand for labor. This further constrains the households, which leads them to consume even less, and so on. A multiplier reaction can thus get started from an initial labor constraint on households. Unemployment in the model can be thought of as the difference between the unconstrained and constrained supply of labor from the households.

2. Disequilibrium of the kind just described can occur because firms do not necessarily set the correct prices and wages, which comes about from expectation errors. In order for a firm to form correct expectations, it would have to know the maximization problems of all the other firms and of the households. It would also have to know the exact way that transactions take place once the decisions have been solved for. Firms are not assumed to have this much knowledge, and so they can make expectation errors. Note that this explanation of disequilibrium does not rely on price and wage rigidities, although if there are such rigidities, this is another reason for the existence of disequilibrium.

3. Once the economy begins to contract, the interest rate is one of the key variables that prevents it from contracting indefinitely. As unemployment increases, the interest rate is lowered by the reaction function of the monetary authority. A fall in the interest rate results in a capital gain on stocks. Both the lower interest rate and the higher wealth have a positive effect on consumption. The lower interest rate may also lead firms to switch to more expensive, less labor intensive machines, which increases investment expenditures.

4. The unemployment rate is a positive function of the supply of labor, which in turn is a function of such variables as the after tax real wage and the level of transfer payments. The effects of a policy change on the unemployment rate thus depend in part on the labor supply response to the policy change. For example, increasing the income tax rate lowers labor supply (assuming the substitution effect dominates), whereas decreasing the level of transfer payments raises it. Given the many fac-

tors that affect labor supply, there is no stable relationship in the model between the unemployment rate and real output and between the unemployment rate and the rate of inflation. There is, in other words, no stable Okun's law and no stable Phillips curve in the model.

This completes the outline of the single country theoretical model. Simulation results for the model can be found in Fair (1984), Chapter 3. To complete the theory, the model needs to be opened to the outside world. A straightforward way of doing this is to link one single country model to another, and this will now be done. The two country model that is developed is sufficient to capture the main links among the countries that exist in the MC model. It accounts for the trade, price, interest rate, and exchange rate links among the countries.

2.2 Two Countries

2.2.1 Background

The theoretical two country model that has guided the specification of the MC model was first presented in Fair (1979). This model was in part a response to the considerable discussion in the literature that had taken place in the 1970s as to whether the exchange rate is determined in a stock market or in a flow market. [See, for example, Frenkel and Rodriguez (1975), Frenkel and Johnson (1976), Dornbusch (1976), Kouri (1976), and the survey by Myhrman (1976).] The monetary approach to the balance of payments stressed the stock market determination of the exchange rate, which was contrasted with "the popular notion that the exchange rate is determined in the flow market so as to assure a balanced balance of payments" [Frenkel and Rodriguez (1975, p. 686)]. In the model in Fair (1979), on the other hand, there is no natural distinction between stock market and flow market determination of the exchange rate. The exchange rate is merely one endogenous variable out of many, and in no rigorous sense can it be said to be *the* variable that clears a particular market. In other words, there is no need for a stock-flow distinction in the model; stock and flow effects are completely integrated. [Other studies in the 1970s in which the stock-flow distinction was important included Allen (1973), Black (1973), Branson (1974), and Girton and Henderson (1976).] The reason there is no stock-flow distinction in the model is the accounting for all flow of funds and balance-sheet constraints. These constraints are accounted for in the single country model, and they are also accounted for when two single country models are put together to form a two country model.

The main features of the model in Fair (1979) that are relevant for the construction of the MC model were discussed in Fair (1984), Section 3.2. Contrary to the case for the single country theoretical model, however, the two country theoretical model was not analyzed by simulation techniques in Fair (1984). In this section a version of the two country model is presented that will be analyzed by simulation techniques. This should help in understanding the properties of the theoretical model before it is used to guide the specification of the MC model. Again, the simulation of the theoretical model is not meant to be a test of the model in any sense.

2.2.2 Notation

In what follows capital letters denote variables for country 1, lower case letters denote variables for country 2, and an asterisk (*) on a variable denotes the other country's holdings or purchase of the variable. There are three sectors per country: private non financial (h), financial (b), and government (g). The private non financial sector includes both households and firms. It will be called the "private sector." Members of the financial sector will be called "banks." Each country specializes in the production of one good (X, x). Each country has its own money (M, m) and its own bond (B, b). Only the private sector of the given country holds the money of the country. The bonds are one period securities. If a sector is a debtor with respect to a bond (i.e., a supplier of the bond), then the value of B or b for that sector is negative. The interest rate on B is R and on b is r. The price of X is P and of x is p. e is the price of country 1's currency in terms of country 2's currency, so that, for example, and increase in e is a depreciation of country 1's currency. The government of each country holds a positive amount of the international reserve (Q, q), which is denominated in the units of country 1's currency, and collects taxes (T, t) as a proportion of income (Y, y). The government of a country does not hold the bond of the other country and does not buy the good of the other country. f_{ij} is the derivative of f_i with respect to argument j.

2.2.3 Equations

There are 17 equations per country and one redundant equation. The equations for country 1 are as follows. (The derivative indicates the expected effect of the particular variable on the left hand side variable.) The demands for the two goods by the private sector of country 1 are

$$X_h = f_1(P, e \cdot p, R', Y - T), \quad f_{11} < 0, f_{12} > 0, f_{13} < 0, f_{14} > 0 \quad (2.1)$$

$$x_h^* = f_2(P, e \cdot p, R', Y - T), \quad f_{21} > 0, f_{22} < 0, f_{23} < 0, f_{24} > 0 \quad (2.2)$$

R' is the real interest rate, $R - (EP_{+1} - P)$, where EP_{+1} is the expected value of P for the next period based on current period information. The equations state that the demands are a function of the two prices, the real interest rate, and after tax income. X_h is the purchase of country 1's good by the private sector of country 1, and x_h^* is the purchase of country 2's good by the private sector of country 1. The domestic price level is assumed to be a function of demand pressure as measured by Y and of the level of import prices, $e \cdot p$:

$$P = f_3(Y, e \cdot p), \quad f_{31} > 0, f_{32} > 0 \quad (2.3)$$

There is assumed to be no inventory investment, so that production is equal to sales:

$$Y = X_h + X_g + X_h^* \quad (2.4)$$

where X_g is the purchase of country 1's good by its government and X_h^* is the purchase of country 1's good by country 2. Taxes paid to the government are

$$T = TX \cdot Y \quad (2.5)$$

where TX is the tax rate.

The demand for real balances is assumed to be a function of the interest rate and income:

$$\frac{M_h}{P} = f_6(R, Y), \quad f_{61} < 0, f_{62} > 0 \quad (2.6)$$

Borrowing by the banks from the monetary authority (BO) is assumed to be a function of R and of the discount rate RD:

$$BO = f_7(R, RD), \quad f_{71} > 0, f_{72} < 0 \quad (2.7)$$

Since the private sector is assumed to be the only sector holding money,

$$M_b = M_h \quad (2.8)$$

where M_b is the money held in banks. Equation 2.8 simply says that all money is held in banks. Banks are assumed to hold no excess reserves, so that

$$BR = RR \cdot M_b \quad (2.9)$$

where BR is the level of bank reserves and RR is the reserve requirement rate.

Let Ee_{+1} be the expected exchange rate for the next period based on information available in the current period. Then from country 1's perspective, the expected (one period) return on the bond of country 2, denoted Er, is $\frac{Ee_{+1}}{e}(1+r)-1$, where r is the interest rate on the bond of country 2. The demand for country 2's bond is assumed to be a function of R and Er :

$$b_h^* = f_{10}(R, Er), \quad f_{10,1} < 0, f_{10,2} > 0 \tag{2.10}$$

b_h^* is the amount of country 2's bond held by country 1. Equation 2.10 and the equivalent equation for country 2 are important in the model. If capital mobility is such as to lead to uncovered interest parity almost holding (i.e., R almost equal to Er), then large changes in b_h^* will result from small changes in the difference between R and Er. If uncovered interest parity holds exactly, which is not assumed here,[3] then equation 2.10 and the equivalent equation for country 2 drop out, and there is effectively only one interest rate in the model.

The next three equations determine the financial saving of each sector:

$$S_h = P \cdot X_g + P \cdot X_h^* - e \cdot p \cdot x_h^* - T + R \cdot B_h + e \cdot r \cdot b_h^* \tag{2.11}$$

$$S_b = R \cdot B_b - RD \cdot BO \tag{2.12}$$

$$S_g = T - P \cdot X_g + R \cdot B_g + RD \cdot BO \tag{2.13}$$

Equation 2.11 states that the saving of the private sector is equal to revenue from the sale of goods to the government, plus export revenue, minus import costs, minus taxes paid, plus interest received (or minus interest paid) on the holdings of country 1's bond, and plus interest received on the holdings of country 2's

[3]It was incorrectly stated in Fair (1984), pp. 154–155, that the version of the model that is used to guide the specification of the MC model is based on the assumption of perfect substitution of the two bonds. The correct assumption is that uncovered interest parity does not hold. As will be seen in Chapter 6, the MC model consists of estimated interest rate and exchange rate equations (reaction functions) for a number of countries (all exchange rates are relative to the U.S. dollar). If there were uncovered interest parity between, say, the bonds of countries A and B, it would not be possible to estimate interest rate equations for countries A and B plus an exchange rate equation. There is an exact relationship between the expected future exchange rate, the two interest rates, and the spot exchange rate if uncovered interest parity holds, and so given a value of the expected future exchange rate, only two of the other three values are left to be determined. It would not make sense in this case to estimate three equations. Covered interest parity, on the other hand, does roughly hold in the data used here. This will be seen in Chapter 6 in the estimation of the forward rate equations.

bond. If the private sector is a net debtor with respect to the bond of country 1, then B_h is negative and $R \cdot B_h$ measures interest payments. Remember that the private sector (h) is a combination of households and firms, and so transactions between households and firms net out of equation 2.11. Equation 2.12 states that the saving of banks is equal to interest revenue on bond holdings (assuming B_b is positive) minus interest payments on borrowings from the monetary authority. Equation 2.13 determines the government's surplus or deficit. It states that the saving of the government is equal to tax revenue, minus expenditures on goods, minus interest costs (assuming B_g is negative), and plus interest received on loans to banks.

The next three equations are the budget constraints facing each sector:

$$0 = S_h - \Delta M_h - \Delta B_h - e \cdot \Delta b_h^* \tag{2.14}$$

$$0 = S_b - \Delta B_b + \Delta M_b - \Delta(BR - BO) \tag{2.15}$$

$$0 = S_g - \Delta B_g + \Delta(BR - BO) - \Delta Q \tag{2.16}$$

Equation 2.14 states that any nonzero value of saving of the private sector must result in the change in its money or bond holdings. Equation 2.15 states that any nonzero value of saving of the financial sector must result in the change in bond holdings, money deposits (which are a liability to banks), or nonborrowed reserves. Equation 2.16 states that any nonzero value of saving of the government must result in the change in bond holdings, nonborrowed reserves (which are a liability to the government), or international reserve holdings.

There is also a constraint across all sectors, which says that someone's asset is someone else's liability with respect to the bond of country 1:

$$0 = B_h + B_b + B_g + B_h^* \tag{2.17}$$

These same 17 equations are assumed to hold for country 2, with lower case and upper case letters reversed except for Q and with $1/e$ replacing e. Q is replaced by q/e. (Remember that Q and q are in the units of country 1's currency.) The last equation of the model is

$$0 = \Delta Q + \Delta q$$

which says that the change in reserves across countries is zero. This equation is implied by equations 2.11–2.17 and the equivalent equations for country 2, and so it is redundant. There are thus 34 independent equations in the model.

It will be useful in what follows to consider two equations that can be derived from the others. First, let S denote the financial saving of country 1, which is the sum of the saving of the three sectors:

$$S = S_h + S_b + S_g$$

S is the balance of payments on current account of country 1. Summing equations 2.14–2.16 and using 2.17 yields the first derived equation:

$$0 = S + \Delta B_h^* - e \cdot \Delta b_h^* - \Delta Q \qquad (i)$$

This equation simply says that any nonzero value of saving of country 1 must result in the change in at least one of the following three: country 2's holdings of country 1's bond, country 1's holding of country 2's bond, and country 1's holding of the international reserve. The second derived equation is obtained by summing equations 2.11–2.13 and using 2.17:

$$S = P \cdot X_h^* - e \cdot p \cdot x_h^* - R \cdot B_h^* + e \cdot r \cdot b_h^* \qquad (ii)$$

This equation says that the saving of country 1 is equal to export revenue, minus import costs, minus interest paid to country 2, and plus interest received from country 2.

2.2.4 Closing the Model

The exogenous government policy variables are: X_g, government purchases of goods; TX, the tax rate; RD, the discount rate; RR, the reserve requirement rate; and the same variables for country 2. Not counting these variables, there are 40 variables in the model: B_b, B_g, B_h, B_h^*, BO, BR, M_b, M_h, P, Q, R ,S_b, S_g, S_h, T, X_h, X_h^*, Y, these same 18 variables for country 2, e, Ee_{+1}, EP_{+1}, and Ep_{+1}. In order to close the model one needs to make an assumption about how the three expectations are determined and to take three other variables as exogenous. (Remember there are 34 independent equations in the model.)

Assume for now that exchange rate expectations are static in the sense that $Ee_{+1} = e$ always. (This implies that $Er = r$ and $ER = R$. Remember that R does not necessarily equal r since uncovered interest parity is not necessarily assumed to hold.) Assume also that the two price expectations are static, $EP_{+1} = P$ and $Ep_{+1} = p$. The model can then be closed by taking B_g, b_g, and Q as exogenous. These are the three main tools of the monetary authorities. Taking these three tools of the monetary authorities as exogenous thus closes the model.

Instead of taking the three tools to be exogenous, however, one can assume that the monetary authorities use the tools to manipulate R, r, and e. If reaction functions for these three variables are used (or the three variables are taken to be exogenous), then B_g, b_g, and Q must be taken to be endogenous. The solution values of B_g, b_g, and Q are whatever is needed to have the target values of R, r, and e met.

Note that in closing the model no mention was made of stock versus flow effects. The exchange rate e is just one of the many endogenous variables, and it is determined, along with the other endogenous variables, by the overall solution of the model.

2.2.5 Links in the Model

The trade links in the model are standard. Country 1 buys country 2's good (x_h^*), and country 2 buys country 1's good (X_h^*). The price links come through equation 2.3 and the equivalent equation for country 2. Country 2's price affects country 1's price, and vice versa. The interest rate and exchange rate links are less straightforward, and these will be discussed next in the context of the overall properties of the model.

2.2.6 Properties of the Model

As will be discussed in the next section, the exchange rate and interest rate equations in the MC model are based on the assumption that the monetary authorities manipulate R, r, and e. (Thus, from above, B_g, b_g, and Q are endogenous in the MC model.) The interest rate and exchange rate equations are interpreted as reaction functions, where the explanatory variables in the equations are assumed to be variables that affect the monetary authorities' decisions. The key question in this work is what variables affect the monetary authorities' decisions. If capital mobility is high in the sense that uncovered interest parity almost holds, it will take large changes in the three tools to achieve values of R, r, and e much different from what the market would otherwise achieve. Since the monetary authorities are likely to want to avoid large changes in the tools, they are likely to be sensitive to and influenced by market forces. In other words, they are likely to take market forces into account in setting their target values of R, r, and e. Therefore, one needs to know the market forces that affect R, r, and e in the theoretical model in order to guide the choice of explanatory variables in the estimated reaction functions in the MC model.

In order to examine the market forces on R, r, and e in the theoretical model, a simulation version has been analyzed. Particular functional forms and coefficients have been chosen for equations 2.1, 2.2, 2.3, 2.6, 2.7, and 2.10 and the equivalent equations for country 2. The five equations for country 1 are:

$$\log X_h = a_1 - .25 \cdot \log P + .25 \cdot \log e \cdot p - 1.0 \cdot R' + .75 \cdot \log(Y - T) \quad (2.1)'$$

$$\log x_h^* = a_2 + 1.0 \cdot \log P - 1.0 \cdot \log e \cdot p - 1.0 \cdot R' + .75 \cdot \log(Y - T) \quad (2.2)'$$

$$\log P = a_3 + .1 \cdot \log e \cdot p + .1 \cdot \log Y \quad (2.3)'$$

$$\log \frac{M_h}{P} = a_6 - 1.0 \cdot R + .5 \cdot \log Y \quad (2.6)'$$

$$BO = a_7 + 50 \cdot R - 50 \cdot RD \quad (2.7)'$$

$$b_h^* = a_{10} - 100 \cdot R + 100 \cdot Er \quad (2.10)'$$

The same functional forms and coefficients were used for country 2. The a_i coefficients were chosen so that when the model was solved using the base values of all the variables, the solution values were the base values.[4] The model was solved using the Gauss-Seidel technique.[5]

The properties of the model can be examined by changing one or more exogenous variables, solving the model, and comparing the solution values to the base values. The following experiments were chosen with the aim of learning about the market forces affecting R, r, and e in the model. Unless otherwise noted, the experiments are based on the assumption that $Ee_{+1} = e$. This means from equation 2.10 and the equivalent equation for country 2 that b_h^* and B_h^* are simply a function of R and r. The experiments are also based on the assumptions that $EP_{+1} = P$ and $Ep_{+1} = p$.

In all but the last experiment, e is endogenous and Q is exogenous. Taking Q to be exogenous means that the monetary authorities are not manipulating e. This is a way of examining the market forces on e without intervention. The solution value of e for each experiment is the value that would pertain if the monetary authorities did not intervene at all in the foreign exchange market in response to whatever change was made for the experiment. B_g and b_g are always endogenous for the experiments because all the experiments

[4]The base values were $X_h = x_h = 60$, $X_h^* = x_h^* = 20$, $X_g = x_g = 20$, $Y = y = 100$, $TX = tx = .2$, $T = t = 20$, $M_h = M_b = m_h = m_b = 100$, $RR = rr = .2$, $BR = br = 20$, $e = 1$, all prices = 1, all interest rates = .07, and all other variables, including lagged values when appropriate, equal to zero.

[5]See Fair (1984), Section 7.2, for a discussion of the Gauss-Seidel technique.

either have R and r exogenous or M_b and m_b exogenous. In other words, it is always assumed that the monetary authorities either keep interest rates or money supplies unchanged in response to whatever change was made for the experiment. When R and r are exogenous, M_b and m_b are endogenous, and vice versa. All shocks in the experiments are for country 1.

The results of all the experiments are reported in Table 2.1, and the following discussion of the experiments relies on this table. Only signs are presented in the table because the magnitudes mean very little given that the coefficients and base values are not empirically based. The simulation experiments are simply meant to be used to help in understanding the qualitative effects on various variables. Even the qualitative results, however, are not necessarily robust to alternative choices of the coefficients. At least some of the signs in Table 2.1 may be reversed with different coefficients. The simulation work is meant to help in understanding the theoretical model, but the results from this work should not be taken as evidence that all the signs in the table hold for all possible coefficient values. In two cases it is necessary to know which interest rate (R or r) changed the most, and these cases are noted in Table 2.1 and discussed below.

Experiment 1: R decreased, r unchanged

For this experiment the interest rate for country 1 was lowered (from its base value) and the interest rate for country 2 was assumed to remain unchanged. (Both interest rates are exogenous in this experiment.) This change resulted in a depreciation of country 1's currency.[6] The fall in R relative to r led to an increase in the demand for the bond of country 2 by country 1 (b_h^* increased) and a decrease in the demand for the bond of country 1 by country 2 (B_h^* decreased). From equation i in Section 2.2.3 it can be seen that this must result in an increase in S, country 1's balance of payments, since Q is exogenous and unchanged. S is increased by increasing country 1's exports and decreasing its imports—equation ii—which is accomplished by a depreciation. Another way of looking at this is that the fall in R relative to r led to a decreased demand for country 1's currency because of the capital outflow, which resulted in a depreciation of country 1's currency. Output for country 1 (Y) increased because of the lower interest rate and the depreciation, and the demand for money increased because of the lower interest rate and the higher level of in-

[6] Remember that a rise in e is a depreciation of country 1's currency. The $+$ in Table 2.1 for e for experiment 1 thus means that country 1's currency depreciated.

Table 2.1
Simulation Results for the Two Country Model

	Experiment					
	1	2	3	4	5	6
	$R(-)$	$M_b(+)$	Eq2.3(+)	Eq2.3(+)	Eq2.2(+)	$R(-)$
e	+	+	+	+	+	0
R	–	–[a]	0	+[b]	+	–
r	0	–	0	+	–	0
S	+	+	0	–	–	–
s	–	–	0	+	+	+
b_h^*	+	+	0	–	–	+
B_h^*	–	–	0	+	+	–
x_h^*	–	–	0	+	–	+
X_h^*	+	+	0	–	+	+
Y	+	+	0	–	+	+
y	–	–	0	+	–	+
P	+	+	+	+	+	+
p	–	–	0	+	–	+
M_h	+	+	+	0	0	+
m_h	–	0	0	0	0	+
Q	0	0	0	0	0	–
q	0	0	0	0	0	+
B_g	+	+	0	–	+	+
b_g	–	–	0	+	–	–

Q is exogenous exept for experiment 6.
Size of changes:
1. R lowered by .001, r exogenous
2. M_b raised by 1.0, m_b exogenous
3. Equation 2.3 shocked by .10, R and r exogenous
4. Equation 2.3 shocked by .10, M_b and m_b exogenous
5. Equation 2.2 shocked by .10, M_b and m_b exogenous
6. R lowered by .001, r and e exogenous

[a] R decreased more than did r.
[b] R increased more than did r.

come. The monetary authority of country 1 bought bonds to achieve the reduction in R (B_g increased).

Although not shown in Table 2.1, experiments with alternative coefficients in the equations explaining b_h^* and B_h^*—equation 2.10 and the equivalent equation for country 2—showed that the more sensitive are the demands for the foreign bonds to the interest rate differential, the larger is the depreciation of the exchange rate and the larger is the increase in B_g for the same drop in R. In other words, the higher is the degree of capital mobility, the larger is the size of open market operations that is needed to achieve a given target value of the interest rate.

Remember that the above experiment is for the case in which exchange rate expectations are static, i.e. where $Ee_{+1} = e$. If instead expectations are formed in such a way that Ee_{+1} turns out to be less than e, which means that the exchange rate is expected to appreciate in the next period relative to the value in the current period (i.e., reverse at least some of the depreciation in the current period), then the depreciation in the current period is less. This is because if Ee_{+1} is less than e, the expected return on country 2's bond (Er) falls. The differential between R and Er thus falls less as a result of the decrease in R, which leads to a smaller increase in b_h^* and a smaller decrease in B_h^*. There is thus less downward pressure on country 1's currency and thus a smaller depreciation. If expectations are formed in such a way that Ee_{+1} turns out to be greater than e, which means that the exchange rate is expected to depreciate further in the next period, there is more of a depreciation in the current period. The expected return on country 2's bond rises, which leads to greater downward pressure on country 1's exchange rate.

Experiment 2: M_b increased, m_b unchanged

For this experiment the monetary authorities are assumed to target the money supplies (M_b and m_b are exogenous), and the money supply of country 1 was increased. The increase in M_b led to a decrease in R, both absolutely and relative to r, which led to a depreciation of country 1's currency. The results of this experiment are similar to those of experiment 1. The monetary authority of country 1 bought bonds to increase the money supply (B_g increased). Country 1's output increased as a result of the depreciation and the fall in R. Note that the effect of a change in the money supply on the exchange rate works through the change in relative interest rates. The interest rate of country 1 falls relative to that of country 2, which decreases the demand for country 1's bond and increases the demand for country 2's bond, which leads to a depreciation of country 1's exchange rate.

Experiment 3: Positive price shock, R and r unchanged

For this experiment the price equation for country 1 was shocked positively. The monetary authorities were assumed to respond to this by keeping interest rates unchanged. The positive price shock resulted in a depreciation of country 1's currency. Given the coefficients and base values that are used for the simulation model, the exchange rate depreciated by the same percent that P increased, and there was no change in any real magnitudes. The reason for the exchange rate depreciation is the following. Other things being equal, a positive price shock leads to a decrease in the demand for exports and an increase in the demand for imports, which puts downward pressure on S. If, however, interest rates are unchanged, then b_h^* and B_h^* do not change, which means from equation i that S cannot change. Therefore, a depreciation must take place to decrease export demand and increase import demand enough to offset the effects of the price shock.

Experiment 4: Positive price shock, M_b and m_b unchanged

This experiment is the same as experiment 3 except that the money supplies rather than the interest rates are kept unchanged. The positive price shock with the money supplies unchanged led to an increase in R. Even though R increased relative to r, country 1's currency depreciated. The negative effects of the price shock offset the positive effects of the interest rate changes.

Experiment 5: Positive import demand shock, M_b and m_b unchanged

For this experiment the import demand equation of country 1 was shocked positively. The increased demand for imports led to a depreciation of country 1's currency, since there was an increased demand for country 2's currency. The depreciation led to an increase in Y and P, which with an unchanged money supply, led to an increase in R. R also increased relative to r, which increased B_h^* and decreased b_h^*. The balance of payments, S, worsened. It may at first glance seem odd that a positive import shock would lead to an increase in Y, but remember that the shock does not correspond to any shock to the demand for the domestic good. The experiment is not a substitution away from the domestic good to the imported good, but merely an increase in demand for the imported good. The latter results in an increase in Y because of the stimulus from the depreciation.

Experiment 6: R decreased, r unchanged, e unchanged

This experiment is the same as experiment 1 except that e rather than Q is exogenous. In this case the monetary authorities choose B_g, b_g, and Q so as to lower R and keep r and e unchanged. One of the key differences between the results for this experiment and the results for experiment 1 is that the

Table 2.2
Summary of the Experiments

Experiment	Effect on: Domestic Interest Rate	Effect on: Exchange Rate
1. Interest rate lowered	—	Depreciation
2. Money supply raised	Lowered	Depreciation
3. Positive price shock; interest rates unchanged	—	Depreciation
4. Positive price shock; money supply unchanged	Raised	Depreciation
5. Positive import shock; money supply unchanged	Raised	Depreciation

balance of payments, S, decreases rather than increases. In experiment 1 S had to increase because of the increase in the demand for country 2's bond by country 1 and the decrease in the demand for country 1's bond by country 2. In experiment 1 S must increase because Q is exogenous—equation i. The increase in S is accomplished by a depreciation. In the present experiment there is still an increase in the demand for country 2's bond and a decrease in the demand for country 1's bond—because R falls relative to r—but S does not necessarily have to increase because Q can change. The net effect is that S decreases (and thus Q decreases). The reason for the decrease in S is fairly simple. The decrease in R is an expansionary action in country 1, and among other things it increases the country's demand for imports. This then worsens the balance of payments. There is no offsetting effect from a depreciation of the currency to reverse this movement.

This completes the discussion of the experiments. They should give one a fairly good idea of the properties of the model. Of main concern here are the effects of the various changes on the domestic interest rate and the exchange rate. Table 2.2 presents a summary of these effects in the model (experiment 6 is not included in the table because both R and e are exogenous in it).

2.2.7 The Use of Reaction Functions

As noted in the previous section, reaction functions for interest rates and exchange rates have been estimated in the MC model. To put this approach in perspective, it will help to consider an alternative approach that in principle could have been followed. If equations 2.1, 2.2, 2.3, 2.6, 2.7, 2.10, and the equivalent equations for country 2 were estimated, one could solve the model for R, r, and e (and the other endogenous variables) by taking B_g, b_g, and Q as exogenous. R, r, and e would thus be determined without having to estimate any direct equations for them. Their values would be whatever is needed to clear the two bond markets and the market for foreign exchange. In doing this, however, one would be making the rather extreme assumption that the monetary authorities' choices of B_g, b_g, and Q are never influenced by the state of the economy, i.e. are always exogenous.

If one believes that monetary authorities intervene at least somewhat, there are essentially two options open. One is to estimate equations with B_g, b_g, and Q on the left hand side, and the other is to estimate equations with R, r, and e on the left hand side. If the first option is followed, then the B_g, b_g, and Q equations are added to the model and the model is solved for R, r, and e. If the second option is followed, the R, r, and e equations are added to the model and the model is solved for B_g, b_g, and Q. The first option is awkward because one does not typically think of the monetary authorities having target values of the instruments themselves. It is more natural to think of them having target values of interest rates (or money supplies[7]) and exchange rates, and this is the assumption made for the MC model.

There is also a practical reason for taking the present approach. If B_g, b_g, and Q are taken to be exogenous or equations estimated for them, equations like 2.10, which determine the bilateral demands for securities, must be estimated. In practice it is very difficult to estimate such equations. One of the main problems is that data on bilateral holdings of securities either do not exist or are not very good. If instead equations for interest rates and exchange rates are estimated, one can avoid estimating equations like 2.10 in order to determine interest rates and exchange rates if one is willing to give up determining B_g, b_g, and Q. For many applications one can get by without knowing the amounts

[7]It is in the spirit of the present approach to estimate money supply reaction functions rather than interest rate reaction functions. In either case B_g is endogenous. No attempt has been made in the construction of the MC model to try to estimate money supply reaction functions. The present work is based on the implicit assumption that interest rate reaction functions provide a better approximation of the way monetary authorities behave than do money supply reaction functions.

of government bonds outstanding and government reserve holdings. One can simply keep in mind that the values of these variables are whatever is needed to have the interest rate and exchange rate values be met.

2.2.8 Further Aggregation

Data on bilateral security holdings were not collected for the MC model, and so data on variables like B_h^* and b_h^* are not available. Instead, a net asset variable, denoted A in the MC model, was constructed for each country. In terms of the variables in the theoretical model, $\Delta A = -\Delta B_h^* + e \cdot \Delta b_h^* + \Delta Q$. Equation i thus becomes

$$0 = S - \Delta A \qquad (i)'$$

Data on S are available for each country, and A was constructed as $A_{-1} + S$, where an initial value for A for each country was first chosen.

This aggregation is very convenient because it allows A to be easily constructed. The cost of doing this is that capital gains and losses on bonds from exchange rate changes are not accounted for. Given the current data, there is little that can be done about this limitation.