Testing the NAIRU Model for 27 Countries

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Abstract

The NAIRU model is tested in this paper using data for 27 countries. The results are generally not supportive of the dynamics implied by the model.

1 Introduction

In Fair (1997) the NAIRU model was tested using U.S. data. In this paper the model is tested for 27 other countries. The following equation is taken to represent the standard NAIRU view:

$$\pi_t = \sum_{i=1}^{n} \delta_i \pi_{t-i} - \beta(u_t - u_t^*) + s_t, \quad \sum_{i=1}^{n} \delta_i = 1 \tag{1}$$

where π is the rate of inflation, u is the unemployment rate, u^* is the nonaccelerating inflation rate of unemployment (the NAIRU), and s is a measure of supply shocks. As was done for the United States, the test of equation (1) is in two steps. The first step is to present and test structural price and wage equations for each country that do well in tests and seem to be good approximations. These equations are in "level" form. From these two equations the final form of the price equation can be

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derived. The second step is to compare equation (1) to the final form price equation. If the two structural equations are correctly specified, equation (1) is a misspecified version of the final form price equation. In particular, equation (1) excludes some explanatory variables that are in the final form, and so a test of equation (1) is to add these variables to it and see if they are significant.

As discussed in the Conclusion, if equations like (1) are not good approximations, then the standard long-run unemployment-inflation story must be changed. The new story, however, does not have to imply that unemployment can be driven close to zero with only a modest long-run effect on the price level. There may be (and seems likely to be) a nonlinear relationship between the price level and unemployment at low levels of unemployment, where pushing unemployment further and further below some low level results in larger and larger increases in the price level. This nonlinearity would in effect bound unemployment above a certain level. An attempt is made in this paper to estimate this nonlinearity, but, as will be seen, there do not seem to be enough observations at very low unemployment rates to provide good estimates.

2 The Structural Price and Wage Equations

Empirical Specification

The theory that has guided the specification of the price and wage equations in this section was first presented in Fair (1974), and more recent discussions are in Fair (1984, Chapter 3), Fair (1994, Chapter 2), and Fair (1997). This theory will not be reviewed here. The empirical specification of the price and wage equations is as follows:

$$p_t = \beta_0 + \beta_1 p_{t-1} + \beta_2(w_t - \lambda_t) + \beta_3 s_t + \beta_4 D_t + \beta_5 t + \epsilon_t \tag{2}$$

$$w_t - \lambda_t = \gamma_0 + \gamma_1(w_{t-1} - \lambda_{t-1}) + \gamma_2 p_t + \gamma_3 p_{t-1} + \gamma_4 D_t + \gamma_5 t + \mu_t \tag{3}$$

p is the log of the price level, and w is the log of the wage rate. s is the log of the import price level minus p lagged once; it is a measure of relative import prices. D is some measure of demand pressure. λ is the log of Λ, where Λ is an estimate of the potential level of output per worker. In the empirical work Λ is estimated from peak-to-peak interpolations of output per worker. The growth rate of Λ is an estimate of the growth rate of potential productivity. The change in $w - \lambda$ is the growth rate of the nominal wage rate less the growth rate of potential productivity. ϵ and μ are error terms.

The lagged price variable in equation (2) can be thought of as picking up expectational effects, the wage variable and the relative import price variable as picking up cost effects, and the demand variable as picking up demand effects. All these effects are in the theoretical specification mentioned above.

The time trend in equation (2) is meant to pick up any trend effects on the price level not captured by the other variables. Adding the time trend to an equation like (2) is similar to adding the constant term to an equation specified in terms of changes rather than levels. The time trend will also pick up any trend mistakes made in constructing λ_t. If, for example, $\lambda_t = \lambda_t^a + \theta t$, where λ_t^a is the correct variable to subtract from w_t to adjust for potential productivity, then the time trend will absorb this error.

In the wage equation, equation (3), the wage rate is a function of the lagged wage rate, the current and lagged price level, the demand variable, and the time trend.

It is an equation in which the wage rate adjusts to the price level over time. The price equation is identified because of the inclusion of the lagged wage in the wage equation, and the wage equation is identified because of the inclusion of the relative import price variable in the price equation.

When price and wage equations are specified, one has to be careful regarding what they imply about the determination of the real wage, which is $w_t - \lambda_t - p_t$ in the present notation. Solving equations (2) and (3) for $w_t - \lambda_t - p_t$ yields:

$$\begin{aligned} w_t - \lambda_t - p_t = \tfrac{1}{1-\beta_2\gamma_2}\{&(1-\beta_2)\gamma_1(w_{t-1}-\lambda_{t-1}) + [(1-\beta_2)\gamma_3 - (1-\gamma_2)\beta_1]p_{t-1} \\ &-(1-\gamma_2)\beta_0 + (1-\beta_2)\gamma_0 - (1-\gamma_2)\beta_3 s_t \\ &-[(1-\gamma_2)\beta_4 D_t + (1-\beta_2)\gamma_4] - [-(1-\gamma_2)\beta_5 + (1-\beta_2)\gamma_5]t \\ &-(1-\gamma_2)\epsilon_t + (1-\beta_2)\mu_t\} \end{aligned} \tag{4}$$

Unless the coefficient of $w_{t-1} - \lambda_{t-1}$ equals the negative of the coefficient of p_{t-1}, equation (4) implies that in the long run the real wage depends on the level of p, which is not sensible. Consequently, the restriction that the two coefficients are equal in absolute value and of opposite signs is imposed in the estimation. The restriction on the structural coefficients is

$$\gamma_3 = \frac{\beta_1}{1-\beta_2}(1-\gamma_2) - \gamma_1 \tag{5}$$

The Demand Pressure Variables

As noted in Section 1, there seems likely to be a nonlinear relationship between p_t and the unemployment rate at low levels of the latter, and an attempt was made to estimate this nonlinearity. Two functional forms were tried for the unemployment rate. In addition, two other activity variables, both measures of the output gap, were tried in place of the unemployment rate, and two functional forms were tried for each gap variable.

Let u_t denote the unemployment rate, and let $u_t' = u_t - u^{min}$, where u^{min} is the minimum value of the unemployment rate in the sample period ($t = 1, \ldots, T$). The first form tried was linear, namely $D_t = u_t'$. The other was $D_t = 1/(u_t' + .02)$. For the second form D_t is infinity when u_t' equals $-.02$, and so this form says that as the unemployment rate approaches 2.0 percentage point below the smallest value it reached in the sample period, the price level approaches infinity.[1]

For the first output-gap variable, a potential output series, denoted Y_t^*, was constructed from peak-to-peak interpolations of the level of output per worker and the number of workers per working-age population. (The peak-to-peak interpolation of output per worker is Λ_t mentioned above.) Define the gap, denoted G_t, as $(Y_t^* - Y_t)/Y_t^*$, where Y_t is the actual level of output, and let $G_t' = G_t - G^{min}$, where G^{min} is the minimum value of G_t in the sample period. For this variable the first form was linear, and the other was $D_t = 1/(G_t' + .02)$.

For the second output-gap variable, a potential output series was constructed by regressing, over the sample period, $\log Y_t$ on a constant and t. The gap G_t is then defined to be $\widehat{\log Y_t}$ - $\log Y_t$, where $\widehat{\log Y_t}$ is the predicted value from the regression. The rest of the treatment is the same as for the first output-gap variable.

Two functional forms for the unemployment rate and two each for the output-gap variables yields 6 different variables to try. In addition, each variable was tried both

[1]In earlier work values other than .02 were tried for D_t, including .005, .01, .015, and .05. The value that resulted in the best fit for a country tended to be around .02, and so for present purposes the formal searching was done using only .02 and the linear form. As discussed below, the fits tend to be similar across functional forms, and the data do not discriminate well among different forms, including the linear form.

unlagged and lagged once separately, giving 12 different variables. The searching was done using equation (2) under the assumption of a first order autoregressive error term and with three variables added. The three added variables are p_{t-2}, $w_{t-1} - \lambda_{t-1}$, and s_{t-1}. The demand pressure variable chosen was the one with the highest t-statistic. No demand pressure variable was chosen if the coefficient estimates of all the demand pressure variables were of the wrong sign.[2]

Once the demand pressure variable was chosen, three further specification decisions were made. The first is whether $w_t - \lambda_t$ or $w_{t-1} - \lambda_{t-1}$ should be included in the final specification, the second is whether s_t or s_{t-1} should be included, and the third is whether the autoregressive assumption about the error term should be retained. For each of the first two decisions the variable with the higher t-statistic was chosen provided its coefficient estimate was of the expected sign, and for the third decision the autoregressive assumption was retained if the autoregressive coefficient estimate was significant at the five percent level. If when tried separately both $w_t - \lambda_t$ and $w_{t-1} - \lambda_{t-1}$ had coefficient estimates of the wrong sign, neither was used, and similarly for s_t and s_{t-1}.[3]

χ^2 Tests of the Price Equation

The final specification of the price equation was subjected to a number of χ^2 tests. Each test consists of adding variables to the equation and testing whether the addition

[2] Data mining is, of course, a potential problem when searching like this. There is a bias in favor of finding significant demand pressure variables when none in fact belong. The main aim of this paper, however, is to compare different dynamic specifications, and this comparison is not likely to be affected much by the searching for demand pressure variables. The reason the three additional variables were added to equation (2) for the searching was to lessen the possibility that the choice of the best demand pressure variable depends on a particular dynamic specification.

[3] When $w_{t-1} - \lambda_{t-1}$ is chosen, the coefficient restriction in (5) becomes $\gamma_3 = (\beta_1 + \beta_2)(1 - \gamma_2) - \gamma_1$.

is significant. An insignificant χ^2 value means the equation has passed the test. A χ^2 value will be said to be significant if its p-value is less than .05—a five percent confidence level.

The first test is to add the lagged value of each variable in the equation: p_{t-2}, $w_{t-1} - \lambda_{t-1}$, s_{t-1}, and D_{t-1}.[4] Adding these values encompasses many different types of dynamic specifications,[5] and so it is a fairly general test of the dynamic specification of the equation.

The second test is to add even more lagged values, namely the above four plus p_{t-3}, $w_{t-2} - \lambda_{t-2}$, s_{t-2}, and D_{t-2}.

The third test concerns the autoregressive properties of the error term. The search for the best demand pressure variable assumed a first order autoregressive error term. If for the final demand pressure variable chosen the autoregressive parameter was significant, the first order assumption was retained; otherwise, the error term was assumed not to be autoregressive. The third test is to estimate the equation under the assumption of a fourth order autoregressive error term and see if the extra autoregressive coefficients are jointly significant. If the first order assumption has been used in the basic estimation, three additional autoregressive coefficients are estimated for the third test; otherwise, four are. This third test is to see if the serial correlation properties of the error term have been properly accounted for.

For the fourth test the value of the wage rate *led* one period was added to the equation. This can be looked upon as a test of the expectation mechanism. If

[4]For simplicity, the following discussion will assume that $w - \lambda$, s, and D in equation (2) are unlagged. If the wage variable is in fact lagged, then the wage lags in the discussion should all be increased by one, and similarly if the import price variable is lagged or the demand pressure variable is lagged.

[5]See Hendry, Pagan, and Sargan (1984) for a general discussion.

the future value is significant, this is evidence in favor of the rational expectations hypothesis.[6]

The fifth test is a stability test due to Andrews and Ploberger (1994) and discussed in Fair (1994, Chapter 4). This test does not require that a break point be chosen *a priori*, just a range in which the structural break occurred if there was one. Depending on whether the data for a country were quarterly or annual, the break period was assumed to begin 32 quarters or 10 years after the beginning of the estimation period and to end 32 quarters or 10 years before the end of the estimation period.

Estimation and Tests of the Wage Equation

The same searching for the best demand pressure variable was done for the wage equation (3) as was done for the price equation. This searching was done without imposing the coefficient restriction in (5) and under the assumption of a first order autoregressive error term. Once the demand pressure variable was chosen, one further specification decision had to be made for the wage equation, namely whether the autoregressive assumption of the error term should be retained. The same decision criterion was used here as was used for the price equation.

The first test for the wage equation is of the coefficient restriction in (5). All the remaining tests were performed with this restriction imposed. In imposing the restriction, the values used for β_1 and β_2 were taken to be the estimated values from the price equation. Given values for β_1 and β_2, the restriction in (5) is simply a linear restriction on the γ coefficients.

[6] See Fair (1994, Chapter 4) for a discussion of this. When future values are added to an equation, consistent estimates can be obtained using Hansen's (1982) method of moments estimator. In the present context, with only one lead, this estimator is just 2SLS, which, as discussed below, is the method used for equations (2) and (3).

The second test adds the lagged values $w_{t-2} - \lambda_{t-2} - p_{t-2}$, $w_{t-3} - \lambda_{t-3} - p_{t-3}$, $w_{t-4} - \lambda_{t-4} - p_{t-4}$, and $w_{t-5} - \lambda_{t-5} - p_{t-5}$. Again, this is a fairly general test of the dynamic specification. Adding the lagged values in this form preserves the real wage restriction discussed above.

The third test is to estimate the equation under the assumption of a fourth order autoregressive process of the error term and see if the extra autoregressive coefficients are jointly significant. The same procedure was followed here as was followed for the price equation.

The fourth test is the Andrews-Ploberger stability test. Again, the same procedure was followed here as was followed for the price equation.

The Data and Estimation Technique

The data are described in Fair (1994), and this description will not be repeated here. Quarterly data were collected for 13 countries (to be called the "quarterly" countries) and annual data were collected for 14 others (to be called the "annual" countries). The main sources of data are IFS and OECD. The price variable is the GDP deflator,[7] and the wage variable is the nominal wage variable from the IFS.

The quality of the data varies across countries, and the results for the individual countries should not necessarily be weighted equally. In particular, the results for the countries with only annual data should probably be weighted less. Also, the wage data are probably not in general as good as the price data. The reason there are

[7] The GDP deflator is not the ideal price index to use because it includes prices of government output and indirect business taxes, which are not decision variables of firms. For the U.S. results in Fair (1997) a private non farm price index was used, but this type of a price index is not available for every country and so for present purposes only the GDP deflators have been used. The rejections of the NAIRU model in Fair (1997) were stronger using the non farm price index than using the GDP deflator.

fewer countries with estimated wage equations than estimated price equations below is simply because of data limitations.[8]

The estimation technique was 2SLS for the quarterly countries and OLS for the annual countries. For 2SLS, the endogenous variables were taken to be p_t, w_t, D_t, and s_t. This means that the price and wage equations were assumed to be imbedded in a larger model, where D_t and s_t are endogenous. The variables used for the first stage regressors are the main predetermined variables in the individual country models in Fair (1994). The list of these variables and a complete description of all the data are available from the website mentioned in the introductory footnote.

The value computed for each χ^2 test is $(S^{**} - S^*)/\sigma^2$, where S^{**} is the value of the minimand (2SLS or OLS) before the addition, S^* is the value of the minimand after the addition, and σ^2 is the estimated variance of the error term after the addition. Under fairly general conditions, as discussed in Andrews and Fair (1988), this value is distributed as χ^2 with k degrees of freedom, where k is the number of variables added.

Four dummy variables were used for Germany for all its estimated equations in an attempt to account for the effects of the reunification of the country. The first had a value of one in 1990:3 and zero otherwise; the second a value of one in 1990:4 and zero otherwise; the third a value of one in 1991:1 and zero otherwise; and the fourth a value of one in 1991:2 and zero otherwise. Because of the use of the dummy variables, no stability tests were performed for Germany. To save space, the

[8]Two of the tests were slightly different for the annual countries because of the smaller number of observations. First, when searching for the best demand pressure variable, the equations were not assumed to have a serially correlated error and the demand pressure variables lagged once were not tried. Second, when testing for serial correlation, the autoregressive process was taken to be of order 3 rather than 4.

coefficient estimates for the dummy variables have not been reported in the tables below.

The Results

The results for the price equation are presented in Tables 1 and 2. The coefficient estimates of the final specification are presented in Table 1, and the test results are presented in Table 2. Tables 3 and 4 are similar tables for the wage equation. The estimation periods are presented in Table 1, and they are the same for both the price and wage equations.

Turn first to Table 1. Of the 18 countries for which a demand pressure variable was used,[9] the functional form was linear for 10 of them. The chosen variable was the unemployment rate for 4 of them, the first output-gap variable for 8 of them, and the second output-gap variable for the remaining 6. There is thus no strong pattern here, although a slight edge for the linear form and the first output-gap variable. The good showing for the linear form shows the difficulty of estimating the point at which the relationship between the price level and demand becomes nonlinear. Also, although not shown in Table 1, the fits of the equations tended not to be very sensitive to the use of alternative functional forms, such as those mentioned in footnote 1, and no clear winner emerged.

Of the 9 countries with no demand pressure variable in Table 1, two of them—the Netherlands and the United Kingdom—have wage equations in Table 3 with demand pressure variables. For these two countries demand pressure affects prices by affecting wages, which affect prices. South Africa is the only quarterly country

[9]Remember, no demand pressure variable was included if the coefficient estimates of all the demand pressure variables were of the wrong sign.

Table 1
Estimates of the Price Equation

$$p_t = \beta_0 + \beta_1 p_{t-1} + \beta_2(w_t - \lambda_t) + \beta_3 s_t + \beta_4 D_t + \beta_5 t$$

	Best D	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	$\hat{\rho}$	SE	DW	Sample
Quarterly											
CA	$G2_{-1}$(lin)	-0.070	0.947	0.012	0.021	[a]-0.13469	0.00047	0.499	0.0053	2.25	1966.1-1996.1
		(-0.67)	(17.53)	(0.25)	(1.44)	(-5.16)	(1.99)	(5.43)			
JA	$G2$(lin)	-0.765	0.742	0.139	0.028	-0.24050	0.00152	0.688	0.0074	2.15	1967.3-1995.4
		(-3.09)	(10.23)	(2.73)	(2.06)	(-3.36)	(3.07)	(7.06)			
AU	$G1$(.02)	-0.734	0.840	[a] 0.095	[a]0.041	0.00023	0.00086	-0.397	0.0104	1.99	1971.1-1994.1
		(-2.40)	(13.00)	(2.13)	(2.57)	(1.04)	(2.26)	(-3.64)			
FR	U_{-1}(lin)	-0.742	0.848	0.099	[a]0.019	[a]-0.06777	0.00050	0.291	0.0047	1.79	1976.1-1995.2
		(-2.74)	(18.14)	(2.76)	(1.35)	(-0.66)	(2.14)	(2.41)			
GE	$G2_{-1}$(lin)	-0.469	0.877	[a]0.047	0.018	[a]-0.07823	0.00053	b	0.0031	1.88	1969.1-1994.4
		(-6.26)	(57.14)	(5.51)	(4.65)	(-4.91)	(5.05)				
IT	$G2$(lin)	-0.157	0.941	0.018	0.042	-0.17374	0.00114	b	0.0069	1.69	1971.1-1995.3
		(-2.01)	(29.46)	(0.64)	(6.23)	(-5.62)	(4.97)				
NE	none	-0.730	0.714	[a]0.130	0.075	–	0.00091	b	0.0080	1.57	1978.2-1995.4
		(-1.77)	(9.30)	(1.30)	(4.53)		(2.05)				
ST	$G1_{-1}$(lin)	0.002	0.979	c	[a]0.015	[a]-0.13828	0.00016	0.575	0.0031	1.64	1971.1-1994.4
		(0.04)	(27.67)		(1.36)	(-4.42)	(0.42)	(5.78)			
UK	none	-0.398	0.856	0.164	0.064	–	-0.00045	b	0.0108	0.99	1966.1-1995.2
		(-4.06)	(23.78)	(3.75)	(7.35)		(-1.63)				
FI	U(.02)	-0.157	0.879	[a]0.090	0.028	0.00057	0.00061	b	0.0076	1.92	1976.1-1993.3
		(-1.92)	(12.01)	(1.12)	(2.47)	(3.78)	(1.41)				
AS	$G1_{-1}$(.02)	0.055	1.001	c	0.020	[a]0.00039	-0.00036	b	0.0105	2.06	1971.1-1995.4
		(1.52)	(79.51)		(1.54)	(3.08)	(-1.56)				
SO	none	-0.127	0.970	c	0.034	–	0.00099	b	0.0176	2.18	1962.1-1995.3
		(-3.31)	(116.75)		(3.03)		(4.09)				
KO	$G2_{-1}$(.02)	-0.665	0.696	0.329	0.100	[a]0.00107	-0.00548	-0.256	0.0367	1.87	1964.1-1995.4
		(-3.42)	(8.65)	(3.80)	(3.07)	(1.58)	(-3.76)	(-2.36)			

Table 1 (continued)

	Best D	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	SE	DW	Sample
Annual										
BE	$G2$(.02)	-1.220	0.577	0.219	0.030	0.00056	0.01095	0.0126	1.16	1966-1992
		(-3.79)	(5.28)	(3.61)	(1.09)	(1.43)	(3.33)			
DE	U(.02)	-2.061	0.634	0.372	0.062	0.00044	-0.00259	0.0079	2.03	1967-1992
		(-9.05)	(13.34)	(10.34)	(2.89)	(1.61)	(-1.13)			
NO	U(lin)	-0.346	0.892	d	0.349	-0.71895	0.01262	0.0256	1.26	1966-1993
		(-1.88)	(11.56)		(3.99)	(-1.15)	(2.07)			
SW	$G1$(lin)	-1.878	0.619	[a]0.273	0.180	-0.31560	0.01097	0.0176	1.54	1966-1993
		(-2.51)	(5.38)	(2.00)	(6.64)	(-1.75)	(2.23)			
GR	$G1$(.02)	-0.165	0.9310	0.046	0.220	0.00103	0.00143	0.0236	1.53	1964-1993
		(-0.90)	(19.32)	(0.76)	(3.98)	(1.51)	(0.26)			
IR	none	-0.462	0.668	0.331	[a]0.093	–	0.00007	0.0258	1.67	1972-1991
		(-1.58)	(4.39)	(1.80)	(0.81)		(0.01)			
SP	$G1$(.02)	-0.832	0.739	0.233	[a]0.004	0.00099	-0.00690	0.0151	1.40	1964-1994
		(-6.26)	(19.83)	(11.92)	(0.17)	(2.36)	(-1.75)			
NZ	none	-1.178	0.742	0.252	[a]0.147	–	0.00120	0.0290	1.48	1962-1992
		(-4.59)	(14.27)	(3.21)	(3.03)		(0.21)			
CO	$G1$(lin)	-3.131	0.527	c	0.098	-0.34885	0.10494	0.0195	2.37	1972-1994
		(-3.33)	(3.86)		(2.41)	(-1.89)	(3.56)			
JO	none	-0.070	0.947	c	0.212	–	0.00486	0.0386	1.82	1971-1995
		(-0.40)	(13.85)		(4.12)		(0.89)			
SY	none	-0.549	0.851	c	0.011	–	0.02017	0.0748	1.38	1965-1994
		(-1.43)	(7.61)		(0.16)		(1.67)			
PA	none	-0.257	0.805	c	0.170	–	0.01077	0.0215	1.57	1976-1993
		(-0.67)	(5.25)		(2.37)		(0.89)			
PH	none	-0.128	0.924	c	0.213	–	0.00605	0.0542	1.53	1962-1993
		(-0.45)	(12.22)		(4.60)		(0.67)			
TH	$G1$(lin)	-0.647	0.519	c	0.315	-0.17183	0.02169	0.0251	1.35	1962-1994
		(-6.11)	(7.57)		(7.75)	(-0.82)	(6.33)			

t-statistics are in parentheses.
[a]Variable lagged once. [b]ρ taken to be 0. [c]No wage data. [d]Coefficient taken to be 0.
ρ is not estimated for the annual countries.
U = unemployment rate, $G1$ = first output-gap variable, $G2$ = second output-gap variable.
The expression in parentheses following U, $G1$, and $G2$ is .02 if the nonlinear form is used and lin if the linear form is used.
$\hat{\beta}_4$ is expected to be negative when the linear form is used and positive when the nonlinear form is used.

CA=Canada, JA=Japan, AU=Austria, FR=France, GE=Germany, IT=Italy, NE=Netherlands, ST=Switzerland, UK=United Kingdom, FI=Finland, AS=Australia, SO=South Africa, KO=Korea, BE=Belgium, DE=Denmark, NO=Norway,SW=Sweden, GR=Greece, IR=Ireland, SP=Spain, NZ=New Zealand, CO=Colombia, JO=Jordan,SY=Syria, PA=Pakistan, PH=Philippines, TH=Thailand

Table 2
Test Results for the Price Equation

	Lags p-val	More Lags p-val	RHO+ p-val	Lead p-val	Stability AP	(df)	λ
Quarterly							
CA	0.363	0.764	0.358	0.115	*17.85	(7)	7.415
JA	0.217	0.011	0.019	0.217	*35.40	(7)	6.289
AU	d	d	0.095	0.149	1.88	(7)	3.465
FR	0.401	0.762	0.420	0.009	*10.52	(7)	1.960
GE	0.228	0.251	0.087	0.269		–	
IT	0.045	0.201	0.075	0.004	*12.36	(6)	4.187
NE	0.101	0.263	0.074	0.287	*7.98	(5)	1.403
ST	0.000	0.319	0.033	c	2.64	(6)	3.818
UK	0.000	0.000	0.000	0.000	*50.21	(5)	6.921
FI	0.211	0.536	0.384	0.995	*9.83	(6)	1.403
AS	0.468	0.013	0.007	c	1.00	(5)	4.314
SO	0.543	0.711	0.123	c	*96.84	(4)	9.947
KO	0.044	0.824	0.003	0.112	*22.22	(7)	8.635
Annual							
BE	0.000	0.000	0.000	0.158	*30.16	(6)	2.469
DE	0.524	0.497	0.067	0.016	6.40	(6)	2.179
NO	0.086	0.230	0.972	0.697	*20.38	(5)	2.778
SW	0.000	0.008	0.103	0.005	7.14	(6)	2.778
GR	0.000	0.000	0.612	0.311	*15.66	(6)	3.449
IR	0.846	0.590	0.707	0.203	3.10	(5)	1.000
SP	0.065	0.035	0.149	0.610	*52.70	(6)	3.812
NZ	0.005	0.012	0.437	0.149	*8.10	(5)	3.812
CO	0.011	0.000	0.208	c	7.25	(5)	1.417
JO	0.261	0.229	0.355	c	*14.67	(4)	1.621
SY	0.022	0.035	0.031	c	4.82	(4)	3.449
PA	0.479	0.808	0.665	c	2.36	(4)	1.000
PH	0.004	0.007	0.016	c	*30.98	(4)	4.193
TH	0.189	0.000	0.000	c	3.14	(5)	4.592

*Significant at the one percent level.
[c]No wage data. [d]Collinearity problems.
AP=Andrews-Ploberger statistic, df=degrees of freedom.
λ depends on the observations chosen for the first and last possible break points.

for which there are no demand pressure effects on the price level.

The relative import price variable, s_t, does well in Table 1. All 27 coefficient estimates are positive, and 19 estimates have t-statistics greater than 2.0. The wage rate also does fairly well. Of the 17 estimates in Table 1, 12 have t-statistics greater than or equal to 2.0.

The price equation does fairly well in the tests in Table 2, especially for the quarterly countries. For the first lags test the added lags are significant at the five percent level for 4 of the 12 quarterly countries and 7 of the 14 annual countries. (At the one percent level there are only 2 significant cases for the quarterly countries and 5 for the annual countries.) The price equation thus "passes" the first lags test for 8 of 12 quarterly countries and 7 of 14 annual countries. For the second lags test there are 9 of 12 quarterly passes and 5 of 14 annual passes. Overall, this is a fairly strong showing. If the price equation had bad dynamics, one would not expect it to do well when lagged values are added, but it does fairly well, especially if the quarterly results are weighted more than the annual results.

For the autoregressive error term test, there are 5 of 13 quarterly rejections and 4 of 14 annual rejections. There are 3 of 10 quarterly rejections and 2 of 8 annual rejections for the leads test. Again, this is a fairly good showing.

The results are not good for the stability test. There are 9 of 12 quarterly rejections and 7 of 14 annual rejections at the one percent level. I have found the Andrews-Ploberger stability test hard to pass for macroeconometric equations, including price and wage equations. One problem is that the period of the 1970s was a volatile time for prices and wages, and equations that are estimated excluding this period can be quite different from those that are estimated including it. In order for a price equation to pass the stability test for sample periods like those in Table 1, it must not be too sensitive to the 1970s, and most equations do not seem to meet this requirement. Whether this is a small sample problem or the sign of a true structural break is hard to know without more observations. Estimating equations using sample periods that begin after the 1970s is problematic because of the small number of observations, and no attempt was made to do this here.

Turn now to the results for the wage equation in Table 3. Of the 11 countries for which a demand pressure variable was used, the functional form was linear for 7 of them. The chosen variable was the unemployment rate for 4 of the 11 and the second output-gap variable for the other 7. There is thus an edge for the linear form and the second output-gap variable. The good showing for the linear form further shows the difficulty of estimating nonlinearities between demand pressure and price and wage levels.

The results of testing the real wage restriction in Table 4 show that the restriction is rejected at the five percent level for 4 of the 10 quarterly countries and none of the 5 annual countries. (At the one percent level there are 2 of 10 quarterly rejections.) For the lags test the added lags are significant at the five percent level for 4 of the 10 quarterly countries and none of the 5 annual countries. Again, this is a fairly strong showing. If the wage equation had bad dynamics, one would not expect it to do well when lagged values are added, but it does fairly well. For the autoregressive error term test, there are 6 of 10 quarterly rejections and 2 of 5 annual rejections.

The stability-test results for the wage equation are not good. The are 7 of 9 quarterly rejections and 2 of 5 annual rejections. The same issues pertain here that were discussed above for the price equation.

Table 3
Estimates of the Wage Equation
$$w_t - \lambda_t = \gamma_0 + \gamma_1(w_{t-1} - \lambda_{t-1}) + \gamma_2 p_t + \gamma_3 p_{t-1} + \gamma_4 D_t + \gamma_5 t$$

	Best D	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_4$	$\hat{\gamma}_5$	$\hat{\rho}$	$\hat{\gamma}_3$	SE	DW
Quarterly										
CA	none	0.089	0.958	1.097	–	-0.00002	b	-1.050	0.0081	1.64
		(1.61)	(34.06)	(11.90)		(-0.48)				
JA	none	0.431	0.903	1.031	–	-0.00025	b	-0.930	0.0107	1.99
		(2.46)	(23.76)	(9.67)		(-1.70)				
AU	$G2_{-1}$(lin)	2.084	0.680	0.392	[a]-0.15830	0.00039	-0.661	-0.112	0.0157	1.66
		(4.13)	(8.96)	(1.50)	(-2.61)	(2.31)	(-7.58)			
FR	none	0.575	0.924	1.348	–	-0.00022	b	-1.252	0.0092	1.61
		(1.80)	(21.09)	(4.46)		(-1.97)				
GE	U_{-1}(lin)	0.684	0.914	0.922	[a]-0.20253	0.00038	-0.312	-0.843	0.0119	2.16
		(2.69)	(30.15)	(3.27)	(-2.39)	(2.11)	(-3.12)			
IT	U_{-1}(lin)	0.188	0.923	1.244	[a]-0.25124	-0.00026	b	-1.157	0.0139	1.94
		(1.80)	(22.81)	(6.74)	(-1.42)	(-0.91)				
NE	$G2_{-1}$(.020)	1.638	0.596	-0.025	[a]0.00020	0.00147	0.412	0.269	0.0055	1.96
		(5.76)	(9.06)	(-0.25)	(1.40)	(10.23)	(3.17)			
UK	$G2_{-1}$(.020)	0.263	0.912	0.790	0.00050	-0.00007	b	-0.697	0.0114	2.22
		(3.03)	(29.51)	(8.83)	(2.44)	(-1.02)				
FI	U_{-1}(lin)	0.149	0.813	0.534	[a]-0.09613	-0.00015	-0.339	-0.361	0.0096	1.96
		(2.13)	(10.06)	(2.43)	(-2.52)	(-1.10)	(-2.37)			
KO	$G2$(.020)	0.272	0.952	0.267	0.00197	-0.00024	b	-0.192	0.0283	2.19
		(3.15)	(21.10)	(3.20)	(2.43)	(-0.31)				
Annual										
DE	U(lin)	0.461	0.911	1.353	-0.61265	0.00290		-1.268	0.0139	2.25
		(0.58)	(6.29)	(6.49)	(-3.45)	(2.28)				
SW	$G2$(.020)	2.945	0.487	0.396	0.00162	0.00092		0.052	0.0224	2.03
		(3.51)	(3.49)	(2.21)	(3.13)	(0.48)				
GR	$G2$(lin)	0.261	0.953	0.912	-0.16925	0.00022		-0.867	0.0398	1.53
		(0.78)	(9.96)	(4.20)	(-1.70)	(0.05)				
IR	none	0.192	0.968	0.521	–	-0.00471		-0.489	0.0256	1.64
		(0.64)	(5.32)	(2.52)		(-2.40)				
SP	$G2$(lin)	0.642	0.845	1.365	-0.14801	0.00281		-1.197	0.0198	2.14
		(3.64)	(16.27)	(8.37)	(-2.41)	(1.46)				

t-statistics are in parentheses.
[b] ρ taken to be 0.
See the notes to Table 1.
$\hat{\gamma}_4$ is expected to be negative when the linear form is used and positive when the nonlinear form is used.
The sample periods are the same as those in Table 1.

Table 4
Test Results for the Wage Equation

	Real Wage Restr. p-val	Lags p-val	RHO+ p-val	Stability AP	(df)	λ
Quarterly						
CA	0.150	0.005	0.000	*44.94	(4)	7.415
JA	0.001	0.205	0.000	*7.77	(4)	4.314
AU	0.200	0.001	0.009	8.40	(6)	3.516
FR	0.879	0.214	0.127	*20.85	(4)	1.960
GE	0.157	0.097	0.012		–	
IT	0.160	0.026	0.054	*10.28	(5)	4.187
NE	0.753	0.805	0.746	2.91	(6)	1.403
UK	0.012	0.445	0.238	*11.18	(5)	6.921
FI	0.017	0.005	0.031	*17.35	(6)	1.403
KO	0.001	0.435	0.009	*16.47	(5)	8.635
Annual						
DE	0.868	0.355	0.045	5.53	(5)	2.179
SW	0.240	0.416	0.937	5.76	(5)	2.778
GR	0.387	0.334	0.467	*43.60	(5)	3.449
IR	0.600	0.314	0.001	1.07	(4)	1.000
SP	0.862	0.217	0.137	*26.05	(5)	3.812

*Significant at the one percent level.
See the notes to Table 2.

3 Tests of the NAIRU Specification

The Tests

The final form of the price equation in the previous section can be derived by lagging equation (2) one period, multiplying through by γ_1, subtracting this expression from equation (2), and then using equation (3) to substitute out the wage rate. The final form of the price equation is:[10]

$$\begin{aligned} p_t = \tfrac{1}{1-\beta_2\gamma_2}[&(\beta_0 + \beta_2\gamma_0 - \beta_0\gamma_1 + \beta_5\gamma_1) + (\beta_1 + \beta_2\gamma_3 + \gamma_1)p_{t-1} \\ &+\beta_3 s_t - \beta_3\gamma_1 s_{t-1} + (\beta_4 + \beta_2\gamma_4)D_t - \beta_4\gamma_1 D_{t-1} \\ &+(\beta_5 - \beta_5\gamma_1 + \beta_2\gamma_5)t + (\epsilon_t - \gamma_1\epsilon_{t-1} + \beta_2\mu_t)] \end{aligned} \tag{6}$$

Now consider equation (1), the NAIRU equation. Assume that the relevant cost shock variable in the equation is s_t and that the relevant demand pressure variable is D_t. Assume also that u_t^* is equal to a constant term plus a coefficient times the time trend, and add an error term to the equation. Given these assumptions, equation (1) can be written

$$\pi_t = \sum_{i=1}^{n} \delta_i \pi_{t-i} + \theta_0 + \theta_1 t + \theta_2 D_t + \theta_3 s_t + \nu_t, \quad \sum_{i=1}^{n} \delta_i = 1 \tag{7}$$

where ν_t is the error term.

How does equation (7) compare to equation (6)? Since $\pi_t = p_t - p_{t-1}$ and n is greater than 0, equation (7) has more lagged price levels in it than does equation (6), but with the restriction that each price level is subtracted from the previous price level and the restriction that the δ_i's sum to one. The restriction that each price level is subtracted from the previous price level will be called the "first derivative" restriction, and the restriction that the δ_i's sum to one will be called the "second derivative" restriction. Equations (6) and (7) also differ in that equation (7) has excluded from it s_{t-1} and D_{t-1}, which equation (6) includes.

If equation (7) is correctly specified, adding p_{t-1}, p_{t-2}, s_{t-1}, and D_{t-1} to it should not result in a significant increase in fit. Equation (6), on the other hand, implies that these variables belong in the equation. A simple test is thus to add

[10] I am indebted to Phil Howrey for suggesting the use of the final form price equation for the tests.

these four variables to equation (7) and do a χ^2 test of their joint significance. The two key variables in this addition are p_{t-1} and p_{t-2}. Adding one of them breaks the second derivative restriction, and adding both of them breaks both the first and second derivative restrictions.

Equation (7) was estimated for the following results by OLS under the assumption that the error term ν_t is iid. Note that the error term in equation (6) is not iid (assuming that ϵ_t and μ_t are either iid or autoregressive), and this is another way in which equations (6) and (7) may differ. Since most estimates of equations like (7) assume that the error term is iid, the iid assumption for (7) is used here. Also, s_t and D_t are treated as exogenous in the estimation of (7), whereas these were treated as endogenous in the previous section. Again, most estimates of equations like (7) use OLS, and so this is done here.

Two tests of equation (7) were made for values of n of 8 and 12 for the quarterly countries and values of n of 2 and 3 for the annual countries. For each n, the first test is adding the four variables p_{t-1}, p_{t-2}, s_{t-1}, and D_{t-1} to the equation and seeing if they are jointly significant. This is a test of both the first and second derivative restrictions. The second test for each n takes as the base equation equation (7) *without* the summation restriction imposed. The test is adding the three variables p_{t-1}, s_{t-1}, and D_{t-1} to this equation and seeing if they are jointly significant. This is a test of the first derivative restriction conditional on the second derivative restriction not being imposed.

Given the time-series nature of equation (7) it is not obvious that the standard asymptotic distributions that are used for hypothesis testing are good approximations of the exact distributions. For the U.S. results in Fair (1997) the χ^2 distribution did not approximate well the exact distributions of the computed “χ^2” values, and this is also true for the present results. The appendix describes how stochastic simulation can be used to obtain the exact distribution of the “χ^2” values for each test, and this was done here. For all the hypothesis testing in this section, the exact distributions are used.

The Results

The computed χ^2 values and the computed five and one percent critical values are presented in Table 5 for each test for each country.[11] Consider first the results for the quarterly countries. For the first test (summation restriction imposed and four variables added), the added variables are significant at the five percent level in 10 of 13 cases for $n = 8$ and in 8 of 13 cases for $n = 12$. The added variables are significant for Canada, Japan, and all the major European countries except France. Overall, this is fairly strong evidence against the NAIRU specification.

The second test is a test of the first derivative restriction conditional on the second derivative restriction not being imposed. The results are presented in the second half of Table 5. For the quarterly countries the added variables are significant at the five percent level in 6 of 13 cases for both $n = 8$ and $n = 12$. The evidence is thus mixed regarding the first derivative restriction. It is rejected at the five percent level a little less than half the time.

[11] If for a given country the price and wage equations had different demand variables, both variables were used in equation (7) and the lagged values of both variables were added for the tests in Table 5. If no demand variables were used for a country in Tables 1 and 3, then no demand variable was used in equation (7). The column labelled “df” in Table 5 shows how many variables were added per test per country.

Table 5
Tests of Equation (7)
p_{t-1}, p_{t-2}, s_{t-1}, D_{t-1} added
Summation restriction imposed

	$n = 8$ or 2			$n = 12$ or 3			
	"χ^2"	Critical 5% Value	Critical 1% Value	"χ^2"	Critical 5% Value	Critical 1% Value	df
Quarterly							
CA	*27.97	24.80	31.37	*27.57	22.74	29.40	4
JA	*29.64	24.57	32.09	*25.80	23.68	30.14	4
AU	**33.42	26.09	31.97	*30.26	24.22	31.16	5
FR	2.47	18.44	25.90	5.42	17.12	23.51	4
GE	**36.12	22.95	33.06	*22.23	20.71	27.41	5
IT	**69.17	27.78	35.02	**57.71	26.33	32.87	5
NE	*22.55	22.01	28.15	*25.24	19.07	26.93	4
ST	**47.77	23.41	30.06	**51.59	22.14	29.30	4
UK	**32.74	20.77	25.42	**27.68	21.39	27.61	4
FI	14.61	28.55	37.34	12.25	27.17	37.28	5
AS	17.20	21.26	26.68	16.98	20.74	27.87	4
SO	**30.28	20.65	25.46	16.19	20.75	27.10	3
KO	*25.97	20.67	26.57	16.22	21.47	25.36	4
Annual							
BE	**97.77	31.14	45.33	**105.90	29.63	46.15	4
DE	12.62	32.55	47.33	12.12	30.08	44.40	5
NO	23.06	23.80	35.15	17.00	20.97	31.24	4
SW	20.94	28.26	42.14	18.24	27.41	40.46	5
GR	19.25	27.70	45.04	15.49	26.71	42.64	5
IR	*33.17	26.63	39.83	22.13	23.15	40.73	3
SP	10.61	26.16	40.21	10.68	25.37	39.28	5
NZ	*27.93	24.83	33.21	*20.58	20.19	28.10	3
CO	**78.00	37.32	60.58	**50.73	31.96	48.88	4
JO	**39.43	25.61	34.04	*34.65	27.76	43.60	3
SY	21.15	25.13	39.04	22.90	23.41	33.06	3
PA	21.96	26.67	45.63	6.78	17.97	27.85	3
PH	**77.60	21.75	28.19	**46.80	17.96	26.81	3
TH	**71.55	30.24	43.35	**55.51	31.64	44.92	4

Table 5 (continued)
Tests of Equation (7)
p_{t-1}, s_{t-1}, D_{t-1} added
Summation restriction not imposed

	n = 8 or 2			n = 12 or 3			
	"χ^2"	Critical 5% Value	Critical 1% Value	"χ^2"	Critical 5% Value	Critical 1% Value	df
Quarterly							
CA	*11.88	10.94	15.94	8.47	10.02	13.97	3
JA	12.37	12.91	19.48	*14.84	12.15	18.46	3
AU	*17.18	12.65	17.23	**20.17	13.89	19.47	4
FR	1.32	13.17	17.25	2.28	12.32	18.29	3
GE	10.34	11.32	16.19	8.55	11.76	17.45	4
IT	**16.95	11.02	16.29	*12.45	12.05	16.62	4
NE	*15.06	14.27	20.65	*18.17	13.18	18.67	3
ST	7.99	11.96	17.95	6.67	12.75	20.27	3
UK	*11.32	10.22	14.97	*10.79	10.25	14.99	3
FI	3.46	14.84	21.61	2.42	14.40	21.43	4
AS	4.22	8.93	13.16	1.01	9.76	13.63	3
SO	**13.45	7.14	11.46	*10.31	9.36	13.93	2
KO	10.30	12.70	17.53	7.30	12.50	16.12	3
Annual							
BE	**38.65	13.27	21.96	*47.32	14.19	23.92	3
DE	8.36	15.35	23.21	9.85	16.13	24.06	4
NO	3.61	12.36	20.42	3.48	12.58	18.79	3
SW	7.14	17.37	27.06	8.00	18.05	27.30	4
GR	5.79	16.71	24.08	6.11	16.88	27.01	4
IR	0.62	8.87	13.56	0.24	9.08	14.34	2
SP	3.37	16.54	25.03	4.09	17.75	30.66	4
NZ	7.59	10.29	15.41	9.09	9.76	18.87	2
CO	*22.13	21.30	33.29	17.42	21.68	35.07	3
JO	1.44	7.64	14.57	0.37	9.79	18.01	2
SY	6.46	13.17	22.67	*14.00	13.88	21.05	2
PA	0.89	14.71	25.74	0.62	12.37	27.36	2
PH	7.49	8.93	15.31	6.29	7.63	13.28	2
TH	**31.54	16.27	25.00	**29.47	14.98	25.12	3

*Significant at the five percent level.
**Significant at the one percent level.

For the annual countries and the first test, the added variables are significant at the five percent level in 7 of 14 cases for $n = 2$ and in 6 of 14 cases for $n = 3$. The evidence against the NAIRU specification is thus not as strong for the annual countries as it is for the quarterly countries. The specification is rejected about half the time.

For the annual countries and the second test, the added variables are significant at the five percent level in 3 of 14 cases for both $n = 2$ and $n = 3$. There is thus little evidence against the first derivative restriction conditional on the second derivative restriction not being imposed for the annual countries.

In summary, if one weights the quarterly results more, the evidence in Table 5 is clearly against the NAIRU specification (i.e., against both the first and second derivative restrictions). The results are mixed regarding the first derivative restriction conditional on the second derivative restriction not being imposed, and no strong conclusion can be drawn regarding it.

4 Policy Implications

If equations (2) and (3) are correctly specified but for policy analysis one used equation (7) instead, how much difference would this make? Results that pertain to this question are presented in Table 6 for Japan and Germany.

The following experiment was performed for Japan, first using equations (2) and (3) and then using equation (7). The output-gap variable was decreased by one percentage point from its base path beginning in 1996:1, and the effects of this change on the predicted values of the price level were examined. For all the predictions the actual values for 1995:4 back were used as initial conditions, and s was taken to

Table 6
Policy Implications: Japan

Quar.	Eqs. (2)&(3) $\frac{P^{new}}{P^{base}}$	Eqs. (2)&(3) $\Delta p^{new} - \Delta p^{base}$	Eq.(7),no t,no $\sum$ $\frac{P^{new}}{P^{base}}$	Eq.(7),no t,no $\sum$ $\Delta p^{new} - \Delta p^{base}$	Eq.(7),no t,yes $\sum$ $\frac{P^{new}}{P^{base}}$	Eq.(7),no t,yes $\sum$ $\Delta p^{new} - \Delta p^{base}$
1	1.0028	1.12	1.0002	0.08	1.0001	0.06
2	1.0053	0.97	1.0005	0.12	1.0003	0.08
3	1.0074	0.84	1.0010	0.18	1.0007	0.12
4	1.0092	0.73	1.0015	0.22	1.0010	0.15
5	1.0108	0.63	1.0021	0.25	1.0015	0.18
6	1.0122	0.55	1.0028	0.26	1.0020	0.20
7	1.0134	0.48	1.0035	0.28	1.0026	0.23
8	1.0145	0.41	1.0042	0.30	1.0032	0.26
9	1.0154	0.36	1.0050	0.31	1.0039	0.29
10	1.0162	0.31	1.0058	0.33	1.0047	0.32
11	1.0168	0.27	1.0066	0.33	1.0056	0.35
12	1.0174	0.24	1.0075	0.34	1.0066	0.38
100	1.0215	0.00	1.0863	0.34	1.4496	2.91
∞	1.0215	0.00	∞	0.34	∞	∞

Policy Implications: Germany

Quar.	Eqs. (2)&(3) $\frac{P^{new}}{P^{base}}$	Eqs. (2)&(3) $\Delta p^{new} - \Delta p^{base}$	Eq.(7),no t,no $\sum$ $\frac{P^{new}}{P^{base}}$	Eq.(7),no t,no $\sum$ $\Delta p^{new} - \Delta p^{base}$	Eq.(7),no t,yes $\sum$ $\frac{P^{new}}{P^{base}}$	Eq.(7),no t,yes $\sum$ $\Delta p^{new} - \Delta p^{base}$
1	1.0000	0.00	1.0000	0.00	1.0000	0.00
2	1.0008	0.31	1.0010	0.39	1.0003	0.11
3	1.0016	0.33	1.0021	0.46	1.0007	0.16
4	1.0024	0.33	1.0035	0.56	1.0012	0.21
5	1.0033	0.34	1.0053	0.68	1.0019	0.28
6	1.0041	0.34	1.0067	0.57	1.0027	0.30
7	1.0050	0.34	1.0084	0.66	1.0036	0.36
8	1.0058	0.34	1.0101	0.67	1.0046	0.42
9	1.0067	0.33	1.0116	0.63	1.0058	0.46
10	1.0075	0.33	1.0133	0.67	1.0071	0.53
11	1.0083	0.32	1.0149	0.61	1.0086	0.57
12	1.0091	0.31	1.0164	0.59	1.0101	0.62
100	1.0239	0.00	1.1524	0.57	1.9353	5.23
∞	1.0239	0.00	∞	0.57	∞	∞

P = price level, $p = 400 \log P$

remain unchanged from 1995:4 on. The base prediction path for each experiment took the output-gap variable to be equal to its 1995:4 value for all future periods, and the new prediction path took the output gap to be one percentage point lower than this value for all future periods.

A similar experiment was performed for Germany. In this case the beginning period was 1995:1 and there were two variables to change, the output-gap variable, which appears in the price equation, and the unemployment rate, which appears in the wage equation. Both were lowered by one percentage point.

The first two columns in Table 6 for each country present the results using equations (2) and (3). The coefficient estimates used for these equations are those presented in Tables 1 and 3, and the equations were solved by the Gauss-Seidel iterative technique. After 12 quarters the predicted Japanese price level is 1.74 percent higher in the new case than in the base case and the predicted German price level is 0.91 percent higher. Inflation in the first year is about one percentage point higher (at an annual rate) for Japan and about a third of a percentage point higher for Germany. After 100 quarters the price level is 2.15 percent higher for Japan and 2.39 percent higher for Germany, and the inflation rates are back to those in the base case.

The results in the next two columns are for equation (7) estimated for $n = 12$ with the time trend excluded[12] and without the summation restriction imposed. For Japan the predicted price level after 12 quarters, new versus base, is 0.75 percent

[12]Even if u_t^* in equation (1) has a trend within the estimation period (thus requiring the use of t in equation (7)), it would not be sensible in policy experiments to extrapolate this far into the future. For long-run policy experiments one could either use the equation with the estimated trend and stop the trend at some future point or estimate the equation without the trend. For present purposes it was decided to estimate the equation without the trend.

higher and the inflation rate is 0.34 percentage points higher. After 100 quarters the price level is 8.63 percent higher, and as time marches on the differences in the price levels become larger and larger. The long-run implication regarding the inflation rate is that it is 0.34 percentage points higher. For Germany the predicted price level after 12 quarters is 1.64 percent higher and the inflation rate is 0.59 percentage points higher. In the long run the inflation rate is 0.57 percentage points higher.

The results in the last two columns are for equation (7) estimated for $n = 12$ with the time trend excluded and with the summation restriction imposed. This is the NAIRU specification. For Japan the predicted price level after 12 quarters, new versus base, is 0.66 percent higher and the inflation rate is 0.38 percentage points higher. After 100 quarters the inflation rate 2.91 percentage points higher and the price level is 44.96 percent higher. For Germany the predicted price level after 12 quarters is 1.01 percent higher and the inflation rate is 0.62 percentage points higher. After 100 quarters the inflation rate is 5.23 percentage points higher and the price level is 93.53 percent higher.

An interesting feature of the results in Table 6 is that their implications after, say, 12 quarters are fairly close, even though the long-run implications are vastly different. One would be hard pressed to choose between equations (2) and (3) versus equation (7) on the basis of which short-run implications seem more "reasonable." Instead, one needs tests of the kind performed in this paper. It is, of course, stretching the trustworthiness of any equation or set of equations to examine what it implies about the effects of a current policy change many quarters in the future. The results in this paper suggest that the dynamics in the Japanese and German economies are closer to the values for equations (2) and (3) in Table 6 than to the values for equation

(7), but the exact long-run implications should be taken with considerable caution.

Finally, note that if the demand pressure variables used for Japan and Germany were nonlinear (which they are not), the size of the changes in Table 6 would depend on the their initial values. Even though the best-fitting form was the linear form, it would not be sensible to use this form to extrapolate the equations to very high levels of demand.

5 Conclusion

The results in Fair (1997) using U.S. data overwhelmingly reject both the NAIRU specification and the first derivative restriction. The results in this paper are not as decisive, but there is a fairly strong rejection of the NAIRU specification. The results testing the first derivative restriction are mixed, and no conclusion is possible. It would be easy in future work for others to test NAIRU specifications in the manner done in this paper. The main test is simply to add a few variables, especially the log of the price level lagged one and two periods, to the equation and examine their significance.

As noted in Section 1, if the NAIRU specification is rejected, it changes the way one thinks about the trade-off between inflation and unemployment, but it does not have to imply that unemployment can be driven to very low levels with only a modest effect on the price level. There may be a strongly nonlinear relationship between the price level and unemployment at low levels of unemployment. Unfortunately, as discussed above, it is hard to estimate where this nonlinear zone begins.

Given the difficulty of estimating where the severe nonlinear zone begins, policy makers are faced with a hard problem. There are too few high-activity observations for any confidence to be placed on the point at which output should not be pushed further without severe price-level consequences. The results in this paper are of little help regarding this question. The main point of this paper for policy makers is that they should not think there is some unemployment rate below which inflation forever accelerates and above which it forever decelerates. They should think instead that the price level is a negative function of the unemployment rate, where at some point the function begins to become severely nonlinear. How bold a policy maker is in pushing the unemployment rate into uncharted waters will depend on how fast he or she thinks the nonlinearity becomes severe.

Appendix

The stochastic simulation procedure that can be used to compute the exact distributions is as follows. Consider the test of adding the four variables p_{t-1}, p_{t-2}, s_{t-1}, and D_{t-1} to equation (7). First, estimate equation (7) without the variables added (with the restriction that the δ_i's sum to one imposed). Record the coefficient estimates and the estimated variance of the error term. Call this the "base" equation. Second, assume that the error term is normally distributed with mean zero and variance equal to the estimated variance. The rest of the procedure is then as follows:

1. Using the normality assumption and the estimated variance, draw a value of the error term for each quarter in the estimation period. Add these error terms to the base equation and solve it dynamically to get new data for p. Given the new data for p and the other necessary data (which have not changed), test the hypothesis that the four variables belong in the equation. This is done by estimating the equation (by OLS) with and without the variables added and computing the χ^2 value. Record this value.

2. Do step 1 J times, which give J χ^2 values. Call the distribution of these values the "exact" distribution.

3. Sort the χ^2 values by size, choose the value above which k percent of the values lie, and compare this value to the critical k percent value of the actual χ^2 distribution.

These calculations were done for $J = 1000$ for each country and each of the two values of n, and the computed five and one percent critical values are presented in Table 5. These values are noticeably larger than the critical values from the actual

χ^2 distribution.[13] The exact distributions appear to have much fatter tails than does the actual χ^2 distribution.

The test of adding the three variables to the equation without the summation restriction imposed is handled similarly. In this case equation (7) is first estimated without the three variables added and without the summation restriction imposed to get the base equation. The rest of the procedure is the same as above, where in step 1 the test is adding three rather than four variables to the equation. These calculations were also done for $J = 1000$ for each country and each of the two values of n, and the computed five and one percent critical values are also presented in Table 5. Again, these values are larger than the critical values from the actual χ^2 distribution. In this case, however, the computed values are closer to the critical values from the actual χ^2 distribution than they are in the previous case.

As noted in the paper, all the hypothesis testing concerning Table 5 was done using the computed critical values.

Finally, some initial experimentation was done using the above procedure to obtain exact distributions for the tests for equations (2) and (3), and for these cases the exact distributions were close to the actual distributions. No adjustments were thus made for the tests in Tables 2 and 4.

[13]The five percent critical values for 2, 3, 4, and 5 degrees of freedom, respectively, are 5.99, 7.82, 9.49, and 11.07. The corresponding one percent critical values are 9.21, 11.34, 13.28, and 15.09.

References

[1] Andrews, Donald W. K., and Ray C. Fair, 1988, "Inference in Nonlinear Econometric Models with Structural Change," *Review of Economic Studies*, 55, 615-640.

[2] Andrews, Donald W. K., and Werner Ploberger, 1994, "Optimal Tests When a Nuisance Parameter is Present Only Under the Alternative," *Econometrica*, 62, 1383-1414.

[3] Fair, Ray C., 1974, *A Model of Macroeconomic Activity. Volume I: The Theoretical Model*, Ballinger Publishing Co.

[4] ________, 1984, *Specification, Estimation, and Analysis of Macroeconometric Models*, Cambridge, MA: Harvard University Press.

[5] ________, 1994, *Testing Macroeconometric Models*, Cambridge, MA: Harvard University Press.

[6] ________, 1997, "Testing the NAIRU Model for the United States," April.

[7] Hansen, Lars Peter, 1982, "Large Sample Properties of Generalized Method of Moments Estimators," *Econometrica*, 50, 1029-1054.

[8] Hendry, David F., Adrian R. Pagan, and J. Denis Sargan, 1984, "Dynamic Specifications," in Z. Griliches and M. D. Intriligator, eds., *Handbook of Econometrics*, Amsterdam: North-Holland, 1023-1100.