Physical Decline Rates: Men versus Women

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March 2024

Abstract

This paper uses world records by age in running, swimming, and rowing to estimate a biological frontier of decline rates for both men and women. Decline rates are assumed to be linear in percent terms up to a certain age and then quadratic after that, where the transition age is estimated. The decline rates are smallest for rowing, followed by swimming and then running. Decline rates for women are roughly the same as those for men for the swimming events. They are slightly larger for the rowing events. They are largest for running. The age at which there is a 50 percent decline from age 30 ranges from 69 to 89, an optimistic result for humans. The estimated decline rates can be used by non physically elite people under the assumption that their decline rates in percentage terms are similar to those of the elite athletes and that they are in peak shape for their age.

1 Introduction

An important economic policy question is what to assume about the physical abilities of people as they age. In setting a retirement age one question is how much decline there is up to the chosen age? What can one expect, say, from a 70 year old versus a 65 year old? In medicine an important question is how much exercise to recommend as people age. The key question here is a biological one: how fast do

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people's physical abilities decline with age. In previous work, Fair(1994, 2007) and Fair and Kaplan (2018), world records by age were used to estimate decline rates in track and field, road racing, swimming, and chess. Except for swimming, only data for men were used. More recent and better data are now available for both men and women, and Concept 2 rowing data are available for both men and women. It is now possible to compare decline rates in men versus women, which is what this paper does.

Nearly a hundred years ago Hill (1925) pointed out the potential usefulness of athletic records to study the physiology of muscular exercise. The present paper is in this tradition. The advantage of using world records to estimate the biological frontier is that each record is based on many tries, where the best try is used. The sample in this sense is very large. This being said, some of the records are likely "soft" at the older ages because not enough elite people at these ages have participated in the event for the world record to be a good estimate of the biological minimum. More will be said about this later. Studies that have used world records are reviewed in Fair (2007). As far as I am aware there are no other studies that compare the performances of men versus women as is done in this paper.

The model used in this paper focuses on two restrictions that seem sensible biologically. The first is that after a certain age the rate of decline is non decreasing with age. This is the "first derivative" restriction. The second is that the change in the rate of decline is non decreasing with age. This is the "'second derivative" restriction. In short, after decline begins, nothing gets better with age. The linear-quadratic (LQ) model used here automatically meets these restrictions. The LQ model postulates that decline rates are linear in percent terms up to a certain age and then quadratic after that, where the transition age is estimated.

It will be seen that for both men and women the decline rates are smallest for rowing, followed by swimming and then running. Decline rates for women are roughly the same as those for men for the swimming events. They are slightly larger for the rowing events. They are largest for running. The age at which there is a 50 percent decline from age 30 ranges from 69 to 89, an optimistic result for humans. The estimated decline rates can be used by non physically elite people under the assumption that their decline rates in percentage terms are similar to those of the elite athletes and that they are in peak shape for their age.

2 The Data

Data for five running events were obtained from the site of the Association of Road Racing Statisticians (AARS): *arrs.net/SARec.htm*. The data are AARS recognized world records by age. Four of the events are road racing events: 5K, 10K, Half Marathon, and Marathon, and the fifth event is 5,000 meters outdoor track. Data for both men and women were obtained. The AARS data end in 2019, and more recent data were obtained from two Wikipedia sites: $https : //en.wikipedia.org/wiki/List_of_world_records_in_masters_athletics$ and $https : //en.wikipedia.org/wiki/List_of_masters_world_records_in_road_running$. The data were obtained on October 22, 2023. For men there were 19 world records set after 2019, and for women there were 21. One of the more impressive records was age 60, women's 10K, where the record dropped from 39:10 to 36:43. This shows the softness of some of the women's records.

World records by age for swimming were obtained from the World Aquatics site: *worldaquatics.com/masters/records*. Results for six long course meters (LCM) freestyle events were obtained: 50, 100, 200, 400, 800, and 1500 meters. Data were only available in five-year intervals, 30-34, 35-39, ..., 100-104. For each interval the age was taken to be the youngest age, 30, 35, ..., 100. Data for both men and women were obtained. The data were obtained on September 2, 2023.

World records by age for Concept 2 rowing were obtained from the site: *concept2.com/indoor-rowers/racing/records/world*. The machine was RowErg; the weight was heavyweight; and the events were 1000, 2000, 5000, 6000, and 10000 meters. Data were also only available in five-year intervals, but in this case the age of the record holder was available. Data for both men and women were obtained. The data were obtained on September 7, 2023.

As noted above, some of the data are likely "soft" at the older ages. This is particularly true for running. To adjust for this, the oldest age for the half marathon for both men and women was taken to be 85. For the marathon the oldest age for men was taken to be 85 and the oldest age for women was taken to be 88. For swimming, the age category 100-104 was not used. For rowing, the age 95 record for men for 1000 meters rowing was excluded. Also for rowing, the data appeared soft for the 21,097 and 42,195 meter rowing events, especially for women, and these two events were not used. The age 80 observation for women's rowing 10000 meters was also excluded. Finally, the 100 meter and 500 meter rowing events were excluded. For men 100 meters and 500 meters and women 500 meters the world records were at ages in the mid 40's, and so the times in the 30's are likely soft. For women 100 meters the time at age 54 was close to the world record.

Observations with dominated times were also excluded. A time is dominated if there is a lower time at an older age. A dominated time is thus soft, which is the reason for its exclusion. There was one dominated record for rowing and three for swimming. There were a number for running, primarily because there were records at each age rather then in just five year intervals.

Regarding possible changes over time, it may be that the estimated curves are shifting down over time as nutrition, knowledge, technology, and the like improve. For this paper it is assumed that the curves do not shift over time. The world record data are primarily since 2000. For rowing the oldest record was 2011 for women and 2010 for men. For swimming all of the records were set after 2000. For running there were only 14 records out of 271 observations used that were set before 1990, with the two earliest being in 1977.

Table 1 lists the notation for the 16 events plus one pooling case.

Table 1
The Events

Notation	Description
RU5000	Running, 5000 meters, outdoor track
RU5K	Running, 5K
RU10K	Running, 10K
RUHMA	Running, half marathon
RUMA	Running, marathon
SW50	Swimming, LCM, freestyle, 50 meters
SW100	Swimming, LCM, freestyle, 100 meters
SW200	Swimming, LCM, freestyle, 200 meters
SW400	Swimming, LCM, freestyle, 400 meters
SW800	Swimming, LCM, freestyle, 800 meters
SW1500	Swimming, LCM, freestyle, 1500 meters
RO1000	Rowing, RowErg, heavyweight, 1000 meters
RO2000	Rowing, RowErg, heavyweight, 2000 meters
RO5000	Rowing, RowErg, heavyweight, 5000 meters
RO6000	Rowing, RowErg, heavyweight, 6000 meters
RO10000	Rowing, RowErg, heavyweight, 10000 meters
ROPOOL	Rowing, RowErg, heavyweight, pooled 1000–10000 meters

In the text a “M” after the name is men, and a “‘W” after the name is women,

3 The Linear/Quadratic (LQ) Model

Consider first decline from age 40 on. It may be that there is some decline between, say, ages 30 and 39, but this decline may be less in perecentage terms than decline from 40 on. The following model is for decline starting at age 40. The estimation of decline between 30 and 39 is discussed at the end of this section.

Ages 40 on

Let r_k denote the log of the record time for age k. Using logs means that all decline rates are in percentage terms. b_k will be used to denote log of the (unobserved) biological minimum time for age k. By definition,

$$r_k = b_k + \epsilon_k \quad , \tag{1}$$

where ϵ_k is the gap between the record time and the true biological minimum time. It will be close to zero if the record time is close to the biological minimum. Otherwise it is positive.

The LQ model postulates that the decline rate (in percentage terms) is linear up to a transition age and then quadratic after that. The transition age is one of the estimated parameters. At the transition age the linear and quadratic segments are constrained to touch and to have the same first derivative. The formula for b_k is

$$b_k = \begin{cases} \beta + \alpha k, & 40 \leq k \leq k^*, \quad \alpha > 0 \\ \gamma + \theta k + \delta k^2, & k > k^*, \quad \delta > 0 \end{cases} \tag{2}$$

with the restrictions

$$\begin{aligned} \gamma &= \beta + \delta k^{*2} \\ \theta &= \alpha - 2\delta k^* \end{aligned} \tag{3}$$

k^* is the transition age. The two restrictions force the linear and quadratic segments to touch and to have the same first derivative at k^*. The unrestricted parameters to estimate are the intercept, β, the slope of the linear segment, α, the transition age,

k^*, and the quadratic parameter, δ. The first derivative of b_k with respect to k is α up to the transition age and then increases by a constant amount (2δ) after that. The second derivative is zero up to the transition age and then constant (2δ) after that.

The equation that is estimated is then

$$r_k = \beta + \alpha k + \delta d_k (k^{*2} - 2k^* k + k^2) + \epsilon_k, \tag{4}$$

where $d_k = 0$ if $k \leq k^*$ and $d_k = 1$ if $k > k^*$. ϵ_k is greater than or equal to zero, so it has a positive mean. A positive mean poses no problem in the estimation because it is simply absorbed in the estimate of the constant term. This means that the constant β is not identified, but this is of no concern here because the derivatives do not depend on β. The equation can be estimated by non linear least squares, NLS.

The equation can also, however, be estimated under the restriction that $\epsilon_k \geq 0$ for all k. The procedure is common in the estimation of frontier production functions—see, for example, Aigner and Chu (1968) and Schmidt (1976). The added complication here is that equation (4) is nonlinear in coefficients. For linear equations the estimation problem can be set up as a quadratic programming problem and solved by standard methods.

The procedure used here is the following. In the NLS case the coefficients in equation (4) are estimated by minimizing the sum of squared residuals, $\sum_{k=1}^{K} \hat{\epsilon}_k^2$, where K is the total number of observations. Instead, one can minimize a weighted sum, $\sum_{k=1}^{K} \lambda_k \hat{\epsilon}_k^2$, where λ_k is equal to 1 if $\hat{\epsilon_k} \geq 0$ and is equal to a number greater than 1 if $\hat{\epsilon_k} < 0$. This penalizes negative errors more than non-negative ones. For the results here a value of 1000 was used for λ_k when $\hat{\epsilon}_k$ was less than zero.

It will be seen that the use of the frontier procedure instead of NLS has generally small effects on the slope coefficients and k^* and thus on the estimated derivatives. The use of the procedure primarily affects the estimate of the constant term β, which is not of concern here.

Some events have very similar coefficient estimates, and for these events pooling was done. The assumption is that the curve for each event is the same except for the intercept. The equation estimated is (n is the number of events pooled):

$$r_{ik} = \beta_1 D_{1ik} + \cdots + \beta_n D_{nik} + \alpha k + \delta d_{ik}(k^{*2} - 2k^* k + k^2) + \epsilon_{ik}, \qquad (5)$$

$$i = 1, \ldots, n; k = 40 \ldots, K_i,$$

where r_{ik} is the log of the observed record for event i and age k, D_{jik} is a dummy variable that is equal to 1 when event i is equal to event j and 0 otherwise $(j = 1 \ldots n)$, $d_{ik} = 1$ if $k \leq k^*$ and $d_{ik} = 0$ if $k \geq k^*$, ϵ_{ik} is the error for event i and age k, and K_i is the oldest age used for event i. The n β coefficients are the n different constant terms.

Ages 30-39

Data were collected for each event from age 30 on. The overall world record for each event and gender was also collected. In estimating the decline rate between 30 and 39 the time at age 30 was taken to be the overall world record even if the actual time was higher. In other words, the times at age 30 were assumed to be soft if they were not the overall world record, and the overall world record was used. This assumes that decline does not begin before 30.

From the above estimation for each event and gender the predicted value of b_k is available for age 40, $\hat{b}_{40}$. (Remember that the times are in logs.) Then the values of b_k between 30 and 39 were assumed to lie on a straight line between the world record (age 30 time) and $\hat{b}_{40}$. One would expect the slope of this line to be less than $\hat{\alpha}$ if the percent decline before age 40 is less. In the tables below the ratio of the slope to $\hat{\alpha}$ is presented. Note that the above estimation from age 40 on is not affected by this treatment for ages 30 through 39.

For the results below ''age factors,'' denoted R_k, are presented. They are computed as follows. Let $\hat{b}_k$ denote the predicted value of b_k using the estimated values of β, α, k^*, and δ for $k = 40, \ldots$. Let $\hat{b}_k$ denote the predicted value of b_k for $k = 30, \ldots, 39$ using the above procedure for ages 30-39. Then R_k is

$$R_k = e^{\hat{b}_k}/e^{\hat{b}_{30}}, \quad k = 30, \ldots. \tag{6}$$

R_k is an estimate of the percent decline at age k from age 30. This estimate does not depend on the estimate of β, so the estimate of the constant term in the equation does not matter. It does depend on the overall world record for the event and gender because $\hat{b}_{30}$ is the overall world record.

4 The Results

There are five running events, six swimming events, and five rowing events, for a total of 16 cases per gender. The estimates for these 32 cases are presented in Table 2. The coefficient estimates for five rowing events for each gender are close enough to warrant pooling, and the pooling estimates are presented at the bottom of Table 2.

Table 2 presents the estimates of α, k^*, δ, the slope divided by α, the implied age factors for ages 70, 80, and 90, the number of observations, the maximum age in the estimation period, the age at which the decline is 50 percent from age 30 (denoted "Half"), and the estimated standard error of the estimate of k^*. For each case the men's results are presented and then the women's. Although not shown, the coefficient estimates are highly significantly different from zero. Only one estimate of α has a t-statistic less than 2.0, 1.85 for rowing 1000 meters women, and no estimate of δ has a t-statistic less than 2.0. This is, of course, not surprising since there is obvious decline in the data. The estimated standard errors of the estimates of k^* are presented to give a sense of the precision of the estimates of the transition age. There is collinearity between the estimate of the transition age

Table 2
NLS Estimates

Event	$\hat{\alpha}$	$\hat{k}^*$	$\hat{\delta}$	$slope/\hat{\alpha}$	R_{70}	R_{80}	R_{90}	No. Obs.	Max Age	Half	SE $\hat{k}^*$
RU5000M	0.0100	73.9	0.00121	0.64	1.44	1.66	2.40	32	96	75	1.0
RU5000W	0.0107	65.8	0.00061	0.55	1.48	1.84	2.58	26	96	71	3.5
RU5KM	0.0084	66.9	0.00085	0.67	1.37	1.71	2.52	30	95	76	2.4
RU5KW	0.0105	66.5	0.00078	0.34	1.43	1.82	2.70	28	95	73	1.8
RU10KM	0.0097	76.0	0.00266	0.36	1.38	1.59	2.83	30	92	78	0.6
RU10KW	0.0135	82.7	0.02124	0.13	1.53	1.75	6.24	29	88	69	0.6
RUHMAM	0.0078	58.6	0.00032	0.82	1.40	1.68	2.15	29	85	75	3.0
RUHMAW	0.0100	56.2	0.00031	0.41	1.49	1.85	2.44	20	85	71	5.3
RUMAM	0.0107	73.3	0.00123	0.23	1.41	1.66	2.48	27	85	76	1.8
RUMAW	0.0137	74.2	0.00221	0.10	1.53	1.89	3.49	20	85	69	2.4
SW50M	0.0050	64.5	0.00043	0.74	1.22	1.41	1.76	12	95	84	1.7
SW50W	0.0072	76.6	0.00145	0.03	1.24	1.36	1.87	12	95	85	2.5
SW100M	0.0068	67.8	0.00053	0.55	1.28	1.47	1.89	12	95	81	2.6
SW100W	0.0067	68.4	0.00075	0.16	1.24	1.46	2.01	11	95	81	3.6
SW200M	0.0057	62.6	0.00044	0.63	1.26	1.49	1.92	12	95	81	2.7
SW200W	0.0050	63.0	0.00054	0.84	1.25	1.49	1.99	12	95	81	4.1
SW400M	0.0044	57.9	0.00038	0.73	1.25	1.48	1.90	12	95	81	2.2
SW400W	0.0057	58.5	0.00038	0.30	1.27	1.53	1.98	12	95	80	4.4
SW800M	0.0038	53.7	0.00030	1.36	1.28	1.51	1.89	12	95	80	4.0
SW800W	0.0065	60.0	0.00042	0.31	1.29	1.56	2.06	12	95	79	4.8
SW1500M	0.0054	59.7	0.00035	0.98	1.29	1.52	1.91	12	95	80	4.7
SW1500W	0.0094	69.3	0.00060	-0.07	1.32	1.55	2.05	10	95	79	4.2
RO1000M	0.0052	63.7	0.00023	0.31	1.20	1.33	1.55	10	91	89	4.0
RO1000W	0.0058	57.1	0.00021	0.31	1.26	1.44	1.71	10	91	83	13.9
RO2000M	0.0049	65.6	0.00028	0.55	1.20	1.32	1.55	11	95	89	1.5
RO2000W	0.0058	66.4	0.00030	0.73	1.24	1.39	1.64	10	90	86	2.5
RI5000M	0.0051	65.4	0.00032	0.39	1.20	1.34	1.60	10	90	87	1.4
RO5000W	0.0052	66.4	0.00059	0.97	1.24	1.45	1.90	9	93	82	2.1
RO6000M	0.0041	65.8	0.00038	0.91	1.18	1.32	1.59	11	95	88	3.1
RO6000Wf	0.0053	65.3	0.00052	1.08	1.26	1.47	1.91	8	80	81	2.5
RO10000M	0.0042	63.6	0.00031	0.57	1.18	1.32	1.57	10	90	88	2.0
RO10000W	0.0037	60.8	0.00043	1.51	1.23	1.44	1.84	8	90	83	2.7
ROPOOLM	0.0047	64.6	0.00030	0.53	1.19	1.33	1.57	57	96	88	1.4
ROPOOLW	0.0051	62.8	0.00037	0.73	1.23	1.42	1.76	49	93	83	3.2

and the estimate of the quadratic coefficient. A larger estimate of k^* tends to result in a larger estimate of δ.

Table 3 is the same as Table 2 except that the estimates are obtained from the frontier method, where all the estimated residuals are forced to be non negative.

Consider Table 2 first. The estimates of the transition age vary from 53.7 to 82.7. The mean across the 16 events (not counting the pooled results) is 64.9 for men and 66.1 for women. The estimates of α, the percent decline per year up to the transition age, vary from 0.0037 to 0.0137. The mean across the 16 events is 0.0063 for men and 0.0078 for women. The slope between 30 and 40 as a fraction of $\hat{\alpha}$ is more erratic. In three cases it is greater than one, and in one case it is negative but essentially zero (-0.07 for SW1500W). Otherwise, the range is from 0.03 to 0.97. A predicted value greater than one means that the linear decline rate is larger between 30 and 39 than it is from 40 on. A predicted value less than zero means that the predicted value at age 40 is less than the overall world record. The age factors at age 80 vary from 1.32 to 1.89. The age at which there is a 50 percent decline from age 30 varies from 69 for the women's running 10K and marathon to 89 for some of the men's rowing events. One of the estimated standard errors of the estimate of k^* is high: 13.9 for RO1000W.

The results in Table 3 for the frontier estimates are similar to those in Table 2. The mean of the estimates of the transition age for men is 67.1 versus 64.9 in Table 2. For women it is 67.6 versus 66.1 in Table 2. The mean of the estimates of α is 0.0063 for men, the same as in Table 2, and 0.0075 for women versus 0.0078 in Table 2. Three of the estimates of ratio of the slope to $\hat{\alpha}$ are still greater than one, and five of the estimates are now slightly less than zero.

There are two problematic results for women in the two tables. One is for the women's marathon, RUMAW. The estimate of δ is fairly large, and the age-90 age factor is unrealistically large—3.49 versus 2.48 for men in Table 2. This likely reflects the soft data problem for women in the marathon, and more time

Table 3
Frontier Estimates

Event	$\hat{\alpha}$	$\hat{k}*$	$\hat{\delta}$	$slope/\hat{\alpha}$	R_{70}	R_{80}	R_{90}	No. Obs.	Max Age	Half
RU5000M	0.0095	71.9	0.00106	0.31	1.37	1.61	2.34	32	96	77
RU5000W	0.0105	70.6	0.00080	0.26	1.41	1.68	2.35	26	96	75
RU5KM	0.0083	70.1	0.00086	0.48	1.34	1.58	2.23	30	95	78
RU5KW	0.0100	69.1	0.00100	0.14	1.37	1.71	2.60	28	95	76
RU10KM	0.0091	75.5	0.00244	0.23	1.34	1.54	2.69	30	92	79
RU10KW	0.0123	81.9	0.01898	-0.27	1.40	1.58	6.18	29	88	76
RUHMAM	0.0078	61.8	0.00044	0.59	1.36	1.66	2.19	29	85	76
RUHMAW	0.0093	57.6	0.00037	0.17	1.42	1.78	2.40	20	85	73
RUMAM	0.0107	74.4	0.00168	-0.04	1.37	1.61	2.56	27	85	78
RUMAW	0.0132	74.5	0.00262	-0.18	1.45	1.79	3.56	20	85	73
SW50M	0.0051	65.6	0.00047	0.46	1.20	1.38	1.74	12	95	85
SW50W	0.0054	73.9	0.00120	0.00	1.18	1.30	1.79	12	95	86
SW100M	0.0065	72.1	0.00076	0.57	1.26	1.41	1.83	12	95	83
SW100W	0.0061	69.0	0.00077	0.13	1.21	1.41	1.92	11	95	83
SW200M	0.0057	63.5	0.00047	0.39	1.24	1.46	1.89	12	95	82
SW200W	0.0061	68.2	0.00068	0.26	1.22	1.43	1.90	12	95	83
SW400M	0.0051	61.6	0.00044	0.43	1.23	1.45	1.87	12	95	82
SW400W	0.0070	66.5	0.00056	-0.19	1.23	1.45	1.91	12	95	82
SW800M	0.0039	59.0	0.00040	1.11	1.23	1.45	1.86	12	95	82
SW800W	0.0076	70.4	0.00080	-0.12	1.24	1.44	1.96	12	95	82
SW1500M	0.0063	66.5	0.00051	0.40	1.24	1.44	1.85	12	95	82
SW1500W	0.0094	71.7	0.00078	-0.34	1.29	1.49	2.02	10	95	81
RO1000M	0.0053	65.1	0.00027	0.13	1.19	1.32	1.55	10	91	89
RO1000W	0.0052	58.8	0.00023	0.34	1.22	1.39	1.65	10	91	85
RO2000M	0.0051	67.5	0.00031	0.35	1.19	1.31	1.54	11	95	89
RO2000W	0.0064	67.8	0.00030	0.34	1.24	1.38	1.64	10	90	86
RI5000M	0.0048	65.1	0.00033	0.37	1.19	1.33	1.59	10	90	88
RO5000W	0.0054	67.7	0.00067	0.59	1.22	1.42	1.88	9	93	83
RO6000M	0.0046	72.3	0.00060	0.40	1.17	1.27	1.55	11	95	89
RO6000Wf	0.0030	54.0	0.00024	2.56	1.26	1.43	1.71	8	80	83
RO10000M	0.0036	61.4	0.00031	0.65	1.16	1.31	1.57	10	90	88
RO10000W	0.0036	60.3	0.00044	1.15	1.21	1.43	1.83	8	90	83
ROPOOLM	0.0047	64.7	0.00029	0.20	1.17	1.30	1.53	57	96	89
ROPOOLW	0.0057	64.5	0.00031	0.21	1.21	1.37	1.64	49	93	86

will be needed to obtain an accurate biological frontier for the women's marathon, at least at the older ages. A similar problem occurs for the women's 10K, RU10KW. This case shows the collinearity between the estimate of the transition age and the estimate of δ. Both are large, likely reflecting a soft data problem. .

Some of the results in Table 2 are summarized in Table 4, where the five rowing events, 1000 through 10000 meters, are summarized by the pooled results. Presented are the percent declines from age 30 to 80. The following is a discussion of this table.

Which sport has the smallest decline rates? The pooled rowing events have remarkably small decline rates at age 80, 33 percent for men and 42 percent for women. Next comes swimming. Running has by far the largest decline rates, roughly double compared to rowing for each gender.

How do men and women compare? Women are on par with men for swimming, a little better for the shorter distances and a little worse for the longer ones. For rowing the difference for the pooled results is 9 percentage points, 33 for men and 42 for women. The differences for running are the largest. The differences for the five events are respectively 18, 11, 16, 17, and 23 percentage points. As noted above, the results for women for the 10K and marathon are problematic, and dropping these two the differences are 18, 11, and 17.

Another way of examining the differences between men and women is to plot the values by age for each. In Figure 1 the predicted values of b_k are plotted for men and women for the running half marathon. (The marathon is not used because of the problematic nature of the results for women at the older ages.) Both of these curves obviously have similar shapes—the linear/quadratic estimates—but the gap between women and men is widening with age. This is better seen in Figure 1a, where the percent decline since 30 is plotted. The gap at age 80 is 0.17 (from Table 3), and it gradually gets larger. Figures 2 and 2a plot the same variables for swimming 200 meters. Here the plots are very similar. The only main difference is that women have a larger constant term. Figures 3 and 3a do the same for pooled

Table 4
Summary: Percent Decline 30 to 80

Event	Men	Women	Difference
RU5000	66	84	18
RU5K	71	82	11
RU10K	59	75	16
RUHMA	68	85	17
RUMA	66	89	23
SW50	41	36	-5
SW100	47	46	-1
SW200	49	49	0
SW400	48	53	5
SW800	51	56	5
SW1500	52	55	3
ROPOOL	33	42	9

Results taken from Table 2.

rowing. (The constant term for each gender in Figure 3 is for the first pooled event, 1000 meters.) The gap widens with age, as in Figure 1a.

Overall, one would say that the differences in decline rates between men and women for swimming and rowing are zero or modest, but more pronounced for running. There is also evidence that the differences widen slightly with age for running and rowing.

Regarding the economic and medical issues mentioned at the beginning of this paper, how bad is aging? Overall, it seems not too bad. Table 2 shows the age at which the decline is 50 percent from age 30. As noted above, the values range from 69 to 89. Rowing is remarkable in showing the high 80's for men and the low 80's for women. In general quadratic decline does not begin until the mid 60's, and even after it begins it is modest for many years.

Figure 1

Running, Half Marathon

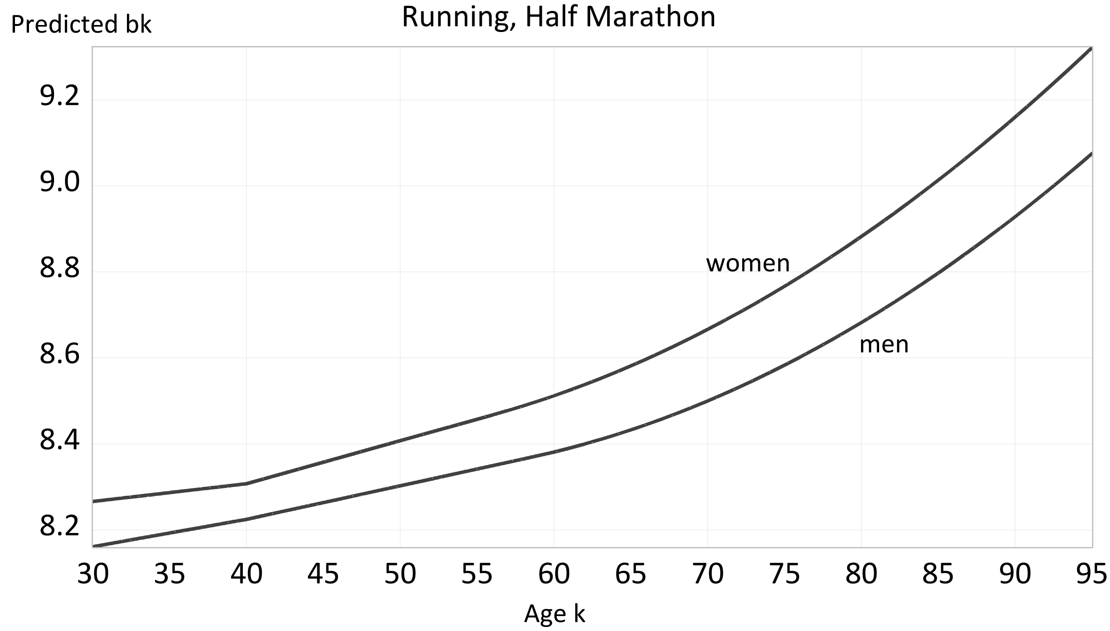


Figure 1a

Running, Half Marathon

Percent Decline Since 30

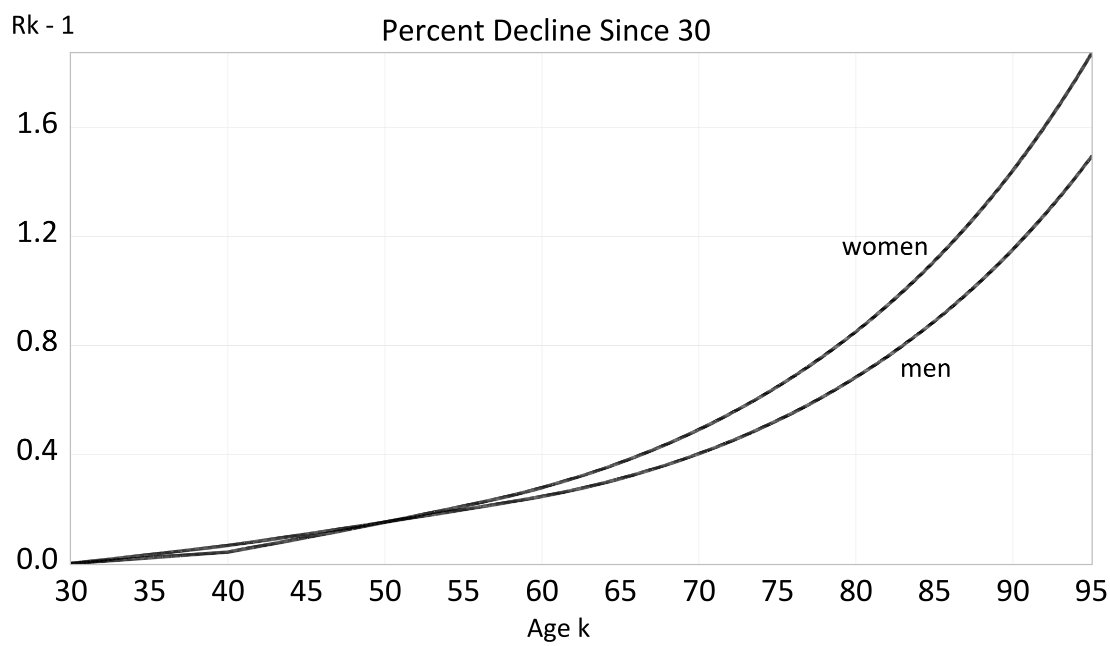


Figure 2

Swimming 200 meters

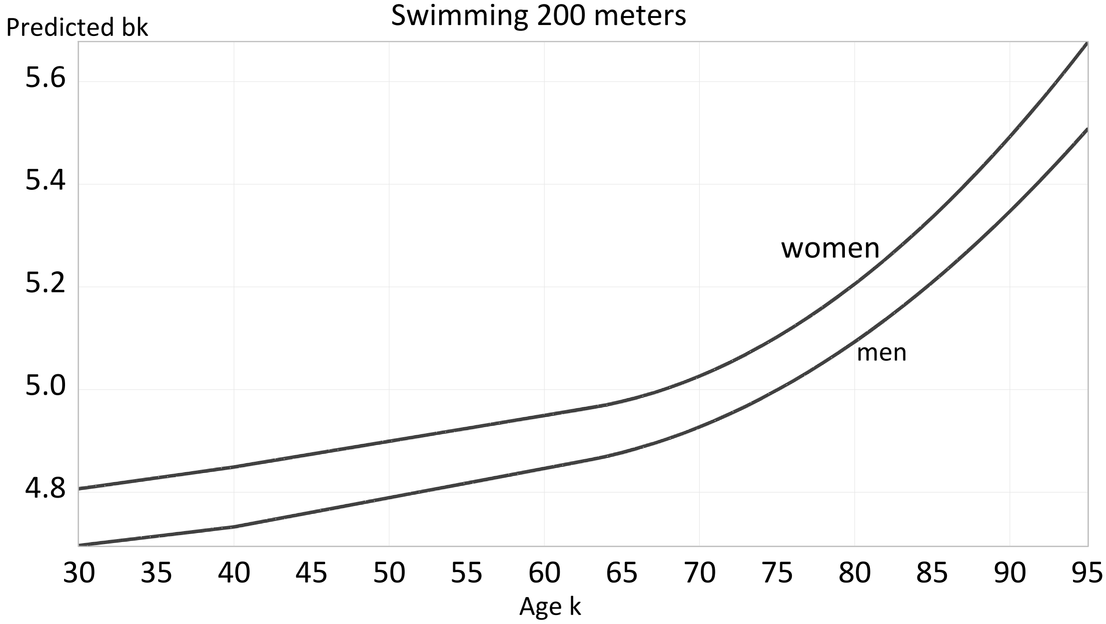


Figure 2a

Swimming, 200 meters

Percent Decline Since 30

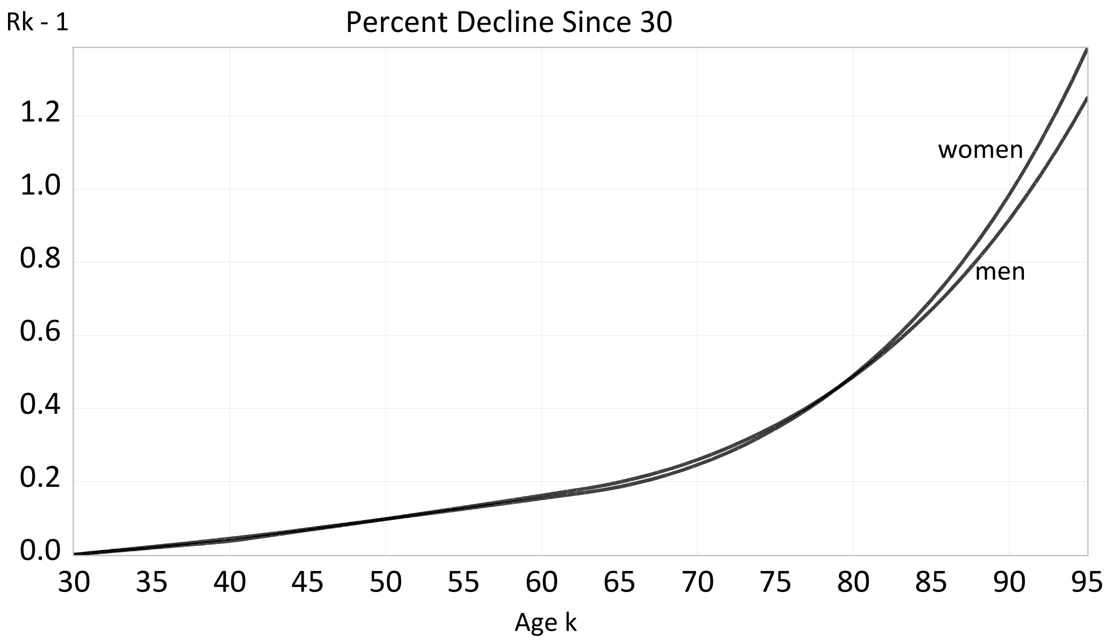


Figure 3

Rowing, Pooled 1000-10000 meters

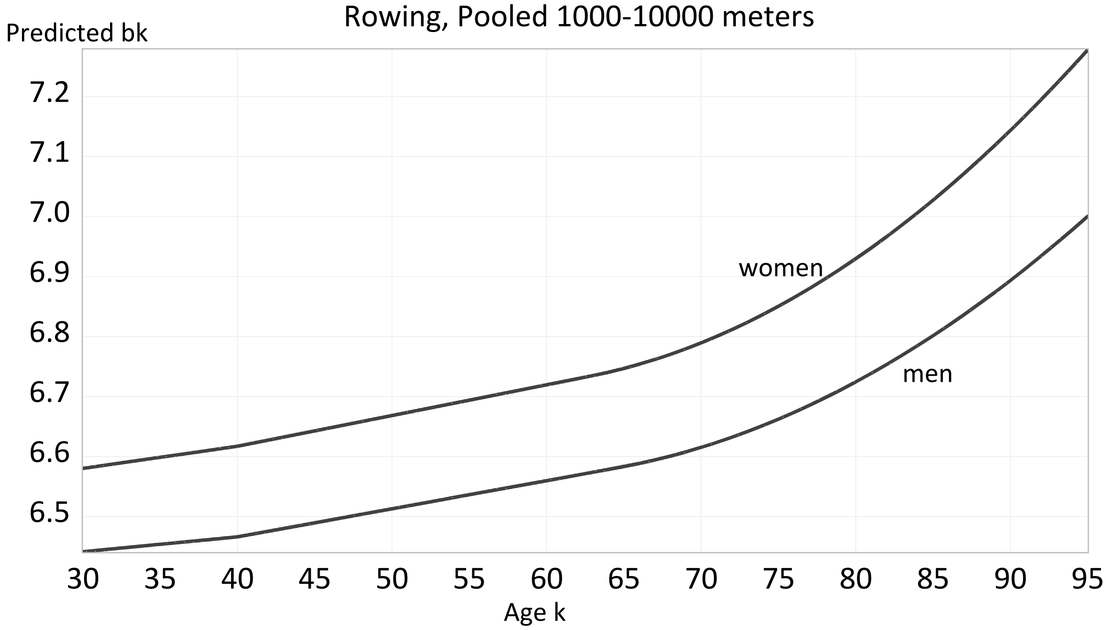
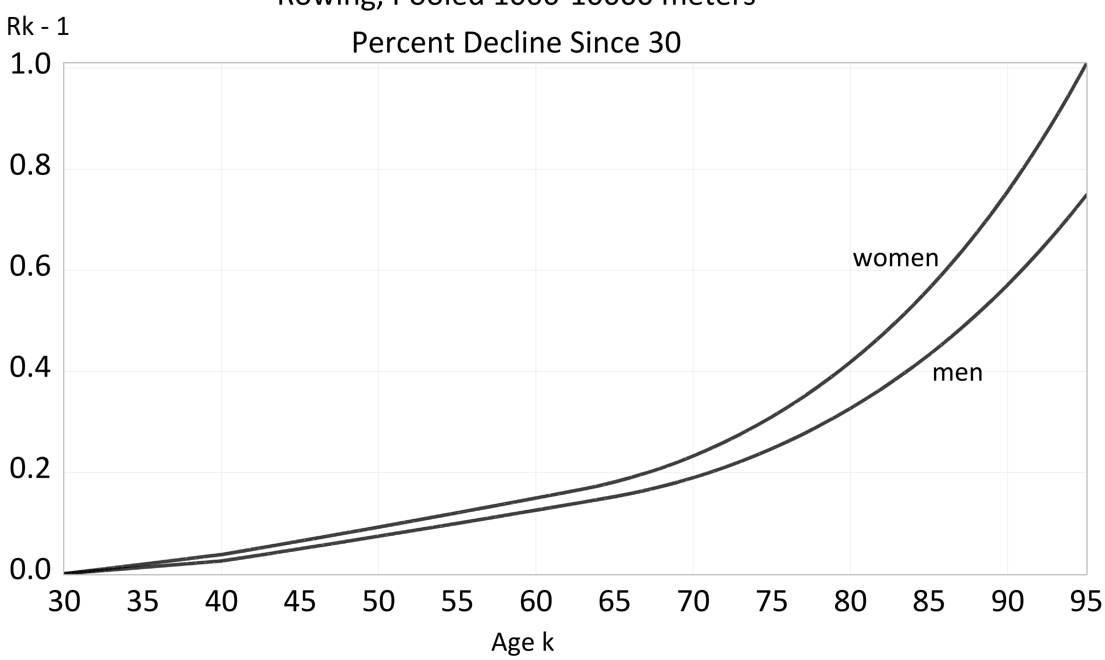


Figure 3a

Rowing, Pooled 1000-10000 meters

Percent Decline Since 30



5 Robustness

The estimates per gender for rowing for the five events 1000 meters through 10000 meters are remarkably similar in Table 2, which is why they were pooled. This is support for the specification. As noted above, the decline rates for rowing are low, which is true for all five estimates per gender.

The estimated standard errors for the estimates of the transition age are small with one exception as discussed above. The also adds support for the specification.

6 Use of the Estimated Decline Rates

A world age record is the best that anyone at that age has done, and so it is a good estimate of the biological frontier aside from the soft data problem. The decline rates are in percent terms, and they can be used by non physically elite people under the assumption that their decline rates are the same percentages as those for the elite athletes. In other words, the decline rates can be used if one is on the biological frontier regarding percentage decline rates. To be on the line requires that one is not sick or injured and is in peak shape age corrected, a severe requirement. My experience is that some are on their line and some are not. But at least it is something to aim for.

In general, as noted above, the decline rates are modest and are encouraging for people having an active life well into the older ages. These results support the recent move in medicine to focus on active lifestyles as people age. See, for example, Attia (2023).

7 Conclusion

There are three main conclusions from the results in this paper, two more conclusive than the third. The first is that the decline rates are modest into the older ages. In most cases the decline is less than 1 percent per year between age 30 and the mid 60's. For rowing it is about a half a percent per year. In many cases the age at which the decline is 50 percent from age 30 is greater than 80. These results suggest that on physical grounds there is no compelling reason for retirement at age 65 for healthy and fit individuals. They also suggest that exercise need not be cut back much as people age, even into the older ages.

The second conclusion is that decline rates are larger for running than for swimming and rowing. Although less strong, there is evidence that the decline in rowing is less than the decline in swimming.

As noted in the text, this is the first study that estimates decline rates for men versus women. The third conclusion is that except for the swimming events there is more decline for women than for men, with the largest differences for the running events. This conclusion is, however, tentative because of the soft data problem. If the data are softer for older women than for older men, there will be in the future more records broken by women than by men, which in the estimation is likely to lower the decline rates more for women than for men. Will this be enough to eliminate the differences? It seems unlikely that the current estimates are this biased, but time will tell. One of the key events where more time is needed for both men and women is the marathon.

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