Physical Decline Rates: Men versus Women Ray C. Fair* November 2024

Abstract

This paper uses world records by age in running, swimming, and rowing to estimate a biological frontier of decline rates for both men and women. Decline rates are assumed to be linear in percent terms up to a certain age and then quadratic after that, where the transition age is estimated. The use of world records avoids the possible problem of survivor bias in a sample.

The decline rates are smallest for rowing, followed by swimming and then running. Decline rates for women are roughly the same as those for men for the swimming events. They are slightly larger for the rowing events. They are largest for running. The age at which there is a 50 percent decline from age 30 ranges from 70 to 89, an optimistic result for humans. Ten year decline rates from age 40 to about the mid 60's are about 10 percent for running and 5 percent for swimming and rowing.

1 Introduction

An important economic policy question is what to assume about the physical abilities of people as they age. In setting a retirement age one question is how much decline there is up to the chosen age? What can one expect, say, from a 70 year old versus a 65 year old? In medicine an important question is how much exercise to recommend as people age. The key question here is a biological one: how fast do people's physical abilities decline with age.

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This paper uses world records by age to estimate biological frontiers. About a hundred years ago Hill (1925) pointed out the potential usefulness of athletic records to study the physiology of muscular exercise. Moore (1975) was the first to use best-performance records by age to examine how athletic performance changes with age. This was followed by Salthouse (1976). Stones and Kozma (1980) used records by five-year age intervals to examine performance changes by age—see also Stones and Kozma (1981, 1982, 1986a, 1986b). The next study after Salthouse (1976) to use records by one-year age intervals was Fair (1994). Later studies using athletic records include Baker, Tang, and Turner (2003), Donato et al. (2003), Tanaka and Seals (1997, 2003), and Baker and Tang (2010). The results in Fair (1994) were updated in Fair (2007) and Fair and Kaplan (2019).

An advantage of using best athletic records to examine human performance is that most of the records are based on very large samples. For example, many 60year old men have run a marathon, and so the fastest marathon time ever recorded by a 60-year old man is based on a very large sample of attempts, much larger than would ever be feasible in an experimental setting. In addition, the age range for which records exist is large, again much larger than is feasible in an experimental setting.

A serious problem with many cross-section and longitudinal studies is survivor bias. Weaker subjects tend to drop out from a sample as they age at a faster rate, and so more talented people may be over represented at the older ages (Brant and Fozard (1990), Colshen and Wallace (1991), Tanaka and Seals (1997), Lindenberger and Baltes (1997), Anstey, Hofer, and Luszcz (2003), Hertzog and Nesselroade (2003), and Singer, Lindenberger, and Baltes (2003)). If this is not accounted for, the estimated decline rates will be too small. Recent studies that are likely to have this bias include Tuna et al. (2009), Milanović et al. (2013), and Bagley et al. (2019), given the sample that each used. These studies have not accounted for the fact that older participants are likely more talented than younger ones simply because they have survived to the older ages. The use of world records avoids this kind of bias, although there still may be a small sample problem as discussed below.

This paper uses world records by age to avoid survivor bias. It makes five main contributions to the literature. First, it is not restricted in using a a particular functional form, usually a quadratic, throughout the entire age range. The model that is estimated assumes a linear decline rate in percentage terms up to some transition age and then quadratic after that. The transition age is estimated. Second, it uses a large number of events, and so comparisons can be made across events. Third, there is now fairly good data on women, and so comparisons can be made of men versus women on how they decline. Fourth, the sensitivity of the estimates to forcing all the errors to lie above or on the biological frontier is examined using a frontier estimation method. Finally, only non dominated times are used in the estimation, as discussed below.

The model uses two restrictions that seem sensible biologically. The first is that after a certain age (age 30 is used here) the rate of decline is non decreasing with age. This is the "first derivative" restriction. The second is that the change in the rate of decline is non decreasing with age. This is the "second derivative" restriction. In short, after decline begins, nothing gets better with age. The linearquadratic (LQ) model used here automatically meets these restrictions.

It will be seen that for both men and women the decline rates are smallest for rowing, followed by swimming and then running. Decline rates for women are roughly the same as those for men for the swimming events. They are slightly larger for the rowing events. They are largest for running. The age at which there is a 50 percent decline from age 30 ranges from 70 to 89, an optimistic result for humans. Ten year decline rates from age 40 to about the mid 60's are about 10 percent for running and 5 percent for swimming and rowing.

2 The Data

Data for five running events were obtained from the site of the Association of Road Racing Statisticians (AARS): *arrs.net/SARec.htm*. The data are AARS recognized world records by age. Four of the events are road racing events: 5K, 10K, Half Marathon, and Marathon, and the fifth event is 5,000 meters outdoor track. Data for both men and women were obtained. The AARS data end in 2019, and more recent data were obtained from two Wikipedia sites: *https* : *//en.wikipedia.org/wiki/List_of_world_records_in_masters_athletics*, *https* : *//en.wikipedia.org/wiki/List_of_masters_world_records_in_road_running*. The data were obtained on November 17, 2024. For men there were 22 world records set after 2019, and for women there were 26. One of the more impressive records was age 60, women's 10K, where the record dropped from 39:10 to 36:43.

World records by age for swimming were obtained from the World Aquatics site: *worldaquatics.com/masters/records*. Results for six long course meters (LCM) freestyle events were obtained: 50, 100, 200, 400, 800, and 1500 meters. Data were only available in five-year intervals, 30-34, 35-39, ..., 100-104. For each interval the age was taken to be the youngest age, 30, 35, ..., 100. Data for both men and women were obtained. The data were obtained on September 2, 2023.

World records by age for Concept 2 rowing were obtained from the site: *concept2.com/indoor-rowers/racing/records/world*. The machine was RowErg; the weight was heavyweight; and the events were 1000, 2000, 5000, 6000, and 10000 meters. Data were also only available in five-year intervals, but in this case the age of the record holder was available. Data for both men and women were obtained. The data were obtained on September 7, 2023.

Some of the data are likely "soft" at the older ages in the sense that not enough people have participated in the event to have the best time be a good estimate of the biological frontier. This means that over time world records are likely to fall more at the older ages than at the younger ones. An example is the women's 10K event at age 60 discussed above.

To adjust for possible softness, the oldest age for the half marathon and marathon for both men and women was taken to be 85. Also, the oldest age for women for the 10K was taken to be 85. For swimming, the age category 100-104 was not used. For rowing, the age 95 record for men for 1000 meters rowing was excluded. Also for rowing, the data appeared soft for the 21,097 and 42,195 meter rowing events, especially for women, and these two events were not used. The age 80 observation for women's rowing 10000 meters was also excluded. Finally, the 100 meter and 500 meter rowing events were excluded. For men 100 meters and 500 meters the world records were at ages in the mid 40's, and so the times in the 30's are likely soft. For women 100 meters the time at age 54 was close to the world record.

Observations with dominated times were also excluded. A time is dominated if there is a lower time at an older age. A dominated time is thus soft, which is the reason for its exclusion. There was one dominated record for rowing and three for swimming. There were a number for running, primarily because there were records at each age rather then in just five year intervals.

It is possible that over time there is technical progress in the various events that lowers times. Examples might be better equipment, better diets, better training. If this progress affects each age in the same percentage terms, the percentage decline rates will not be affected. A decline curve will just be shifting parallel down over time. There could, however, be a timing issue in that progress may afect some age performances faster than others. Nothing can be done about this, but the world records used in this study are mostly recent. The world record data are primarily since 2000. For rowing the oldest record was 2011 for women and 2010 for men. For swimming all of the records were set after 2000. For running there were only 14 records out of 271 observations used that were set before 1990, with the two earliest being in 1977.

Table 1 lists the notation for the 16 events plus one pooling case.

The Events							
Notation	Description						
Run							
5000	5000 meters, outdoor track						
5K	5К						
10K	10K						
Half MA	half marathon						
MA	marathon						
Swim							
50	LCM, freestyle, 50 meters						
100	LCM, freestyle, 100 meters						
200	LCM, freestyle, 200 meters						
400	LCM, freestyle, 400 meters						
800	LCM, freestyle, 800 meters						
1500	LCM, freestyle, 1500 meters						
Row							
1000	RowErg, heavyweight, 1000 meters						
2000	RowErg, heavyweight, 2000 meters						
5000	RowErg, heavyweight, 5000 meters						
6000	RowErg, heavyweight, 6000 meters						
10000	RowErg, heavyweight, 10000 meters						
POOL	RowErg, heavyweight, pooled 1000-10000 meters						

Table 1

3 The Linear/Quadratic (LQ) Model

Consider first decline from age 40 on. It may be that there is some decline between, say, ages 30 and 39, but this decline may be less in percentage terms than decline from 40 on. The following model is for decline starting at age 40. The estimation of decline between 30 and 39 is discussed at the end of this section.

Ages 40 on

Let r_k denote the log of the record time for age k. Using logs means that all decline rates are in percentage terms. b_k will be used to denote log of the (unobserved) biological minimum time for age k. By definition,

$$r_k = b_k + \epsilon_k \quad , \tag{1}$$

where ϵ_k is the gap between the record time and the true biological minimum time. It will be close to zero if the record time is close to the biological minimum. Otherwise it is positive.

The LQ model postulates that the decline rate (in percentage terms) is linear up to a transition age and then quadratic after that. The transition age is one of the estimated parameters. At the transition age the linear and quadratic segments are constrained to touch and to have the same first derivative. The formula for b_k is

$$b_k = \begin{cases} \beta + \alpha k, & 40 \le k \le k^*, \quad \alpha > 0\\ \gamma + \theta k + \delta k^2, & k > k^*, \quad \delta > 0 \end{cases}$$
(2)

with the restrictions

$$\gamma = \beta + \delta k^{*2}$$

$$\theta = \alpha - 2\delta k^{*}$$
(3)

 k^* is the transition age. The two restrictions force the linear and quadratic segments to touch and to have the same first derivative at k^* . The unrestricted parameters to estimate are the intercept, β , the slope of the linear segment, α , the transition age, k^* , and the quadratic parameter, δ . The first derivative of b_k with respect to k is α up to the transition age and then increases by a constant amount (2δ) after that. The second derivative is zero up to the transition age and then constant (2δ) after that.

The equation that is estimated is then

$$r_k = \beta + \alpha k + \delta d_k (k^{*2} - 2k^* k + k^2) + \epsilon_k,$$
(4)

where $d_k = 0$ if $k \le k^*$ and $d_k = 1$ if $k > k^*$. ϵ_k is greater than or equal to zero, so it has a positive mean. A positive mean poses no problem in the estimation because it is simply absorbed in the estimate of the constant term. This means that the constant β is not identified, but this is of no concern here because the derivatives do not depend on β . The equation can be estimated by non linear least squares, NLS.

The equation can also, however, be estimated under the restriction that $\epsilon_k \ge 0$ for all k. The procedure is common in the estimation of frontier production functions—see, for example, Aigner and Chu (1968) and Schmidt (1976). The added complication here is that equation (4) is nonlinear in coefficients. For linear equations the estimation problem can be set up as a quadratic programming problem and solved by standard methods.

The procedure used here is the following. In the NLS case the coefficients in equation (4) are estimated by minimizing the sum of squared residuals, $\sum_{k=1}^{K} \hat{\epsilon}_k^2$, where K is the total number of observations. Instead, one can minimize a weighted sum, $\sum_{k=1}^{K} \lambda_k \hat{\epsilon}_k^2$, where λ_k is equal to 1 if $\hat{\epsilon}_k \ge 0$ and is equal to a number greater than 1 if $\hat{\epsilon}_k < 0$. This penalizes negative errors more than non-negative ones. For the results here a value of 1000 was used for λ_k when $\hat{\epsilon}_k$ was less than zero.

It will be seen that the use of the frontier procedure instead of NLS has generally small effects on the slope coefficients and k^* and thus on the estimated derivatives. The use of the procedure primarily affects the estimate of the constant term β , which is not of concern here.

The rowing events had very similar coefficient estimates, and for these events pooling was done. The assumption is that the curve for each event is the same except for the intercept. The equation estimated is (n is the number of events pooled):

$$r_{ik} = \beta_1 D_{1ik} + \dots + \beta_n D_{nik} + \alpha k + \delta d_{ik} (k^{*2} - 2k^* k + k^2) + \epsilon_{ik}, \quad (5)$$
$$i = 1, \dots, n; k = 40 \dots, K_i,$$

where r_{ik} is the log of the observed record for event *i* and age *k*, D_{jik} is a dummy variable that is equal to 1 when event *i* is equal to event *j* and 0 otherwise (j = 1 ... n), $d_{ik} = 1$ if $k \le k^*$ and $d_{ik} = 0$ if $k \ge k^*$, ϵ_{ik} is the error for event *i* and age *k*, and K_i is the oldest age used for event *i*. The *n* β coefficients are the *n* different constant terms.

Ages 30-39

Data were collected for each event from age 30 on. The overall world record for each event and gender was also collected. In estimating the decline rate between 30 and 39 the time at age 30 was taken to be the overall world record even if the actual time was higher. In other words, the times at age 30 were assumed to be soft if they were not the overall world record, and the overall world record was used. This assumes that decline does not begin before 30.

From the above estimation for each event and gender the predicted value of b_k is available for age 40, \hat{b}_{40} . (Remember that the times are in logs.) Then the values of b_k between 30 and 39 were assumed to lie on a straight line between the world record (age 30 time) and \hat{b}_{40} . One would expect the slope of this line to be less than $\hat{\alpha}$ if the percent decline before age 40 is less. In the tables below the ratio of the slope to $\hat{\alpha}$ is presented. Note that the above estimation from age 40 on is not affected by this treatment for ages 30 through 39.

For the results below 'age factors," denoted R_k , are presented. They are computed as follows. Let \hat{b}_k denote the predicted value of b_k using the estimated values of β , α , k^* , and δ for $k = 40, \ldots$. Let \hat{b}_k denote the predicted value of b_k for $k = 30, \ldots, 39$ using the above procedure for ages 30-39. Then R_k is

$$R_k = e^{b_k} / e^{b_{30}}, \quad k = 30, \dots$$
(6)

 R_k is an estimate of the percent decline at age k from age 30. This estimate does not depend on the estimate of β , so the estimate of the constant term in the equation

does not matter. It does depend on the overall world record for the event and gender because \hat{b}_{30} is the overall world record.

4 The Results

There are five running events, six swimming events, and five rowing events, for a total of 16 cases per gender. The estimates for these 32 cases are presented in Table 2. The coefficient estimates for five rowing events for each gender are close enough to warrant pooling, and the pooling estimates are presented at the bottom of Table 2.

Table 2 presents the estimates of α , k^* , δ , the slope divided by α , the implied age factors for ages 70, 80, and 90, the number of observations, the maximum age in the estimation period, the age at which the decline is 50 percent from age 30 (denoted "Half"), and the estimated standard error of the estimate of k^* . For each case the men's results are presented and then the women's. Although not shown, the coefficient estimates are highly significantly different from zero. Only one estimate of α has a t-statistic less than 2.0, 1.85 for row 1000 meters women, and no estimate of δ has a t-statistic less than 2.0. This is, of course, not surprising since there is obvious decline in the data. The estimated standard errors of the estimates of the transition age. There is collinearity between the estimate of the transition age and the estimate of δ .

Table 3 is the same as Table 2 except that the estimates are obtained from the frontier method, where all the estimated residuals are forced to be non negative.

					NLS Esti	imates						
Event	m/ w	$\hat{\alpha}$	$\hat{k^*}$	$\hat{\delta}$	$slope/\hat{lpha}$	R_{70}	R_{80}	R_{90}	No. Obs.	Max Age	Half	${\displaystyle \mathop{\rm SE}_{\hat{k^{*}}}}$
Run 5000 5000	m w	0.0095 0.0107	72.8 65.8	0.00114 0.00061	0.71 0.55	1.42 1.48	1.66 1.84	2.41 2.58	30 26	96 96	76 71	1.1 3.5
5K	m	0.0084	66.9	0.00085	0.67	1.37	1.71	2.52	30	95	76	2.4
5K	w	0.0101	66.0	0.00078	0.39	1.43	1.82	2.70	27	95	73	1.7
10K	m	0.0101	76.2	0.00268	0.22	1.38	1.59	2.83	28	92	78	0.6
10K	w	0.0096	58.6	0.00035	0.45	1.46	1.80	2.38	22	85	72	0.6
HMA	m	0.0083	61.5	0.00039	0.63	1.39	1.68	2.18	23	85	75	3.5
HMA	w	0.0112	58.9	0.00033	0.19	1.49	1.85	2.46	21	85	71	5.9
MA	m	0.0107	73.2	0.00123	0.22	1.41	1.66	2.47	27	85	76	1.8
MA	w	0.0124	65.9	0.00074	0.20	1.51	1.95	2.93	19	85	70	4.3
Swim 50 50	m w	0.0050 0.0072	64.5 76.6	0.00043 0.00145	0.74 0.03	1.22 1.24	1.41 1.36	1.76 1.87	12 12	95 95	84 85	1.7 2.5
100	m	0.0068	67.8	0.00053	0.55	1.28	1.47	1.89	12	95	81	2.6
100	w	0.0067	68.4	0.00075	0.16	1.24	1.46	2.01	11	95	81	3.6
200	m	0.0057	62.6	0.00044	0.63	1.26	1.49	1.92	12	95	81	2.7
200	w	0.0050	63.0	0.00054	0.84	1.25	1.49	1.99	12	95	81	4.1
400	m	0.0044	57.9	0.00038	0.73	1.25	1.48	1.90	12	95	81	2.2
400	w	0.0057	58.5	0.00038	0.30	1.27	1.53	1.98	12	95	80	4.4
800	m	0.0038	53.7	$0.00030 \\ 0.00042$	1.36	1.28	1.51	1.89	12	95	80	4.0
800	w	0.0065	60.0		0.31	1.29	1.56	2.06	12	95	79	4.8
1500	m	0.0054	59.7	$0.00035 \\ 0.00060$	0.98	1.29	1.52	1.91	12	95	80	4.7
1500	w	0.0094	69.3		-0.07	1.32	1.55	2.05	10	95	79	4.2
Row 1000 1000	m w	0.0052 0.0058	63.7 57.1	0.00023 0.00021	0.31 0.31	1.20 1.26	1.33 1.44	1.55 1.71	10 10	91 91	89 83	4.0 13.9
2000	m	0.0049	65.6	0.00028	0.55	1.20	1.32	1.55	11	95	89	1.5
2000	w	0.0058	66.4	0.00030	0.73	1.24	1.39	1.64	10	90	86	2.5
5000	m	0.0051	65.4	0.00032	0.39	1.20	1.34	1.60	10	90	87	1.4
5000	w	0.0052	66.4	0.00059	0.97	1.24	1.45	1.90	9	93	82	2.1
6000	m	0.0041	65.8	0.00038	0.91	1.18	1.32	1.59	11	95	88	3.1
6000	w	0.0053	65.3	0.00052	1.08	1.26	1.47	1.91	8	80	81	2.5
10000	m	0.0042	63.6	0.00031	0.57	1.18	1.32	1.57	10	90	88	2.0
10000	w	0.0037	60.8	0.00043	1.51	1.23	1.44	1.84	8	90	83	2.7
POOL	m	0.0047	64.6	0.00030	0.53	1.19	1.33	1.57	57	96	88	1.4
POOL	w	0.0051	62.8	0.00037	1 0.73	1.23	1.42	1.76	49	93	83	3.2

Table 2 NLS Estimates

Consider Table 2 first. The estimates of the transition age vary from 53.7 to 76.6. The mean across the 16 events (not counting the pooled results) is 65.1 for men and 64.2 for women. The estimates of α , the percent decline per year up to the transition age, vary from 0.0037 to 0.0124. The mean across the 16 events is 0.0064 for men and 0.0075 for women. The slope between 30 and 40 as a fraction of $\hat{\alpha}$ is more erratic. In three cases it is greater than one, and in one case it is negative but essentially zero (-0.07 for swim 1500 meters women). Otherwise, the range is from 0.03 to 0.98. A predicted value greater than one means that the linear decline rate is larger between 30 and 39 than it is from 40 on. A predicted value less than zero means that the predicted value at age 40 is less than the overall world record. The age factors at age 80 vary from 1.32 to 1.95. The age at which there is a 50 percent decline from age 30 varies from 71 to to 89. Only one of the estimated standard errors of the estimate of k^* is high: 13.9 for row 1000 meters women.

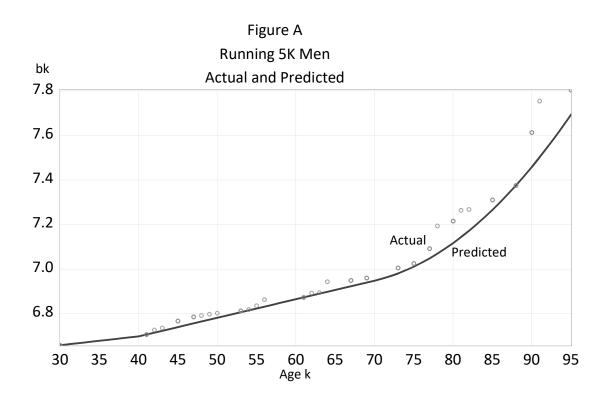
The 5000 meters and 5K results are interesting, since they are the same distance. The results show, for example, that at age 90 the decline is slightly less for 5000 meters: 2.41 versus 2.52 for men and 2.58 versus 2.70 for women.

The results in Table 3 for the frontier estimates are similar to those in Table 2. The mean of the estimates of the transition age for men is 67.5 versus 65.1 in Table 2. For women it is 66.7 versus 64.2 in Table 2. The mean of the estimates of α is 0.0065 for men versus 0.0064 in Table 2 and 0.0074 for women versus 0.0075 in Table 2. Three of the estimates of ratio of the slope to $\hat{\alpha}$ are still greater than one, and five of the estimates are now slightly less than zero.

Figure A shows what is involved in using the frontier method. The event is running, 5K, men. The dots are the actual non dominated times and the line is the predicted line. All the points are on or above the line.

Frontier Estimates											
Event	m/ w	\hat{lpha}	$\hat{k^*}$	$\hat{\delta}$	$slope/\hat{lpha}$	R_{70}	R_{80}	R_{90}	No. Obs.	Max Age	Half
Run 5000 5000	m w	0.0095 0.0105	71.9 70.6	0.00106 0.00080	0.31 0.26	1.37 1.41	1.61 1.68	2.34 2.35	30 26	96 96	77 75
5K	m	0.0083	70.1	0.00086	0.48	1.34	1.58	2.23	30	95	78
5K	w	0.0100	69.1	0.00100	0.14	1.37	1.71	2.60	27	95	76
10K	m	0.0099	76.1	0.00256	0.04	1.35	1.55	2.69	28	92	79
10K	w	0.0094	66.6	0.00075	0.26	1.37	1.71	2.47	22	85	76
HMA	m	0.0096	68.1	0.00071	0.04 0.06	1.34	1.63	2.29	23	85	77
HMA	w	0.0099	58.7	0.00038		1.42	1.78	2.40	21	85	73
MA MA Swim	m w	0.0107 0.0132	74.4 74.4	0.00168 0.00261	-0.04 -0.17	1.37 1.45	1.61 1.79	2.56 3.55	27 19	85 85	78 73
50	m	0.0051	65.6	0.00047	0.46	1.20	1.38	1.74	12	95	85
50	w	0.0054	73.9	0.00120	0.00	1.18	1.30	1.79	12	95	86
100	m	0.0065	72.1	0.00076	0.57	1.26	1.41	1.83	12	95	83
100	w	0.0061	69.0	0.00077	0.13	1.21	1.41	1.92	11	95	83
200 200	m w	0.0057 0.0061	63.5 68.2	0.00047 0.00068	0.39 0.26	1.24 1.22	1.46 1.43	1.89 1.90	12 12	95 95 05	82 83
400	m	0.0051	61.6	0.00044	0.43	1.23	1.45	1.87	12	95	82
400	w	0.0070	66.5	0.00056	-0.19	1.23	1.45	1.91	12	95	82
800	m	0.0039	59.0	0.00040	1.11	1.23	1.45	1.86	12	95	82
800 800 1500	m w	0.0039 0.0076 0.0063	70.4 66.5	0.00040 0.00080 0.00051	-0.12 0.40	1.23 1.24 1.24	1.43 1.44 1.44	1.80 1.96 1.85	12 12 12	95 95 95	82 82 82
1500 Row	W	0.0094	71.7	0.00078	-0.34	1.29	1.49	2.02	10	95	81
1000	m	0.0053	65.1	0.00027	0.13	1.19	1.32	1.55	10	91	89
1000	w	0.0052	58.8	0.00023	0.34	1.22	1.39	1.65	10	91	85
2000	m	0.0051	67.5	0.00031	0.35	1.19	1.31	1.54	11	95	89
2000	w	0.0064	67.8	0.00030	0.34	1.24	1.38	1.64	10	90	86
5000	m	0.0048	65.1	0.00033	0.37	1.19	1.33	1.59	10	90	88
5000	w	0.0054	67.7	0.00067	0.59	1.22	1.42	1.88	9	93	83
6000	m	0.0046	72.3	0.00060	0.40	1.17	1.27	1.55	11	95	89
6000 10000	w m	0.0040 0.0030 0.0036	54.0 61.4	0.00024 0.00031	2.56 0.65	1.17 1.26 1.16	1.27 1.43 1.31	1.55 1.71 1.57	8 10	80 90	83 88
10000	w	0.0036	60.3	0.00044	1.15	1.21	1.43	1.83	8	90	83
POOL	m	0.0047	64.7	0.00029	0.20	1.17	1.30	1.53	57	96	89
POOL	W	0.0057	64.5	0.00031	19.21	1.21	1.37	1.64	49	93	86

Table 3 Frontier Estimates



Some of the results in Table 2 are summarized in Table 4, where the five rowing events, 1000 through 10000 meters, are summarized by the pooled results. Presented are the estimates of α and the percent declines from age 30 to 80.

 α is the percent decline per year between age 40 and the estimated transition age, which is usually in the mid 60's. The estimates of α hover around 1.0 for running and 0.5 for swimming and rowing. The ten-year rates of decline are thus about 10 percent for running and 5 percent for swimming and rowing, although there are obvious some differences between men and women and among the events.

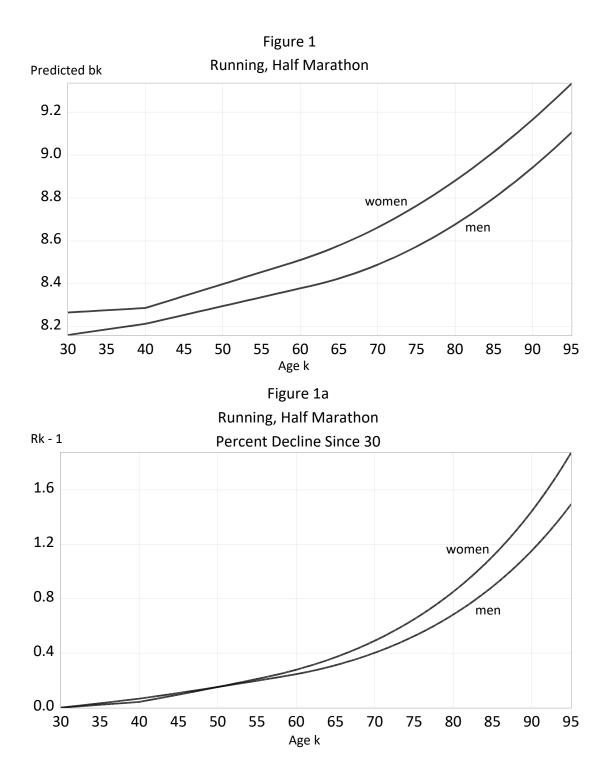
The pooled rowing events have remarkably small decline rates at age 80, 33 percent for men and 42 percent for women. Next comes swimming. Running has by far the largest decline rates, roughly double compared to rowing for each gender.

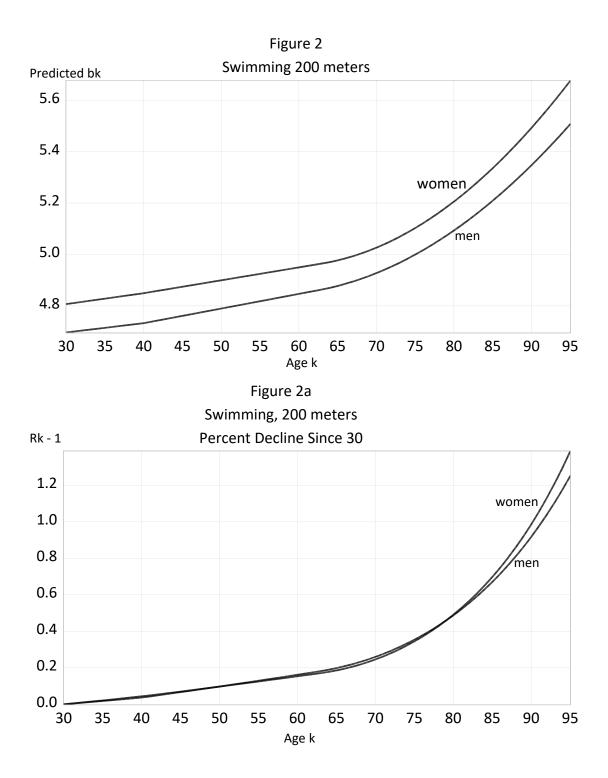
Summary from Table 2									
	1	$00\hat{lpha}$	Percent Decline 30 to 80						
Event	Men	Women	Men	Women	Diff.				
Run									
5000	0.95	1.07	66	84	18				
5K	0.84	1.01	71	82	11				
10K	1.01	0.96	59	80	21				
HMA	0.83	1.12	68	85	17				
MA	1.07	1.24	66	95	29				
Swim									
50	0.50	0.72	41	36	-5				
100	0.68	0.67	47	46	-1				
200	0.57	0.50	49	49	0				
400	0.44	0.57	48	53	5				
800	0.38	0.65	51	56	5				
1500	0.54	0.94	52	55	3				
Row									
POOL	0.47	0.51	33	42	9				

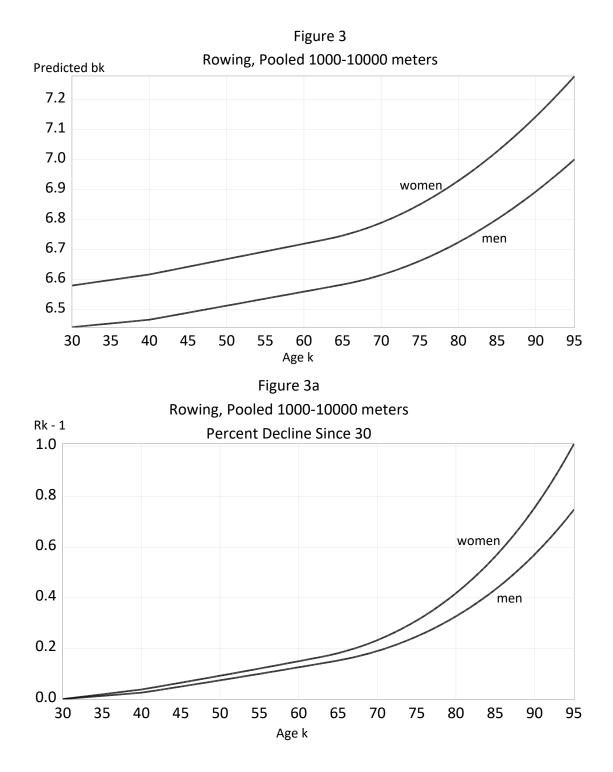
Table 4

	POOL	0.47	0.51	33	42	9	
			· · · ·				
How do m	nen and w	omen co	mpare? W	/omen ar	e on par wit	h men for s	wimming,
a little better	for the sl	horter di	stances ar	nd a little	worse for	the longer	ones. For
rowing the di	fference f	for the po	oled resu	lts is 9 p	ercentage p	oints, 33 fo	r men and
42 for women	n. The dif	fferences	for runni	ng are th	e largest. T	he differen	ces for the
five events ar	e respecti	ively 18,	11, 21, 1	7, and 29	percentage	points.	

Another way of examining the differences between men and women is to plot the values by age for each. In Figure 1 the predicted values of b_k are plotted for men and women for run half marathon. Both of these curves obviously have similar shapes—the linear/quadratic estimates—but the gap between women and men is widening with age. This is better seen in Figure 1a, where the percent decline since 30 is plotted. The gap at age 80 is 0.17 (from Table 4), and it gradually gets larger. Figures 2 and 2a plot the same variables for swim 200 meters. Here







the plots are very similar. The only main difference is that women have a larger constant term. Figures 3 and 3a do the same for pooled rowing. (The constant term for each gender in Figure 3 is for the first pooled event, 1000 meters.) The gap widens with age, as in Figure 1a.

Overall, one would say that the differences in decline rates between men and women for swimming and rowing are zero or modest, but more pronounced for running. There is also evidence that the differences widen slightly with age for running and rowing.

Regarding the economic and medical issues mentioned at the beginning of this paper, how bad is aging? Overall, it seems not too bad. Table 2 shows the age at which the decline is 50 percent from age 30. As noted above, the values range from 70 to 89. Rowing is remarkable in showing the high 80's for men and the low 80's for women. In general quadratic decline does not begin until the mid 60's, and even after it begins it is modest for many years. The results are encouraging for people having an active life well into the older ages. They support the recent move in medicine to focus on active lifestyles as people age. See, for example, Attia (2023).

5 Robustness

The estimates per gender for rowing for the five events 1000 meters through 10000 meters are remarkably similar in Table 2, which is why they were pooled. This is support for the specification. As noted above, the decline rates for rowing are low, which is true for all five estimates per gender.

The estimated standard errors for the estimates of the transition age are small with one exception as discussed above. The also adds support for the specification.

The use of world records by age to avoid survivor bias is likely quite important. In an early study of walking speed by age Himann et al. (1988) using a cross section study of 289 males and 149 females estimated that there was a 1 to 2 percent decline per decade up to age 63. In the present study, as noted above, the decade decline is between about 5 and 10 percent. Most of this difference is likely survivor bias.

It is the case, as discussed in Ronto (2024), that the time difference between men and women of the same age decreases with the length of the race. where women are actually faster than men at about 195 miles. The comparisons of men and women in the present study are, of course, different. The comparisons are for decline rates, not times between men and women of the same age.

6 Personal Use of the Estimated Decline Rates

A world age record is the best that anyone at that age has done, and so it is a good estimate of the biological frontier aside from the soft data problem. The decline rates are in percent terms, and they can be used by non physically elite people under the assumption that their decline rates are the same percentages as those for the elite athletes. In other words, the decline rates can be used if one is on the biological frontier regarding percentage decline rates even through one is slower than the elite athletes. To be on the line requires that one is not sick or injured and is in peak shape age corrected, a severe requirement.

My experience is that some non elite individuals are on their line, so the line is at least relevant for some. On my website, *https://fairmodel.econ.yale.edu/aging3/indexne3.htm*, are examples of individuals who have stayed on their line for most of their career. One is Amy Burfoot, who has run the Manchester Road Race, 4.748 miles, 61 times in a row! But this is not to say that the line is relevant for everyone. Even if someone stays in peak shape age corrected throughout their entire life and is not sick or injured, they may still not be genetically capable of staying on their line.

An example of how to think about the line is the following. Joan Benoit Samuelson ran a 2:21:21 marathon at age 28, her best time. She ran 3:02:21 at age 62. Assuming she could have done 2:21:21 at age 30 (the beginning of the

lines), using this time as her base, and using the estimates in this study of decline rates in the women's marathon, her line gives a 3:10:29 at age 62. She was thus faster at 3:02:21 at age 62 than this line says. If one takes a line as the biological frontier, the base should now be 3:02:21 at age 62. This is her best age corrected time. This line says she should have been able to run a 2:15:19 at age 30. Whether Ms. Samuelson changes the base and frets that she could have run faster at age 30 or does not change the base and glows in the knowledge that she is slowing down less that other elite runners is, of course, a personal decision.

7 Conclusion

There are three main conclusions from the results in this paper, two more conclusive than the third. The first is that the decline rates are modest into the older ages. The decline is about 1 percent per year for running between age 40 and the mid 60's. For swimming and rowing it is about a half a percent per year. In many cases the age at which the decline is 50 percent from age 30 is greater than 80. These results suggest that on physical grounds there is no compelling reason for retirement at age 65 for healthy and fit individuals. They also suggest that exercise need not be cut back much as people age, even into the older ages.

The second conclusion is that decline rates are larger for running than for swimming and rowing. Although less strong, there is evidence that the decline in rowing is less than the decline in swimming.

As noted in the text, this is the first study that estimates decline rates for men versus women. The third conclusion is that except for the swimming events there is more decline for women than for men, with the largest differences for the running events. This conclusion is, however, tentative because of the soft data problem. If the data are softer for older women than for older men, there will be in the future more records broken by women than by men, which in the estimation is likely to lower the decline rates more for women than for men. Will this be enough

to eliminate the differences? It seems unlikely that the current estimates are this biased, but time will tell. One of the key events where more time is needed for both men and women is the marathon.

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